RT&A, No 2 (84)
Volume 20, June 2025

ON A CLASS OF INDEFINITE KENMOTSU MANIFOLDS ADMITTING QUARTER-SYMMETRIC METRIC CONNECTION

K. L. Sai Prasad[1], S. Sunitha Devi[2,*]

•

[1] Department of Mathematics, Gayatri Vidya Parishad College of Engineering for Women, Visakhapatnam, 530 048, INDIA

[2,*] Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur District, Andhra Pradesh, 522 302, INDIA

klsprasad@yahoo.com[1] sunithamallakula@yahoo.com[2,*]

Abstract

In this present paper, a class of almost contact metric manifolds equipped with an indefinite metric, termed indefinite Kenmotsu manifolds (known as, ϵ-Kenmotsu), is considered that accepts a connection of quarter-symmetric. We obtain some typical identities for the Riemannian curvature tensor and the Ricci tensor of indefinite Kenmotsu manifolds with respect to the quarter-symmetric metric connection. Furthermore, we derive the scalar curvature of locally symmetric indefinite Kenmotsu manifold which admits a quarter-symmetric metric connection. Finally in this article, in particular, we study and have shown the non-existence of generalised recurrent, ϕ-recurrent and the pseudo-symmetric indefinite Kenmotsu manifolds with respect to quarter-symmetric metric connection.

Keywords: Quarter-symmetric metric connection, indefinite Kenmotsu manifold, locally symmetric manifold, generalised recurrent manifold, ϕ-recurrent manifold, pseudo-symmetric manifold

1. Introduction

In 1969, Takahashi [16] introduced an almost contact manifold equipped with an associated indefinite metric and explored some geometrical properties of almost contact manifolds (particularly, Sasakian manifolds) with indefinite metrics [1]. Later on, it was shown by Xufeng and Xiaoli [20] that those manifolds are real hypersurface of indefinite Kaehlerian manifolds.

On the other hand, in 1972, Kenmotsu [8] established a new class of almost contact manifold known as Kenmotsu manifolds. A Kenmotsu manifold admitting an indefinite metric is termed as an ϵ-Kenmotsu manifold, which was proposed by De and Sarkar [4] in 2009. They showed that the existence of new structure on an indefinite metrics influences the curvatures.

Since the index of the metric generates variety of vector fields such as space-like, time-like and light-like, the study of indefinite structures on manifolds becomes very interesting and of great importance. In view of this, these manifolds have been widely studied by many geometers such as Haseeb, Khan and Siddiqi [7], Rajendra Prasad and Shashikant Pandey [11], Siddiqi and Chaubey [12], Venkatesha and Vishnuvardhana [19] and obtained several results on these manifolds. Recently the authors Sunitha and Sai Prasad established a new class of indefinite almost paracontact metric manifolds, termed (ϵ)-para Kenmotsu manifolds [14].

Let ∇ be a linear connection in an n-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇, respectively given by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y], \tag{1}$$

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z. \tag{2}$$

The connection ∇ is said to be symmetric if its torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is said to be metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and also metric if the connection is of Levi-Civita [7].

A linear connection ∇ in an n-dimensional differentiable manifold is said to be quarter-symmetric [6] if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)\phi(X) - \eta(X)\phi(Y), \tag{3}$$

where η is a 1-form and ϕ is a tensor field of type (1, 1). If we change ϕX by X then the connection is said to be semi-symmetric metric connection [21]. Both quarter-symmetric and semi-symmetric metric connections are widely studied by several researchers, for instance [10, 13, 15, 17, 18].

A non-flat n-dimensional Riemannian manifold $M, n > 3$, is called generalised recurrent [3] if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y,Z)W = \alpha(X)R(Y,Z)W + \beta(X)[g(Z,W)Y - g(Y,W)Z], \tag{4}$$

where α and β are two 1-forms, $(\beta \neq 0)$. If $\beta = 0$ and $\alpha \neq 0$ then M is called recurrent.

A non-flat n-dimensional Riemannian manifold $M, n > 3$, is called ϕ-recurrent [5] if its curvature tensor R satisfies the condition

$$\phi^2((\nabla_X R)(Y,Z)W) = \alpha(X)R(Y,Z)W. \tag{5}$$

A non-flat n-dimensional Riemannian manifold $M, n > 3$, is called pseudo-symmetric [2] if there exists a 1-form α on M such that

$$\begin{aligned}(\nabla_X R)(Y,Z,W) = {} & 2\alpha(X)R(Y,Z)W + \alpha(Y)R(X,Z)W + \alpha(Z)R(Y,X)W \\ & + \alpha(W)R(Y,Z)X + g(R(Y,Z)W,X)A,\end{aligned} \tag{6}$$

where $X, Y, Z \in \chi(M)$ are arbitrary vector fields and $A \in \chi(M)$ is the vector field, given by $g(X,A) = \alpha(X)$. If $\nabla R = 0$ then the manifold M is said to be locally symmetric [9].

In the present paper, we study indefinite Kenmotsu manifolds which accepts a connection of quarter-symmetric. The present paper is organised as follows:

Section 2 is equipped with some prerequisites about indefinite Kenmotsu manifolds. In Section 3, We obtain some typical identities for the Ricci and Riemannian curvature tensors of indefinite-Kenmotsu manifolds with respect to the quarter-symmetric metric connection. In Section 4, we derive the scalar curvature of locally symmetric indefinite Kenmotsu manifold which admits a quarter-symmetric metric connection. In Section 5, we study and have shown the non-existence of generalised recurrent, ϕ-recurrent and the pseudo-symmetric indefinite Kenmotsu manifolds with respect to quarter-symmetric metric connection.

2. Preliminaries

An n-dimensional differentiable manifold (M, g) is said to be an indefinite almost contact metric manifold if it admits a (1, 1) tensor field ϕ, a structure vector field ξ, a 1-form η and an indefinite metric g such that

$$\phi^2 X = -X + \eta(X)\xi, \tag{7}$$

$$\eta(\xi) = 1, \tag{8}$$

$$g(\xi, \xi) = \epsilon, \tag{9}$$

$$\eta(X) = \epsilon g(X, \xi), \tag{10}$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y); \tag{11}$$

for every $X, Y \in \chi(M)$, and $\chi(M)$ is a collection of differentiable vector fields on M. Since the structure vector field ξ which has been vector field that is either space-like or time-like, and the rank of that tensor filed ϕ is $(n-1)$, in this case, (ϵ) is either 1 or -1.

If $g(X,\ Y)$ is positive definite, that is

$$d\eta(X,\ Y) = g(X,\ \phi Y), \tag{12}$$

the manifold $M(\phi,\ \xi,\ \eta,\ g,\ \epsilon)$ is referred as an indefinite almost contact metric manifold. Evidently, on M, we have

$$\phi\ \xi = 0, \qquad \eta(\phi X) = 0. \tag{13}$$

If an indefinite contact metric manifold satisfies

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi - \epsilon\eta(Y)\phi X, \tag{14}$$

the manifolds M is said to be an indefinite Kenmotsu manifold [4]. An indefinite almost contact metric manifold is an indefinite Kenmotsu if and only if

$$\nabla_X \xi = \epsilon(X - \eta(X)\xi). \tag{15}$$

Moreover, the curvature tensor R, the Ricci tensor S and the Ricci operator Q in an indefinite Kenmotsu manifold M with respect to the Levi-Civita connection satisfies [4] the following :

$$(\nabla_X \eta)Y = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{16}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{17}$$

$$R(\xi, X)\xi = X - \eta(X)\xi, \tag{18}$$

$$\eta(R(X, Y)Z) = \epsilon[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)], \tag{19}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{20}$$

$$Q\xi = -\epsilon(n-1)\xi, \ \textit{where}\ g(QX, Y) = S(X, Y). \tag{21}$$

It yields

$$S(\phi X, \phi Y) = S(X, Y) + \epsilon(n-1)\eta(X)\eta(Y). \tag{22}$$

An indefinite Kenmotsu manifold M is said to be η-Einstein manifold if its Ricci tensor is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{23}$$

where a and b are scalar functions of ϵ.

3. Expression of $\widetilde{R}$(X, Y)Z in terms of R(X, Y)Z

Let M be an n-dimensional indefinite Kenmotsu manifold and ∇ be the Levi-Civita connection on M. The quarter-symmetric metric connection $\widetilde{\nabla}$ on M is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \tag{24}$$

The curvature tensor $\widetilde{R}$ of M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ is defined by

$$\widetilde{R}(X,Y)Z = \widetilde{\nabla}_X \widetilde{\nabla}_Y Z - \widetilde{\nabla}_Y \widetilde{\nabla}_X Z - \widetilde{\nabla}_{[X,Y]} Z. \tag{25}$$

By Virtue of (24) and (25), we have

$$\begin{aligned}\widetilde{R}(X,Y)Z =& R(X,Y)Z + (2+\epsilon)\big[g(X,Z)Y - g(Y,Z)X\big] \\ & (1+\epsilon)\big[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\big]\xi \\ & (1+\epsilon)\big[\eta(Y)X - \eta(X)Y\big]\eta(Z),\end{aligned} \tag{26}$$

which is a relation between the curvature tensor of M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ and Levi-Civita cpnnection ∇.

Then from (26) and (17), we get

$$\widetilde{R}(X,\xi)Y = (1+\epsilon)\big[g(X,Y)\xi - \eta(Y)X\big], \tag{27}$$

$$\widetilde{R}(X,Y)\xi = (1+\epsilon)\big[\eta(X)Y - \eta(Y)X\big], \tag{28}$$

and

$$\widetilde{R}(\xi,X)\xi = (1+\epsilon)\big[X - \epsilon\eta(X)\xi\big]. \tag{29}$$

By taking the inner product of (26) with W, we have

$$\begin{aligned}\widetilde{R}(X,Y,Z,W) = R(X,Y,Z,W) &+ (2+\epsilon)\big[g(X,Z)g(Y,W) - g(Y,Z)g(X,W)\big] \\ &+ (1+\epsilon)\big[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\big]g(\xi,W) \\ &+ (1+\epsilon)\big[\eta(Y)g(X,W) - \eta(X)g(Y,W)\big]\eta(Z).\end{aligned} \tag{30}$$

On the contraction of the above expression with respect to X and W, we get

$$\widetilde{S}(Y,Z) = S(Y,Z) + g(Y,Z)\big[(\epsilon+2)(\epsilon-n)+2\big] + \eta(Y)\eta(Z)\big[(1+\epsilon)(n-2\epsilon)\big], \tag{31}$$

where $\widetilde{S}$ and S are the Ricci tensors of the connections $\widetilde{\nabla}$ and ∇, respectively. It shows that in an indefinite Kenmotsu manifold, the Ricci tensor of the quarter-symmetric metric connection is not symmetric.

Further on contracting (31) with regard to Y and Z, results in

$$\widetilde{r} = r + n\big[(\epsilon+2)(\epsilon-n)+2\big] + \big[(1+\epsilon)(n-2\epsilon)\big], \tag{32}$$

where $\widetilde{r}$ and r are the scalar curvatures of the connections $\widetilde{\nabla}$ and ∇, respectively.
From the above, we state the following theorem:

Theorem 1. For an indefinite Kenmotsu manifold M which admits the quarter-symmetric metric connection $\widetilde{\nabla}$, for every $X, Y, Z \in \chi(M)$, we obtain the following:

(a) The curvature tensor $\widetilde{R}$ is given by (26)
(b) The Ricci tensor $\widetilde{S}$ is given by (31)
(c) $\widetilde{R}(X,Y,Z,W) + \widetilde{R}(X,Y,W,Z) = 0$
(d) $\widetilde{R}(X,Y,Z,W) + \widetilde{R}(Y,X,W,Z) = 0$
(e) $\widetilde{S}(Y,\xi) = (1-n)(1+\epsilon)\eta(Y)$
(f) $\widetilde{r} = r + n\ \big[(\epsilon+2)(\epsilon - n) + 2\ \big] + \big[(1+\epsilon)(n-2\epsilon)\big]$
(g) The Ricci tensor $\widetilde{S}$ is not symmetric.

4. Locally symmetric indefinite Kenmotsu manifolds with respect to quarter-symmetric metric connection

In this section, we consider locally symmetric indefinite Kenmotsu manifold with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$.

Theorem 2. Let M be a locally symmetric indefinite Kenmotsu manifold with respect to the quarter symmetric metric connection $\widetilde{\nabla}$. Then the scalar curvature of the Levi-Civita connection of M is $r = (1-n)[1-(n-3)\epsilon]$.

Proof. Assume that M is a locally symmetric indefinite Kenmotsu manifold with respect to quarter symmetric metric connection $\widetilde{\nabla}$. Then $(\widetilde{\nabla}_X \widetilde{R})(Y,Z)W = 0$.

By a suitable contraction of this equation, we get

$$\widetilde{\nabla}_X(\widetilde{S}(Z,W)) = \widetilde{\nabla}_X \widetilde{S}(Z,W) - \widetilde{S}(\widetilde{\nabla}_X Z, W) - \widetilde{S}(Z, \widetilde{\nabla}_X W) = 0.$$

For $W = \xi$ in the above equation, we have

$$\widetilde{\nabla}_X(\widetilde{S}(Z,\xi)) = \widetilde{\nabla}_X \widetilde{S}(Z,\xi) - \widetilde{S}(\widetilde{\nabla}_X Z, \xi) - \widetilde{S}(Z, \widetilde{\nabla}_X \xi) = 0.$$

By making use of (15), (20), (24) and (31), we get

$$-S(Z,X) + g(Z,X)[n\epsilon - 1 - 2\epsilon] + \eta(X)\eta(Z)\big[(1-n)(\epsilon-1) - (1+\epsilon)(n-2\epsilon)\big] = 0. \tag{33}$$

Then on contracting the above equation over X and Z, we obtain

$$r = (1-n)[1-(n-3)\epsilon], \tag{34}$$

which completes the proof of the theorem. ■

5. Non-existence of certain kinds of indefinite Kenmotsu manifolds with respect to the quarter-symmetric metric connection

Theorem 3. There is no generalised recurrent indefinite Kenmotsu manifold with respect to quarter-symmetric metric connection $\widetilde{\nabla}$.

Proof. Suppose that there exists a generalised recurrent indefinite Kenmotsu manifold M with respect to quarter-symmetric metric connection $\widetilde{\nabla}$. Then from (4), we have

$$(\widetilde{\nabla}_X \widetilde{R})(Y,Z)W = \alpha(X)\widetilde{R}(Y,Z)W + \beta(X)\big[g(Z,W)Y - g(Y,W)Z\big], \tag{35}$$

for all vector fields $X, Y, Z, W \in \chi(M)$.

For $Y = W = \xi$ in (35), we have

$$(\widetilde{\nabla}_X \widetilde{R})(\xi,Z)\xi = \alpha(X)\widetilde{R}(\xi,Z)\xi + \beta(X)\big[\epsilon\eta(Z)\xi - \epsilon Z\big]. \tag{36}$$

Now by using (30) and (17), we have, from the above equation

$$(\widetilde{\nabla}_X \widetilde{R})(\xi,Z)\xi = \big[Z - \eta(Z)\xi\big]\alpha(X) + \epsilon\big[Z - \eta(Z)\xi\big]\big[\alpha(X) - \beta(X)\big]. \tag{37}$$

On the other hand, in view of (15), (17), (24), (27), (28) and (29), we have

$$(\widetilde{\nabla}_X \widetilde{R})(\xi,Z)\xi = 0, \tag{38}$$

and hence from the equations (37) and (38), we have

$$\big[Z - \eta(Z)\xi\big]\alpha(X) + \epsilon\big[Z - \eta(Z)\xi\big]\big[\alpha(X) - \beta(X)\big] = 0. \tag{39}$$

Replacing Z by ϕZ in (39) we get

$$\phi Z\,\alpha(X) + \epsilon\,\phi Z\big[\alpha(X) - \beta(X)\big] = 0, \tag{40}$$

which implies $\alpha = \beta = 0$. This contradicts the condition $\beta \neq 0$. Therefore the statement of the theorem follows. ■

Theorem 4. There is no ϕ-recurrent indefinite Kenmotsu manifold with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$.

Proof. Suppose that there exists a ϕ-recurrent indefinite Kenmotsu manifold with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$. Then from (5), we have

$$\phi^2[(\widetilde{\nabla}_X\widetilde{R}(Y,Z)W] = \alpha(X)\widetilde{R}(Y,Z)W,$$

for all vector fields $X, Y, Z, W \in \chi(M)$.

Then on using (7), we get

$$-(\widetilde{\nabla}_X\widetilde{R})(Y,Z)W + \eta((\widetilde{\nabla}_X\widetilde{R}(Y,Z)W)\xi = \alpha(X)\widetilde{R}(Y,Z)W. \tag{41}$$

By replacing Y and W with ξ in (41), we have

$$-(\widetilde{\nabla}_X\widetilde{R})(\xi,Z)\xi + \eta((\widetilde{\nabla}_X\widetilde{R}(\xi,Z)\xi)\xi = \alpha(X)\widetilde{R}(\xi,Z)\xi. \tag{42}$$

On the other hand, from (38) we have

$$(\widetilde{\nabla}_X\widetilde{R})(\xi,Z)\xi = 0.$$

Therefore the above equation (42) turns into

$$\alpha(X)\widetilde{R}(\xi,Z)\xi = 0.$$

Then by virtue of (29), it is obvious that

$$\alpha(X)(1+\epsilon)[Z - \epsilon\eta(Z)\xi] = 0.$$

This implies $\alpha(X) = 0$ for any vector field $X \in \chi(M)$.

It contradicts the condition that $\alpha \neq 0$ and hence the theorem is proved. ∎

Theorem 5. There is no pseudo-symmetric indefinite Kenmotsu manifold with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$.

Proof. Suppose that there exists a pseudo-symmetric indefinite Kenmotsu manifold M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$. Then from (6) we have

$$\begin{aligned}(\widetilde{\nabla}_X\widetilde{R})(Y,Z)W = {} & 2\alpha(X)\widetilde{R}(Y,Z)W + \alpha(Y)\widetilde{R}(X,Z)W \\ & + \alpha(Z)\widetilde{R}(Y,X)W + \alpha(W)\widetilde{R}(Y,Z)X \\ & + g(\widetilde{R}(Y,Z)W,X)A.\end{aligned} \tag{43}$$

By a suitable contraction of (43), we get

$$\begin{aligned}(\widetilde{\nabla}_X\widetilde{S})(Z,W) = {} & 2\alpha(X)\widetilde{S}(Z,W) + \alpha(\xi)\widetilde{R}(X,Z,W,\xi) \\ & + \alpha(Z)\widetilde{S}(X,W) + \alpha(W)\widetilde{S}(Z,X) + g(\widetilde{S}(Z,W),X)A.\end{aligned} \tag{44}$$

Taking $W = \xi$ in (44) and on using the equations (20), (27), (28) and (31), we obtain

$$\begin{aligned}(\widetilde{\nabla}_X\widetilde{S})(Z,\xi) = {} & 2\alpha(X)\widetilde{S}(Z,\xi) + \alpha(\xi)\widetilde{R}(X,Z,\xi,\xi) \\ & + \alpha(Z)\widetilde{S}(X,\xi) + \alpha(\xi)\widetilde{S}(Z,X) + g(\widetilde{S}(Z,\xi),X)A.\end{aligned} \tag{45}$$

On the other hand, by the covariant differentiation of the Ricci tensor $\widetilde{S}$ with respect to the quarter symmetric metric connection $\widetilde{\nabla}$, we have

$$(\widetilde{\nabla}_X\widetilde{S})(Z,W) = \widetilde{\nabla}_X\widetilde{S}(Z,W) - \widetilde{S}(\widetilde{\nabla}_XZ,W) - \widetilde{S}(Z,\widetilde{\nabla}_XW). \tag{46}$$

Now by putting $W = \xi$ in (46) and on using (31), (24) and (15), we get

$$\begin{aligned}(\widetilde{\nabla}_X\widetilde{S})(Z,\xi) = &-S(Z,X) + g(Z,X)[n\epsilon - 1 - 2\epsilon]\\ &+ \eta(X)\eta(Z)[(1-n)(\epsilon-1) - (1+\epsilon)(n-2\epsilon)].\end{aligned} \tag{47}$$

Then by comparing the right hand sides of the equations (45) and (47), we have

$$\begin{aligned}-S(Z,X) &+ g(Z,X)[n\epsilon - 1 - 2\epsilon] + \eta(X)\eta(Z)[(1-n)(\epsilon-1) - (1+\epsilon)(n-2\epsilon)]\\ &= 2(1-n)(1+\epsilon)\alpha(X)\eta(Z) + (1-n)(1+\epsilon)\eta(X)\alpha(Z)\\ &+ \alpha(\xi)\big[S(Z,X) + g(Z,X)\big((\epsilon+2)(\epsilon-n)+2\big) + \eta(X)\eta(Z)(1+\epsilon)(n-2\epsilon)\big]\\ &+ (1-n)(1+\epsilon)\eta(Z)\eta(X).\end{aligned} \tag{48}$$

Replacing X and Z with ξ in the above equation, we get (since $n > 3$)

$$3(1+\epsilon)\alpha(\xi) + (1-\epsilon) = 0, \tag{49}$$

which implies

$$\alpha(\xi) = 0. \tag{50}$$

Now we show that $\alpha = 0$ holds for any vector field on M.

By taking $Z = \xi$ in (43) and on using (50), we get

$$(\widetilde{\nabla}_X\widetilde{S})(\xi,W) = 2\alpha(X)\widetilde{S}(\xi,W) + \alpha(W)\widetilde{S}(\xi,X) + g(\widetilde{S}(\xi,W),X)A. \tag{51}$$

Using the aforementioned equation along with (20), (27), (28), (46) and (50), we now obtain

$$\begin{aligned}-S(X,W) &+ g(X,W)[n\epsilon - 1 - 2\epsilon] + \eta(X)\eta(W)(n - n\epsilon + 2\epsilon)\\ &= 2(1-n)(1+\epsilon)\alpha(X)\eta(W) + (1-n)(1+\epsilon)\eta(X)\alpha(W)\\ &+ \alpha(\xi)\big[S(X,W) + g(X,W)\big((\epsilon+2)(\epsilon-n)+2\big) + \eta(X)\eta(W)(1+\epsilon)(n-2\epsilon)\big].\end{aligned} \tag{52}$$

Further, by taking $w = \xi$ in (52), we have

$$2(1-n)(1+\epsilon)\alpha(X) + \eta(X)(1-n)(1-\epsilon) = 0, \tag{53}$$

which implies $\alpha(X) = 0$ for any vector field $X \in \chi(M)$. This is a contradiction to the definition of pseudo-symmetry. Thus the theorem is proved. ■

6. Conclusion

This paper defines a class of almost contact metric manifolds equipped with an indefinite metric, termed indefinite Kenmotsu manifolds. Since the index of the metric generates variety of vector fields such as space-like, time-like and light-like, the geometrical features of these manifolds are widely applied in a variety of physical and geometrical fields, including the construction of super resolution sensors in electronic and communication systems, in electrical engineering, and in the general theory of relativity.

Acknowledgements: The G. V. P. College of Engineering for Women in Visakhapatnam's Senior Assistant professor, Dr. A. Kameswara Rao, is appreciated by the authors for his insightful advice during the manuscript's preparation.

Declarations of conflicting interests: The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] A. Bejancu, A. and Duggal, K. L. (1993). Real hypersurfaces of Indefinite Kaehler manifolds. *Int. J. Math. Math. Sci.*, 16(3):545-556.

[2] Chaki, M. C. (1987). On pseudo symmetric manifolds. *An. Stiint. Univ. Al. I. Cuza Iasi Sect. I a Mat.*, 33:53-58.

[3] De, U. C. and Guha, N. (1991). On generalised recurrent manifolds. *Proc. Math. Soc.*, 7:7-11.

[4] De, U. C. and Sarkar, A. (2009). On ϵ-Kenmotsu manifold. *Hardonic J.*, 32(2): 231-242.

[5] De, U. C., Shaikh, A. A. and Biswas, S. (2003). On ϕ-recurrent Sasakian manifolds. *Novi Sad J. Math.*, 33(2):43-48.

[6] Golab, S. (1975). On semi-symmetric and quarter-symmetric linear connections. *Tensor (N.S.)*, 29:249-254.

[7] Haseeb, A., Khan, M. A. and Siddiqi, M. D. (2016). Some more results on an ϵ-Kenmotsu manifold with a semi-symmetric metric connection. *Acta Math. Univ. Comenianae*, 85(1):9-20.

[8] Kenmotsu, K. (1972). A class of almost contact Riemannian manifold. *Tohoku Math. J.*, 24:93-103.

[9] Neill, B. O. (1983). Semi-Riemannian geometry with applications to relativity. *Pure and Applied Mathematics*, Academic Press, Inc., New York, 103:1-483.

[10] Pathak, G. and De, U. C. (2002). On a semi-symmetric connection in a Kenmotsu manifold. *Bull. Calcutta Math. Soc.*, 94(4):319-324.

[11] Rajendra Prasad and Shashikant Pandey. (2017). An indefinite Kenmotsu manifold endowed with quarter symmetric metric connection. *Global Journal of Pure and Applied Math.*, 13(7):3477-3495.

[12] Siddiqi, M. D. and Chaubey, S. K. (2020). η-Einstein solitons on ϵ-Kenmotsu manifolds. *Kyungpook Math. J.*, 60:805–819.

[13] Sunitha Devi, S. and Sai Prasad, K. L. (2023). On a class of Lorentzian para-Kenmotsu manifolds admitting quarter-symmetric metric connection. *Reliability: Theory and Applications*, 18(4):525-532.

[14] Sunitha Devi, S. and Sai Prasad, K. L. (2025). A class of indefinite almost paracontact metric manifolds. *International Journal of Maps in Mathematics*, 8(1):247-257.

[15] Sunitha Devi, S., Satyanarayana, T. and Sai Prasad, K. L. (2021). Curvature tensors in SP-Kenmotsu manifolds with respect to quarter-symmetric metric connection. *Reliability: Theory and Applications*, 16(4):210-218.

[16] Takahashi, T. (1969). Sasakian manifold with pseudo-Riemannian metric. *Tohoku Math. J. Second Ser.*, 21:271–290.

[17] Tripathi, M. M. (1999). On a semi symmetric metric connection in a Kenmotsu manifold. *J. Pure Math.*, 16:67-71.

[18] Tripathi, M. M. (2001). On a semi symmetric non-metric connection in a Kenmotsu manifold. *Bull. Calcutta Math. Soc.*, 93:323-330.

[19] Venkatesha and Vishnuvardhana, S. V. (2017). ϵ-Kenmotsu manifolds admitting a semi-symmetric metric connection. *Ital. J. Pure Appl. Math.*, 38:615–623.

[20] Xufeng, X. and Xiaoli, C. (1998). Two theorem on ϵ-Sasakian manifolds. *Int. J. Math. and Math. Sci.*, 21(2):249-254.

[21] Yano, K. (1970). On semi-symmetric metric connection. *Rev. Roumaine Math. Pures Appl.*, 15:1579-1586.