ATTRIBUTE CONTROL CHART APPROACH FOR LIFE TESTING USING EXPONENTIAL-POISSON DISTRIBUTION

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Abstract

Manufacturing processes are often monitored using control charts, visual representations of collected data. These charts help determine if a production process is stable. Various control chart methods have been developed to address diverse production scenarios. For some products, lifetime is a crucial quality attribute. Life testing, while valuable for evaluating production processes and classifying products as conforming or non-conforming, can be time-consuming. Consequently, censoring techniques are indispensable in such situations. Life testing often employs censoring methods like Type I, Type II, and Hybrid censoring. Compounding techniques create new distributions, expanding existing mixture models. These compound distributions are crucial in life testing, especially when dealing with processes exhibiting significant early failures. This article introduces a novel attribute np control chart for monitoring the median lifetime of products under a hybrid censoring plan, assuming an Exponential-Poisson lifetime distribution. Optimal control chart parameters are derived to minimize deviations from a target Average Run Length (ARL) for an in-control process. The chart's parameters are calculated for different scenarios, and its performance is evaluated using ARL analysis. Numerical examples and simulated data illustrate the chart's practical application and demonstrate its utility.

Keywords: Exponential-Poisson Distribution, Hybrid Censoring Scheme, Attribute Control Chart, Average Run Length.

I. Introduction

Statistical process control (SPC) is a method for understanding and managing process quality. It involves regularly collecting and analyzing data on key quality attributes, then reacting appropriately to any deviations from established standards or specifications. Visual representations are powerful tools in statistical analysis. Control charts are particularly valuable for maintaining processes at target specifications. A control chart is a graph depicting a quality characteristic, calculated from sample data (such as the mean) and plotted against the sample's collection time or number. This chart comprises three horizontal lines: the center line, upper control limit, and lower control limit. Control charts are categorized into two types: attribute and variable charts.

Attribute charts distinguish between conforming and non-conforming items, while variable charts are employed when data arises from a measurement process.Product longevity is a crucial quality attribute for some items. Manufacturing processes are monitored through life testing, which categorizes products as conforming or non-conforming based on test results. These tests are often time-consuming, necessitating the critical application of censoring techniques. Common methods include Type-I, Type-II, and Hybrid censoring. Hybrid censoring terminates the test upon reaching either a predefined time (t) or the (UCL + 1)th failure. Statistical control of the production process is maintained if the observed number of failures during the test falls within the lower (LCL) and upper (UCL) control limits at time t; otherwise, the process is deemed out of control.

The non-conforming rate, typically tracked using attribute control charts like the np chart, assumes a normal distribution for the quality characteristic. However, real-world quality characteristics often deviate from normality. This discrepancy can lead industrial engineers to erroneous conclusions and potentially an escalation in the number of non-conforming products if the existing control chart remains in use.

This paper [9] proposed an enhanced attribute control chart for more effectively monitoring the proportion of non-conforming items in high-quality manufacturing processes. They Presented [4] attribute control charts based on the Weibull distribution for monitoring product quality under truncated life testing conditions. Compounded univariate distributions [14] and introduces new generalized classes to enhance flexibility in statistical modeling. An attribute control chart based on the Pareto distribution of the second kind for time-truncated life tests [5] evaluating its performance using average run length and simulation studies to monitor non-conforming items in industrial applications. A control chart for gamma-distributed quality characteristics using multiple dependent state sampling [6] introduced demonstrating improved detection of process shifts compared to traditional Shewhart charts. A control chart combining successive sampling with multiple dependent state repetitive sampling [3] presented significantly improving sensitivity to process mean shifts and outperforming existing charts in average run length performance. A n attribute control chart based on the Inverse Rayleigh distribution under Type-I censoring [15] proposed to monitor product lifetimes in life testing scenarios. An attribute control chart for time-truncated life tests using Burr X, Burr XII, inverse Gaussian, and exponential distributions, [16] developed evaluating their performance through average run length analysis. An attribute control chart based on the Exponentiated Half Logistic distribution for time-truncated life tests, [18] showed improved performance through ARL analysis and simulations for industrial quality monitoring. An attribute np control chart utilizing repetitive group sampling under truncated life tests [11] introduced to enhance process monitoring efficiency. An attribute np control chart using multiple deferred state sampling [8] presented to monitor mean life in time-truncated life tests, showing improved detection of process shifts and enhanced performance over existing models. An attribute control chart based on the Dagum distribution for time-truncated life tests [20] proposed demonstrating effective performance in detecting non-conforming items through ARL analysis and simulations. An attribute control chart based on the Exponentiated Exponential distribution under Type-I censoring [12] introduced to monitor product lifetimes in life testing scenarios. The study evaluates the chart's performance using the average run length (ARL) metric and provides numerical examples to illustrate its practical application. An attribute control chart based on the log-logistic distribution for time-truncated life tests, [19] introduced evaluating its performance using average run length (ARL) through Monte Carlo simulations and providing extensive tables for practical implementation. A control chart utilizing repetitive sampling to monitor process variation in log-normal distributions, [17] introduced demonstrating enhanced sensitivity to small process shifts. An attribute control chart utilizing the neutrosophic Weibull distribution [2] introduced to monitor manufacturing processes under uncertainty, demonstrating enhanced efficiency in detecting process shifts compared to existing charts. An attribute control chart based on the generalized exponential distribution for time-truncated life tests, [1] introduced assessing its performance using the average run length (ARL) metric. An attribute control chart based on the Inverse Weibull distribution for time-truncated life tests, [7] presented demonstrating its effectiveness through ARL analysis and simulation for industrial quality monitoring. He [10] introduced an attribute np control chart based on the Exponentiated Exponential distribution under accelerated life tests with hybrid censoring, optimized using ARL analysis and validated through simulations. The paper [13] introduced a flexible two-parameter lifetime distribution with a decreasing failure rate, derived from compounding exponential and zero-truncated Poisson distributions, and analyses its properties and parameter estimation using the EM algorithm.

The Exponential-Poisson distribution finds practical use in diverse areas, including network traffic modeling, manufacturing quality control, service queue management, and stock price analysis. Its application in modeling product lifetimes is particularly significant within the realm of lifetime and reliability studies. Prior research has not investigated control charts for life tests involving non-normal distributions, such as the Exponential-Poisson with hybrid censoring.

This paper introduces a novel attribute control chart specifically designed for this type of data. The paper's structure is as follows: Section 2 details the proposed control chart methodology. Also presents formulas for calculating in-control and out-of-control average run lengths. A practical industrial application, along with a simulation study, is detailed in Section 3. Finally, Section 4 summarizes the findings and conclusions.

II. Design of the control chart

Let t represent the lifetime of a product in a manufacturing process. t follows an Exponential-Poisson distribution with parameters $\lambda > 0$ and $\beta > 0$. The probability density function for this distribution is specified by

$$f(t) = \frac{\lambda\beta e^{-\lambda-\beta t+\lambda e^{-\beta t}}}{1-e^{-\lambda}}\ , t, \lambda, \beta > 0 \tag{1}$$

then the cdf of the EP model proposed by Kus(2007) can be obtained as

$$F(t) = \frac{1-e^{-\lambda+\lambda e^{-\beta t}}}{1-e^{-\lambda}}, t, \lambda, \beta > 0 \tag{2}$$

The lifetimes of the units follow Exponential-Poisson Distribution with a median life defined as

$$m_0 = \left(-\frac{1}{\beta}\right)\log\{1+\log[1-2^{-1}(1-e^{-\lambda})]\lambda^{-1}\} \tag{3}$$

This truncation, with a fixed time point t_0, was implemented to manage time constraints in the life testing process.In the simulation, for each subgroup, failures occurring within or before the truncation time $\leq t_0$ were counted and where the truncation time is determined by

$$t_0 = am_0 \tag{4}$$

Where t_0 is the truncated time, a is the truncation coefficient and m_0 is the desired median life of the product.The probability of failure before t_0 is given by

$$p = \frac{1-e^{-\lambda+\lambda e^{-\beta t_0}}}{1-e^{-\lambda}} \tag{5}$$

Using Equation(3),the probability of failure of a product when the process is in control is

$$p_0 = \frac{1-e^{-\lambda+\lambda^{1-a}\left[\lambda+\log\left(\frac{1+e^{-\lambda}}{2}\right)\right]^a}}{1-e^{-\lambda}} \tag{6}$$

When the median shifts to m_1, the probability of failure of a product becomes
Let $m_1 = fm_0$,where f is the shift coefficient,then

$$p_1 = \frac{1 - e^{-\lambda + \lambda^{1-\frac{a}{f}} \left[\lambda + \log\left(\frac{1+e^{-\lambda}}{2}\right)\right]^{\frac{a}{f}}}}{1 - e^{-\lambda}} \tag{7}$$

We propose an np control chart for the Exponential-Poisson distribution, using Hybrid censoring, personalized to the number of products within each subgroup.

Step 1: Randomly sample n products from the production line.

Step 2: Subject the chosen products to a life test, concluding the test at time t_0. Record the number of failures (denoted as D).

Step 3: End the life test when either the time reaches t_0 or D exceeds the Upper Control Limit (UCL), whichever occurs first.

Step 4: Declare the process as out of control if D>UCL or D<LCL. Declare the process as in control if LCL≤D≤UCL.

We are interested in observing the number of failures D in each sample. If the units have failure times that are less than the truncation time t_0, then the units are considered nonconforming or defective. The binomial probability mass function is given by

$$p(D = x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, 2, 3... \tag{8}$$

The Lower and Upper control limits are given by

$$LCL = \max(0, np_0 - k\sqrt{np_0\,(1-p_o)}) \tag{9}$$

$$UCL = np_0 + k\sqrt{np_0(1-p_o)} \tag{10}$$

Where k is the coefficient of the control limits and p_0 is the probability of "nonconforming" when the process is in control. The probability of stating that the process is in control when it is truly in control is obtained using the binomial probability given in Equation(8)

$$p^0_{\text{in}} = P(LCL \le D \le UCL \mid p_0) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_0^d (1-p_0)^{n-d} \tag{11}$$

The probability of stating that the process is in control when the median life of the product has shifted to m_1 is given by

$$p^1_{\text{in}} = P(LCL \le D \le UCL \mid p_1) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_1^d (1-p_1)^{n-d} \tag{12}$$

It follows from the values p^0_{in} and p^1_{in} given in Equation (11) and (12) that the in-control ARL is given by

$$ARL_0 = \frac{1}{1 - p^o_{in}} \tag{13}$$

and the out of control ARL are given by

$$ARL_1 = \frac{1}{1 - p^1_{in}} \tag{14}$$

To construct the tables for the proposed control chart, we applied the following algorithm.

Step 1: Specify the values of ARL, say r_0 and sample size n.

Step 2: Determine the control chart parameter and truncated time constant a values for which the ARL from equation (13) approach r_0.

Step 3: By using the values of the control chart parameters obtained in step 2, determine the ARL_1 in accordance with shift constant f by using equation(14).

Table 1: *The Values of ARLs when λ = 1.0*

n	25			30			35		
r_0	260	320	420	260	320	420	260	320	420
a	0.704	0.592	0.574	0.869	0.9	0.719	0.573	0.757	0.597
k	3.0003	3.0017	2.98	2.99	3.110	3.159	2.965	3.086	3.077
LCL	3	2	1	6	6	4	4	6	4
UCL	17	16	15	22	22	20	20	23	21
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	259.52	321.23	421.30	260.19	320.49	419.59	260.07	319.80	420.25
0.90	264.308	347.558	199.184	349.632	300.581	322.237	206.078	237.493	364.749
0.80	124.887	173.658	78.631	156.556	113.714	118.843	78.775	81.322	132.093
0.70	44.100	61.177	29.634	47.148	35.108	37.446	25.562	24.719	39.108
0.60	15.050	20.268	11.171	14.119	11.102	11.972	8.528	7.934	11.801
0.50	5.384	6.888	4.416	4.650	3.907	4.191	3.195	2.943	3.968
0.40	2.214	2.622	1.992	1.894	1.717	1.799	1.519	1.434	1.691
0.30	1.218	1.309	1.179	1.125	1.094	1.110	1.055	1.040	1.080
0.20	1.007	1.012	1.005	1.002	1.001	1.001	1.000	1.000	1.001
0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

For various values of r_0,n and λ we determine the control chart parameters and ARL_1, which are shown in Tables 1,2,3,4 shows that the ARLs tend to get smaller when the shift constant f gets less.

Table 2: *The Values of ARLs when* $\lambda = 0.5$

n	25			30			35		
r_0	260	320	420	260	320	420	260	320	420
a	0.71	0.59	0.58	0.97	0.57	0.50	0.61	0.55	0.65
k	3.002	2.99	2.98	3.027	2.909	3.001	2.902	2.964	3.007
LCL	3	2	1	7	2	1	4	4	4
UCL	17	16	15	23	17	16	20	20	21
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	260.33	320.09	419.69	260.40	323.92	421.29	260.23	320.21	419.99
0.90	262.437	346.255	194.951	342.785	145.774	177.804	133.168	152.809	186.836
0.80	120.159	168.626	75.505	139.129	53.958	65.698	46.710	53.522	60.511
0.70	41.241	57.888	27.949	39.886	19.411	23.622	15.789	18.116	19.210
0.60	13.759	18.767	10.376	11.662	7.202	8.646	5.673	6.439	6.493
0.50	4.858	6.282	4.068	3.855	2.946	3.421	2.362	2.606	2.550
0.40	2.014	2.395	1.850	1.646	1.485	1.627	1.291	1.360	1.328
0.30	1.157	1.238	1.134	1.070	1.054	1.080	1.020	1.029	1.023
0.20	1.003	1.006	1.002	1.000	1.000	1.001	1.000	1.000	1.000
0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3: *The Values of ARLs when λ = 2.0*

n	25			30			35		
r_0	260	320	420	260	320	420	260	320	420
a	0.515	0.831	0.868	0.566	0.784	0.753	0.568	0.537	0.592
k	2.885	3.051	3.175	2.931	3.056	2.99	2.966	2.964	3.077
LCL	1	4	4	3	5	4	4	3	4
UCL	14	19	19	18	21	20	20	19	21
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	259.50	319.80	420.08	260.58	320.23	419.52	260.30	320.26	420.64
0.90	140.451	509.357	476.965	276.405	347.133	217.930	207.243	156.951	367.202
0.80	60.559	309.230	218.449	128.875	145.433	77.690	80.508	56.819	135.566
0.70	24.505	102.767	72.067	43.714	45.713	26.335	26.507	19.802	40.815
0.60	9.882	31.137	22.980	14.404	14.360	9.237	8.949	7.194	12.491
0.50	4.161	9.691	7.637	5.050	4.909	3.576	3.371	2.925	4.232
0.40	1.974	3.368	2.869	2.085	2.025	1.688	1.589	1.483	1.789
0.30	1.194	1.506	1.398	1.184	1.170	1.104	1.074	1.058	1.106
0.20	1.008	1.038	1.027	1.005	1.005	1.002	1.001	1.000	1.001
0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4: *The Values of ARLs when λ = 5*

n	25			30			35		
r_0	260	320	420	260	320	420	260	320	420
a	0.706	0.594	0.577	0.87	0.789	0.721	0.576	0.545	0.6
k	3.0003	2.999	2.986	2.996	3.055	3.159	2.967	2.964	3.079
LCL	3	2	1	6	5	4	4	3	4
UCL	17	16	15	22	21	20	20	19	21
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	259.50	319.90	419.96	260.36	319.75	419.71	260.44	320.33	421.41
0.90	263.399	347.646	196.845	348.792	343.275	319.973	204.106	154.273	360.916
0.80	123.060	172.391	77.105	154.203	137.990	116.711	77.033	54.662	128.765
0.70	43.150	60.196	28.890	46.241	42.074	36.553	24.828	18.709	37.848
0.60	14.694	19.831	10.858	13.885	12.952	11.675	8.267	6.714	11.398
0.50	5.277	6.734	4.300	4.619	4.404	4.111	3.111	2.728	3.852
0.40	2.195	2.582	1.957	1.909	1.855	1.787	1.497	1.411	1.664
0.30	1.223	1.306	1.174	1.140	1.127	1.114	1.053	1.042	1.077
0.20	1.009	1.014	1.006	1.003	1.003	1.002	1.000	1.000	1.001
0.10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

II.1. Illustration 1

Assuming a Exponential-Poisson lifetime distribution, characterized by with parameter λ= 2. Consider the following values for the products: m_0 = 1000 hours, ARL_0 = 260 and n = 25. Table 3, provided the following control chart parameters: k=2.885, a=0.515, a lower control limit (LCL) of 1, and an upper control limit (UCL) of 14. This led to the design and implementation of the control chart as described.

Step 1: Select a sample of 25 products from each subgroup and submit them to the life test.
Step 2: During the testing, count the number of failed items (D) and fix the time period.
Step 3: Terminate the test if either first failures occur or time elapse whichever comes first.
Step 4: Declare that the process is in control if $1 \leq D \leq 14$
Step 5: Declare that it is out of control if D > 14 or D < 1

II.2. Illustration 2

Assume that the product lifetimes follow an exponential poisson distribution with parameters λ=5 , Consider the following product values: m_0 is 1000 hours, ARL is 260, and n is 35. The following control chart parameters were presented in Table 4: k = 2.967, a = 0.576, LCL = 4 and UCL = 20. As a result, the control chart was established in the following manner: From each subgroup, choose a sample of 35 products, and put them through a life test . Count the number of failed items (D) which is minimum failure during testing or maximum time reached whichever comes first. Now monitor failures and time. Stop the process if the value of D is between 4 and 20 or the time elapse before reaching maximum time then the process is in control; otherwise, it is out of control.

III. Simulation study

In this section, the application of the proposed control chart is demonstrated using simulated data. The data are generated from an Exponential Poisson Distribution when the process is in-control with the parameters following Exponential Poisson Distribution and the targeted Average lifetime of 1000 hours. We consider a random sample of size n=25 for each sample batch. The first fifteen samples are generated from the in-control process, and the next fifteen samples are from a shifted process with f = 0.40 to achieve 30 sample batches. Then from table 3, when n=25 and $ARL0$ = 260, we have constants k=2.885, a=0.515, UCL=14 and LCL=1. The number of products having the lifetime below 515 hours is noted ("D") and the Values are : 7,7,10,3,6,7,9,8,7,6,5,12,7,10,9,12,13,16,16,17,13,16,15,17,14,15,14,19,10,11. The values of the nonconforming items are plotted with two control limits (LCL =1 and UCL= 14) in Figure 1.

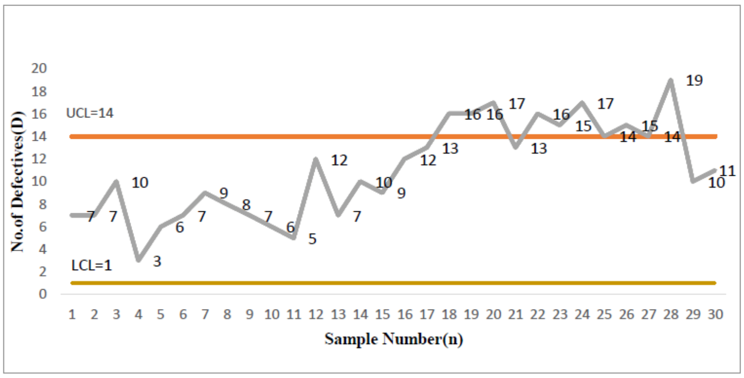

Figure 1: *Control chart for Simulated Data*

IV. Conclusion

In this article, a new attribute control chart based on Exponential-Poisson distribution under hybrid censoring scheme is proposed to ensure the median lifetime of the product as the quality criterion. The newly established control chart is highly adaptable and may be used to monitor the lifetime of quality products. The tables are provided for industrial usage and are explicated with the use of simulated data. It is generated using R software from an Exponential-Poisson distribution. The performance of the proposed control chart is expressed in terms of ARLs for various shift constants (f). It should be noted that if the hybrid censoring scheme is used to carry out the life test, executing the sampling inspection will reduce the time and cost of conducting the life test. The developed attribute control chart has the potential to be extended for use with various other statistical distributions as part of a continuing research study.

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