RELIABILITY ESTIMATION AND PARAMETERS FOR THE LIFESPAN DISTRIBUTION BASED ON FAILURE TIME DATA FOR ELECTRICAL COMPONENTS

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Abstract

The objective of the present study is to analyse the failure times of electrical components in order to identify the optimal distribution model for their longevity. Fifteen components failure data were analysed using the Weibull, Lognormal, and Exponential lifespan distribution algorithms. The Akaike Information Criterion (AIC), Kolmogorov-Smirnov (KS) test, likelihood ratio test (LRT), and log-likelihood function were among the statistical tests used to estimate and assess the goodness-of-fit of each distribution's essential parameters. The results demonstrate that, with better LRT and AIC values, the exponential distribution best fits this electrical component failure times. Real-world issues are used to illustrate the importance of the suggested model.

Keywords: Weibull, Lognormal, Exponential distribution, Reliability Measures, electrical components

I. Introduction

Reliability theory and survival analysis both heavily rely on lifetime distributions. Weibull, log-normal, exponential, and gamma distributions are a few of the distributions that are frequently used to represent lifetime data. The reason why exponential and Weibull distributions are more often used than Gamma and log-Normal distributions is that the survival function of the latter two does not have a closed form. Studies on survival and reliability have recently made extensive use of mixture distributions. It has been receiving a lot of interest because it allows for the simultaneous modelling of numerous causes of failure and mixture models are more appropriate. Because survival mixture models are so flexible, they are a preferable option for analysing survival or reliability data when the data are thought to be diverse and a single parametric distribution might not be enough. Using power series distributions to compound the exponentiated half-logistic distribution, Esmaeili and Niaparast [1] established the exponentiated half logistics-power series. In order to meet the mean lifetime of the products under Birnbaum-Saunders and Weibull distributions, Aslam et al. [2] devised and created a variation of the sampling plan for multiple dependent states. Using the lognormal distribution, Shaheen et al. [3] created a control chart to track variation under a repeated sampling approach. A new monitoring system for the intervals between events under exponential and gamma distributions was proposed by Shah et al. [4]. The Weibull-Lindly Distribution was introduced by Chacko et al. [5] to model failure rate data with a bathtub shape. By introducing a new parameter into the exponential distribution, Mahdavi and Kundu [6] created a novel alpha power transformation technique that yielded new probability distributions. The transmuted exponential distribution's performance rating was applied to a real-world dataset by Enahoro et al. [7]. Weibull distributions with two or three parameters (shape, scale, and location) can be used to estimate the reliability of a system or sub-system [8].

The two most common approaches used to estimate the values of these characteristics are graphic and analytical procedures. See Barreto-Souza et al. [10] for a new combination of an exponential and binomial distribution, which was part of the exponential power series (EPS) family of distributions created by Chahkandi and Ganjali [9]. The maintenance engineers can use the resulting best-fit data to assist them make strategic decisions about which essential components are most likely to cause machine failure [11]. The distribution that most closely matches the failure times is chosen for analysis [12]. The most popular distributions for modelling failure times are Weibull, Gamma, Exponential, and Lognormal, according to Haviaras [13]. Lawless [14] provides a thorough analysis of Statistical Models and Methods for Lifetime Data. According to [15], the most popular failure distributions used in reliability analysis—exponential, Weibull, normal, and lognormal are critically reviewed in this study. Studying the suggested parameter estimation techniques (Probability Plotting, Least Square Estimation, and Maximum Likelihood Estimation) requires a thorough understanding of the failure rate distributions. In order to determine which distribution best fits a real set of field data, the second section of the research aims to evaluate the benefits and drawbacks of each approach.

In this paper, a new lifetime distribution by considering a series system such that the components are Exponential, log-normal and Weibull distribution is introduced. This lifetime model was introduced for the following main reasons: (i) to propose a new flexible lifetime distribution that can be used to model lifetime data in a broader class of reliability problems; (ii) to extend the log-normal, exponential, and Weibull distributions; (iii) to assess the goodness of fit and estimate distribution parameters using statistical tests like the Kolmogorov-Smirnov and log-likelihood tests.

The following is how the remainder of the article goes: Section 2 introduces the revised distribution. Goodness-of-fit statistics and parameter estimation are covered in Section 3. Section 4 offers applications to actual data. The study concludes with some conclusions in Section 5.

II. Methods

I. Exponential Distribution

The Exponential distribution is commonly known as completely random failure time. It is widely used in the domain of reliability. Exponential distribution shows the "lack of memoryless property". It has only one parameter, which is the scale parameter. Exponential distribution is a constant failure rate and it does not suitable consistent failure rate, typically the components that fatigue or enhances its performance over time.
Recollected equations for this distribution are as:

$$\mathrm{f(f)} = \frac{1}{\theta} e^{-\frac{f}{\theta}} = \lambda e^{-\lambda f}, f \geq 0, \text{ where f is the time to failure.}$$

Reliability function denoted as:

$$\mathrm{R(f)} = e^{-\frac{f}{\theta}} = e^{-\lambda f}, f \geq 0, \text{Where } \theta = \frac{1}{\lambda} > 0 \text{ is an MTTF'S parameter and } \lambda \geq 0$$

is a constant failure rate. The failure rate for the exponential density function is:

$$\text{h(f)} = \frac{f(f)}{R(f)} = \frac{1}{\theta} = \lambda$$

This distribution is favored due to its basic mathematical structure and quick manipulation. The long flat "intrinsic failure" part of the bathtub curve is well modelled by the exponential because of its constant failure rate characteristic. This encourages widespread use since a majority of systems and parts lose the majority of their lifespans in this area of the bathtub curve of the exponential. Inter-arrival times are a good fit for the exponential model. The exponential lifespan model can be used when these occurrences trigger failures.

II. Weibull Distribution

This distribution, which Wallodi Weibull first suggested in 1939, is mostly used in dependability to determine when items fail. An expanded form of the exponential distribution is the Weibull distribution. Forecasting the parts or system of the life span that are being studied is also helpful. Finding the best fit distributions is more commonly accomplished with the Weibull distribution. The cumulative distribution function (CDF) of the two parameter Weibull distribution is

$$F(f, a, b) = 1 - exp\left[-(\frac{f}{a})^b\right]$$

where f is the time to failure, a is the scale parameter, b is the shape parameter. Reliability refers to the likelihood that an item or thing will fulfil its intended function consistently throughout time under given conditions. The reliability function for the two-parameter Weibull distribution is as follows.
Reliability function denoted as:

$$R(f) = exp\left[-(\frac{f}{a})^b\right]$$

The Weibull failure rate function is defined as the number of failures per unit time that can expected to occur for the product. It is given as:

$$\lambda(f) = \frac{b}{a} \times (\frac{f}{a})^{b-1}$$

The two-parameter Weibull probability density function f(t) is given as

$$PDF = f(f) = \frac{b}{a} \times (\frac{f}{a})^{b-1} \times e^{-\left(\frac{f}{a}\right)^b}$$

$$CDF = F(f) = 1 - exp\left[-(\frac{f}{a})^b\right]$$

III. Lognormal Distribution

The lognormal distribution is a distribution of a variable whose logarithm is similar to the normal distribution. The lognormal distribution hypothesis was introduced by McAlister in 1879 and is used in many areas of medicine. The density function for the two-parameter lognormal distribution is:

$$f(f) = \frac{1}{af\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{logf - \theta}{a}\right)^2}$$

Where f is the time to failure,

Reliability function for Lognormal Distribution:

$$R(f) = P(T>f) = 1-F(f)$$

Where F(f) is the CDF of the lognormal Distribution.
Cumulative Distribution function is denoted as:

$$F(f) = P(T > f) = \phi(\frac{log f - \theta}{a})$$

Φ is the standard normal cumulative distribution function.
Reliability function:

$$R(f) = 1- \phi(\frac{log f - \theta}{a})$$

IV. Akaike Information Criteria

The Akaike information criteria (AIC), which are an estimator of the out-of-sample prediction error, help determine the relative quality of statistical models for a given collection of data. AIC estimates the proportionate amount of information lost by a particular model. It should be noted that the more information a model keeps, the better it is. When determining how much information the model loses, AIC takes into account the risk of both over-fitting and under-fitting. The formula for AIC that was employed to compare the parametric model is

$$AIC = -2LL + 2p$$

Where, p is the number of parameters in the model, LL is the log-likelihood and AIC values that are smaller indicate superior models for data adaptation.

IV. Kolmogorov-Simron Test

In reliability analysis, the Kolmogorov-Smirnov (K-S) test is used to determine whether a sample conforms to a particular distribution (Weibull, exponential, lognormal, etc.), which is essential for forecasting product reliability. The real data is shown to be closer to the fitted distribution by the smaller KS Statistic.

$$D = max \mid Fn(f) - F(f) \mid$$

$Fn\,(f)$ is the empirical distribution function and $F(f)$ is the cumulative distribution function of the proposed model.

III. Results and Discussions

The user needs to enter the data in a column containing the equipment failure times in order to begin the reliability study using the code. Figure 1 illustrates how a histogram relating to the frequency of these failures over time is created from these timings. Failure time data for a component must be supplied in order to run the RELPF function. The information must be organised in columns. In order to alter and analyse the data in order to suit the different distribution models, reliability analysis can be performed using R software. Packages like MASS, fidistrplus, and stats4 have been used for this study's setup and initialisation. Installing the MASS, fidistrplus, and stats4 packages from the RELPF function will allow you to estimate all parameters, including shape and scale, based on the distribution that has to be computed. The parameters that the dependability function calculated for the distributions are shown in Table 1.

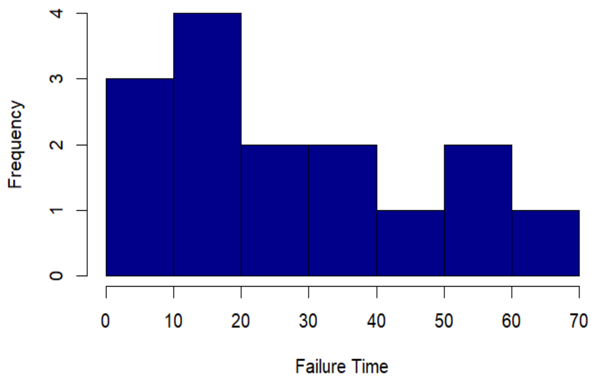

Figure 1: *Histogram generated by the software R of the failure time of electronic components.*

Table 1: *Values of the shape and scale parameters generated by the RELPF function of the failure time of electronic components*

Distributions	Shape parameter	Scale parameter
Lognormal	2.93059	1.02527
Exponential	0.03630	-
Weibull	1.30560	29.7645

From Table 1, in comparison to the other two distributions, the Lognormal distribution has a significant skew and spread, as shown by its scale parameter of 1.02527 as well as shape parameter of 2.93059. The exponential distribution has a modest scale (rate) parameter of 0.03630, indicating a continuous failure rate, due to its memoryless quality. The Weibull distribution's shape parameter of 1.30560 and scale parameter of 29.7645, which show a declining failure rate over time (since the shape is >1 but close to 1), make it more flexible than the other distributions for modelling life data.

Table 2: *Table obtained by the RELPF function showing the LRV, AIC, and Loglik results for the electrical component failure time*

Distributions	Loglik	AIC	LRT	p-value
Lognormal	-65.61742	135.23	2.8629	0.0906
Exponential	-64.73822	131.47	1.1045	0.2937
Weibull	-64.0202	132.04	1.4360	0.2307

The result is obtained in the Table 2 display that the Lognormal model has highest AIC = 135.23, the Weibull model was shown to be the second highest AIC = 132.04 and the Exponential model has the least AIC = 131.47.

Table 3: *Result of the KS test for the failure time of the electronic components.*

Distribution	Statistics KS	D-critic	Hypothesis
Lognormal	0.161	0.3511	It cannot be rejected
Exponential	0.155	0.3511	It cannot be rejected
Weibull	0.098	0.3511	It cannot be rejected

Table 3 indicate the all three models have D-statistics (0.161, 0.155, and 0.098, respectively) below the critical value (0.3511) when the three distributions (Lognormal, Exponential, and Weibull) are compared using the KS test. This means that none of the models can be dismissed. The Weibull distribution, which fits the data the best among them, has the lowest D-statistic (0.098), followed by the exponential and lognormal distributions. As a result, it seems that the Weibull distribution is the best fit for this dataset. The CDFM estimation approach is the best estimation method to produce the estimates of the Lognormal, Exponential, and Weibull parameters. The p value of the K–S test in Table 3 shows that the Lognormal, Exponential, and Weibull can well match this data set. Furthermore, the sample histogram and the fitted PDFs of the Weibull distributions using the parameter estimate findings in Table 3 are displayed in Figure 2.

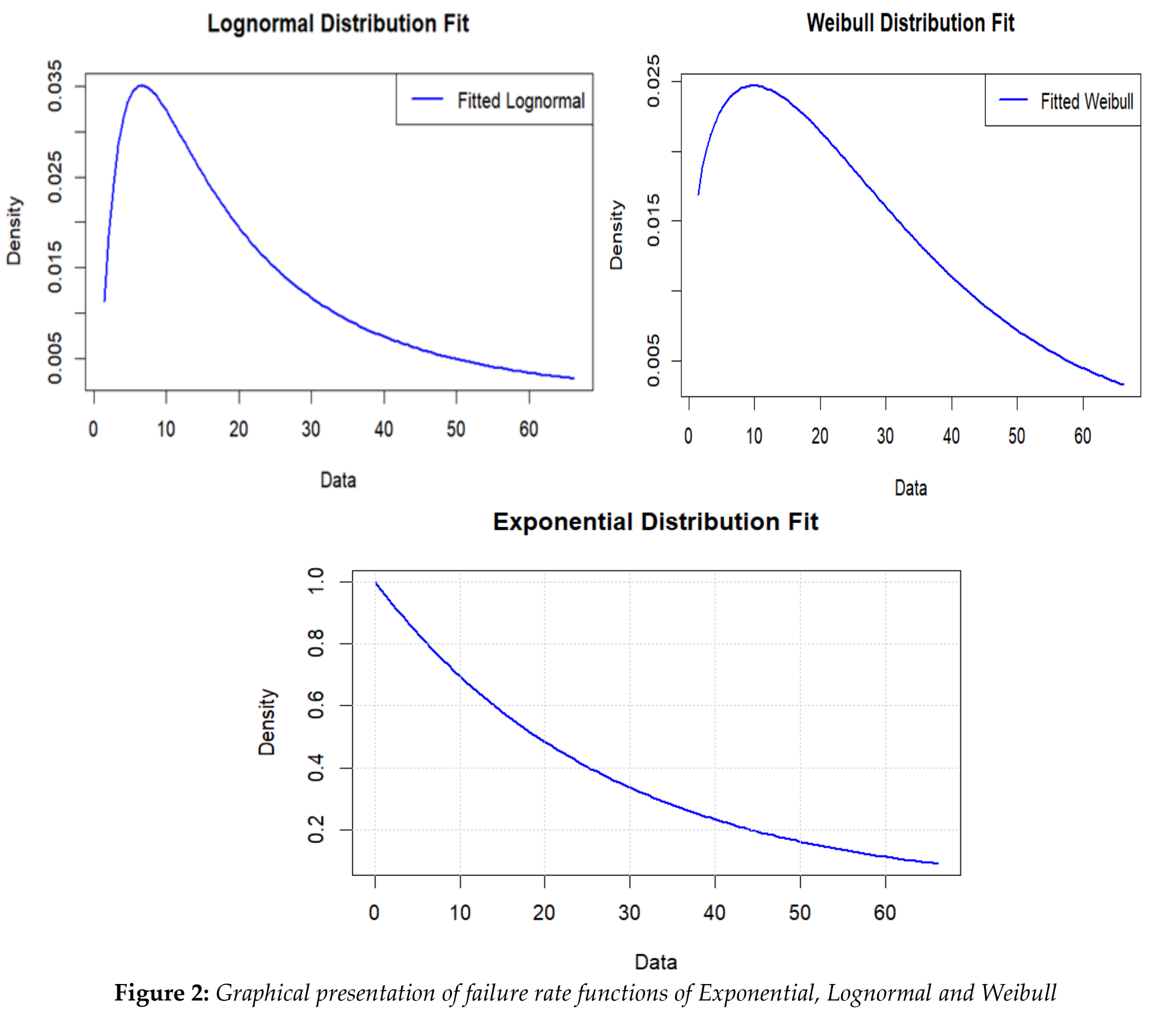

Figure 2: *Graphical presentation of failure rate functions of Exponential, Lognormal and Weibull*

IV. Conclusion

In this paper, a new lifetime distribution by considering a series system such that the components are Exponential, log-normal and Weibull distribution is introduced. The proposed distribution has successfully attained the density function features in the graphical analysis of the exponential, weibull, and lognormal distributions under different functions with varying parameter values. For the lifetime distribution, the statistical and mathematical properties are used. The lifespan distribution satisfies the conditions mentioned above. Using the maximum likelihood estimation method, the parameters of the lifespan distributions are computed. The Akaike Information Criterion (AIC), Kolmogorov-Smirnov (KS) test, likelihood ratio test (LRT), and log-likelihood function were used to verify the goodness of fit test. We also looked at the application of real-time failure time electronic components to lifetime distribution. The random samples were generated from the Exponential, Weibull, and lognormal distributions. The Weibull distribution can better suit the failure time electronic components in actual life, according to the results mentioned above. In various fields of study, including reliability analysis, medical, engineering, and economics, among others, we anticipate that the suggested distribution would attract more extensive applications.

References

[1] Esmaeili, L. and Niaparast, M. (2022). New compounding lifetime distributions with applications to real data. *International Journal of Modeling, Simulation, and Scientific Computing*, 13(05): 2250038.

[2] Aslam, M. Jeyadurga, P. Balamurali, S. Azam, M. and Al-Marshadi, A. (2021). Economic determination of modified multiple dependent state sampling plan under some lifetime distributions. *Journal of Mathematics*, 2021(1): 7470196.

[3] Shaheen, U. Azam, M. and Aslam, M. (2020). A control chart for monitoring the lognormal process variation using repetitive sampling. *Quality and Reliability Engineering International*, 36(3): 1028-1047.

[4] Shah, M. T. Azam, M. Aslam, M. and Sherazi, U. (2021). Time between events control charts for gamma distribution. *Quality and Reliability Engineering International,* 37(2): 785-803.

[5] Chacko, V. M. Deepthi, K. S. Thomas, B. and Rajitha, C. (2018). Weibull-Lindley Distribution: A Bathtub Shaped Failure Rate Model. *Reliability: Theory and Applications*, 13(4): 9-20.

[6] Mahdavi, A. and Kundu, D. (2017). A new method for generating distributions with an application to exponential distribution. *Communications in Statistics-Theory and Methods*, 46(13): 6543-6557.

[7] Owoloko, E. A. Oguntunde, P. E. and Adejumo, A. O. (2015). Performance rating of the transmuted exponential distribution: an analytical approach. *SpringerPlus*, 4: 1-15.

[8] Dolas, D. R. Jaybhaye, M. D. and Deshmukh, S. D. (2014). Estimation the system reliability using Weibull distribution. *International Proceedings of Economics Development and Research*, 75(29): 144-148.

[9] Chahkandi, M. and Ganjali, M. (2009). On some lifetime distributions with decreasing failure rate. *Computational Statistics and Data Analysis*, 53(12): 4433-4440.

[10] Barreto-Souza, W. Santos, A. H. and Cordeiro, G. M. (2010). The beta generalized exponential distribution. *Journal of statistical Computation and Simulation*, 80(2): 159-172.

[11] Herbert, G. J. Iniyan, S. and Goic, R. (2010). Performance, reliability and failure analysis of wind farm in a developing country. *Renewable energy*, 35(12): 2739-2751.

[12] Agete, A. Ayalew, M. M. Admassu, S. and Dessie, Z. G. (2024). Prevalence and associated factors of teenage childbearing among Ethiopian women using semi-parametric and parametric proportional hazard and accelerated failure time models. *BMC Women's Health*, 24(1): 342.

[13] Yang, L. Cheng, J. Luo, Y. Zhou, T. Zhang, X. Shi, L. and Xu, Y. (2025). Domain adaptation via gamma, Weibull, and lognormal distributions for fault detection in chemical and energy processes. *The Canadian Journal of Chemical Engineering*, 103(1): 359-372.

[14] Lawless, J. F. Statistical Models and Methods for Lifetime Data. John Wiley and Sons, New York, 2003.

[15] Rausand, M. A. Hoyland, System Reliability Theory, John Wiley & Sons Inc. Publication, 2004.