DESIGNING AN ATTRIBUTE CONTROL CHART BASED ON EXPONENTIAL-RAYLEIGH DISTRIBUTION UNDER HYBRID CENSORING

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Abstract

Statistical Process Control (SPC) is an approach to quality assurance that leverages statistical tools to analyse, monitor, and enhance processes. A control chart is an essential tool for observing process performance; it provides a visual mechanism to identify significant deviations caused by assignable factors. This chart compares the values of a quality attribute against specified control limits. Statistical distributions play a crucial role in parametric inferences and are commonly utilized to characterize real-world phenomena. Researchers have directed their attention towards the advancement of these widely used distributions to generate models that are progressively more realistic and versatile for data interpretation. The new Exponential-Rayleigh distribution can be derived by combining the cumulative distribution functions of both the Exponential and Rayleigh distributions. In this paper, we present a new attribute np control chart aimed at monitoring the median lifetime of products within a hybrid censoring scheme, based on the premise that the lifetimes of these products follow an Exponential-Rayleigh distribution. We determine the optimal parameters for the construction of the proposed control chart to achieve an average run length for the in-control process that is as near as possible to the designated average run length. Various sets of values are utilized to estimate the parameters of the control chart, and the performance of the developed control chart is analyzed through the average run length. Numerical examples are presented to illustrate the proposed control chart, and its applicability is validated with simulated data.

Keywords: exponential-Rayleigh distribution, attribute control chart, hybrid censoring scheme, average run length

1. Introduction

The control chart stands out as one of the most prevalent and effective methods for quality control, providing a visual representation of data collected from the manufacturing industry. It typically includes a central line (CL) that signifies the average, an upper line that marks the upper control limit (UCL), and a lower line that indicates the lower control limit (LCL). To enhance the monitoring of production processes in a variety of situations, many advanced control charting techniques have been introduced. Several varieties of control charts are similar and have been designed to correspond with certain aspects of the quality attribute under consideration. For some products, lifetime is considered as a quality characteristic. Life tests are utilized to evaluate the manufacturing process of these products. The classification of a product as conforming or non-conforming is based on the results obtained from the life test. This testing approach is notably time-intensive due to the prolonged nature of the tests. Consequently, the use of censoring techniques is imperative and should not be disregarded. Some of the censoring techniques that are commonly employed in life testing include Type-I censoring, Type-II censoring, and hybrid censoring.

Attribute control charts, including the np control chart, are constructed on the premise that the fraction of non-conforming items is based on a normal distribution of the quality characteristic. Nevertheless, the actual distribution of these characteristics may not be normal. Therefore, the use of the current control chart in such scenarios could lead to misconceptions for industrial engineers and an increase in the occurrence of non-conforming items. Numerous studies focusing on the development of attribute control charts utilizing various lifetime distributions are documented in the literature. Notable contributions include [1] on the Weibull distribution, [2] addressing the Pareto distribution, and [3] exploring the Birnbaum-Saunders distribution. Further research by [4] revisits the Weibull distribution, while [5] examines the Inverse Rayleigh distribution. Reference [6] investigates the Burr X and XII, Inverse Gaussian, and Exponential distributions. Additional studies include [7] on the Lognormal distribution, [8] on the Exponentiated Half Logistic distribution, and [9] on the Dagum distribution. Reference [10] contributes to the literature on the Pareto distribution, while [11] discusses the Weibull-Pareto combination. Reference [12] focuses on the Exponentiated Exponential distribution, and [13] analyzes the Gamma distribution. Reference [14] presents findings on the Log-Logistic distribution, and [15] investigates the Weibull distribution. Reference [16] explores the Generalized Log Logistic distribution, while [17] examines the Generalized Exponential distribution. Reference [18] focuses on the Inverse Weibull distribution, and [19] studies the Length-Biased Weighted Lomax distribution. Reference [20] contributes insights on the Rayleigh distribution, and [21] investigates the Half Normal and Half Exponential Power distributions. Recently, [22] proposed an attribute np control chart that is based on the Exponentiated Exponential distribution, implemented in the framework of an accelerated life test with a hybrid censoring approach.

The Exponential and Rayleigh distributions are fundamental in the fields of life testing and reliability theory. Various methodologies have been employed to generalize these established distributions, enhancing their flexibility to better accommodate real-world data. One notable approach found in statistical literature for model development is the T-X family. Introduced by [23], the T-X families of distributions facilitate the creation of a new class of distributions that provide increased adaptability for modelling diverse data sets. Numerous researchers have derived different classes within these distributions, and several have proposed generalizations of standard distributions based on the T-X families. Recently, [24] presented a novel class of distribution known as the Exponential Rayleigh distribution, which utilizes the Rayleigh distribution as its foundational distribution and the Exponential distribution as its generator through the generator technique. This Exponential Rayleigh distribution exhibits greater flexibility compared to some related models. A review of the literature reveals a lack of research on control charts that incorporate life tests for non-normal distributions, specifically the Exponential-Rayleigh distribution with Hybrid censoring.

This study establishes an attribute control chart based on the assumption that an Exponential-Rayleigh distribution characterizes the lifetime, taking into account the influence of hybrid censoring. The control chart coefficient has been calculated, and the performance of the proposed chart has been analysed concerning the average run length (ARL). The design of the control chart is responsive to changes in the process median. The applicability of the proposed control chart is illustrated using simulated data. Section 2 focuses on the design of the new control chart and presents a procedure for deriving the control chart for a specified strength. The real-world application of the proposed control chart, supported by a simulation study, is discussed in Section 3. The final section offers a summary of the overall findings.

2. Construction of a Control Chart based on Exponential-Rayleigh Distribution

Consider "T" to represent the lifetime of the products manufactured in a production process, which conforms to an Exponential–Rayleigh distribution with parameters λ and β, valid for $t > 0$, as indicated by its density function.

$$f(t) = \lambda \beta t e^{\frac{\beta}{2}t^2} \{e^{-\lambda(e^{\frac{\beta}{2}t^2}-1)}\}, t, \lambda, \beta > 0 \tag{1}$$

The cumulative distribution function for the Exponential – Rayleigh distribution is expressed as follows:

$$F(t) = 1 - \{e^{-\lambda(e^{\frac{\beta}{2}t^2}-1)}\}, t, \lambda, \beta > 0 \tag{2}$$

The median lifetime of the product, as determined by the Exponential – Rayleigh distribution, is expressed as follows:

$$m = \sqrt{\frac{2}{\beta} log\{1 - \frac{log(\frac{1}{2})}{\lambda}\}} \tag{3}$$

where,

$$\beta = \frac{2}{m^2} log\{1 - \frac{log(\frac{1}{2})}{\lambda}\}$$

The expected average (median) lifetime of the product is indicated by m_0 when the process is stable. The termination time for the test, t_0, can be formulated in terms of the median of the in-control process, expressed as $t_0 = a * m_0$, with "a" representing a constant related to test termination. Let p be the probability of an item's failure before reaching the test termination time t_0, which can be defined as follows.

$$p_0 = 1 - \{e^{-\lambda(e^{\frac{\beta t_0^2}{2}}-1)}\} \tag{4}$$

The expression in equation (4) may be modified by representing the parameter β as a function of m.

$$p_0 = 1 - \{e^{-\lambda(e^{a^2 log\{1 - \frac{log(\frac{1}{2})}{\lambda}\}}-1)}\} \tag{5}$$

We propose the following attribute control chart specifically for the Exponential – Rayleigh distribution, which is based on a hybrid censoring scheme that takes into account the number of products present in each subgroup.

- Randomly select a group of n products from the manufacturing process.
- Perform a life test on the chosen items, using t_0 as the test termination time.
- Record the number of items that have failed (denote this as D)
- End the life test either when time t_0 is reached or if the number of failures surpasses the upper control limit (UCL) before reaching t_0, depending on which event occurs first.
- If the number of failures is below the lower control limit (LCL) or above the UCL, the process should be deemed out of control. Conversely, if neither condition is satisfied, the production process is considered to be in control.

This procedure is identified as the np control chart, which focuses on plotting the number of failures rather than the defective fraction (p). The number of defective products adheres to a binomial distribution characterized by parameters n and p_0 in a stable process, where n represents the sample size and p_0 denotes the probability of a product failing before t_0, as specified in equation (5).

Therefore, the subsequent equations are utilized to ascertain the control limits for the proposed control chart.

$$UCL = np_0 + k\sqrt{np_0(1 - p_0)} \tag{6}$$

$$LCL = max[0, np_0 - k\sqrt{np_0(1 - p_0)}] \tag{7}$$

Here, "k" denotes the coefficient associated with control limits, and the failure probability p_0 of the product before the time t_0 is computed through equation (5).

In certain instances, the likelihood of product failure, denoted as p_0, may be indeterminate; therefore, the control limits for the number of defective products are established for practical applications as follows.

$$UCL = \bar{D} + k\sqrt{\bar{D}(1 - \frac{\bar{D}}{n})} \tag{8}$$

$$LCL = max[0, \bar{D} - k\sqrt{\bar{D}(1 - \frac{\bar{D}}{n})}] \tag{9}$$

where $\bar{D}$ represents the average number of defective products that arise during the production process across the entire subgroup.
The probability of the process being recognized as in control is indicated by p_{in}^0 and given as

$$p_{in}^0 = P(LCL \leq D \leq UCL|p_0)$$

$$p_{in}^0 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1 - p_0)^{n-d} \tag{10}$$

The ARL is typically employed to assess the effectiveness of the control chart. The in-control average run length of the proposed control chart, referred to as ARL_0, is established by.

$$ARL_0 = \frac{1}{1 - [\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1 - p_0)^{n-d}]} \tag{11}$$

Let us assume that the average (median) of the process has shifted from m_0 to $m_1 = f * m_0$, where f is a constant representing the degree of shift. Therefore, the failure probability of an item in a state of process instability denoted as p_1, is formulated as.

$$p_1 = 1 - \{e^{-\lambda(e^{(\frac{a}{f})^2 log\{1 - \frac{log(\frac{1}{2})}{\lambda}\}} - 1)}\} \tag{12}$$

The probability of the process being to be in control following the transition to m_1 is now established by

$$p_{in}^1 = P(LCL \leq D \leq UCL|p_1)$$

$$p_{in}^1 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1 - p_1)^{n-d} \tag{13}$$

The ARL for the shifted process, which is considered out of control, is referred to as ARL_1 and can be determined by the following method:

$$ARL_1 = \frac{1}{1 - [\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1 - p_1)^{n-d}]} \tag{14}$$

To evaluate the effectiveness of the proposed control chart in identifying process shifts, we will establish the optimal parameters LCL, UCL, a, and k based on the specified values of in-control ARL. In this study, we examined four scenarios of in-control ARL set at 200, 300, 400, and 500, maintaining a fixed sample size of $n = 25$. Additionally, we analyzed four sample sizes of 20, 25, 30, and 35, while keeping the in-control ARL constant at 370. The values of λ considered were 0.1, 1, and 5. The selection of optimal parameters aimed to ensure that the in-control ARL closely matched the predetermined ARL values, and the out-of-control ARLs were recorded for varying shift constants. The results, including the optimal parameters and their associated out-of-control ARLs, are detailed in Table 1 to Table 6, which reveal a trend of decreasing ARLs as the shift constant f is reduced.

Table 1: *ARLs when the process average (median) shifted for the fixed sample size n = 25*

$\lambda = 0.1$				
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	0	0	0	0
UCL	12	12	13	13
a	0.7433	0.7986	0.7711	0.7820
k	3.3901	3.0418	3.4616	3.2941
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	199.9331	299.6661	399.6790	499.7774
0.90	150.4326	22.2081	161.0164	104.8118
0.80	7.4186	2.0473	6.1044	4.4984
0.70	1.1601	1.0036	1.0781	1.0393
0.60	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

Table 2: *ARLs when the process average (median) shifted for the fixed sample size n = 25*

$\lambda = 1.0$				
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	0	0	0	0
UCL	12	12	13	13
a	0.6069	0.635	0.6423	0.6566
k	3.3888	3.1694	3.4606	3.2932
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	200.2535	300.3884	400.2921	500.3088
0.90	114.5821	190.5079	266.8718	340.8119
0.80	59.8863	29.2275	62.3957	42.6413
0.70	8.4136	4.9009	7.7912	5.8724
0.60	1.8474	1.4247	1.6791	1.4753
0.50	1.0233	1.0053	1.0110	1.0049
0.40	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

Table 3: *ARLs when the process average (median) shifted for the fixed sample size $n = 25$*

$\lambda = 5.0$				
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	0	0	0	0
UCL	12	12	13	13
a	0.6671	0.6417	0.6041	0.6188
k	2.7768	3.0422	3.4614	3.2935
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	200.0398	299.8027	399.8401	500.1496
0.90	44.3556	77.7719	218.1062	394.6173
0.80	9.7549	15.4763	91.5669	62.7539
0.70	2.7615	3.7734	13.3523	9.9351
0.60	1.2569	1.4351	2.7475	2.3034
0.50	1.0076	1.0197	1.1502	1.1001
0.40	1.0000	1.0000	1.0005	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

Table 4: *ARLs when the process average (median) shifted for the fixed $r_0 = 370$*

$\lambda = 0.1$				
Parameter	$n = 20$	$n = 25$	$n = 30$	$n = 35$
LCL	0	0	1	2
UCL	12	13	15	18
a	0.8191	0.8322	0.8124	0.8552
k	3.2103	2.9882	2.9812	2.9722
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	370.2829	369.972	370.1011	370.4807
0.90	106.7354	17.1442	28.1197	9.7886
0.80	4.4606	1.6244	1.8481	1.1557
0.70	1.0343	1.0004	1.0006	1.0000
0.60	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0005	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

Table 5: *ARLs when the process average (median) shifted for the fixed $r_0 = 370$*

	$\lambda = 1.0$			
Parameter	$n = 20$	$n = 25$	$n = 30$	$n = 35$
LCL	0	0	1	2
UCL	12	13	15	18
a	0.7071	0.7257	0.7102	0.7594
k	3.2106	2.8440	3.0584	2.9379
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	370.1107	370.1636	370.0825	369.7833
0.90	350.4508	57.8725	75.3012	41.1492
0.80	48.7114	9.0932	9.8727	5.2281
0.70	6.6761	2.1124	2.0630	1.3939
0.60	1.5712	1.0642	1.0485	1.0044
0.50	1.0072	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

Table 6: *ARLs when the process average (median) shifted for the fixed $r_0 = 370$*

	$\lambda = 5.0$			
Parameter	$n = 20$	$n = 25$	$n = 30$	$n = 35$
LCL	0	0	0	1
UCL	12	13	13	16
a	0.6713	0.6908	0.6060	0.6561
k	3.2101	2.9888	2.7447	2.8279
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	370.3710	370.3930	370.1113	369.8262
0.90	200.7384	70.7588	80.2528	61.7222
0.80	74.9887	13.2549	14.8150	10.0207
0.70	12.2362	3.2301	3.4992	2.4188
0.60	2.7234	1.3122	1.3605	1.1419
0.50	1.1623	1.0092	1.0124	1.0012
0.40	1.0008	1.0000	1.0002	1.0000
0.30	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000

An analysis of Table 1 to Table 6 reveals that the out-of-control ARL, specifically ARL_1, diminishes as the shift constant f is reduced. This indicates the proposed control chart's effectiveness in promptly identifying shifts in the process. To establish the optimal parameters for the design of this control chart, an algorithm can be employed, accommodating various combinations of the specified in-control ARL and sample size.

1. Define the parameters of the ARL, specifically r_0, λ, and the sample size n.
2. Assign the values $a = 0.001$ and $k = 0.001$.
3. Calculate the failure probability of the product (denoted as p_0) under the condition that the process is in control, utilizing equation (5). Additionally, compute the control chart parameters,

namely the Lower Control Limit (LCL) and Upper Control Limit (UCL), by inserting the values of n, p_0, and k into equations (6) and (7).

4. Determine the in-control ARL using equation (11) and compare the result with r_0. Repeat this procedure for various combinations of a and k until the in-control ARL closely approximates the specified ARL, r_0.

When an ARL is identified, the relevant parameters (LCL, UCL, a, k) are deemed the optimal parameters required. Following this, apply these optimal parameters in equation (14) to determine the out-of-control ARL, known as ARL_1.

3. Application of Proposed Control Chart

3.1. Real-Life Application 1

This section presents a design example to illustrate a practical application of the proposed control chart. Consider a manufacturer aiming to improve the quality of its product. It is established that the failure time of the product adheres to the Exponential-Rayleigh distribution with a parameter of $\lambda = 1$, and the desired median lifetime of the product is set at 1000 hours. A sample of size $n = 25$ will be drawn from each subgroup and will undergo a censored life test. The objective for the in-control ARL, denoted as r_0, is 370. According to Table 5, the parameters are established as $a = 0.7257$, $k = 2.8440$, with a lower control limit (LCL) of 0 and an upper control limit (UCL) of 13. Consequently, the manufacturer will implement the control chart as follows:

Step 1: A sample of 25 products will be selected from each subgroup, followed by a life test lasting 726 hours. The number of products that fail during this test will be recorded and referred to as "D."

Step 2: The life test will conclude either upon reaching 726 hours or if the number of failed products exceeds thirteen before the completion of 726 hours, whichever occurs first.

Step 3: If D exceeds 13, the process will be deemed out of control. Conversely, if the value of D falls within the range of 0 to 13, the process will be considered in control.

3.2. Real-Life Application 2

It is assumed that the lifetimes of the products follow an Exponential-Rayleigh distribution characterized by the parameter $\lambda = 5$. The product specifications are as follows: m_0 equals 1000 hours, r_0 is 500, and n is 25. The control chart parameters, as detailed in Table 3, are $k = 3.2935$, $a = 0.6188$, with a lower control limit (LCL) of 0 and an upper control limit (UCL) of 13. Consequently, the control chart was constructed as follows: For each subgroup, a sample of 25 products is selected and subjected to a life test lasting a minimum of 619 hours. The number of failures recorded during this testing phase (D) is then counted. If the value of D falls within the range of 0 to 13, the process is deemed to be in control; if not, it is considered out of control.

3.3. Simulation Study

In this section, the implementation of the proposed control chart is demonstrated through the use of simulated data. The data originates from an Exponential-Rayleigh distribution, assuming the process is in control with parameters $\lambda = 1$ and a desired average (median) lifetime of 1000 hours. Each sample batch consists of a random sample of size $n = 20$. The first fifteen samples are generated from the in-control process, while the following fifteen samples are derived from a shifted process with $f = 0.60$. From Table 5, with $n = 20$ and $ARL_0 = 370$, the constants are identified as $k = 3.2106$, $a = 0.7071$, UCL = 0, and LCL = 12. The number of products exhibiting a lifetime below 707 hours is recorded ("D") and are: 7, 3, 4, 6, 9, 4, 6, 9, 5, 6, 4, 6, 5, 5, 7, 14, 7, 13, 9, 11, 14, 17, 13, 12, 16, 13, 12, 12, 14, 13. The values of the nonconforming items are plotted against two control limits (LCL = 0 and UCL = 12) in Figure 1.

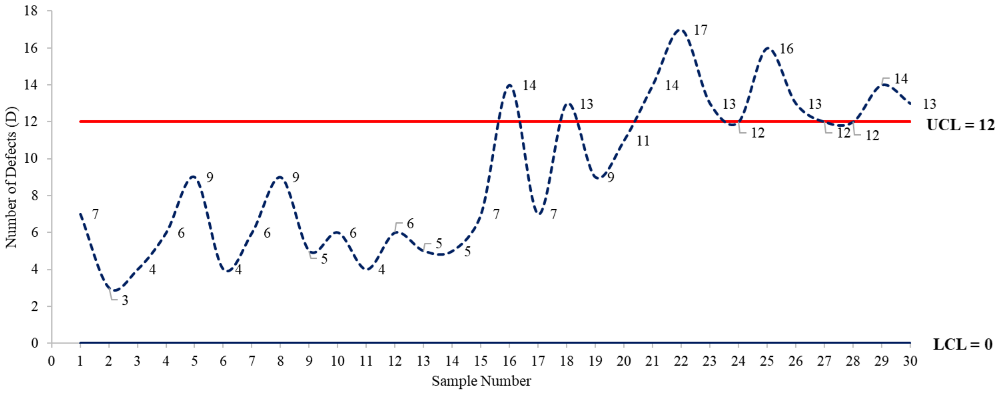

Figure 1: *Control Chart for Simulation Data*

4. Discussion

This paper presents a novel methodology for an attribute control chart designed to assess the lifetime of products. Life tests are conducted using a hybrid censoring approach, which assumes an Exponential-Rayleigh distribution for product lifetimes. The primary objective of the proposed control chart is to maintain the median lifetime of the product as a key quality criterion. This newly developed control chart is highly versatile and can effectively monitor the longevity of quality products. Accompanying tables are provided for practical industrial application and are illustrated using simulated data generated through *R* software based on the Exponential-Rayleigh distribution. The efficacy of the proposed control chart is evaluated in terms of *ARLs* for various shift constants (f). It is important to highlight that employing the hybrid censoring scheme for life testing can significantly reduce both the time and costs associated with sampling inspections. Furthermore, the developed attribute control chart holds promise for adaptation to various other statistical distributions in future research endeavours.

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