RELIABILITY ANALYSIS OF AAC BLOCK PLANT USING BOOLEAN FUNCTION AND PATH TRACING METHOD

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Abstract

This study offers a comprehensive reliability analysis of Autoclaved Aerated Concrete (AAC) block plants, employing Boolean function technique and path tracing method. The analysis addresses a non-repairable system comprising five interconnected subsystems arranged in series. The configuration includes a single component in the first subsystem, two parallel components in the second, single components in subsystems three and four and three parallel components in the fifth subsystem. The complexity of the system is heightened by the presence of non-identical components with failure rates modelled by the Weibull distribution. Utilizing these mathematical approaches, we derive expressions for reliability and mean time to system failure (MTSF). The study meticulously evaluates system reliability, with a focus on the Weibull distribution, especially the exponential distribution case. The numerical and graphical results, highlighting the performance and durability of the AAC block plant. This research provides critical insights into the reliability dynamics of AAC block production, offering a pathway to enhanced operational efficiency and optimized performance in the industry.

Keywords: Autoclaved Aerated Concrete (AAC) Block, Boolean Technique, Path Tracing Method, Weibull Distribution

I. Introduction

In today's world, Industrialization and infrastructure development have long been recognized as cornerstones of economic progress. However, these advancements often bring significant environmental challenges, with industrial by-products such as fly ash from thermal power plants posing severe threats to ecosystems worldwide. In response to such concerns, the invention of Autoclaved Aerated Concrete (AAC) by Swedish architect and inventor John Axel Eriksson in the early 20th century marked a pivotal moment in construction innovation. AAC blocks, distinguished by their lightweight structure, superior thermal insulation, and fire resistance, represent a sustainable alternative to traditional building materials. Their manufacturing process, which involves curing concrete under high pressure and temperature, not only enhances performance but also contributes to reducing industrial waste. This process enhances their performance, making AAC blocks an essential material in both residential and commercial construction. Autoclaved Aerated Concrete (AAC) blocks have gained significant traction in the construction industry due to their unique properties, including lightweight nature, thermal insulation, and fire resistance. Malhotra [3] demonstrated that the fire resistance of AAC blocks is twice that of normal bricks, highlighting their superior safety profile. Valore [10] affirmed that AAC blocks possess all desirable characteristics, making them a highly favorable construction material.

Over the years, scholars have proposed various system configurations such as series, parallel, series-parallel, parallel-series, and k-out-of-n to ensure the optimal functioning of non-repairable systems with minimal risks and manufacturing expenses. These configuration strategies are designed to maximize reliability while keeping costs under control. Malik et al. [5] conducted the reliability analysis of NSP system. Narwal et al. [7] considered different mathematical approaches for the reliability analysis of parallel system of order (3,3,3,1). Malik et al. [6] evaluated the fuzzy reliability and MTSF of NSP system under time varying failure rates. Yadav et al. [12] determined the reliability analysis of a three-unit parallel repairable system using Markov approach. Yadav et al. [13] analysed the reliability of a four-unit repairable system. Rathi et al. [9] conducted the reliability modelling of fly ash bricks manufacturing system. The reliability analysis of Autoclaved Aerated Concrete (AAC) block plants can be effectively approached using Boolean function and path tracing method. These methodologies allow for a systematic evaluation of the operational reliability of AAC production systems, which are critical in ensuring the structural integrity and performance of AAC blocks in construction applications. Therefore, integrating the reliability analysis of the production process with the material properties of AAC blocks is essential for a comprehensive assessment. Boolean function can be utilized to model the various components and processes involved in the AAC production system. By defining the operational states of different components (e.g., mixers, autoclaves, and curing chambers) as binary variables, one can create a logical representation of the system's reliability. Agarwal et al. [1] employed the Boolean function technique to discuss the reliability analysis of a sugar manufacturing plant, setting a benchmark for reliability studies. Pervaiz et al. [8] further advanced this field by deriving the reliability of a paper plant using the same Boolean technique. Chandra et al. [2] examined the reliability factors in a chocolate manufacturing plant using the Boolean function technique, adding valuable knowledge to the manufacturing sector. This approach is particularly useful in identifying critical failure paths and understanding how different components interact within the production process. The application of Boolean algebra enables the simplification of complex reliability expressions, facilitating the assessment of the overall system reliability. Path tracing methods complement this analysis by allowing for the identification and quantification of failure paths within the AAC production system. By tracing the paths from the initial state (successful operation) to the final state (failure), one can determine the likelihood of various failure scenarios occurring. For instance, if a failure occurs in the autoclaving process due to temperature fluctuations, path tracing can help identify how this failure impacts the quality of the AAC blocks produced and the subsequent structural reliability of the blocks in construction. Vishnevskiy & Kapustin [11] discussed that the reliability of AAC blocks themselves is influenced by various factors, including the materials used in their production, such as fly ash and aluminum powder, and the conditions under which they are cured. Malik et al. [4] analyzed the reliability and Mean Time to System Failure (MTSF) of a fast-food manufacturing system using Boolean Function and Path Tracing Method for various time values and parameters, offering crucial data for optimizing the system performances.

Here, the reliability analysis of Autoclaved Aerated Concrete (AAC) Blocks plants has been meticulously carried out employing the well-established Boolean function technique and path tracing method. The system is characterized as non-repairable, featuring five subsystems interconnected in series. Within this configuration, the first subsystem comprises a single component, while the second subsystem entails two parallel components. Subsystems three and four are arranged in series, each encompassing a single component. The fifth subsystem, connected in series, accommodates three components operating in parallel. Importantly, all components within the system are non-identical, adding complexity to the analysis. The failure rates of these components adhere to the Weibull distribution. Expressions for reliability and mean time to system failure (MTSF) have been derived employing these mathematical approaches. Additionally, the reliability measures of the system have been meticulously evaluated, focusing on the specific case of the Weibull distribution, particularly the exponential distribution. Through comprehensive analysis, the reliability of the system has been determined for specific parameter values, highlighting the results both numerically and graphically. This comprehensive study provides valuable insights into the reliability dynamics of AAC Blocks plants, facilitating enhanced operational efficiency and performance optimization.

II. Assumptions and State Descriptions

1. All units in the system are operational.
2. There are no repair facilities available within the system.
3. The failure of each component is statistically independent, with known reliability rates.
4. Initially, the entire system is in good and operable condition.
5. Each component of the system can only be in either a good or bad state.
6. The failure of any single subsystem can cause the entire system to fail.
7. Primary Mixer (X_{11}): All raw materials except aluminum are processed through the primary mixer.
8. Main Mixer (X_{21}, X_{22}): The second subsystem consists of two components connected in parallel. This subsystem mixes the raw materials properly, and aluminum powder is added in the proper ratio.
9. Block Casting (X_{31}): This stage involves casting the blocks in molds.
10. De-molding and Cutting (X_{41}): The fourth subsystem handle the de-molding and wire cutting of blocks to the desired size.
11. High-Pressure Steam Curing (X_{51}, X_{52}, X_{53}): The fifth subsystem consists of three components connected in series. Blocks are kept in this subsystem for 12 hours for high-pressure steam curing to gain strength.

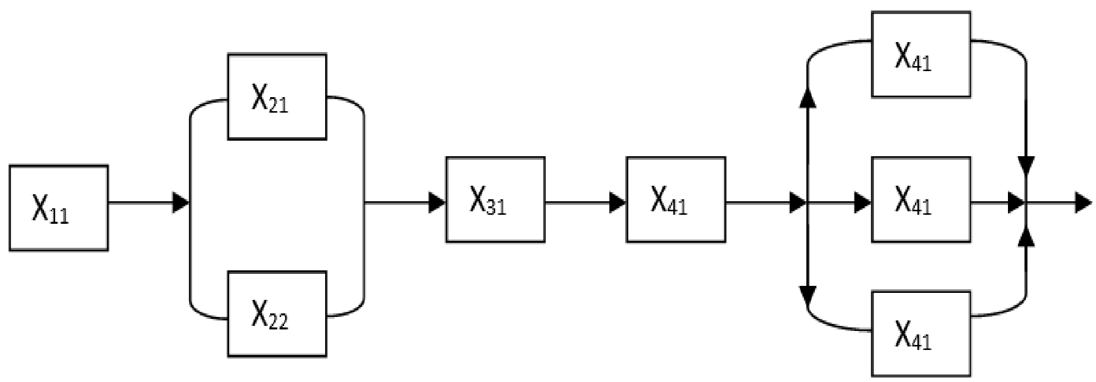

Figure 1: *State Transition Diagram*

III. Solution Using Boolean Function Technique

a) Reliability

The conditions of capability of successful working of the system in terms of logical matrix are shown below:

$$F(x_{11}, x_{21}, x_{22}, \dots \dots, x_{51}) = \begin{pmatrix} x_{11} & x_{21} & x_{31} & x_{41} & x_{51} \\ x_{11} & x_{21} & x_{31} & x_{41} & x_{52} \\ x_{11} & x_{21} & x_{31} & x_{41} & x_{53} \\ x_{11} & x_{22} & x_{31} & x_{41} & x_{51} \\ x_{11} & x_{22} & x_{31} & x_{41} & x_{52} \\ x_{11} & x_{22} & x_{31} & x_{41} & x_{53} \end{pmatrix}$$

By algebra of logics. We have

$$F(x_{11}, x_{21}, x_{31}, \dots . x_{71}) = [x_{11}, x_{31}, x_{41}] \Lambda f(x_{21}, x_{22}, x_{51}, x_{52}, x_{53})$$

Where

$$f(x_{21}, x_{22}, x_{51}, x_{52}, x_{53}) = \begin{pmatrix} x_{21} & x_{51} \\ x_{21} & x_{52} \\ x_{21} & x_{53} \\ x_{22} & x_{51} \\ x_{22} & x_{52} \\ x_{23} & x_{53} \end{pmatrix}$$

Let's assume

$$P_1 = [x_{21} \quad x_{51}]$$
$$P_2 = [x_{21} \quad x_{52}]$$
$$P_3 = [x_{21} \quad x_{53}]$$
$$P_4 = [x_{22} \quad x_{51}]$$
$$P_5 = [x_{22} \quad x_{52}]$$
$$P_6 = [x_{22} \quad x_{53}]$$

Now

$$P_1' = \begin{pmatrix} x_{21}' & \\ x_{21} & x_{51}' \end{pmatrix}$$
$$P_1'P_2 = [x_{21}x_{51}'x_{52}]$$
$$P_1'P_2' = \begin{pmatrix} x_{21}' & & \\ x_{21} & x_{51}' & x_{52}' \end{pmatrix}$$
$$P_1'P_2'P_3 = [x_{21}x_{51}'x_{52}'x_{53}]$$
$$P_1'P_2'P_3' = \begin{pmatrix} x_{21}' & & & \\ x_{21} & x_{51}' & x_{52}' & x_{53}' \end{pmatrix}$$
$$P_1'P_2'P_3'P_4 = [x_{21}'x_{22}x_{51}]$$
$$P_1'P_2'P_3'P_4' = \begin{pmatrix} x_{21}' & x_{22}' & & & \\ x_{21} & x_{22}' & x_{51}' & x_{52}' & x_{53}' \\ x_{21}' & x_{22} & x_{51}' & & \\ x_{21} & x_{22} & x_{51}' & x_{52}' & x_{53}' \end{pmatrix}$$
$$P_1'P_2'P_3'P_4'P_5 = [x_{21}'x_{22}x_{51}'x_{52}]$$
$$P_1'P_2'P_3'P_4'P_5' = \begin{pmatrix} x_{21}' & x_{22}' & & & \\ x_{21} & x_{22}' & x_{51}' & x_{52}' & x_{53}' \\ x_{21}' & x_{22} & x_{51}' & x_{52}' & \\ x_{21} & x_{22} & x_{51}' & x_{52}' & x_{53}' \end{pmatrix}$$
$$P_1'P_2'P_3'P_4'P_5'P_6 = [x_{21}'x_{22}x_{51}'x_{52}'x_{53}]$$

$$R_s(t) = R_{11}R_{31}R_{41}[R_{21}R_{51} + R_{21}R_{52}(1 - R_{51}) + R_{21}R_{53}(1 - R_{51})(1 - R_{52}) + R_{22}R_{51}(1 - R_{21}) + R_{22}R_{52}(1 - R_{21})(1 - R_{51}) + R_{22}R_{53}(1 - R_{21})(1 - R_{51})(1 - R_{52})]$$

$$R_s(t) = R_{11}R_{31}R_{41}[R_{21}R_{51} + R_{21}R_{52} - R_{21}R_{51}R_{52} + R_{21}R_{53} - R_{21}R_{51}R_{53} - R_{21}R_{52}R_{53} + R_{21}R_{51}R_{52}R_{53} + R_{22}R_{51} - R_{21}R_{22}R_{51} + R_{22}R_{52} - R_{21}R_{22}R_{52} - R_{22}R_{51}R_{52} + R_{21}R_{22}R_{51}R_{52} + R_{22}R_{53} - R_{22}R_{51}R_{53} - R_{22}R_{52}R_{53} + R_{22}R_{51}R_{52}R_{53} - R_{21}R_{22}R_{53} + R_{21}R_{22}R_{51}R_{53} + R_{21}R_{52}R_{53} - R_{21}R_{22}R_{51}R_{52}R_{53}]$$

b) Reliability with Weibull Failure Laws

$$R_s(t) = e^{-(\lambda_{11}t)^k-(\lambda_{31}t)^k-(\lambda_{41}t)^k}\Big[e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k}$$
$$+ e^{-(\lambda_{21}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k}$$
$$+ e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k}$$
$$+ e^{-(\lambda_{22}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k}$$
$$+ e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k}$$
$$- e^{-(\lambda_{22}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{53}t)^k}$$
$$+ e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k}$$
$$- e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k}\Big]$$

Let's suppose the units are identical. Then reliability of the system is given by

$$R_s(t) = R^3[6R^2 - 8R^3 + 4R^4 - R^5]$$

For Weibull Failure laws, i.e., failure rate is $(\lambda t)^{k-1}$, then reliability of each component is $e^{-(\lambda t)^k}$ and reliability of the system can be given by

$$R_{SW}(t) = e^{-3(\lambda t)^k}[6e^{-2(\lambda t)^k} - 8e^{-3(\lambda t)^k} + 4e^{-4(\lambda t)^k} - e^{-5(\lambda t)^k}]$$

Now, the mean time to system failure is given by

$$MTSF = \frac{1}{\lambda k}\Gamma\left(\frac{1}{k}\right)\left(\frac{6}{5^{1/k}} - \frac{8}{6^{1/k}} + \frac{4}{7^{1/k}} - \frac{1}{8^{1/k}}\right)$$

c) Reliability with Exponential Failure Laws

$$R_s(t) = e^{-(\lambda_{11}t)-(\lambda_{31}t)-(\lambda_{41}t)}\big[e^{-(\lambda_{21}t)-(\lambda_{51}t)} + e^{-(\lambda_{21}t)-(\lambda_{52}t)} - e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{21}t)-(\lambda_{53}t)}$$
$$- e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{53}t)} - e^{-(\lambda_{21}t)-(\lambda_{52}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)}$$
$$+ e^{-(\lambda_{22}t)-(\lambda_{51}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)} + e^{-(\lambda_{22}t)-(\lambda_{52}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{52}t)}$$
$$- e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{22}t)-(\lambda_{53}t)}$$
$$- e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{53}t)} - e^{-(\lambda_{22}t)-(\lambda_{52}t)-(\lambda_{53}t)} + e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)}$$
$$- e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{52}t)-(\lambda_{53}t)}$$
$$- e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)}\big]$$

Let's suppose the units are identical. Then reliability of the system is given by

$$R_s(t) = R^3[6R^2 - 8R^3 + 4R^4 - R^5]$$

For Exponential Failure Laws i.e., a constant failure rate λ, then reliability of each component is $e^{-\lambda t}$.

Now, reliability of the system can be obtained as

$$R_{SE}(t) = e^{-3\lambda t}[6e^{-2\lambda t} - 8e^{-3\lambda t} + 4e^{-4\lambda t} - e^{-5\lambda t}]$$

Also, the mean time to system failure is given by

$$MTSF = \frac{6}{5\lambda} - \frac{8}{6\lambda} + \frac{4}{7\lambda} - \frac{1}{8\lambda}$$

III. Solution Using Path Tracing Method

a) Reliability

The success of any one path out of the paths $P_1, P_2 \ldots P_6$ results into the successful operation of the system.

We have,

$$P(U_{i=1}^{6}P_i) = P(P_1) + P(P_2) + P(P_3) + P(P_4) + P(P_5) + P(P_6) - (P(P_1 \cap P_2) + P(P_1 \cap P_3) + P(P_1 \cap P_4) + P(P_1 \cap P_5) + P(P_1 \cap P_6) + P(P_2 \cap P_3) + P(P_2 \cap P_4) + P(P_2 \cap P_5) + P(P_2 \cap P_6) + P(P_3 \cap P_4) + P(P_3 \cap P_5) + P(P_3 \cap P_6) + P(P_4 \cap P_5) + P(P_4 \cap P_6) + P(P_5 \cap P_6)) + (P(P_1 \cap P_2 \cap P_3) + P(P_1 \cap P_2 \cap P_4) + P(P_1 \cap P_2 \cap P_5) + P(P_1 \cap P_2 \cap P_6) + P(P_1 \cap P_3 \cap P_4) + P(P_1 \cap P_3 \cap P_5) + P(P_1 \cap P_3 \cap P_6) + P(P_1 \cap P_4 \cap P_5) + P(P_1 \cap P_4 \cap P_6) + P(P_1 \cap P_5 \cap P_6) + P(P_2 \cap P_3 \cap P_4) + P(P_2 \cap P_3 \cap P_5) + P(P_2 \cap P_3 \cap P_6) + P(P_2 \cap P_4 \cap P_5) + P(P_2 \cap P_4 \cap P_6) + P(P_2 \cap P_5 \cap P_6) + P(P_3 \cap P_4 \cap P_5) + P(P_3 \cap P_4 \cap P_6) + P(P_3 \cap P_5 \cap P_6) + P(P_4 \cap P_5 \cap P_6)) - (P(P_1 \cap P_2 \cap P_3 \cap P_4) + P(P_1 \cap P_2 \cap P_3 \cap P_5) + P(P_1 \cap P_2 \cap P_3 \cap P_6) + P(P_1 \cap P_2 \cap P_4 \cap P_5) + P(P_1 \cap P_2 \cap P_4 \cap P_6 + P(P_1 \cap P_2 \cap P_5 \cap P_6) + P(P_1 \cap P_3 \cap P_4 \cap P_5) + P(P_1 \cap P_3 \cap P_4 \cap P_6) + P(P_1 \cap P_3 \cap P_5 \cap P_6) + P(P_1 \cap P_4 \cap P_5 \cap P_6) + P(P_2 \cap P_3 \cap P_4 \cap P_5) + P(P_2 \cap P_3 \cap P_4 \cap P_6) + P(P_2 \cap P_3 \cap P_5 \cap P_6) + P(P_2 \cap P_4 \cap P_5 \cap P_6) + P(P_3 \cap P_4 \cap P_5 \cap P_6)) + (P(P_1 \cap P_2 \cap P_3 \cap P_4 \cap P_5) + P(P_1 \cap P_2 \cap P_3 \cap P_4 \cap P_6) + P(P_1 \cap P_2 \cap P_3 \cap P_5 \cap P_6) + P(P_1 \cap P_2 \cap P_4 \cap P_5 \cap P_6)P(P_1 \cap P_3 \cap P_4 \cap P_5 \cap P_6) + P(P_2 \cap P_3 \cap P_4 \cap P_5 \cap P_6) - P(P_1 \cap P_2 \cap P_3 \cap P_4 \cap P_5 \cap P_6)$$

$$R_s(t) = R_{11}R_{31}R_{41}[R_{21}R_{51} + R_{21}R_{52}(1 - R_{51}) + R_{21}R_{53}(1 - R_{51})(1 - R_{52}) + R_{22}R_{51}(1 - R_{21}) + R_{22}R_{52}(1 - R_{21})(1 - R_{51}) + R_{22}R_{53}(1 - R_{21})(1 - R_{51})(1 - R_{52})]$$

$$R_s(t) = R_{11}R_{31}R_{41}[R_{21}R_{51} + R_{21}R_{52} - R_{21}R_{51}R_{52} + R_{21}R_{53} - R_{21}R_{51}R_{53} - R_{21}R_{52}R_{53} + R_{21}R_{51}R_{52}R_{53} + R_{22}R_{51} - R_{21}R_{22}R_{51} + R_{22}R_{52} - R_{21}R_{22}R_{52} - R_{22}R_{51}R_{52} + R_{21}R_{22}R_{51}R_{52} + R_{22}R_{53} - R_{22}R_{51}R_{53} - R_{22}R_{52}R_{53} + R_{22}R_{51}R_{52}R_{53} - R_{21}R_{22}R_{53} + R_{21}R_{22}R_{51}R_{53} + R_{21}R_{52}R_{53} - R_{21}R_{22}R_{51}R_{52}R_{53}]$$

b) Reliability with Weibull Failure Laws

$$R_s(t) = e^{-(\lambda_{11}t)^k-(\lambda_{31}t)^k-(\lambda_{41}t)^k}\Big[e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{52}t)^k} - e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{22}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{53}t)^k} + e^{-(\lambda_{21}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k} - e^{-(\lambda_{21}t)^k-(\lambda_{22}t)^k-(\lambda_{51}t)^k-(\lambda_{52}t)^k-(\lambda_{53}t)^k}\Big]$$

Let's suppose the units are identical. Then reliability of the system is given by

$$R_s(t) = R^3[6R^2 - 8R^3 + 4R^4 - R^5]$$

For Weibull Failure laws, i.e., failure rate is $(\lambda t)^{k-1}$, then reliability of each component is $e^{-(\lambda t)^k}$ and reliability of the system can be given by

$$\mathrm{R_{SW}}\ (\mathrm{t}) = e^{-3(\lambda t)^k}[6e^{-2(\lambda t)^k} - 8e^{-3(\lambda t)^k} + 4e^{-4(\lambda t)^k} - e^{-5(\lambda t)^k}]$$

Now, the mean time to system failure is given by

$$MTSF = \frac{1}{\lambda k}\Gamma\left(\frac{1}{k}\right)\left(\frac{6}{5^{1/k}} - \frac{8}{6^{1/k}} + \frac{4}{7^{1/k}} - \frac{1}{8^{1/k}}\right)$$

c) Reliability with Exponential Failure Laws

$$R_s(t) = e^{-(\lambda_{11}t)-(\lambda_{31}t)-(\lambda_{41}t)}\big[e^{-(\lambda_{21}t)-(\lambda_{51}t)} + e^{-(\lambda_{21}t)-(\lambda_{52}t)} - e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{21}t)-(\lambda_{53}t)} - e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{53}t)} - e^{-(\lambda_{21}t)-(\lambda_{52}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)} + e^{-(\lambda_{22}t)-(\lambda_{51}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)} + e^{-(\lambda_{22}t)-(\lambda_{52}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{52}t)} - e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)} + e^{-(\lambda_{22}t)-(\lambda_{53}t)} - e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{53}t)} - e^{-(\lambda_{22}t)-(\lambda_{52}t)-(\lambda_{53}t)} + e^{-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{53}t)} + e^{-(\lambda_{21}t)-(\lambda_{52}t)-(\lambda_{53}t)} - e^{-(\lambda_{21}t)-(\lambda_{22}t)-(\lambda_{51}t)-(\lambda_{52}t)-(\lambda_{53}t)}\big]$$

Let's suppose the units are identical. Then reliability of the system is given by

$$R_s(t) = R^3[6R^2 - 8R^3 + 4R^4 - R^5]$$

For Exponential Failure Laws i.e., a constant failure rate λ, then reliability of each component is

$e^{-\lambda t}$.

Now, reliability of the system can be obtained as

$$R_{SE}\ (t) = e^{-3\lambda t}[6e^{-2\lambda t} - 8e^{-3\lambda t} + 4e^{-4\lambda t} - e^{-5\lambda t}]$$

Also, the mean time to system failure is given by

$$\text{MTSF} = \frac{6}{5\lambda} - \frac{8}{6\lambda} + \frac{4}{7\lambda} - \frac{1}{8\lambda}$$

IV. Numerical and Graphical Presentation

Here, we evaluate the reliability and MTSF for the arbitrary values of failure rate (λ) with operating time (t) of the components. The numerical and graphical representation of the results is given below: From Figure 2, it is observed that the reliability of components using the Weibull distribution with varying failure rates (λ) and shape parameters (k) at two different times (t = 5 and t = 10). As the failure rate (λ) increases, the reliability of the system decreases across all conditions. For instance, when t=5 and k=0.90, the reliability drops from 0.8565 at λ=0.01 to 0.2066 at λ=0.09. This indicates that as the component becomes more prone to failure (higher λ), it is less likely to perform reliably over time. Here, two values of are used: 0.90 and 0.80, both of which imply decreasing failure rates. As expected, for a lower value of (0.80), the reliability is lower than for k=0.90, reflecting a more aggressive failure behavior.

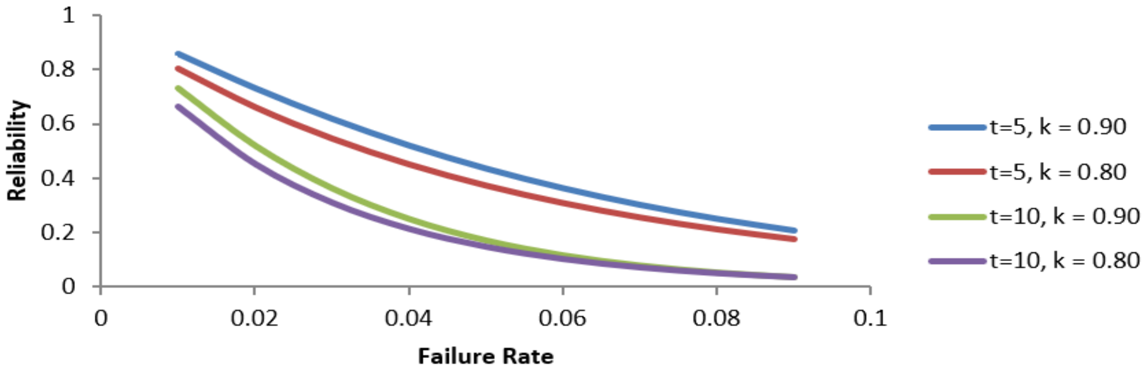

Figure 2: *Reliability V/s Failure Rate(λ) with Operating Time(t) (Boolean Function Technique)*

From Figure 3, it is observed that the MTSF decreases across all shape parameters (k). This indicates that higher failure rates lead to shorter expected operational lifetimes. For example, for k=1, the MTTF decreases from 31.3095 at λ=0.01 to 3.4788 at λ=0.09. This shows that higher failure rates drastically reduce the mean operational time before failure.

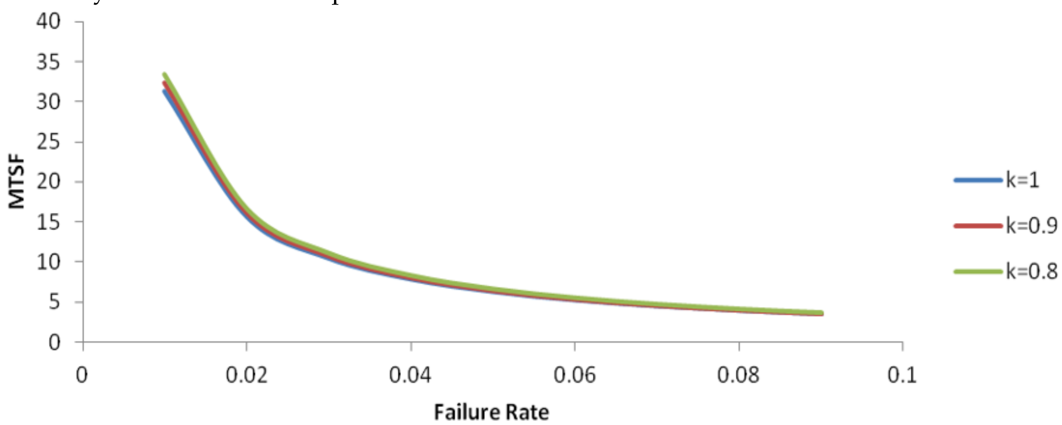

Figure 3: *MTSF V/s Failure Rate(λ) (Boolean Function Technique)*

From Figure 4, as the failure rate (λ) increases, the reliability of the system decreases for both time values. This is expected because a higher failure rate indicates that the component is more prone to failure. For instance, at t=5, when λ=0.01, the reliability is 0.8946, but when λ=0.09, it drops to 0.2388. Similarly, for t=10, the reliability is 0.7857 at λ=0.01, dropping to 0.0371 at λ=0.09. As time increases, the reliability decreases. Components are more likely to fail over a longer period. For example, for λ=0.03, the reliability is 0.6801 at t=5, but it decreases to 0.4154 at t=10. This indicates that the longer the component is in operation, the more its reliability diminishes due to accumulated wear and tear or other degradation factors.

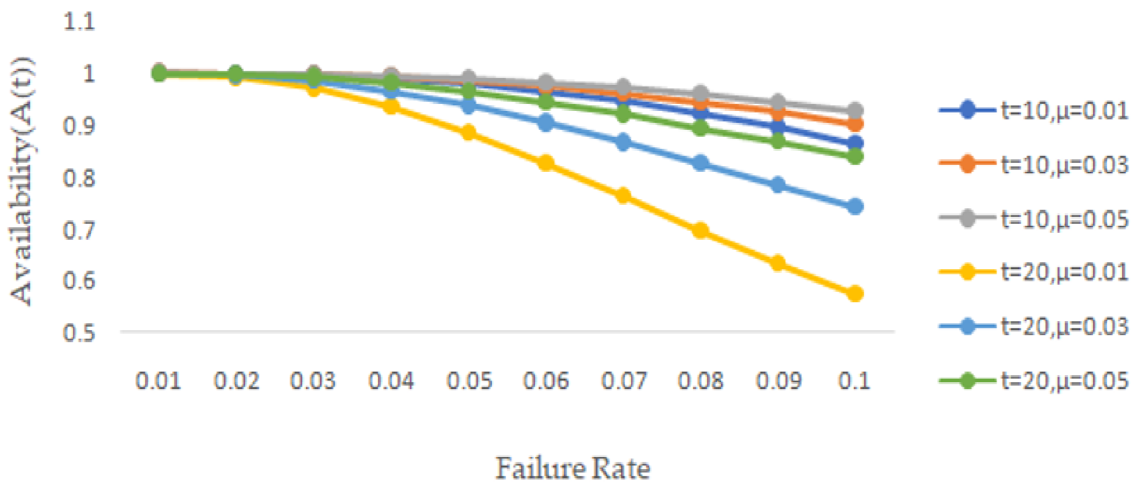

Figure 4: *Reliability V/s Failure Rate(λ) with Operating Time(t) (Boolean Function Technique)*

From Figure 5, we observed that the reliability of components as a function of the failure rate (λ), time (t = 5, t = 10), and the shape parameter (k) for the Weibull distribution. As the failure rate (λ) increases, the reliability of the system decreases for all combinations of t and k. For example, when k=0.90 and t=5, reliability decreases from 0.8565 at λ=0.01 to 0.2066 at λ=0.09. This shows that a higher failure rate reduces the likelihood of the system continuing to function over time. For a lower value of k (e.g., 0.80), reliability tends to be lower than for k=0.90, indicating that a system with more frequent early-life failures will have a reduced overall reliability. For instance, at λ=0.03 and t=5, the reliability for k=0.90 is 0.6177, while for k=0.80, it is 0.5470.

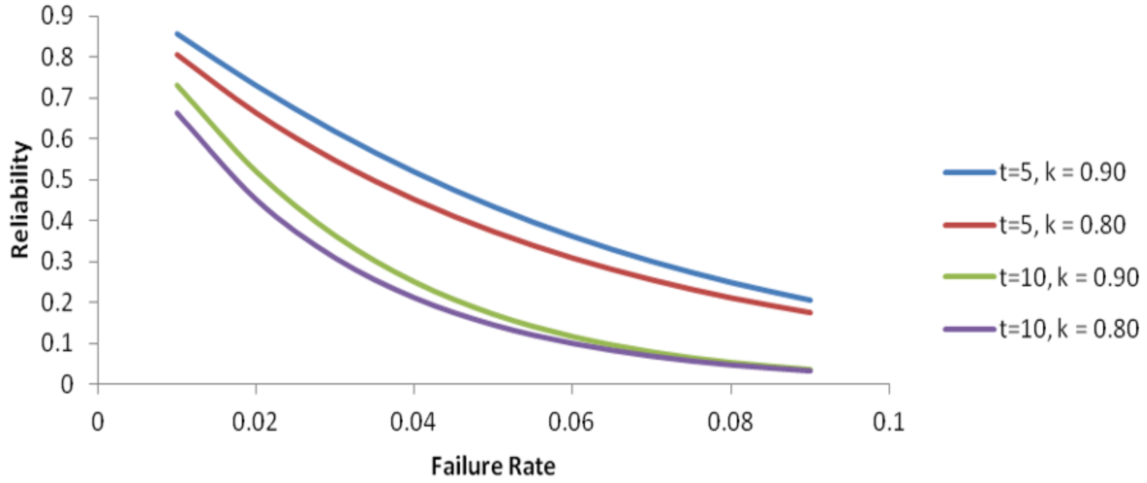

Figure 5: *Reliability V/s Failure Rate(λ) with Operating Time(t)(Path Tracing Method)*

V. Conclusion

The reliability analysis of Autoclaved Aerated Concrete (AAC) block plants has been conducted using the well-established Boolean function technique and path tracing method. By deriving expressions for reliability and Mean Time to System Failure (MTSF) for various failure rates and operating times, and assuming identical units, we have gained significant insights. Our findings indicate that the system's reliability diminishes at a nearly uniform rate under exponential failure laws, while it decreases sharply with a Weibull distribution of failure rates. Furthermore, it is evident that the reliability and MTSF of system continually decline as the failure rate and operating time of the units increase while in case of decreasing the parameter value of k, the MTSF and reliability shows negligible change with the increase of failure rate.

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