A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A REPAIRABLE SYSTEM USING MARKOV APPROACH

A. D. Yadav[*,1], S.C. Malik[2], S. Malik[3] and N. Nandal[4]

•

[*,1,2,3,4]Department of Statistics, M.D. University, Rohtak
[*,1] amitdevstats@gmail.com,
[2]sc_malik@rediffmail.com,
[3]maliksushil1983@gmail.com,
[4]nsinghnandal@gmail.com

Abstract

This study presents a reliability-availability-maintainability (RAM) analysis of a repairable system consisting of two non-identical units, where one unit operates as the main unit while another unit is in cold standby mode. A Markov approach is applied to develop the system model and transition probability equations are derived using the Chapman-Kolmogorov technique. The system is deemed to have failed only when both units are inoperative. Failure and repair times are assumed to follow negative exponential distribution. Utilizing Laplace transform techniques and Mathematica 12.30 software, the system equations are solved to obtain key reliability measures such as reliability, Mean Time to System Failure (MTSF), availability, Mean Time to Repair (MTTR), and maintainability. The cost benefit analysis of the system model has also been carried out for some specific values of the parameters. The study examines the graphical and numerical behavior of these measures and applies the findings to a real-life scenario involving a recently launched Bajaj Freedom 125 CNG bike whose engine that operates on both CNG and petrol. This analysis provides valuable insights into the reliability and economic efficiency of dual-fuel systems in automotive engineering.

Keywords: Markov Model, Chapman-Kolmogorov Equations, Repairable System, Reliability Measures, Cost-Benefit Analysis

I. Introduction

In today's rapidly advancing technological era, the importance of reliability as a measure of system functionality is increasingly recognized within the automobile industry. As automotive technology evolves, the reliability of vehicles becomes crucial not only for consumer satisfaction but also for safety and efficiency. The success of the automobile sector, a cornerstone of economic development, depends on the consistent performance and dependability of its products. To maintain a competitive edge, industry leaders are relentlessly pursuing ways to enhance vehicle reliability. Automotive engineers and researchers play a pivotal role in this endeavor, focusing on innovative strategies such as redundancy, robust repair infrastructures, and resilient maintenance protocols. Their efforts significantly contribute to overall economic progress by ensuring that vehicles operate reliably and efficiently under various conditions. Consequently, both simple and

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

sophisticated automotive systems have been introduced to the market, delivering seamless services to customers without encountering technical obstacles. The advancements in vehicle reliability are evident through the continuous improvements in automotive system design. Structural integrity and the parallel arrangement of critical components are key factors in enhancing the reliability of complex vehicle systems. These designs distribute operational stress and reduce the risk of system failures. Researchers often employ semi-Markov processes and regenerative point technique to develop detailed models of automotive systems and analyze their reliability measures. Statisticians and engineers frequently discuss topics such as mean time to system failure (MTSF) and system availability, which are crucial for both repairable and non-repairable automotive systems. Kadyan et al. [8] conducted an analysis on a non-identical repairable system comprising three units, where a cold standby unit operates simultaneously. El-Sherbany et al. [5] determined the reliability of cold standby system with arbitrary distributions. Behboudi et al. [1] conducted the reliability modelling using periodic switching approach. Rathi et al. [13] examined the reliability characteristics of a parallel system incorporating a priority concept. Lin et al. [10] conducted the reliability analysis of cold standby using simulation method. Eryilmaz et al. [6] assessed the reliability of the priority standby system. Malik et al. [11] conducted the reliability analysis of a complex system.

Scholars have evaluated the reliability of systems with diverse component structures utilizing the Markov approach. Danjuma et al. [4] determined the RAMD analysis of cold standby series-parallel system. Cui et al. [3] evaluated the history dependent up and down states of repairable systems using Markov process. Manglik et al. [12] analyzed the reliability of a two-unit cold standby system using Markov process. Gupta et al. [7] carried out the cost benefit analysis of a two dissimilar unit cold standby system with Weibull failure and repair laws. Wang et al. [18] evaluated the reliability for multi-state Markov repairable system. Chao et al. [2] carried out the reliability of large series system using Markov structure. Kalaiarasi et al. [9] analyzed system reliability using Markov Technique. Umamaheshwari et al. [17] discussed a Markov model with human error and common cause. Saritha et al. [15] considered the reliability and availability for non-repairable & repairable systems using Markov modeling. Shivani et al. [16] analyzed the sensitivity of series parallel system in component failures. Yadav et al. [19] discussed the Markov approach for reliability, availability and maintainability analysis of a three-unit repairable system. Yadav et al. [20] determined the reliability and availability of a four-unit repairable system using the Markovian approach. Rathi et al. [14] conducted the reliability modelling of fly ash bricks manufacturing system. Previous studies have primarily focused on assessing the reliability of non-repairable systems using Markov modeling, covering scenarios with both identical and non-identical units. However, there is a significant research gap in applying Markov methodologies to perform cost-benefit analyses for repairable systems. Exploring this area offers a new avenue for investigation, extending Markov modeling beyond reliability assessment to include repair and maintenance costs. This comprehensive approach can improve decision-making in system management and optimization. By addressing this gap, researchers can gain a deeper understanding of the economic implications of maintaining and operating repairable systems, leading to more informed and effective resource allocation strategies in various industrial and engineering fields.

In this study, we employed a homogeneous continuous-time Markov chain (CTMC) model, where the Markov property remains consistently valid. In such a framework, units strictly follow a negative exponential distribution, ensuring that each transition is independent of past and future transitions, and interstate transition rates remain constant. This key feature ensures that the future behavior of the Markov system is solely determined by its present state, independent of the historical path taken to reach it. Our research focuses on leveraging Chapman-Kolmogorov equations to derive expressions for key reliability measures of a repairable system composed of

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

two non-identical cold standby units. To achieve this, we constructed a state transition diagram based on the Markovian birth and death process. Initially, system starts with a single unit and the other unit (non-identical) is kept as spare in cold-standby. We assume the failure and repair times of the units follow a negative exponential distribution. We solved the system equations using the Laplace transform technique, supported by Mathematica 12.30 software. Through this approach, we evaluated the reliability, availability and maintainability of the system model for non-identical units. Additionally, we conducted a cost-benefit analysis for specific values of failure and repair rates. We observed the behavior of these measures graphically for particular failure and repair rates, along with fixed values of revenue generated per unit uptime and the cost of server busy per unit time. Furthermore, we applied our findings to a real-life scenario involving the newly launched Bajaj Freedom 125 CNG bike, which features an engine that operates on both CNG and petrol. This analysis provides valuable insights into the reliability and economic efficiency of dual-fuel systems in automotive engineering, demonstrating the practical applications of our research.

II. Assumptions and State Descriptions

1. The transition rates of the units follow negative exponential distribution
2. The repair of the failed unit is done immediately at the availability of the repair facility.
3. The system is declared failed at the failure of all units.
4. All the units are operative at time 't' in state '1' of the system.
5. The system is in state '2' at time 't' upon the failure of one unit, the repair is made immediately and another cold standby unit is still in operation.
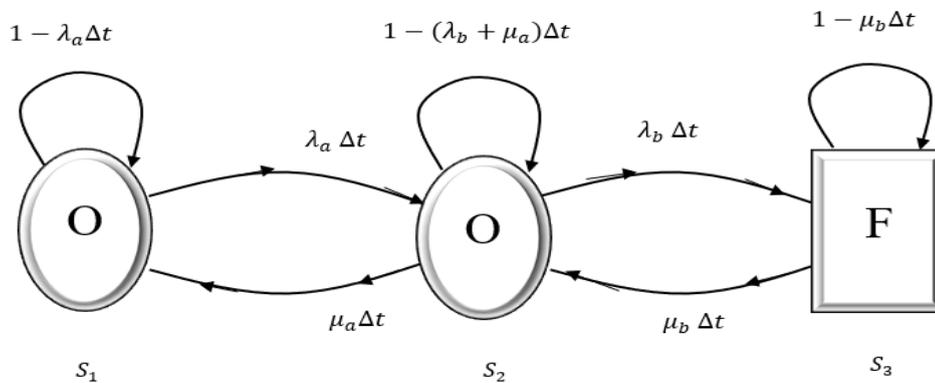6. All the units are failed at time 't' in state '3' of the system.

Figure 1: *State Transition Diagram*

a) Notations and Abbreviations

λ_x Failure rate of the system at state where x units have failed (x= a and b)
μ_y Repair rate of the system at state where y units have failed (y=a and b)
S_x States (x=1,2,3)
t   Time
$P_x(t)$ Probability that the system is in state x at time t (x=1,2,3)
$A(\infty)$ Steady State Availability of the system
R(t)   Reliability of the system
MTSF   Mean Time to System Failure

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

III. Reliability Measures of the System

a) Reliability

In passing from state 'i' at time 's' to state 'j' at time t (s<t), we must pass through some intermediate state 'k' at some intermediate time 'u'. When the continuous-time Markov chain is homogeneous, the Chapman-Kolmogorov equation may be written as:

$$P_{ij}(t + \Delta t) = \sum_{all\ k} P_{ik}(t)P_{kj}(\Delta t) \qquad\qquad for\ t, t \geq 0$$

$$= \sum_{k \neq j} P_{ik}(t)P_{kj}(\Delta t) + P_{ik}(t)P_{kj}(\Delta t) \tag{1}$$

The Chapman- Kolmogorov equations of the system can also be obtained from expression (1) as

$$P_1(t + \Delta t) = P_1(t)(1 - \lambda_a \Delta t) + P_2(t)\mu_a \Delta t$$
$$P_2(t + \Delta t) = P_2(t)(1 - (\mu_a + \lambda_b)\Delta t) + P_1(t)\lambda_a \Delta t$$
$$P_3(t + \Delta t) = P_3(t)\Delta t + P_2(t)\lambda_b \Delta t \tag{2-4}$$

These Markov equations are being developed by taking the probability of each state at time $t + \Delta t$. Converting these equations (2-4) to a differential equation and taking Lim $\Delta t \to 0$, we get

$$P_1{}'(t) = -P_1(t)\lambda_a + P_2(t)\mu_a$$
$$P_2{}'(t) = -P_2(t)(\mu_a + \lambda_b) + P_1(t)\lambda_a$$
$$P_3{}'(t) = -P_3(t) + P_2(t)\lambda_b \tag{5-7}$$

Above equations (5-7) can be solved by using LT method.

$$sp_1(s) - P_1(0) = -\lambda_a p_1(s) + \mu_a p_2(s)$$
$$sp_2(s) - P_2(0) = -(\mu_a + \lambda_b)p_2(s) + \lambda_a p_1(s)$$
$$sp_3(s) - P_3(0) = p_3(s) + \lambda_b p_2(s) \tag{8-10}$$

Boundary conditions are

$$P_1(0) = 0, P_2(0) = 0, P_3(0) = 0$$

So, the system of equations will be

$$(s - \lambda_a)p_1(s) - \mu_a p_2(s) = 1$$
$$-\lambda_a p_1(s) + (s - (\mu_a + \lambda_b))p_2(s) = 0$$
$$-\lambda_b p_2(s) + s\, p_3(s) = 0$$

It can be written as

$$\begin{bmatrix} s + \lambda_a & -\mu_a & 0 \\ -\lambda_a & s + (\mu_a + \lambda_b) & 0 \\ 0 & -\lambda_b & s \end{bmatrix} \begin{bmatrix} p_1(s) \\ p_2(s) \\ p_3(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now, solving for $p_1(s), p_2(s)$ and $p_3(s)$ using Cramer's Rule, we have

$$\Delta_{R0} = \begin{bmatrix} s + \lambda_a & -\mu_a & 0 \\ -\lambda_a & s + (\mu_a + \lambda_b) & -\mu_a \\ 0 & -\lambda_b & s + \mu_b \end{bmatrix}$$

$$\Delta_{R0} = (s - 0)(s - s_1)(s - s_2)$$

Here s_1 and s_2 are roots of Δ_R.

$$s_1 = -\frac{\lambda_a + \lambda_b + \mu_a + \sqrt{-4\lambda_a\lambda_b + (\lambda_a + \lambda_b + \mu_a)^2}}{2}$$

$$s_2 = \frac{\lambda_a + \lambda_b + \mu_a + \sqrt{-4\lambda_a\lambda_b + (\lambda_a + \lambda_b + \mu_a)^2}}{2}$$

$$\Delta_{R1} = \begin{bmatrix} 1 & -\mu_a & 0 \\ 0 & s + (\mu_a + \lambda_b) & 0 \\ 0 & -\lambda_b & s \end{bmatrix}$$

$$\Delta_{R1} = s(\lambda_b + \mu_a + s))$$

$$\Delta_{R2} = \begin{bmatrix} s + \lambda_a & 1 & 0 \\ -\lambda_a & 0 & 0 \\ 0 & 0 & s \end{bmatrix}$$

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

$$\Delta_{R2} = \lambda_a s$$

$$\Delta_{R3} = \begin{bmatrix} s + \lambda_a & -\mu_a & 1 \\ -\lambda_a & s + (\mu_a + \lambda_b) & 0 \\ 0 & -\lambda_b & 0 \end{bmatrix}$$

$$\Delta_{R3} = \lambda_a \lambda_b$$

Now,

$$p_1(s) = \frac{\Delta_{R_1}}{\Delta_R}$$

$$= \frac{(\lambda_b + \mu_a + s)}{(s - s_1)(s - s_2)}$$

$$p_2(s) = \frac{\Delta_{R_2}}{\Delta_R}$$

$$= \frac{\lambda_a}{(s - s_1)(s - s_2)}$$

$$p_3(s) = \frac{\Delta_{R_3}}{\Delta_R}$$

$$= \frac{\lambda_a \lambda_b}{(s - s_1)(s - s_2)}$$

Taking Laplace Inverse of $p_1(s)$, $p_2(s)$ and $p_3(s)$, we get

$$p_0(t) = \frac{e^{s_1 t}(\lambda_b + \mu_a + s_1) - e^{s_2 t}(\lambda_b + \mu_a + s_2)}{(s_1 - s_2)}$$

$$p_2(t) = \frac{\lambda_a(e^{s_1 t} - e^{s_2 t})}{(s_1 - s_2)}$$

$$p_3(t) = \frac{-\lambda_a \lambda_b(e^{s_2 t} s_1 - s_1 + s_2 - e^{s_1 t} s_2)}{(s_1 - s_2)s_1 s_2}$$

Thus, Reliability of the system is

$$R(t) = p_1(t) + p_2(t) = 1 - p_3(t)$$

$$= \frac{e^{s_1 t}(\lambda_a + \lambda_b + \mu_a + s_1) - e^{s_2 t}(\lambda_b - \lambda_a + \mu_a + s_2)}{(s_1 - s_2)} \qquad (11)$$

b) Mean Time to System Failure (MTSF)

On integrating equation (11), we get the expression for MTSF as

$$MTSF = \int_0^\infty R(t) dt$$

$$MTSF = \frac{\lambda_a + \lambda_b + \mu_a}{s_1 s_2}$$

c) Availability

Now, using equation (1), we get the Chapman- Kolmogorov equations for the system to derive the expression for availability as

$$P_1(t + \Delta t) = P_1(t)(1 - \lambda_a \Delta t) + P_2(t)\mu_a \Delta t$$
$$P_2(t + \Delta t) = P_2(t)(1 - (\mu_a + \lambda_b)\Delta t) + P_1(t)\lambda_a \Delta t + P_3(t)\mu_b \Delta t$$
$$P_3(t + \Delta t) = P_3(t)(1 - \mu_b \Delta t) + P_2(t)\lambda_b \Delta t \qquad (12\text{-}14)$$

These Markov equations are being developed by taking the probability of each state at time $t + \Delta t$.
Taking Lim $\Delta t \to 0$ in the above equations (12-14), we get the following differential equations:

$$P_1'(t) = -P_1(t)\lambda_a + P_2(t)\mu_a$$
$$P_2'(t) = -P_2(t)(\mu_a + \lambda_b) + P_1(t)\lambda_a + P_3(t)\mu_a$$
$$P_3'(t) = -P_3(t)\mu_a + P_2(t)(\lambda_b) \qquad (15\text{-}17)$$

Above equations (15-17) can be solved by using LT method.

$$sp_1(s) - P_1(0) = -\lambda_a p_1(s) + \mu_a p_2(s)$$
$$sp_2(s) - P_2(0) = -(\mu_a + \lambda_b)p_2(s) + \lambda_a p_1(s) + \mu_a p_3(s)$$
$$sp_3(s) - P_3(0) = -p_3(s)\mu_b + p_2(s)(\lambda_b) \qquad (18\text{-}20)$$

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

Boundary conditions are

$$P_1(0) = 0, P_2(0) = 0, P_3(0) = 0$$

So, the system of equations will be

$$(s+\lambda_a)p_1(s) - \mu_a p_2(s) = 1$$
$$-\lambda_a p_1(s) + (s +(\mu_a + \lambda_b))p_2(s) - \mu_a p_3(s) = 0$$
$$-\lambda_b p_2(s) + (s + \mu_b)\, p_3(s) = 0$$

It can be written as

$$\begin{bmatrix} s + \lambda_a & -\mu_a & 0 \\ -\lambda_a & s + (\mu_a + \lambda_b) & -\mu_a \\ 0 & -\lambda_b & s + \mu_b \end{bmatrix} \begin{bmatrix} p_1(s) \\ p_2(s) \\ p_3(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now, solving for $p_1(s), p_2(s)$ and $p_3(s)$ using Cramer's Rule, we have

$$\Delta_{A0} = \begin{bmatrix} s + \lambda_a & -\mu_a & 0 \\ -\lambda_a & s + (\mu_a + \lambda_b) & -\mu_a \\ 0 & -\lambda_b & s + \mu_b \end{bmatrix}$$

$$\Delta_{A0} = \lambda_a \lambda_b s + \lambda_a \mu_b s + \mu_a \mu_b s + \lambda_a s^2 + \lambda_b s^2 + \mu_a s^2 + \mu_b s^2 + s^3$$
$$= (s - 0)(s - s_3)(s - s_4)$$

Here,

$$s_3 = -\frac{\lambda_a + \lambda_b + \mu_a + \mu_b + \sqrt{(\lambda_a + \lambda_b + \mu_a + \mu_b)^2 - 4(\lambda_a \lambda_b + \lambda_a \mu_b + \mu_a \mu_b)}}{2}$$

$$s_4 = \frac{\lambda_a + \lambda_b + \mu_a + \mu_b + \sqrt{(\lambda_a + \lambda_b + \mu_a + \mu_b)^2 - 4(\lambda_a \lambda_b + \lambda_a \mu_b + \mu_a \mu_b)}}{2}$$

$$\Delta_{A1} = \begin{bmatrix} 1 & -\mu_a & 0 \\ 0 & s + (\mu_a + \lambda_b) & -\mu_a \\ 0 & -\lambda_b & s + \mu_b \end{bmatrix}$$

$$\Delta_{A1} = \mu_a \mu_b + \lambda_b s + \mu_a s + \mu_b s + s^2$$

$$\Delta_{A2} = \begin{bmatrix} s + \lambda_a & 1 & 0 \\ -\lambda_a & 0 & -\mu_a \\ 0 & 0 & s + \mu_b \end{bmatrix}$$

$$\Delta_{A2} = \lambda_a(\mu_b + s)$$

$$\Delta_{A3} = \begin{bmatrix} s + \lambda_a & -\mu_a & 1 \\ -\lambda_a & s + (\mu_a + \lambda_b) & 0 \\ 0 & -\lambda_b & 0 \end{bmatrix}$$

$$\Delta_{A3} = \lambda_a \lambda_b$$

Now,

$$p_1(s) = \frac{\Delta_{A_1}}{\Delta_A}$$

$$= \frac{\mu_a \mu_b + \lambda_b s + \mu_a s + \mu_b s + s^2}{s(s - s_3)(s - s_4)}$$

$$p_2(s) = \frac{\Delta_{A_2}}{\Delta_A}$$

$$= \frac{\lambda_a(\mu_b + s)}{s(s - s_3)(s - s_4)}$$

$$p_3(s) = \frac{\Delta_{A_3}}{\Delta_A} = \frac{\lambda_a \lambda_b}{s(s - s_3)(s - s_4)}$$

Taking Laplace Inverse of $p_1(s), p_2(s)$ and $p_3(s)$, we get

$$p_1(t) = \frac{e^{s_3 t}(\mu_a \mu_b + \lambda_a s_3 + \mu_a s_3 + \mu_b s_3 + s_3{}^2)}{s_3(s_3 - s_4)} + \frac{\mu_a \mu_b}{s_3 s_4} - \frac{e^{s_4 t}(\mu_a \mu_b + \lambda_b s_4 + \mu_a s_4 + \mu_b s_4 + s_4{}^2)}{s_4(s_3 - s_4)}$$

$$p_2(t) = \lambda_a \left[\frac{e^{s_3 t}(\mu_b + s_3)}{s_3(s_3 - s_4)} + \frac{\mu_b}{s_3 s_4} + \frac{e^{s_4 t}(-\mu_b - s_4)}{(s_3 - s_4)s_4} \right]$$

$$p_3(t) = -\frac{\lambda_a \lambda_b(-s_3 + e^{s_4 t}s_3 + s_4 - e^{s_3 t}s_4)}{s_3(s_3 - s_4)s_4}$$

The availability is given by

$$A(t) = p_1(t) + p_2(t) = 1 - p_3(t)$$

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

$$A(t) = \frac{e^{s_3 t}(\mu_a \mu_b + \lambda_a s_3 + \mu_a s_3 + \mu_b s_3 + s_3{}^2)}{s_3(s_3 - s_4)} + \frac{\mu_a \mu_b}{s_3 s_4} - \frac{e^{s_4 t}(\mu_a \mu_b + \lambda_b s_4 + \mu_a s_4 + \mu_b s_4 + s_4{}^2)}{s_4(s_3 - s_4)}$$

$$+ \lambda_a \left[\frac{e^{s_3 t}(\mu_b + s_3)}{s_3(s_3 - s_4)} + \frac{\mu_b}{s_3 s_4} + \frac{e^{s_4 t}(-\mu_b - s_4)}{(s_3 - s_4)s_4} \right]$$

The steady state availability is given by

$$A(\infty) = \lim_{s \to 0} s(p_1(s) + p_2(s))$$

$$A(\infty) = \frac{\lambda_a \mu_b}{s_3 s_4} + \frac{\mu_a \mu_b}{s_3 s_4}$$

d) Mean Time to Repair

It plays a crucial role in maintenance planning and optimization by providing an estimate of the anticipated downtime and aiding in the scheduling of maintenance activities to minimize disruptions to operations.

$$MTTR = MTSF\left(\frac{1}{Availability} - 1\right)$$

$$MTTR = \frac{\lambda_a + \lambda_b + \mu_a}{s_1 s_2}\left(\frac{s_3 s_4 - \lambda_a \mu_b + \mu_a \mu_b}{\lambda_a \mu_b + \mu_a \mu_b}\right)$$

e) Maintainability

Maintainability refers to how easily and efficiently a system or product can be maintained, repaired, inspected, and serviced throughout its operational life cycle. By prioritizing maintainability during the design, development, and management stages, a product or system can achieve long-term sustainability and peak performance.

$$M(t) = 1 - \exp\left(-\frac{t}{MTTR}\right)$$

$$M(t) = 1 - e^{\left(-\frac{t}{\frac{\lambda_a + \lambda_b + \mu_a}{s_1 s_2}\left(\frac{s_3 s_4 - \lambda_a \mu_b + \mu_a \mu_b}{\lambda_a \mu_b + \mu_a \mu_b}\right)}\right)}$$

f) Busy Period of the Server Due to Repair

The busy period of a server due to repair refers to the duration during which the server is engaged in repairing or servicing tasks. It starts when the server begins the repair process after detecting a failure or issue and ends when the repair is completed, and the server is ready to resume its normal operation.

$$B_0(t) = p_2(t) + p_3(t) = 1 - p_1(t)$$

$$B_0(t) = 1 - \frac{(\lambda_b + \mu_a + s)}{(s - s_1)(s - s_2)}$$

g) Cost Function

If we assume that K_0 be the revenue per unit up-time of the system and K_1 be the cost per unit for which server is busy due to repair, then the following equation helps to find the amount that the system model experiences while it is in steady state.

The expected cost incurred in (0,t] is

$$Cost\ Function = K_0 A(\infty) - K_1 B_0(\infty)$$

IV. Numerical and Graphical Presentation

Here, we evaluate the reliability, availability, maintainability and the cost function for the arbitrary values of failure rate and repair rate of the components with arbitrary time when they are taken as non-identical in nature. The numerical and graphical representations of the findings are provided

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

below:

From Figure 2, it is observed that the reliability of the system declines with the increase of failure rate and operating time. Also, the reliability of the system is increases with an increase in repair rate of the units.
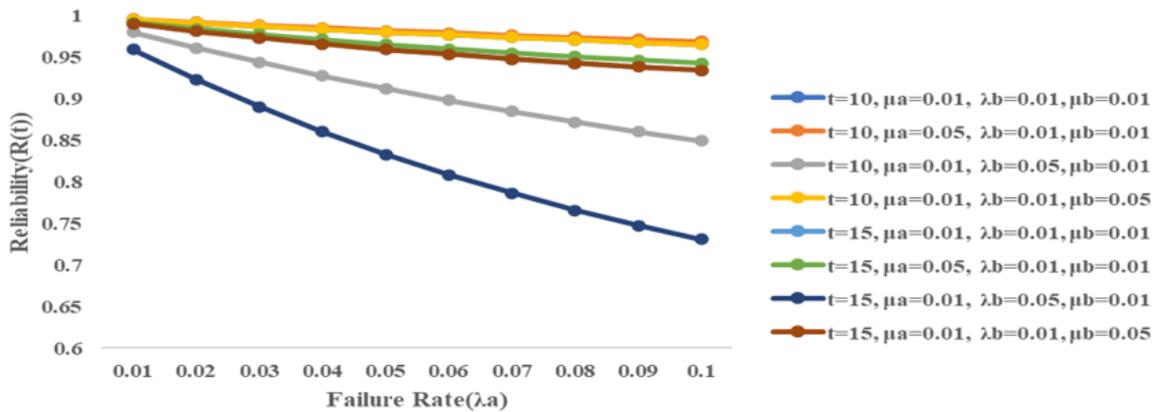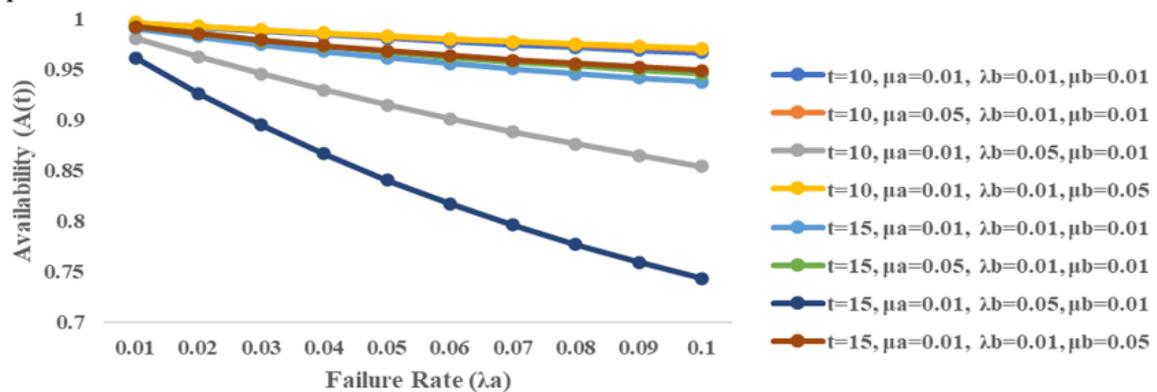

Figure 2: *Reliability V/s Repair Rate (μ) and Failure Rate(λ) with Operating Time(t)*

From Figure 3, it is observed that the availability (A(t)) of the system declines with the increase of failure rate and operating time. Also, the availability of the system is increases with an increase in repair rate of the units.

Figure 3: *Availability V/s Repair Rate (μ) and Failure Rate(λ) with Operating Time(t)*

From Figure 4, it is observed that the availability (A(∞)) of the system declines with the increase of failure rate. Also, the availability of the system is increases with an increase in repair rate of the units.
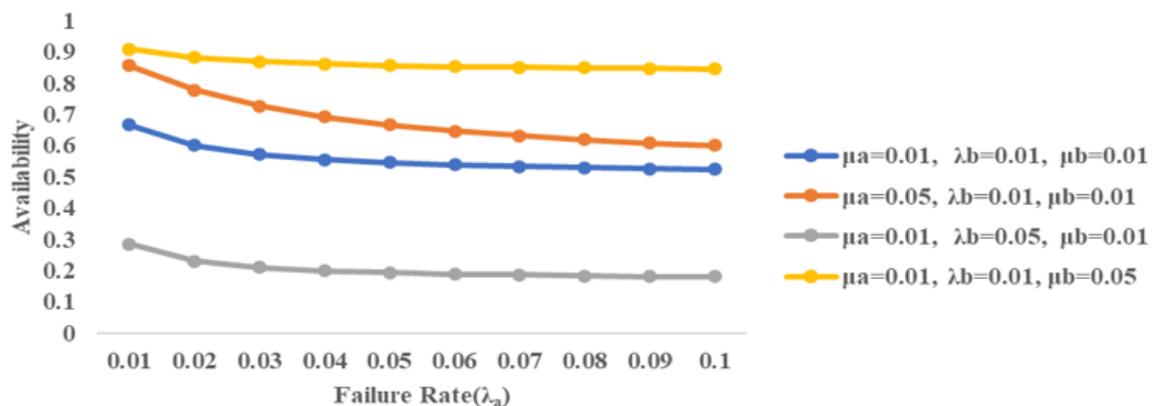

Figure 4: *Availability V/s Repair Rate (μ) and Failure Rate (λ)*

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

From Figure 5, it is observed that the maintainability (M(t)) of the system increases with the increase of failure rate. Also, the maintainability of the system declines with an increase in repair rate of the units.
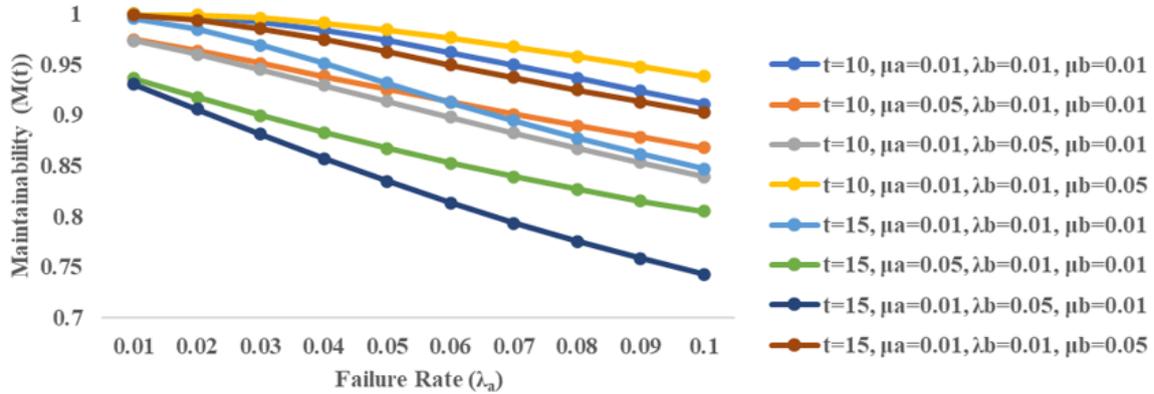

Figure 5: *Maintainability V/s Repair Rate (μ) and Failure Rate (λ)*

From Figure 6, it is observed that the cost function (C(t)) of the system declines with the increase of failure rate. Also, the cost of the system is increases with an increase in repair rate of the units.
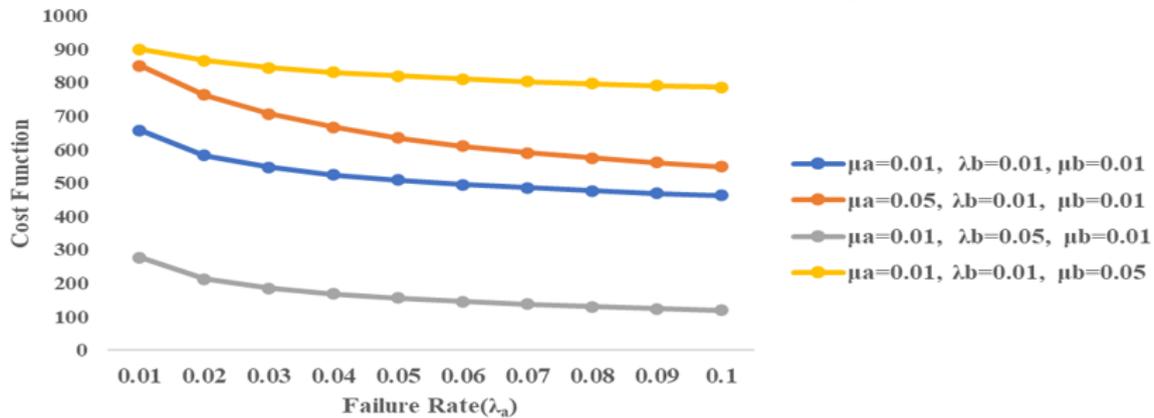

Figure 6: *Cost Function V/s Repair Rate (μ) and Failure Rate (λ)*

V. Application

The present study finds practical application in various fields, including the innovative cold standby system design of the Bajaj Freedom 125 CNG bike. Addressing the reliability and efficiency of dual-fuel motorcycles presents a significant challenge today. While fully electric models face limitations due to battery capacity and infrastructure for long-range travel, a promising solution lies in the cold standby architecture for dual-fuel systems. Specifically, the cold standby model, illustrated in figure 7, has garnered attention from industry leaders like Bajaj Auto. In this configuration, both CNG and petrol are available as fuel options, but only one is active at a time, with the other in standby mode. This setup enhances reliability by ensuring that if one fuel system fails, the other can take over, maintaining the bike's operation. This approach is potentially more efficient and cost-effective compared to single-fuel designs, which may require more frequent refuelling and higher maintenance costs. In our system, illustrated in Figure 7, we consider two units—CNG and petrol—each playing a unique role in ensuring continuous operation and reliability in which main unit is CNG cylinder and the cold standby is in petrol tank. This cold standby approach offers significant advantages in terms of reduced emissions, enhanced

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

fuel efficiency, and lower operating costs, making it a compelling choice for modern automotive engineering. By leveraging the cold standby model, the Bajaj Freedom 125 CNG bike can provide a reliable, efficient, and environmentally friendly solution for today's transportation needs.

Figure 7: *Bajaj Freedom 125 CNG Bike*

VI. Conclusion

This study examines the reliability measures of a cold standby repairable system of two non-identical units using a Markov approach. By employing Chapman-Kolmogorov equations, we derive expressions for key reliability measures, including the system's cost function. We conduct the cost-benefit analysis for the non-identical units, assuming exponential distributions for both failure and repair rates. The results for reliability, availability, maintainability, and the cost function are obtained using arbitrary parameter values. Graphical and numerical behaviors of the system model are illustrated in figures 2, 3, 4, 5, 6 and tables 1, 2, 3, 4, 5. Our observations reveal that as the failure rate increases, steady-state availability, mean time to system failure, and the profit function decrease. Conversely, these measures increase with higher repair rates. Additionally, reliability, availability, and maintainability decrease with both higher failure rates and longer operating times, but increase with higher repair rates. Systems with high maintainability not only offer ease and cost-effectiveness in maintenance but also contribute to reduced downtime, improved reliability, and lower lifecycle costs. These findings underscore the importance of prioritizing maintainability throughout the lifecycle of systems for optimal performance and resource efficiency. The analysis indicates that the system is more reliable, available, and profitable when the units' repair rate is increased.

References

 [1] Behboudi, Z., Borzadaran, G. R. M. and Asadi, M. (2021). Reliability modelling of two-unit cold standby systems: a periodic switching approach. *Applied Mathematical Modeling*, 92:176-195.

[2] Chao, M.T. and James, C. F. (1991). The reliability of a large series system under markov structure. *Advances in Applied Probability*, 23(4):894–908.

[3] Cui, L., Li H. and Li, J. (2007). Markov repairable systems with history-dependent up and down states. *Stochastic Models*, 23:665-681.

[4] Danjuma, M. U., Yusuf, B. and Yusuf, I. (2022). Reliability, availability, maintainability, and dependability analysis of cold standby series-parallel system. *Journal of Computational and Cognitive Engineering*, 1(4):193-200.

[5] El-Sherbeny, M. S., Mahmoud, M. A. W. and Hussien, Z. M. (2020). Reliability analysis of a two-unit cold standby system with arbitrary distributions and change in units. *Life Cycle Reliability Safety Engineering*, 9:261-272.

[6] Eryilmaz, S. and Finkelstein, M. (2021). Reliability of the two-unit priority standby system revisited. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*,

A. D. Yadav, S.C. Malik, S. Malik and N. Nandal
RELIABILITY- AVAILABILITY - MAINTAINABILITY OF A
REPAIRABLE SYSTEM USING MARKOV APPROACH

RT&A, No 2 (84)
Volume 20, June 2025

236(6):1096-1103.

[7] Gupta, R., Kumar, P. and Gupta, A. (2013). Cost Benefit Analysis of a Two Dissimilar Unit Cold Standby System with Weibull Failure and Repair Laws. *International Journal Of System Assurance Engineering and Management*, 4:327-334.

[8] Kadyan, S., Malik, S. C. and Gitanjali (2021). Stochastic analysis of a three-unit non-identical repairable system with simultaneous working of cold standby units. *Journal of Reliability and Statistical Studies*, 13(2):385-400.

[9] Kalaiarasi, S., Merceline, A. A. and Geethanjalii, R. (2017). Analysis of system reliability using markov technique. *Global Journal of Pure and Applied Mathematics*, 13(9):5265-5273.

[10] Lin, Z., Tao, L., Wang, S., Chen, C., Wang, J. and Ge, D. (2023). Reliability analysis of cold standby systems using a self-stratifying subset simulation method. *Journal of Statistical Computation and Simulation Method*, 93:2257-2275.

[11] Malik, S., Malik, S. C., Nandal, N., & Yadav, A. D. (2023). Reliability assessment of a NSP system under constant triangular fuzzy failure rates. *International Journal of Industrial and Systems Engineering*, 1(1).DOI: https://doi.org/10.1504/IJISE .2023.10060854 .

[12] Manglik, M. and Ram, M. (2013). Reliability analysis of a two-unit cold standby system using markov process. *Journal of Reliability and Statistical Studies*, 6(2):65-80.

[13] Rathi, P., Malik, S. C. and Nandal, N. (2021). Reliability characteristics of a parallel cold standby system of four units with priority proviso. *International Journal of Agricultural and Statistical Sciences*, 17(2):563-572.

[14] Rathi, R., Kadyan, M.S. and Nandal, N. (2024). Reliability modelling and profit analysis of a fly ash bricks manufacturing system. *OPSEARCH.* DOI: https://doi.org/10.1007/s12597-024-00890-x .

[15] Saritha, G., Devi, M. T. and Maheswari, T. S. U. (2020). The reliability and availability for non-repairable & repairable systems using markov modelling. *International Journal of Engineering Research and Technology*, 9(2):169-171.

[16] Shivani, Ram, M., Goyal, N. and Kumar, A. (2023). Analysis of series-parallel system's sensitivity in context of component failures. *RAIRO Operations Research*, 57:2131-2149.

[17] Umamaheshwari, K. and Srilakshmi, S. (2020). Markov model for a system with 'n' elements with human error and common cause. *Journal of Xi'an University of Architecture and Technology*, 12(5):2611-2618.

[18] Wang, L., Jia, X. and Zhang, J. (2013). Reliability evaluation for multi-state markov repairable systems with redundant dependencies. *Quality Technology and Quantitative Management*, 10(3):277-289.

[19] Yadav, A. D., Nandal, N. and Malik, S.C. (2023). Markov approach for reliability – availability – maintainability analysis of a three-unit repairable system. *Reliability: Theory and Applications*, 18(1):193-205.

[20] Yadav, A. D., Nandal, N., Malik, S. and Malik, S. C. (2023). Markov approach for reliability and availability analysis of a four-unit repairable system. *OPSEARCH*, 60:1731-1756.