

# APPLICATION OF WEIGHT MEASURE ORDERING FOR INTUITIONISTIC FUZZY CRITICAL PATH ANALYSIS

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## Abstract

*The intuitionistic fuzzy critical path problem is a frequent and important challenge in network optimization, particularly in the planning and management of complex projects. However, the traditional critical path approach often fails to accurately reflect real-world scenarios. To address this, we have developed a critical path strategy within an intuitionistic fuzzy framework. In this study, the critical path is determined using a ranking algorithm, where the edge weights are modeled as triangular intuitionistic fuzzy numbers (ITFN). We have introduced definitions for ranking strategies that help identify the intuitionistic fuzzy critical path. Additionally, a proposed criticality degree measures that significance of each activity. Examples are provided to illustrate the proposed strategy, and the simulation results of the ranking techniques are also presented.*

**Keywords:** Critical path Method, Intuitionistic Fuzzy Triangular Number, Acyclic Network, Project Scheduling.

## 1. Introduction

In today's highly competitive corporate environment, where numerous business activities are managed as projects, the ability of project management to plan operations and monitor progress on schedule is becoming increasingly crucial. The duration of business deals in network operations is often ambiguous and imprecise in practice. In these situations, Zadeh's fuzzy numbers [20] and Atanassov's intuitionistic fuzzy sets [2] are used to represent activity durations, driving advancements in intuitionistic fuzzy project management techniques. Project managers commonly use the concepts of the earliest, latest, and total float tasks [13]. These are valuable when deciding when to introduce a new activity, eliminate an existing one, or adjust the project's due date if the scope changes [15]. With a clear understanding of the activities and their interconnections in the project network, successful execution is achievable. While calculating the fuzzy earliest starting times of activities using the critical path method is straightforward, determining the fuzzy latest starting times is more challenging, as explored by Elizabeth and Sujatha [10]. When activity durations are vaguely understood and best represented by fuzzy sets, Dubois et al. [9] computed the criticality levels, slacks, and latest start times of events using possibility theory by describing the earliest, latest, and slack times of events as a function of the lengths of routes. A novel method was developed to quickly identify the intervals of the latest starting times in intervalued networks. Chen and Huang [7] suggested using the criticality level of activities and paths to identify fuzzy critical paths. Zareei et al. [19] employed the Zadeh extension principle, linear programming, and a relative

path degree criticality technique to evaluate the criticality of each event and path, determining the earliest and latest event timings. Sireesha and Shankar [17] proposed a method for finding the optimal solution in a fuzzy project network based on the highest total float of the path. While approaches based on the fuzzy extension of the crisp approach are significant, they are also time-consuming and of limited interest to practitioners, according to Bonnal et al.'s [5] assessment of fuzzy PERT (Project Evaluation and Review Technique). Stephen Dinakar and Rameshan [18] presented a technique for analyzing the critical path in a project network using octagonal fuzzy numbers. Instead of precise numbers, Balaganesan and Ganesan [3] developed a novel method for determining the critical path in a network architecture using intuitionistic fuzzy triangular numbers. The innovative JOSE Algorithm, which includes 13 parameters and a ranking mechanism known as the Euclidean ranking approach, was introduced by N. Jose Parvin Praveena [12] and colleagues to identify the fuzzy critical path (FCP). They demonstrated that the dynamic encoding recursion of the critical path could be expressed using triskaidecagonal and fuzzy triskaidecagonal numbers. Recent research by the authors also includes works from [1,6,16].

The paper is organized as follows: Preliminaries are in Part 2, the methodology is in Section 3, results and comments are in Section 4, and the conclusion is in Section 5.

## 2. Preliminaries

### Definition 2.1 Operations on IFTFN [2]

Consider  $X = (p_1, q_1, r_1) (p'_1, q_1, r'_1)$  and  $Y = (p_2, q_2, r_2) (p'_2, q_2, r'_2)$  be two IFTFN, then

- i. Addition:  
 $X \oplus Y = (p_1 + p_2, q_1 + q_2, r_1 + r_2) (p'_1 + p'_2, q_1 + q_2, r'_1 + r'_2)$ .
- ii. Subtraction:  
 $X \ominus Y = (p_1 - r_2, q_1 - q_2, r_1 - p_2) (p'_1 - r'_2, q_1 - q_2, r'_1 - p'_2)$ .
- iii. Maximum Operation:

$$M_{\max} = \max(X, Y) \\ = (\max(p_1, p_2), \max(q_1, q_2), \max(r_1, r_2)), (\max(p'_1, p'_2), \max(q_1, q_2), \max(r'_1, r'_2))).$$

- iv. Minimum Operation:

$$M_{\min} = \min(X, Y) \\ = (\min(p_1, p_2), \min(q_1, q_2), \min(r_1, r_2)), (\min(p'_1, p'_2), \min(q_1, q_2), \min(r'_1, r'_2))).$$

### Definition 2.2 Acyclic network [4]

A graph with directed edges is referred to as a directed graph. Consequently, a directed graph without a cycle is called as acyclic directed graph. An acyclic directed graph which forms a network is known as acyclic network.

### Definition 2.3 Signed distance measure (SDM) [11]

For every closed interval  $[x, y]$ ,  $d([x, y], 0) = \frac{1}{2}(x + y)$  is the definition of the SDM of  $[x, y]$  measured from '0'.

### Definition: 2.4 $\alpha$ -cut Ranking [11]

If  $(x, y, z) (x', y, z')$  be the triangular IFN then  $\alpha$ -cut is defined as,

$$\alpha(P) = \int_0^1 [x + \alpha(y - x)]\alpha * d\alpha + [z + \alpha(z - y)]\alpha * d\alpha \\ - \int_0^1 [y + \alpha(y - x')]\alpha * d\alpha - [y + \alpha(z' - y)]\alpha * d\alpha \\ \alpha(P) = \frac{x+4y+z}{6}, -\frac{x'+y+z'}{3} = (\text{membership function, non-membership function})$$

**Definition: 2.5 Euclidean Ranking [11]**

Let  $L_i = (x_i, y_i, z_i)$  ( $x'_i, y_i, z'_i$ ) be the  $i^{\text{th}}$  fuzzy path length,  $L_{\max} = (x, y, z)$  ( $x', y, z'$ ) be the fuzzy longest length, then Euclidean Ranking (ER) is defined as,

$$ER(P) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}, \sqrt{(x' - x'_i)^2 + (y - y_i)^2 + (z' - z'_i)^2}$$

**Definition: 2.6 Similarity Ranking [11]**

Let  $L_i = (x_i, y_i, z_i)$  ( $x'_i, y_i, z'_i$ ) be the  $i^{\text{th}}$  fuzzy path length,  $L_{\max} = (x, y, z)$  ( $x', y, z'$ ) be the fuzzy longest length, then Similarity Ranking (SR) is defined as,

$$SR(L_i, L_{\max}) = \begin{cases} \emptyset, & \text{if } L_i \cap L_{\max} = \emptyset \\ \frac{1}{2}[z_i - x] * Z_i, \frac{1}{2}[z'_i - x'] * Z'_i, & \text{if } L_i \cap L_{\max} \neq \emptyset \end{cases}$$

Where,

$$Z_i = \frac{z_i - x}{(y - x) + (z_i - y_i)}, y_i < a < z_i \text{ and } x < a < y$$

$$Z'_i = \frac{y - y_i}{(z'_i - y_i) + (y - x')}, y_i < a < z'_i \text{ and } x' < a < y$$

### 3. Proposed Methodology

An intuitionistic fuzzy project is represented by an acyclic directed graph where vertices represent the project's nodes, and directed edges act as its links. A typical representation of an intuitionistic fuzzy project network is  $G = (V, E, T)$ , where  $V = \{v_1, v_2, \dots, v_n\}$  denotes the collection of vertices,  $E: V \times V$  are the set of directed edges  $(i, j) = (v_i, v_j)$ , and  $T = \{t(i, j) / 1 \leq i, j \leq n\}$  reflects the fuzzy times for activity  $(i, j)$ .

#### 3.1. Traditional Forward and Backward Calculations for Acyclic Networks

The objective of this study is to identify the critical path, or the sequence of operations with the longest duration, as well as the float for each operation. This allows for easier comparison between actual progress and the project's anticipated completion date. To achieve this, we perform specific calculations that yield the following data:

1. Forward Calculation
2. Backward Calculation

**Forward Calculation:**

- Triangular Intuitionistic Fuzzy Earliest Start (TrIFESTj) for the project network's is determined via forward pass computations.
- $TrIFEST_j = \max \{TrIFEST_i \oplus \text{duration}\}$ ,  $i =$  number of nodes before it.
- $TrIFEST(\text{node } 1) = TrIFLFT(\text{node } 1) = 0$
- $TrIFEFT_j = TrIFEST_i \oplus \text{duration}$ .

**Backward Calculation:**

- Triangular Intuitionistic Fuzzy Latest Finish (TrIFLFTj) for the project network's is determined via backward pass computations.
- $TrIFLFT_i = \min \{TrIFLFT_j \ominus TrIFt_{ij}\}$ ,  $j =$  number of nodes after it.
- $TrIFLFT(n) = TrIFES(n) = n$
- $TrIFLS = TrIFLFT \ominus \text{duration}$

Definition 3.1.1: Triangular Intuitionistic Fuzzy Total Float (TrIFTF)

The Triangular Intuitionistic Fuzzy Total Float (TrIFTF) is the amount of time that can be added to an activity without impacting the project's overall deadline. It is calculated as the difference between the Intuitionistic Fuzzy Backward Latest Time (IFBLT) and the Intuitionistic Fuzzy Backward Earliest Time (IFBET). The formula is expressed as:

$$\text{TrIFTF} = \text{TrIFLF} - \text{TrIFEF} \text{ (or) } \text{TrIFTF} = \text{TrIFLS} - \text{TrIFES}$$

3.2. Ranking for Intuitionistic Fuzzy Triangular Numbers(IFTN)

To perform an intuitionistic fuzzy critical path analysis, it is essential to rank or order intuitionistic fuzzy numbers. Consequently, several fuzzy ordering and evaluation methods have been introduced in the literature. According to Lee and Li (1998) [14], possibility theory serves as the foundation for most fuzzy comparison approaches, which aim to fuzzify uncertainty. Delgado et al. (1998) [8] and Lee and Li (1998) [14] argued that relying on a single value can result in an overly generic presentation of data, which may lack discrimination and sometimes conflict with intuitive thinking from a practical standpoint.

In this subsection, a novel fuzzy number ranking technique called the weight measure is introduced, which computes the metric distance using the left and right membership functions. It is important to note that this technique can rank both positive and negative intuitionistic fuzzy numbers (IFN).

Theorem: Weight Measure for IFTN

If  $(x, y, z)$   $(x', y, z')$ , be the IFTN then weight measure (WM) is defined as

$$\text{WM} = \left( \frac{x+3y-z}{6}, \frac{2x'+3y-2z'}{6} \right) = (\text{Membership function, non-membership function}).$$

Proof:

Let  $S^{(\gamma)} = \{[L(\gamma), R(\gamma)], [L'(\gamma), R'(\gamma)]\}$  be a IFTN, then the signed distance of  $[L(\gamma), R(\gamma)]$  which is measured from zero is defined as  $d([L(\gamma), R(\gamma)], [L'(\gamma), R'(\gamma)], 0) = ([L(\gamma), R(\gamma)], [L'(\gamma), R'(\gamma)])$

Left Measure  $L(\gamma)$ ,

$$\begin{aligned} \frac{a-x}{y-x} &= \gamma, x \leq a \leq y, [0,1] \\ a - x &= \gamma(y - x) \\ a &= x + \gamma(y - x) \end{aligned} \tag{1}$$

Right measure  $R(\gamma)$ ,

$$\begin{aligned} \frac{z-a}{z-y} &= \gamma, y \leq a \leq z, [0,1] \\ z - a &= \gamma(z - y) \\ a &= z - \gamma(z - y) \end{aligned} \tag{2}$$

From equation (1) and (2)

$$\begin{aligned} S^{(\gamma)} &= [L(\gamma), R(\gamma)] = [x + (y - x)\gamma, z - \gamma(z - y)] \\ \text{WM} &= \int_0^1 [L(\gamma)]\gamma \, d\gamma + \int_1^0 [R(\gamma)]\gamma \, d\gamma + \int_0^1 (y)\gamma \, d\gamma \\ &= \int_0^1 [x + \gamma(y - x)]\gamma \, d\gamma + \int_1^0 [z - \gamma(z - y)]\gamma \, d\gamma + \int_0^1 [y]\gamma \, d\gamma \end{aligned}$$

After Integrating and applying the limit we get,  $\text{WM} = \frac{x+3y-z}{6}$

For calculating non-membership function. consider,

Left Measure  $L'(\gamma)$ ,

$$\begin{aligned} \frac{x'-a}{y-x'} &= \gamma \\ x' - a &= \gamma(y - x') \\ a &= x' + \gamma(y - x') \end{aligned} \tag{3}$$

Right measure  $R'(\gamma)$ ,

$$\begin{aligned} \frac{a-y}{z'-y} &= \gamma \\ a - y &= \gamma(z' - y) \\ a &= y + \gamma(z' - y) \end{aligned} \tag{4}$$

From equation (3) and (4),

$$\begin{aligned} S^{(\gamma)} &= [L'(\gamma), R'(\gamma)] = [x' + (y - x')\gamma, y + \gamma(z' - y)] \\ \text{WM} &= \int_1^0 L'(\gamma) \gamma \, d\gamma + \int_0^1 [R'(\gamma)] \gamma \, d\gamma + \int_1^0 (y) \gamma \, d\gamma \\ &= \left[ -\int_0^1 [x' + (y - x')\gamma] \gamma \, d\gamma + (-\int_1^0 [y + \gamma(z' - y)] \gamma \, d\gamma) + (-\int_0^1 [y] \gamma \, d\gamma) \right] \\ &= -\left[ \left(\int_0^1 [x' + (y - x')\gamma] \gamma \, d\gamma\right) + \left(\int_1^0 [y + \gamma(z' - y)] \gamma \, d\gamma\right) + \left(\int_0^1 [y] \gamma \, d\gamma\right) \right] \end{aligned}$$

After Integrating and applying the limit we get,  $\text{WM} = -\left(\frac{-2x'-3y+2z'}{6}\right) = \left(\frac{2x'+3y-2z'}{6}\right)$

Weight measure formula for intuitionistic fuzzy is given in equation 5.

$$\text{WM} = \left(\frac{p+3q-r}{6}, \frac{2p'+3q-2r'}{6}\right) \tag{5}$$

Therefore, the ranking is defined as follows in order to maintain the fuzziness and to analyze the fuzzy numbers:

For any two TFN,  $A = (x_1, y_1, z_1)(x'_1, y_1, z'_1)$  and  $B = (x_2, y_2, z_2)(x'_2, y_2, z'_2)$ ,

- (i) If  $\text{WM}(A) > \text{WM}(B)$ , then  $A > B$
- (ii) If  $\text{WM}(A) = \text{WM}(B)$ , then  $A = B$
- (iii) If  $\text{WM}(A) < \text{WM}(B)$ , then  $A < B$

**Results:**

Some significant findings that support the suggested method are demonstrated in this section.

Let  $A=(x,y,z)(x',y',z')$  be the ITFN then,

- (i) Rank ( $\mathfrak{R}$ ) of  $A = \left(\frac{x+3y-z}{6}, \frac{2x'+3y-2z'}{6}\right)$
- (ii) Left spread (LS) of  $A = \int_0^1 (y - x) \, dx = (y - x)$
- (iii) Right spread (RS) of  $A = \int_0^1 (z - y) \, dx = (z - y)$
- (iv) Divergence (Div) of  $A = \int_0^1 (z - x) \, dx = (z - x)$
- (v) Mean (M) of  $A = \frac{x+y+z}{3}$

**Theorem: A**

If  $X = (x, y, z) (x', y, z')$  and  $U = (u, v, w) (u', v, w')$  be the two IFTN, then

- (i)  $\mathfrak{R}(X) = \mathfrak{R}(U)$
- (ii)  $M(X) = M(U)$
- (iii)  $\text{Div}(X) = \text{Div}(U)$  then
  - a.  $\text{LS}(X) > \text{LS}(U)$  iff  $x < u$
  - b.  $\text{LS}(X) < \text{LS}(U)$  iff  $x > u$
  - c.  $\text{LS}(X) = \text{LS}(U)$  iff  $x = u$

Proof:

$$\begin{aligned} \text{(i)} \quad \mathfrak{R}(X) = \mathfrak{R}(U) &\implies \left(\frac{x+3y-z}{6}\right) = \left(\frac{u+3v-w}{6}\right) \\ &x + 3y - z = u + 3v - w \end{aligned} \tag{6}$$

$$\begin{aligned} \text{(ii)} \quad M(X) = M(U) &\implies \left(\frac{x+y+z}{3}\right) = \left(\frac{u+v+w}{3}\right) \\ &x + y + z = u + v + w \end{aligned} \tag{7}$$

$$\begin{aligned} \text{(iii)} \quad \text{Div}(X) = \text{Div}(U) &\implies \int_0^1 (z - x) \, dx = \int_0^1 (w - u) \, dx \\ &(z - x) = (w - u) \end{aligned} \tag{8}$$

Solving equation (6) and (8), we get

$$\begin{aligned} x + 3y - z &= u + 3v - w \\ 3y - z + x &= 3v - w + u \\ 3y - (z - x) &= 3v - (w - u) \\ \text{Using equation (8), } 3y - (w - u) &= 3v - (w - u) \\ 3y &= 3v \\ y &= v \end{aligned} \tag{9}$$

a.  $LS(X) > LS(U)$  iff  $x < u$

Using equation (9),

$$\begin{aligned} y - x &> v - u \\ y - x &> y - u \\ -x &> -u \\ x &< u \end{aligned} \tag{10}$$

b.  $LS(X) < LS(U)$  iff  $x > u$

Using equation (9),

$$\begin{aligned} y - x &< v - u \\ y - x &< y - u \\ -x &< -u \\ x &> u \end{aligned} \tag{11}$$

c.  $LS(X) = LS(U)$  iff  $x = u$

Using equation (9),

$$\begin{aligned} y - x &= v - u \\ y - x &= y - u \\ -x &= -u \\ x &= u \end{aligned} \tag{12}$$

*Hence Proved*

**Theorem: B**

If  $X = (x, y, z)$  ( $x', y, z'$ ) and  $U = (u, v, w)$  ( $u', v, w'$ ) be the two ITFN,

- (iv)  $\Re(X) = \Re(U)$
- (v)  $M(X) = M(U)$
- (vi)  $\text{Div}(X) = \text{Div}(U)$  then

- a.  $RS(X) > RS(U)$  iff  $z > w$
- b.  $RS(X) < RS(U)$  iff  $z < w$
- c.  $RS(X) = RS(X)$  iff  $z = w$

Proof:

a.  $RS(X) > RS(U)$  iff  $z > w$

$$\begin{aligned} z - y &> w - v \\ \text{Using equation (9)} \quad z - y &> w - y \\ z &> w \end{aligned} \tag{13}$$

b.  $RS(X) < RS(U)$  iff  $z < w$

$$\begin{aligned} z - y &< w - v \\ z - y &< w - y \\ y &< v \end{aligned} \tag{14}$$

c.  $RS(X) = RS(U)$  iff  $z = w$

$$\begin{aligned} z - y &= w - v \\ z - y &= w - y \\ z &= w \end{aligned} \tag{15}$$

*Hence Proved.*

**Theorem: C**

If  $X = (x, y, z)$  ( $x', y, z'$ ) and  $U = (u, v, w)$  ( $u', v, w'$ ) be the two ITFN then,

- (i)  $\Re(X+U) = \Re(X) + \Re(U)$
- (ii)  $\Re(-X) = -\Re(X)$

Proof:

(i)  $X + U = (x + u, y + v, z + w)$

$$\begin{aligned} \mathfrak{R}(X+U) &= \frac{(x+u) + 3(y+v) - (z+w)}{6} \\ &= \frac{x+u + 3y+3v -z-w}{6} \\ &= \frac{x + 3y -z+u+3v-w}{6} \\ &= \frac{x + 3y -z}{6} + \frac{u + 3v -w}{6} \\ \mathfrak{R}(X+U) &= \mathfrak{R}(X) + \mathfrak{R}(U) \end{aligned}$$

(ii)  $\mathfrak{R}(-X) = -\mathfrak{R}(X)$

$$\begin{aligned} X = (x, y, z) \quad (x', y, z') \Rightarrow -X = (-x, -y, -z) \quad (-x', -y, -z') \\ \mathfrak{R}(-X) = \frac{-x-3y+z}{6} \\ \mathfrak{R}(-X) = \frac{-x-3y+z}{6} = -\left(\frac{x+3y-z}{6}\right) = -\mathfrak{R}(X) \\ \mathfrak{R}(-X) = -\mathfrak{R}(X) \end{aligned}$$

Hence proved.

Note: Similarly, we can prove all the above proposition for non-membership function.

### 3.3. Criticality Degree

To calculate the intuitionistic fuzzy criticality degrees (IFCD) of activities and paths, we developed a method for measuring criticality and provided an analytical approach that integrates intuitionistic fuzzy set theory with the PERT methodology. The criticality degree of an activity (i,j)(i, j)(i,j) is defined as follows: if the fuzzy float of the activity (i, j) is IFTF(i,j) = (x<sub>ij</sub>, y<sub>ij</sub>, z<sub>ij</sub>) (x' <sub>ij</sub>, y<sub>ij</sub>, z' <sub>ij</sub>),

Criticality Degree formula for the membership function:

$$\mathbf{IFCD}_{(i,j)}(A) = \begin{cases} \frac{-x}{y-x} & ; \quad x \leq 0 < y \\ \frac{z}{z-y} & ; \quad y < 0 \leq z \end{cases} \quad (16)$$

Criticality Degree formula for the non - membership function:

$$\mathbf{IFCD}_{(i,j)}(A') = \begin{cases} \frac{y}{y-x'} & ; \quad p' \leq 0 < q \\ \frac{-y}{z'-y} & ; \quad q < 0 \leq z' \end{cases} \quad (17)$$

and the criticality of the path is defined as,

$\mathbf{IFCD}(P_i) = \max_{(i,j) \in P} \{ \mathbf{IFCD}_{(i,j)} \}$ , define the membership function and

$\mathbf{IFCD}(P_i) = \min_{(i,j) \in P} \{ \mathbf{IFCD}_{(i,j)} \}$ , define the non-membership function, where p<sub>i</sub> defines the i<sup>th</sup> path across the network and IFCD (P<sub>i</sub>) represents its criticality.

A path  $\hat{P}$  is known as to be critical if  $\mathbf{IFCD}(\hat{P}) = \max \{ \mathbf{IFCD}(\hat{P}_i) \}$  for the membership function and  $\mathbf{IFCD}(\hat{P}) = \min \{ \mathbf{IFCD}(\hat{P}_i) \}$  for the non-membership function, where  $\hat{P}$  defines the set contains every potential path.

Numerical Illustration:

Construct an eight way road from Salem to Chennai in fig 3 and the places were consider as node which was given in table 1.

**Table 1: Network Description**

Node	Description
1	Salem
2	Vellore
3	Polur
4	Thiruvanmalai
5	Cheygar
6	Villupuram
7	Chennai

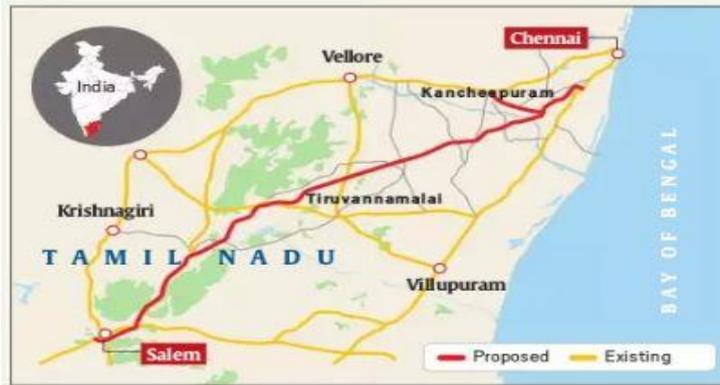


Figure 1: Network

Construct the network.

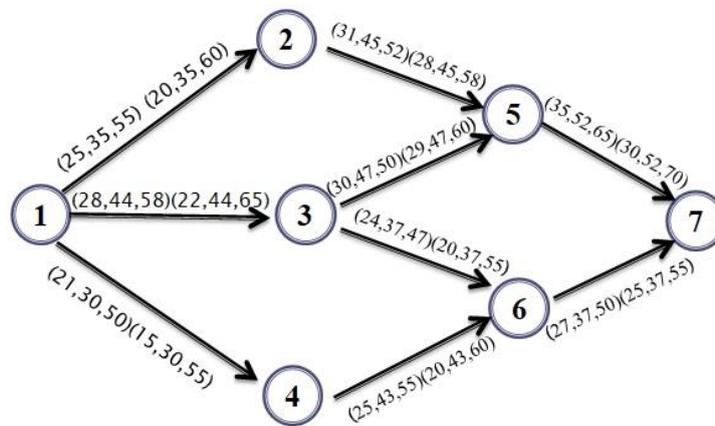


Figure 2: Graphical Network

Table 2: Calculation for IFTI and Defuzzification

Activity	Duration ( $d_{ab}$ )	Defuzzified Value	IFTF	Defuzzified Value
1-2	(25,35,55)(20,35,60)	(12.5,4.167)	(-79,11,82) (-107,11,117)	(-21.33, -69.167)
1-3	(28,44,58)(22,44,65)	(17,7.67)	<b>(-80,0,80) (-114,0,114)</b>	<b>(0, 0)</b>
1-4	(21,30,50)(15,30,55)	(10.167,1.67)	(-62,33,100) (-89,33,135)	(-10.5, -58.167)
2-5	(31,45,52)(28,45,58)	(19,12.5)	(-79,11,82) (-107,11,117)	(-21.33, -69.167)
3-5	(30,47,50)(29,47,60)	(20.167,13.167)	<b>(-80,0,80) (-114,0,114)</b>	<b>(0, 0)</b>
3-6	(24,37,47)(20,37,55)	(14.67,6.83)	(-62,25,94) (-94,25,128)	(-13.5,-61.5)
4-6	(25,43,55)(20,43,60)	(16.5,8.167)	(-62,33,100) (-89,33,135)	(-10.5, -58.167)
5-7	(35,52,65)(30,52,70)	(21,12.67)	<b>(-80,0,80) (-114,0,114)</b>	<b>(0, 0)</b>
6-7	(27,37,50) (25,37,55)	(14.67,8.5)	(-62,25,94) (-94,25,128)	(-13.5, -61.5)

Defuzzified value using equation 5, and IFTF was calculated using definition 3.1., which was displayed in table 2. From table 2, the activity which has (0,0) is identified as Critical path. So, conclude that the critical path is 1-3-5-7 that is Salem – Polur – Cheyyar – Chennai and path length that is total duration is (93, 143, 173) (81, 143, 195) using 3.1.

#### 4. Results and Discussions

##### Verification:

Here, verification is carried out using the Criticality degree for comparison's purpose.

**Table 3:** Calculation for Criticality Degree using equation (17)

Activity	Duration (d <sub>ab</sub> )	IFTF	Defuzzified Value	Criticality Degree
1-2	(25,35,55)(20,35,60)	(-79,11,82)(-107,11,117)	(-21.33, -69.167)	(0.994, 0.040)
1-3	(28,44,58)(22,44,65)	<b>(-80,0,80)(-114,0,114)</b>	<b>(0,0)</b>	<b>(1, 0)</b>
1-4	(21,30,50)(15,30,55)	(-62,33,100)(-89,33,135)	(-10.5, -58.167)	(0.67, 0.12)
2-5	(31,45,52)(28,45,58)	(-79,11,82)(-107,11,117)	(-21.33, -69.167)	(0.885, 0.25)
3-5	(30,47,50)(29,47,60)	<b>(-80,0,80)(-114,0,114)</b>	<b>(0,0)</b>	<b>(1, 0)</b>
3-6	(24,37,47)(20,37,55)	(-62,25,94)(-94,25,128)	(-13.5, -61.5)	(0.67, 0.12)
4-6	(25,43,55)(20,43,60)	(-62,33,100)(-89,33,135)	(-10.5, -58.167)	(0.885, 0.25)
5-7	(35,52,65)(30,52,70)	<b>(-80,0,80)(-114,0,114)</b>	<b>(0,0)</b>	<b>(1, 0)</b>
6-7	(27,37,50)(25,37,55)	(-62,25,94)(-94,25,128)	(-13.5, -61.5)	(0.88, 0.08)

The IFTF was calculated using definition 3.1. The defuzzified value was obtained using equation 5, and the criticality degree was calculated using equations 16 and 17, as shown in Table 3. An activity is identified as part of the Critical Path if it has a defuzzified value of (0,0), a membership function of 1, and a non-membership function of 0 for the criticality degree.

From Table 3, the Intuitionistic Fuzzy Critical Path (IFCP) is identified as Path P1: 1-3-5-7. The comparison of solution yields was made using the proposed procedure, with the criticality degree serving as a verification tool. The outcome of this study is found to be consistent with the results of methods currently in use.

**Table 4:** Verification using various ranking method

Path (P <sub>i</sub> )	α- cut [6]	Magnitude Measure [6]	Similarity Measure [6]	Weight Measure(5)	Ranking
P <sub>1</sub> : 1-2-5-7	14.33,22.7	14.5,22.3	5.538,2.75	3,2	2
P <sub>2</sub> : 1-3-5-7	14.83, 23.5	14.75,22.75	6,3.5	3.5,2.5	1
P <sub>3</sub> : 1-3-6-7	12.83,20.9	12.75,20.5	3.7485,2.5	1.8,1.5	3
P <sub>4</sub> : 1-4-6-7	12.33,20.5	12,20	3.2,1.2	1.5,1	4

From table 4, using different measures we verified that the same critical path 1-3-5-7 is identified as IFCP. Rather than the other measure weight measure gives the optimized value.

**Simulation:**

```

NeuTrON DOS-C++ 0.77, Cpu speed: max 100% cycles, Frameskip 0, Program: TC
ENTER THE ACTIVITY(1,3):28 44 58 22 44 65
ENTER THE ACTIVITY(1,4):21 30 50 15 30 55
ENTER THE ACTIVITY(2,5):31 45 52 28 45 58
ENTER THE ACTIVITY(3,5):30 47 50 29 47 60
ENTER THE ACTIVITY(3,6):24 37 47 20 37 55
ENTER THE ACTIVITY(4,6):25 43 55 20 43 60
ENTER THE ACTIVITY(5,7):35 52 65 30 52 70
ENTER THE ACTIVITY(6,7):27 37 50 25 37 55
L1: 91 132 172 78 132 188
L2: 93 143 173 81 143 195
L3: 79 118 155 67 118 175
L4: 73 110 155 60 110 170
(L1)=398.000000,
(L2)=419.000000,
(L3)=360.000000,
(L4)=340.000000,
The Critical Path is 1-3-5-7
    
```

**Figure 5:** Simulation result using C

```

MATLAB R2020a - academic use
C:\Users\SIMON\Desktop
Command Window
>> cp8
Path1: 91 Path1: 132 Path1: 172 Path1: 78 Path1: 132
Path2: 143
Path2: 173
Path2: 81
Path2: 143
Path2: 195
Path3: 79 Path3: 118 Path3: 155 Path3: 67 Path3: 118
Path4: 110
Path4: 155
Path4: 60
Path4: 110
Path4: 170
The Critical path is Path2 1-3-5-7
The Critical path length is 93
The Critical path length is 143
The Critical path length is 173
The Critical path length is 81
The Critical path length is 143
The Critical path length is 195
fx >>
    
```

Figure 6: Simulation result using MATLAB

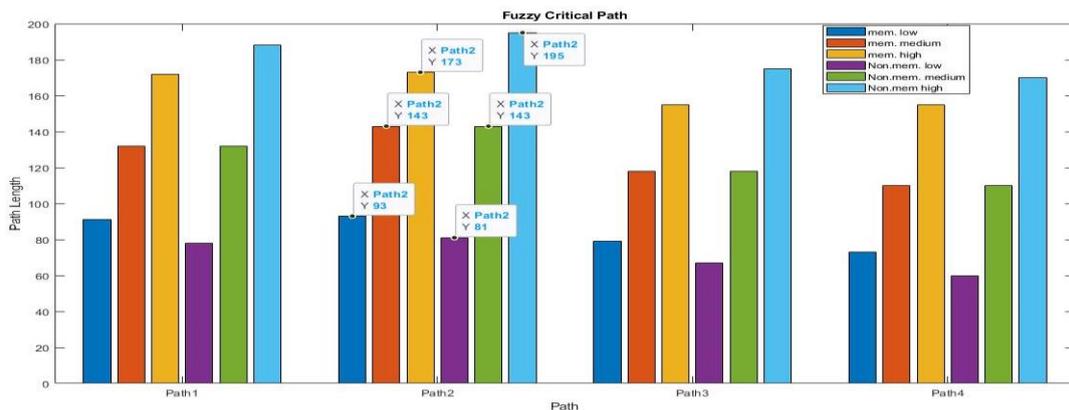


Figure 7: Simulation result using MATLAB

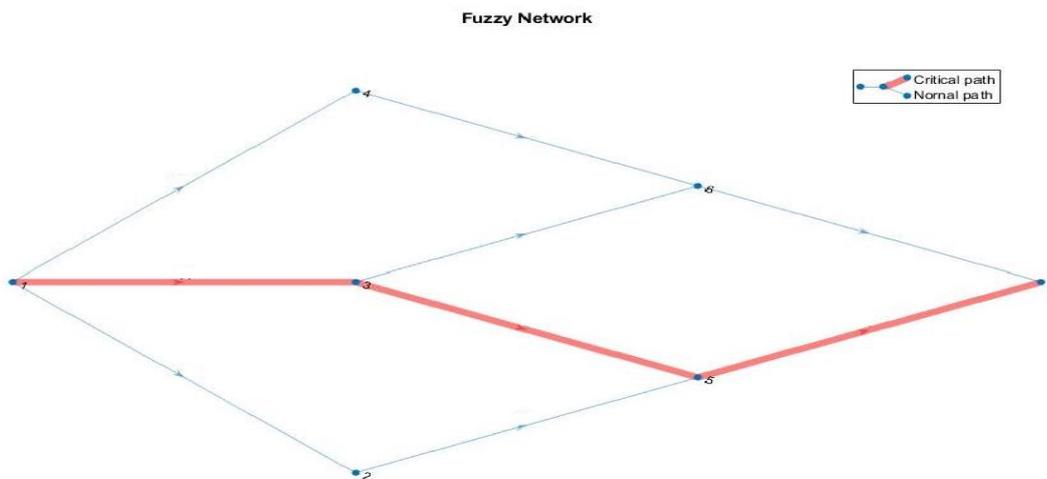


Figure 8: Network diagram using MATLAB

## 5. Conclusion

This study introduces a simple, practical method for ordering fuzzy numbers, specifically designed to rank triangular numbers that are imprecise. Unlike several methods in the literature that cannot handle certain types of fuzzy numbers, this approach can effectively rank crisp numbers, a specific case of fuzzy numbers. The method is easy to use and compute, providing reliable solutions to problems with clear objectives and dependable rankings for issues with ambiguous purposes. Additionally, the ranking measure can be applied to negative integers. Examples are provided to demonstrate the advantages of the proposed strategy. Future research will focus on applying this ranking system to solve fuzzy optimization problems such as network analysis and various fuzzy decision-making challenges like fuzzy risk analysis.

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