

MATHEMATIC SIMULATION OF SERIES CONFIGURED SYSTEM UNDER PREVENTIVE AND CORRECTIVE MAINTENANCE USING PSO

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Abstract

For any industry looking to optimize earnings while incurring minimal maintenance costs, unbroken operating time has become crucial. However, the continuous strain wears out the system's components, which ultimately leads to a reduction in system efficiency and complete failure. Total failure is not always controllable; it could lead to a large loss of revenue or output. In this sense, even with reduced efficiency, preventative maintenance is essential to making sure the sector functions properly. In any industry, preventive maintenance is necessary to meet the demands of high productivity at low cost and maximum profit. This paper addressed the performance of a well-maintained two-unit system by calculating reliability metrics like the average duration of system failure and the available time to perform it by counting the effect of the collapse rate and its reparation facilities. Any machine's output is entirely dependent on its inputs and processing, and achieving increasingly higher profits depends critically on the accuracy of the results. The goal of this paper is to maximize the overall profit using a nature inspired optimization tool, particle swarm optimization by counting the reliability metrics for the performance of a series framework subjected to the influence of routine upkeep and changing of components using the Markov process and Chapman-Kolmogorov differential equations. This study focused on two operational units with preventive maintenance features for one crucial unit. Corrective maintenance is applied by most of the authors, but the application of preventive maintenance reduces the cost of repairing and also the cost of changing parts of a system. Throughout the operation of the system, a variation in the rates of malfunction and repair has been observed. The average time to system failure is counted with preventive maintenance rates and availability is checked out in the effect of both preventive and corrective maintenance rates. For reliability metrics across various rates, the results have been calculated. The most appropriate potential values across all variables that follow the exponential distribution have been examined and explored in order to increase reliability and boost businesses. According to the findings, maintaining the system well is essential to raising its overall efficiency. Here is a solution and graphic representation for a mathematical instance.

Keywords: Preventive maintenance, series system, malfunction rate, recovery rate, availability, Particle Swarm Optimization.

I. Introduction

In modern scenarios, physical work has been reduced due to the advancement of technology. The invention of different machines or equipment leads to less physical work done and more accurate results. Hence, for better output, the efficiency of that equipment becomes an important factor, which completely depends on the working and maintenance of it and its subparts. The likelihood of success at time t may be employed to define reliability, i.e., the probability of a machine designed to complete its task in the allotted time under specified external conditions in successful mode. When a system or device performs an intended task without issue or encounters any problems, it can be considered highly reliable or more dependable. Because constant use of a system leads to part wear and tear, no system can operate at peak efficiency all the time. Due to this, a system's reliability and efficiency are compromised when certain components degrade and fail.

Using semi-Markov processes and the influence of cold standby, the reliability of induced draft fans in thermal plants has been analysed [1]. The upkeep of turbo generators and their availability through vibration monitoring experiments have been covered [2]. To determine the best course of maintenance, Systems composed of multiple parts with distinct repairable elements have been analyzed through the use of reinforcement learning [3]. The reliability metrics has been counted under the concept of parallel cold standby system [4]. Two distinct techniques have been proposed to optimize system reliability under deterioration [5]. The study covered some malfunctioning processes, examined the findings for each kind of failure, and displayed the findings to show the reliability of the system [6]. The affordability and reliability of different subsystems in coal-fired thermal plant have been assessed using the Markov method concept [7]. The behavior of a system that includes a primary unit with the replacement of two sub units upon request following unsuccessful processes organized in a parallel fashion has been understood through the use of a semi-Marco process with regenerative point technique [8]. To maximize the cost of a rubber plant, a nature-inspired algorithm called particle swarm optimization has been explored [9]. A deep learning process was examined [10], in order to optimize the reliability parameters and boost industry revenues and manufacturing of a 2:3 good system. Researcher investigate a failing system by applying a genetic algorithm to ascertain the reliability metrics influenced by the rate of degradation and the rate of preventive maintenance [11]. The study provided an investigation of reliability parameters using the ABC method [12]. The investigation of the two-unit repairable system using fuzzy linguistics has been analysed and discussed overall availability [13]. The analysis of poly-tube manufacturing plant has been done to discuss the availability regarding each unit of plant with the optimizing tool PSO [14]. An evolutionary algorithm is applied on a reliability model to discuss the reliability metrics and have good profit [15]. A review on the methodology studied and implemented by various researchers has been presented [16].

This article's main objective is to show that maintenance, both corrective and preventive, plays a crucial role in the overall profit of a complete industry. Using the optimization tool PSO, the optimum can be achieved more easily, which is very helpful to meet the upcoming demands. The paper comprises different sections to help understand the presented work. The model's details, including the explanation of the model, the outline of the state, and the annotations, are included in Section 2. The mathematical presentation of the framework and reliability metrics like average time to system failure and availability have been counted in Section 3. Section 4 represents the methodology used to optimize the parameters of reliability. Section 5 includes the mathematical simulation. The analysis and conclusion of the results were presented in Sections 6 and 7, respectively.

II. Model Description and Notations

I. System Overview

There are two units P and Q under corrective and preventive maintenance. In this paper Unit P is considered more sensitive therefor undergo two degradation and preventive maintenance is applied on it While Unit Q is covered by corrective maintenance. Unit P first degraded to a small extent, which results into the slow working process up to 50 % and further on second degradation reduce to 70 % that were inefficient. Preventive maintenance is applied to reverse this condition and restore full functioning. After the complete failure, the units are changed with the new units to continue the work. The mathematical formulation based on Markov birth process is computed and first order Chapman–Kolmogorov differential equations are made which are solved for steady state condition i.e. under influence of Markov process using matrix formation. The approach od Particle swarm optimization is used to optimize the parameters and hence outcomes.

II. Assumptions

The two units having the arrangement of series manner, hence both play important role for finding reliability metrics.

- The system is reduced 50% firstly and then to 70% due to degradation in Unit P.
- After the failure of any one component, the system will stop.
- The malfunction and recovery rate are considered exponential distributed.

III. Notations

The various notations that are used during the study is presented in table 1.

Table 1: Different symbols and notations used in present work

Symbols	Explanations or meaning
  	Full functioning state, Failed state and Degraded state respectively.
$P/\bar{P}/\bar{\bar{P}}/p$	100% operational/Deteriorating condition i.e. need of routine upkeeping/ Again Deteriorating condition / collapsed state.
S_i	Various possible state of up and down condition where $i = 0$ to 4
χ_1/χ_2	Completely down rate of P/Q
μ_1/μ_2	Deteriorate rate moving from P to \bar{P} / \bar{P} to $\bar{\bar{P}}$
δ_1/δ_2	Routine upkeeping rate from \bar{P} to P / from $\bar{\bar{P}}$ to \bar{P}
σ_1/σ_2	Recovery rate correspond to P/Q
$U_i(t)$	Represent the likelihood of a variety of various changing states in time t .
$U(t)/U'_0(t)$	The general likelihood vector and its associated differential vector.

The possible visualization of different states is represented in Figure 1.

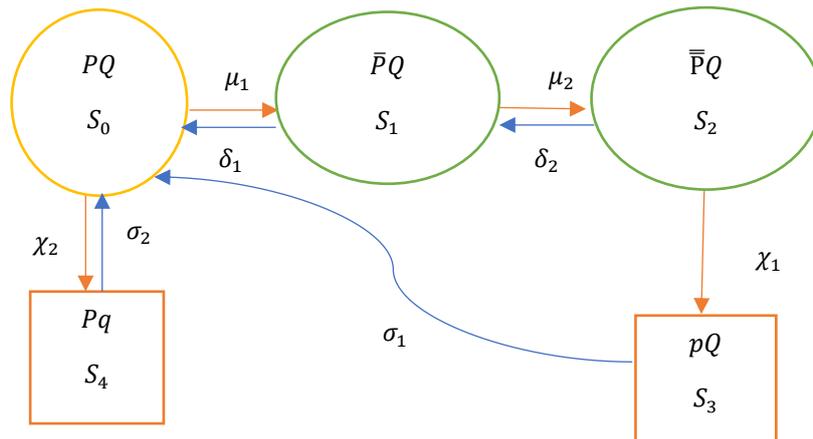


Figure 1: State transition diagram of given system

IV. State Description

Different state that represents the function of system on each situation.

- S_0 : Full functioning state.
- S_1 : 1st degraded state due to degradation of unit P and need a preventive maintenance.
- S_2 : 2nd degraded state due to degradation of unit P further and need a preventive maintenance.
- S_3 : Failed state due to failure of P.
- S_4 : Failed state due to failure of Q

III. Mathematic Simulation

Using straightforward probabilistic principles, Through the use of the Markov birth-death theory, computational simulations are formed associated with the state diagram, i.e., figure 1, the first-order Chapman-Kolmogorov differential equations are generated, through the use of mnemonic rules.

$$U'_0 = -(\chi_2 + \mu_1)U_0 + \delta_1 U_1 + \sigma_1 U_3 + \sigma_2 U_4 \quad (1)$$

$$U'_1 = \mu_1 U_0 - (\delta_1 + \mu_2)U_1 + \delta_2 U_2 \quad (2)$$

$$U'_2 = \mu_2 U_1 - (\delta_2 + \chi_1)U_2 \quad (3)$$

$$U'_3 = \chi_1 U_2 - \sigma_1 U_3 \quad (4)$$

$$U'_4 = \chi_2 U_0 - \sigma_2 U_4 \quad (5)$$

With initial condition

$$U_i(0) = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases} \quad (6)$$

One way to represent the equation from (1) to (5) is in matrix form, i.e.

$$\begin{bmatrix} U'_0(t) \\ U'_1(t) \\ U'_2(t) \\ U'_3(t) \\ U'_4(t) \end{bmatrix} = \begin{bmatrix} -(\chi_2 + \mu_1) & \delta_1 & 0 & \sigma_1 & \sigma_2 \\ \mu_1 & -(\delta_1 + \mu_2) & \delta_2 & 0 & 0 \\ 0 & \mu_2 & -(\chi_1 + \delta_2) & 0 & 0 \\ 0 & 0 & \chi_1 & -\sigma_1 & 0 \\ \chi_2 & 0 & 0 & 0 & -\sigma_2 \end{bmatrix} \begin{bmatrix} U_0(t) \\ U_1(t) \\ U_2(t) \\ U_3(t) \\ U_4(t) \end{bmatrix} \quad (7)$$

I. Average time to system failure (ATTF)

To eliminate the complications for evaluating ATTF, The restriction of elimination of the absorbing

states has been applied and taking transpose of mentioned matrix, We get matrix T.

$$ATTF = P(0)(T^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (8)$$

Where,

$$T = \begin{bmatrix} -(\chi_2 + \mu_1) & \mu_1 & 0 \\ \delta_1 & -(\delta_1 + \mu_2) & \mu_2 \\ 0 & \delta_2 & -(\chi_1 + \delta_2) \end{bmatrix}$$

$$ATTF = \frac{T_1}{Q_1} \quad (9)$$

Where,

$$T_1 = \delta_1 \chi_1 + \delta_1 \delta_2 + \mu_2 \chi_1 + \mu_1 \chi_1 + \delta_2 \mu_1 + \mu_1 \mu_2$$

$$Q_1 = (\delta_1 + \mu_2) \chi_1 \chi_2 + \delta_1 \delta_2 \chi_2 + \chi_1 \mu_1 \mu_2$$

II. Performing time (Availability Analysis)

Together with the system's degraded state(s), the system availability ($A(t)$) in accordance with time is the total of the system's availability when operating at maximum capacity. i.e.

$$A(t) = U_0(t) + U_1(t) + U_2(t) \quad (10)$$

Eq. (10) is utilized for the calculation of the time-sensitive framework accessibility. Ensuring the system's long-term availability is of interest to any management. By applying the steady state condition, the system's steady state probabilities are determined, i.e., $U'(t)$, i.e., $d/dt \rightarrow 0$, whenever $t \rightarrow \infty$. Consequently, Eqs. (1)– (5) are broken down into equations.

$$-(\chi_2 + \mu_1)U_0 + \delta_1 U_1 + \sigma_1 U_3 + \sigma_2 U_4 = 0 \quad (11)$$

$$\mu_1 U_0 - (\delta_1 + \mu_2)U_1 + \delta_2 U_2 = 0 \quad (12)$$

$$\mu_2 U_1 - (\delta_2 + \chi_1)U_2 = 0 \quad (13)$$

$$\chi_1 U_2 - \sigma_1 U_3 = 0 \quad (14)$$

$$\chi_2 U_0 - \sigma_2 U_4 = 0 \quad (15)$$

Long-term stability state availability

$$A(\infty) = U_0(\infty) + U_1(\infty) + U_2(\infty) \quad (16)$$

Applying the subsequent normalizing circumstances

$$U_0(\infty) + U_1(\infty) + U_2(\infty) + U_3(\infty) + U_4(\infty) = 1 \quad (17)$$

The model that is being presented, long-term steady-state availability is computed as

$$A(\infty) = 1 - U_3(\infty) - U_4(\infty) = U_0(\infty) + U_1(\infty) + U_2(\infty) \quad (18)$$

$$U_1(\infty) = \frac{\mu_1 \sigma_1 \sigma_2 (\chi_1 + \delta_2)}{T}$$

$$U_2(\infty) = \frac{\mu_1 \mu_2 \sigma_1 \sigma_2}{T}$$

Where,

$$T = \sigma_1 \sigma_2 \chi_1 (\mu_1 + \mu_2 + \delta_1) + \mu_1 \sigma_1 \sigma_2 (\mu_2 + \delta_2) + \chi_1 \chi_2 \sigma_1 (\mu_2 + \delta_1) + \mu_1 \mu_2 \chi_1 (\sigma_1 + \sigma_2) + \delta_1 \delta_2 \sigma_1 (\chi_2 + \sigma_2)$$

IV. Methodology

I. Introduction

Inspired by swarming behaviors found in nature, such as flocks of fish and birds, Particle Swarm Optimization (PSO) is a potent meta-heuristic optimization algorithm, also called a stochastic search algorithm, based on population dynamics. It is a computational method used to optimize the problem. It performs its task of optimization by improving particle solutions. This algorithm works

with some parameters, like particle size, population, position, velocity, search space, etc. In PSO, the population, like the bird group, represents a swarm, and each member of the swarm represents a particle. Every particle's movement is dictated by its local position. Each particle has velocities that direct the flight of particles. The search space refers to the spectrum in which the technique calculates the most effective regulatory variables. The value will be reset if the searching space is exceeded by any particle's optimal control value.

II. Working procedure of PSO with an example

To better understand how PSO operates, let's look at an example. A flock of birds flying aimlessly in an area, trying to find a single piece of food. Not a single bird is aware of the location of the food, i.e., they are aware of their progress in each iteration, even though they are unsure of the ideal eating position. They launch themselves in different directions and adhere to the PSO's search plan, i.e., swiftly follow that bird that is close to food. Each particle or bird, starting from a randomly selected population, moves through the searching space in randomly chosen directions while recalling its best historical positions and those of its neighbours, i.e., the highest ranking globally. Follow that bird to the global best position to obtain the optimal value, i.e., food.

III. PSO Algorithm fundamentals

Initialization

- Start with a bunch of erratic particles, where each particle represents a solution.
- Every particle seeks the ideal value through generation updates (iterations).
- All particles are updated in every iteration.
 - a) The first-choice one is the best solution (fitness value).
 - b) The second-best is tracked by PSO.
- Particles update their travel speed and orientation.
- Best particle position.
- Best Group Position.

IV. PSO algorithm

Phase 1. Initialization

- Set up the parameters originally.
- Set up the population.
- For every particle, start each one at a random location.
- For every particle, set its initial velocity at random.

Phase 2. Compute Fitness Value. (1st iteration)

- Determine the value of each particle's function or fitness.
- In the event that the fitness value surpasses the best fitness value (gbest), the new value is to be designated as the new (gbest).
- Select the particle with the highest fitness value as gbest.

Phase 3. Determine the velocity and position of every particle.

- Particle Position Calculation
- Compute Velocity

Phase 4. Assess Fitness

- Get the Current Best Here (gbest)

Phase 5. Perform the next iteration until you get the optimum value.

Phase 6. Output the gbest and particle positions.

V. Flow chart to understand the algorithm

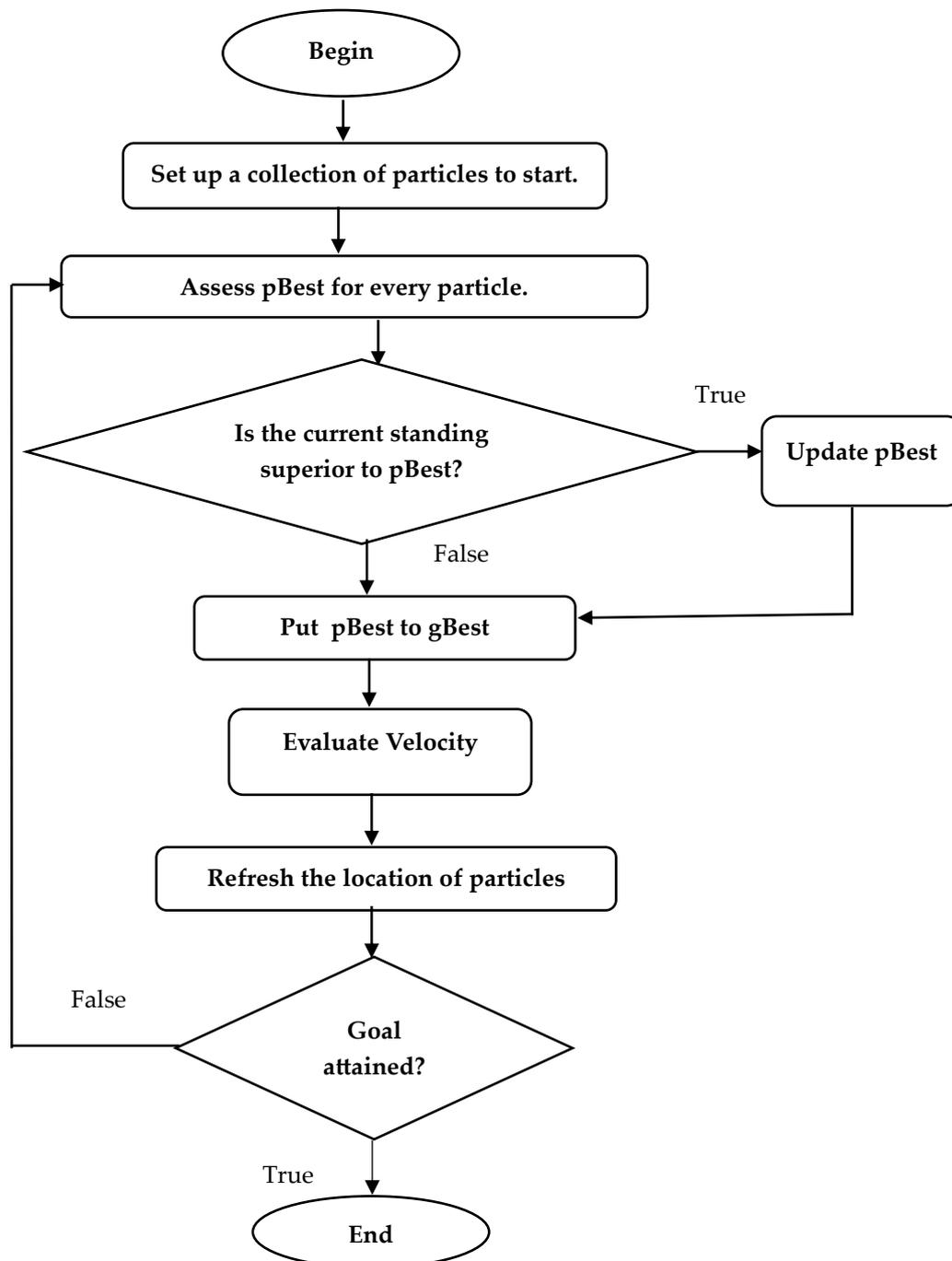


Figure 2: The flowchart of working of PSO

V. Mathematical Computation for Reliability Parameters using PSO

I. Average time to system failure

Through fitness function optimization and analysis of the implications of malfunction and recovery rate variations, the PSO method determines the ATTF of the system.

By having the value of $\chi_1=0.08$ and $\chi_2=0.07$ and by applying 11 generation with the lower limit value 0.05 and upper limit value of 0.09, The optimization tool PSO is applied to obtained the table 2 and figure 3.

Table 2: Variation in ATTF with respect to degraded rate and preventive maintenance rate.

Sr. No.	(μ_1)	(μ_2)	(δ_1)	(δ_2)	ATTF
1	0.0500	0.0972	0.0990	0.0500	17.4521
2	0.0500	0.0990	0.0990	0.0533	17.4650
3	0.0500	0.0980	0.0950	0.0500	17.4988
4	0.0502	0.0901	0.0950	0.0500	17.5055
5	0.0513	0.0979	0.0990	0.0508	17.5151
6	0.0507	0.0963	0.0982	0.0503	17.5235
7	0.0500	0.0941	0.0990	0.0536	17.5704
8	0.0511	0.0950	0.0964	0.0529	17.6440
9	0.0531	0.0980	0.0990	0.0562	17.6717
10	0.0535	0.0958	0.0972	0.0520	17.7170
11	0.0539	0.0902	0.0953	0.0553	17.9585

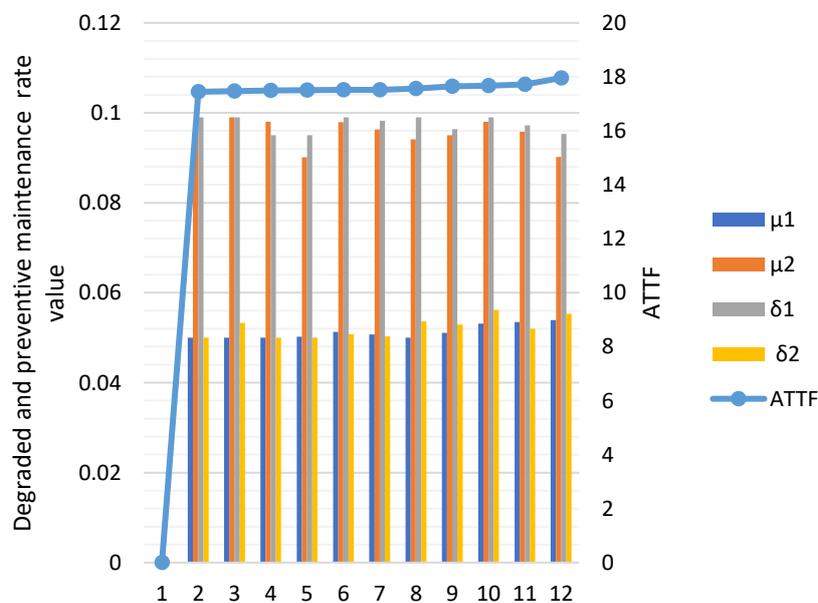


Figure 3: ATTF vs. Different rate.

II. Performing time (Availability Analysis)

Through fitness function optimization and analysis of the implications of malfunction and recovery rate variations, the PSO method determines the Availability of the system.

By having the value of $\sigma_1=0.07$, $\sigma_2= 0.05$, $\chi_1=0.08$ and $\chi_2=0.07$ and by applying 11 generation with the lower bound of value 0.05 and upper bound value of 0.09 for both degraded rate and corresponds preventive maintenance rate, The optimization tool PSO is applied to obtained the table 3 and figure 4.

Table 3: Variation in availability with respect to degraded rate and preventive maintenance rate.

Sr. No.	(χ_1)	(χ_2)	(σ_1)	(σ_2)	Availability
1	0.0981	0.0990	0.0501	0.0500	0.3220
2	0.0975	0.0946	0.0535	0.0500	0.3283
3	0.0990	0.0989	0.0540	0.0500	0.3307
4	0.0955	0.0955	0.0511	0.0500	0.3334
5	0.0970	0.0990	0.0549	0.0500	0.3346
6	0.0941	0.0961	0.0520	0.0500	0.3361
7	0.0949	0.0990	0.0500	0.0545	0.3413
8	0.0990	0.0950	0.0500	0.0540	0.3427
9	0.0958	0.0990	0.0540	0.0540	0.3490
10	0.0990	0.0950	0.0540	0.0536	0.3515
11	0.0941	0.0990	0.0500	0.0575	0.3526

With the value of $\mu_1=0.03$, $\mu_2=0.04$, $\delta_1=0.05$ and $\delta_2=0.07$ and by applying 11 generation with the lower bound of value 0.05 and upper bound value of 0.09 for both failure rate and corresponds corrective maintenance rate, The optimization tool PSO is applied to obtained the table 4 and figure 5.

Table 4: Variation in availability with respect to failure rate and corrective maintenance rate.

Sr. No.	(μ_1)	(μ_2)	(δ_1)	(δ_2)	Availability
1	0.0990	0.0990	0.0500	0.0945	0.3393
2	0.0956	0.0975	0.0500	0.0990	0.3396
3	0.0953	0.0990	0.0500	0.0976	0.3400
4	0.0989	0.0869	0.0500	0.0946	0.3408
5	0.0969	0.0951	0.0520	0.0966	0.3412
6	0.0961	0.0889	0.0536	0.0990	0.3425
7	0.0950	0.0873	0.0500	0.0935	0.3429
8	0.0950	0.0918	0.0534	0.0973	0.3431
9	0.0982	0.0789	0.0534	0.0970	0.3436
10	0.0960	0.0823	0.0532	0.0972	0.3439
11	0.0950	0.0849	0.0546	0.0947	0.3456

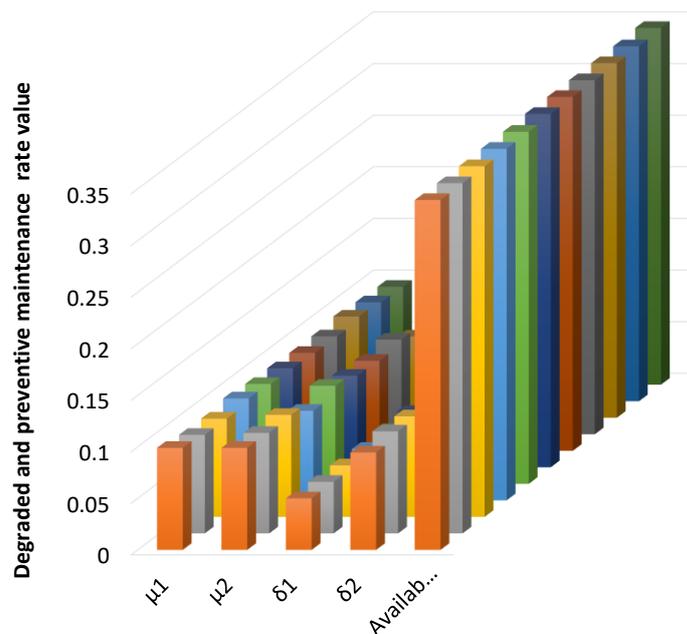


Figure 4: Availability vs. Degraded rate and Preventive Maintenance Rate.

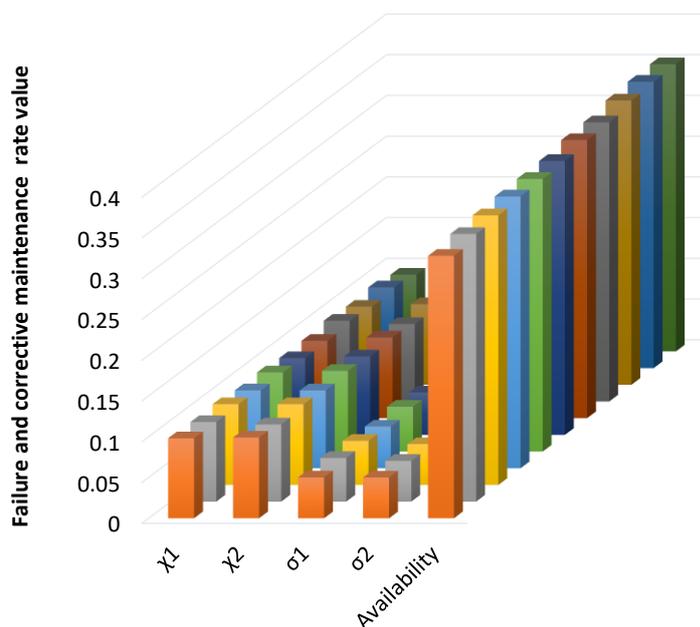


Figure 5: Availability vs. Failure rate and corrective Maintenance Rate.

VI. Results and Discussion

The degraded, failure, preventive and corrective maintenance rate have direct effect on ATTF and Availability which can be understand by the above example using the optimization tool PSO. Table

2 and figure 3 depicted that with the various value of degraded rate and preventive maintenance rate ATTF is increasing. The optimum value is selected to have a good profit in production in any industry.

The greater availability is a key point for any industry to gain more and more financial profit over the cost of infrastructure. The table 3 and figure 4 shown the effect of degraded rate and corresponds preventive maintenance rate. It is clear that the availability is optimized over different values of rates. Similarly, the effect of failure rate and corrective maintenance rate is also checked to count the availability which shown the increasing criteria for the different parameters value in table 4 and figure 5.

VII. Conclusions

The outcomes demonstrated by figure 3,4 and 5 are concluded that preventive and corrective maintenance had an impact on system reliability parameters like ATTF and availability, which in turn affected the system's accuracy and processing efficiency. Using the appropriate preventive or corrective maintenance rate values can lead to an increase in ATTF and availability. Increasing maintenance not only increases the working power of units but also prolongs its performance life. The influence of various maintenance rates on the optimal value was plotted on a graph. The optimal value for all malfunctions and repair rates can be used to improve the system's overall reliability.

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