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GEOMETRIC HAZARD-BASED PREVENTIVE MAINTENANCE AND REPLACEMENT (GHBPMAR) MODEL FOR AGING MECHANICALLY REPAIRABLE MACHINES

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Abstract

This work aims to develop a geometric hazard-based preventive maintenance model to improve the sustainability of aging mechanically repairable machines particularly in developing countries. The goal is to optimize maintenance and replacement schedules to meet user needs effectively. The model utilizes a geometric process as a hazard rate adjustment factor to account for increasing deterioration rates of machines due to aging and prolong usage and applies same model to a cassava grinding machine across three phases of lifespan: average lifespan, beyond average lifespan, and beyond initial replacement age. The Weibull failure distribution is used to characterize the machine's increasing failure rate. The result reveals that the machine's operational time decreases following preventive maintenance, and maintenance costs increase as the machine progresses through many phases of its lifespan. This highlights the model's effectiveness in managing the cost and timing of maintenance activities. The model's implementation can significantly impact communities in developing countries by improving the reliability and lifespan of essential machinery like cassava grinding machines and other industrial machines which can lead to stable local economies. This approach helps in managing costs and improving the sustainability of machines, especially in resource-constrained environments over extended lifespan.

Keywords: Preventive maintenance, replacement model, geometric process, repairable system, aging, Weibull hazard rate

I. Introduction

Preventive maintenance (PM) entails all necessary actions required to keep a system at a working condition before due maintenance time. It reduces the number of failures during operation and is therefore essential in keeping to production target. It may be done at fixed time interval or sequentially at different time points which may be of different lengths. The common assumption of PM is that the machine will requires minimal repair if it breaks down before PM action.

Many PM models have been developed over the years including [1] and [2] which introduced the concept of adjustment/improvement factors for both the hazard rate and effective age of repairable systems after PM. Furthermore, periodic and sequential imperfect PM model was developed by [3] by adopting the adjustment/improvement factors into the hazard rate and effective age for a sequential PM policy. Later, [4] proposed the hazard model which ensures that the hazard rate of the system is reduced after PM action and the age model which ensures that the effective or virtual age of the system is improved after PM. A general sequential PM model was developed by [5] which combines the age reduction model and hazard rate adjustment model of [4] to exploit the advantages of both models. In view of the peculiarity of developing countries of the world, where repairable systems are used longer than the specified period with increased usage, a geometric imperfect preventive maintenance and replacement (GIPMAR) model for aging repairable systems was proposed by [6], which is a generalization of the hybrid model by [5]. The geometric process (GP) was use as the hazard rate adjustment factor to characterize and regulate the increasing hazard rate at various degrees of deterioration denoted by the index of the GP model. The proposed models were used to obtain optimal PM and replacement schedule for repairable systems at three phases of operation; namely: average lifespan, beyond average lifespan and after initial replacement age.

The use of GP in modelling the stochastic increase in repair times after failures and stochastic decrease of the life time of a component of a system after repair was propagated by [7]. Thereafter, authors in [8], [9] and [10] among others have successfully applied the GP models to describe the stochastic increase or decrease in the life time of a system. Hence, this work aims at incorporating the geometric hazard rate adjustment factor of [6] into the hazard rate-based preventive maintenance model of [4] in a bid to generalize the model to meet the needs of developing countries who import and use second-hand (fairly used) products beyond the specified age limits. This model will be used to obtain the optimal PM and replacement schedule for three life phases of a cassava grinding machine which PM and replacement schedule at what is regarded as phase 1 was earlier obtained in [11] but not for the other phases with higher degrees of deterioration consequent upon the peculiarity of developing countries, where equipment is used beyond normal specified lifespan. Hence, this work seeks to extend the hazard model of [4] to cover the maintenance of these items in the extended phases of operation for sustainability.

II. Methods

2.1 Assumptions of ageing mechanically repairable machines

1. The machine undergoes PM at successive times, $x_1 + x_2 + \ldots + x_{N-1}$ and is replaced at x_N.
2. The hazard rate $h(t)$ after the kth PM becomes $\lambda_k h(t)$, when it was $h(t)$ and increases with the number of PMs. In other words, the system becomes "better-than-old" after PM and as-good-as-new at replacement.
3. The repairable machine undergoes only minimal repair at failures between PMs and the hazard rate remains unchanged after minimal repair.
4. Preventive maintenance (PM), repair and replacement times are negligible.
5. The hazard rate, $h(t)$ is continuous and strictly increasing.

2.2 Phases of operation for ageing mechanically repairable machines

The working life of the machine is classified into three phases:

Phase 1: Machine is working within the specified age limit with normal degree of deterioration and low cost of maintenance. The failure degree is denoted by $j = 2$.

Phase 2: Machine is working beyond specified age limit due for initial replacement, it is on extended life span and has high degree of deterioration with associated high cost of maintenance. The failure degree is denoted by $j = 3$.

Phase 3: Machine is working beyond *phase 2*, it is on further extended life span, it also exhibits higher degree of deterioration due to aging and usage, and has associated higher cost of maintenance. The failure degree is denoted by $j = 4$.

2.3 The proposed Geometric hazard-based PM and Replacement (GHBPMAR) model

Let the GP model given by $H_j(t) = \lambda^{j-1}t$; $j \geq 2$, where λ is the GP parameter. In this work, we let λ^{j-1} be the hazard rate adjustment factor for repairable machines with higher degrees of deterioration due to aging and usage beyond age limit at various levels of operation, where $j - 1$

denotes a measure of failure degree of repairable machines due to aging and usage at extended lifespan. Hence, the geometric hazard model is given in (1) as;

$$\lambda_k{}^{j-1} h(t) \tag{1}$$

Equation 1 shows that the hazard rate, $h(t)$ becomes $\lambda_k{}^{j-1} h(t)$ after PM, according to [6], where $\lambda \geq 1, t \in (0, t_1), x \in (0, t_2 - t_1)$. Also, t is the time point and x is the interval between two successive PM actions. The geometric hazard rate adjustment factor, $\lambda_k{}^{j-1}$ in (2) according to [6] is defined as;

$$\lambda_k^{j-1} = \left(\frac{6k+1}{5k+1}\right)^{j-1}, k = 1,2,3, \ldots, j \geq 2 \tag{2}$$

Accordingly, the modified cost function from [4] for the geometric hazard model is given in (3) as;

$$C = C(x_1, x_2 x_3, \ldots x_N) = \frac{C_m \sum_{k=1}^{N} \lambda_k^{j-1} H(x_k) + (N-1)C_p + C_r}{\sum_{k=1}^{N} x_k} \tag{3}$$

where C_m, C_p and C_r are the cost parameters respectively due to minimal repairs, preventive maintenance and corrective maintenance.

2.4 Minimization of the cost function of ageing repairable machines

By applying the minimization condition to (3), we have;

$$\frac{\partial C}{\partial x_k} = \frac{[\sum_{k=1}^{N} x_k] C_m \lambda_k^{j-1} h(x_k) - \left[(N-1)C_p + C_r + C_m \sum_{k=1}^{N} \lambda_k^{j-1} H(x_k)\right]}{[\sum_{k=1}^{N} x_k]^2} = 0$$

$$C_m \lambda_k^{j-1} h(x_k) - C = 0$$

$$C = C_m \lambda_k^{j-1} h(x_k) \tag{4}$$

From (3) we have that;

$$C \sum_{k=1}^{N} x_k = C_m \sum_{k=1}^{N} \lambda_k^{j-1} H(x_k) + (N-1)C_p + C_r \tag{5}$$

Substituting (4) into (5), we have;

$$C_m \lambda_k^{j-1} h(x_k) \sum_{k=1}^{N} x_k = C_m \sum_{k=1}^{N} \lambda_k^{j-1} H(x_k) + (N-1)C_p + C_r$$

By utilizing the hazard and cumulative hazard functions of Weibull failure function, respectively given as;

$$h(x_k) = \alpha\beta^\alpha x_k^{\alpha-1}, H(x_k) = \beta^\alpha x_k^\alpha$$

$$\lambda^{j-1} \alpha\beta^\alpha x_k^{\alpha-1} \sum_{k=1}^{N} x_k - \sum_{k=1}^{N} \lambda^{j-1} \alpha\beta^\alpha x_k^{\alpha-1} = \frac{(N-1)C_p + C_r}{C_m} \tag{6}$$

Let $h(x_k) = \theta$, then, $\alpha\beta^\alpha x_k^{\alpha-1} = \theta$ and;

$$x_k = \left[\frac{\theta}{\alpha\beta^\alpha}\right]^{\frac{1}{\alpha-1}} \tag{7}$$

Substitute (7) into (6);

$$x_k = \left[\frac{\theta}{\alpha\beta^\alpha}\right]^{\frac{1}{\alpha-1}} \quad \lambda_k^{j-1} \alpha\beta^\alpha \left[\frac{\theta}{\alpha\beta^\alpha}\right] \sum_{k=1}^{N} \left[\frac{\theta}{\alpha\beta^\alpha}\right]^{\frac{1}{\alpha-1}} - \sum_{k=1}^{N} \lambda_k^{j-1} \alpha\beta^\alpha \left[\frac{\theta}{\alpha\beta^\alpha}\right]^{\frac{\alpha}{\alpha-1}} = \frac{(N-1)C_p + C_r}{C_m}$$

$$\lambda_k^{j-1}\alpha\beta^\alpha\sum_{k=1}^{N}\left[\frac{\theta^{\frac{\alpha}{\alpha-1}}}{(\alpha\beta^\alpha)^{\frac{1}{\alpha-1}}}\right]-\sum_{k=1}^{N}\lambda_k^{j-1}\alpha\beta^\alpha\left[\frac{\theta^{\frac{\alpha}{\alpha-1}}}{(\alpha\beta^\alpha)^{\frac{1}{\alpha-1}}}\right]=\frac{(N-1)C_p+C_r}{C_m}$$

$$\theta=\left\{\frac{(N-1)C_p+C_r}{C_m\left[\frac{1}{\alpha\beta^\alpha}\right]^{\frac{1}{\alpha-1}}\left\{N\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}}\right\}^{\frac{\alpha}{\alpha-1}} \tag{8}$$

We seek the optimal number, N^* of PM by minimizing (9);

$$B(N)=\frac{(N-1)C_p+C_r}{\left\{N\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}} \tag{9}$$

A necessary condition that there exist a finite N^* minimizing $B(\mathrm{N}^*)$ is that N^*satisfies the inequality: $D(\mathrm{N}^*)=B(\mathrm{N}^*+1)\geq B(\mathrm{N}^*)$ and $D(\mathrm{N}^*)=B(\mathrm{N}^*)<B(\mathrm{N}^*-1)$. We show that $D(\mathrm{N}^*)=B(\mathrm{N}^*+1)\geq B(\mathrm{N}^*)$ as follows:

$$D(\mathrm{N}^*)=\frac{\mathrm{N}^*C_p+C_r}{\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{\mathrm{N}+1}\lambda_k^{j-1}\right\}}\geq\frac{(\mathrm{N}^*-1)C_p+C_r}{\left\{\mathrm{N}^*\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{\mathrm{N}}\lambda_k^{j-1}\right\}};\ k=1,2,\ldots$$

$$=\left\{\mathrm{N}^*\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}\left[\mathrm{N}^*C_p+C_r\right]\geq\left[(\mathrm{N}^*-1)C_p+C_r\right]\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N+1}\lambda_k^{j-1}\right\}$$

$$=\mathrm{N}^*C_p\left\{\mathrm{N}^*\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}+C_r\left\{\mathrm{N}^*\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}\leq(\mathrm{N}^*-1)C_p\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N+1}\lambda_k^{j-1}\right\}$$
$$+C_r\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N+1}\lambda_k^{j-1}\right\}$$

$$D(\mathrm{N}^*)=\frac{\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N+1}\lambda_k^{j-1}\right\}}{\left\{\mathrm{N}^*\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N}\lambda_k^{j-1}\right\}-\left\{(\mathrm{N}^*+1)\lambda_k^{j-1}-\frac{1}{\alpha}\sum_{k=1}^{N+1}\lambda_k^{j-1}\right\}}\geq\frac{C_r}{C_p} \tag{10}$$

Similarly, $$D(\mathrm{N}^*)=B(\mathrm{N}^*)<B(\mathrm{N}^*-1). \tag{11}$$

III. Results

We seek to implement the model in section II by obtaining its parameters such as the optimal number of PM/replacement points and cost ratios in Table 1 which are required for generating the optimal number of PM/replacement schedule.

Table 1: *Cost ratio,* c_r/c_p *for values of* N^**for phases 1, 2 and 3*

Phase	CR	N^*=	1	3	5	7	9	11	13
1:j =2	c_r/c_p		3	10	20	40	80	10 0	110
2:j =3	c_r/c_p		4	20	40	70	300	580	800
3:j = 4	c_r/c_p		6	100	800	1000	3000	8000	15000

The optimal number of PM/replacement points, N^* in Table 1, row 1 were computed from (9) and chosen to satisfy the inequalities in (10) and (11). Also, the cost ratios (CR): $\frac{C_r}{C_p}$ in rows 2, 3 and 4 for phases 1, 2 and 3 were obtained from (10) with the corresponding values of N^* showing the cost implication of succeeding PM/replacement actions associated with prolong usage of mechanically repairable machines. These results in Table 1 are utilized in (7) to obtain the sequential PM/replacement schedule for the machine at the three phases of operation under consideration and the results are shown in Tables 2, 3 and 4 for phases 1, 2 and 3 respectively.

Table 2: *Optimal PM/replacement schedule for the 8HP-PML Gold Engine cassava grinding machine for phase 1 (j = 2)*

N^*	1	3	5	7	9	11	13
c_r/c_p	3	10	20	40	80	100	110
x_1	0.5114	0.7845	0.8188	0.9582	1.1938	1.2501	1.3164
x_2		0.5462	0.4711	0.6389	0.7711	0.9836	1.0123
x_3		0.5461	0.3798	0.5049	0.6064	0.7664	0.9592
x_4			0.3134	0.4093	0.4868	0.6129	0.7642
x_5			0.3010	0.3619	0.4280	0.5361	0.6661
x_6				0.3242	0.3811	0.4750	0.5884
x_7				0.3002	0.3415	0.4237	0.5231
x_8					0.3084	0.3807	0.4689
x_9					0.2796	0.3436	0.4212
x_{10}						0.2615	0.3815
x_{11}						0.3829	0.3457
x_{12}							0.3500
x_{13}							0.3873

Table 3: *Optimal PM/replacement schedule for the 8HP-PML Gold Engine cassava grinding machine for phase 2 (j = 3)*

N^*	1	3	5	7	9	11	13
c_r/c_p	4	20	40	70	300	580	800
x_1	0.6771	0.8345	1.3188	1.5528	1.9761	2.5011	3.4164
x_2		0.6543	0.9210	1.2377	1.7055	1.8836	2.2123
x_3		0.6012	0.8221	1.004	1.6650	1.7244	1.9322
x_4			0.7002	0.8753	1.5454	1.6133	0.9632
x_5			0.6944	0.7765	1.4266	1.5221	0.9433
x_6				0.6596	1.2344	1.4550	0.8944
x_7				0.6272	0.9884	1.4007	0.7432
x_8					0.8323	0.9817	0.6501
x_9					0.7788	0.8426	0.4321
x_{10}						0.8765	0.32109
x_{11}						0.7829	0.1345
x_{12}							0.0940
x_{13}							0.0873

Table 4: *Optimal PM/replacement schedule for the 8HP-PML Gold Engine cassava grinding machine for phase 3 (j = 4)*

N^*	1	3	5	7	9	11	13
c_r/c_p	6	100	800	1000	3000	8000	15000
x_1	0.8771	2.5333	4.2322	6.2329	11.6054	25.7512	50.4597
x_2		0.9121	2.6762	1.2377	9.1555	10.8513	20.1123
x_3		0.9022	1.0221	0.9104	1.7452	5.4544	12.7722
x_4			0.8002	0.6753	1.2311	2.9123	8.9193
x_5			0.7932	0.3455	0.4239	1.0221	6.4554
x_6				0.2546	0.2211	0.8551	3.7444
x_7				0.1828	0.1742	0.7007	2.0127
x_8					0.0143	0.5812	1.251
x_9					0.0119	0.2421	0.7329
x_{10}						0.1065	0.5210
x_{11}						0.00921	0.4344
x_{12}							0.2642
x_{13}							0.1773

3.1 Computation of optimal PM and replacement schedule using the geometric hazard model

The optimal operating time, $x_k; k = 1,2, \dots ,13$ before PM/replacement were computed from (7) for the three phases of operation, viz; *phase 1:* when machine is working within the specified age limit with normal degree of deterioration and low cost of maintenance as shown in Table 2. *Phase 2:* when machine is working beyond specified age limit due for initial replacement. It has extended life span and high degree of deterioration with high cost of maintenance as shown in Table 3. Finally, *phase 3:* when machine is working beyond *phase 2*. It is characterized by further extended lifespan; it has higher degree of deterioration due to aging and prolong usage and it also has associated higher cost of maintenance as shown in Table 4. Row 1 in Tables 2, 3 and 4 contain the optimal number of PM/replacement points with the associated cost ratios in row 2, while the other rows 3 – 15 are the generated operating time, $x_k; k = 1,2, \dots 13$ for the machine at different cost ratios. Each value typifies a working cycle of the machine followed by necessary PM activity and the last one is the replacement point.

3.2 Application of the GHBPMAR Model to obtain optimal PM and replacement schedule for gold cassava grinding machine

In this section, we consider the application of the derived model in section II by adapting the geometric hazard model to the maintenance of a locally manufactured cassava grinding machine which was considered in [11] to provide its maintenance schedules using sequential imperfect PM and replacement model. The cost implication of extended usage of the machine at phases 2 and 3 which exhibits higher degrees of deterioration due to aging and prolong usage are further examined in this work. The failure distribution of the machine was shown by [11] to follow a 2-parameter Weibull distribution with hazard and cumulative hazard function respectively given as; $h(t) = \alpha\beta^{\alpha}(t)^{\alpha-1}, \alpha > 1, \beta > 0$ and $\mathrm{H}(t) = \beta^{\alpha}t^{\alpha}$. The estimate of the shape and scale parameters from failure data of the machine are; $\alpha = 1.3$ and $\beta = 1386$ respectively. Also, the cost ratios (CR); $\frac{C_m}{C_p} =$

3 is a fixed cost ratio by industry standard, based on the assumption that repair time is negligible, while $\frac{C_r}{C_p}$ = 3, 10, 20, 40, 80, 100, 110 for phase 1 in Table 2, for example, are computed from the optimal values of N^* obtained in (9) such that they satisfy (10) and (11). Finally, the optimal PM schedule, x_k for $k = 1,2, \dots 13$ and $j = 2,3,4$ are obtained from (7) as shown in Tables 2 - 4.

IV. Discussion

The obtained optimal preventive and replacement schedules for the cassava grinding machine in phases 1, 2 and 3 are presented in Tables 2, 3 and 4. For instance, column 2 in Table 2 with N^*=1 shows that only one cycle of operation is required with cost ratio of 3 and $x_1 = 0.5114$. This implies that the cassava grinding machine is operated for about x_1 =51hrs, (in '000) and a replacement maintenance is performed thereafter. Further use of the grinding machine requires a move to column 3 with N^*= 3 and cost ratio of 10 where the machine is operated for x_1 = 78hrs before the first PM and then for x_2 = 55hrs before the second PM and can be further operated for x_3 = 30hrs before replacement. This is because this column provides for only two PMs and the third cycle is the replacement point. In the same manner, the entries in columns 4 – 8 show the working of the machine under different N^* with their associated cost ratios. It is observed that the operational time in each column is decreasing which depicts a typical hazard PM/replacement model for repairable systems with increasing hazard rate.

Similarly, optimal PM and replacement schedule for phases 2 and 3, when j = 3 and 4 were obtained in Tables 3 and 4 having higher PM cost implication as shown in the cost ratios in row 2 of Tables 3 and 4.

We observed an increasing cost ratio with frequent PM actions due to the extended usage of the machine and a decreasing trend of successive operating time, x_k of the machine in phases 2 and 3 as shown in Tables 3 and 4. This is characterized by the geometric process in accordance with [8] and [9] having a non-increasing parameter which underscores its usefulness in describing the stochastic decrease of lifetime of machines after repairs as well as the stochastic increase in failure times after failures. These observed features of the model are also in line with the PM and replacement schedules obtained by [4], [6] and [12] for repairable systems and in [13] under different PM policies.

The geometric hazard-based preventive maintenance model effectively addresses the increasing deterioration rates of aging machinery by using a geometric process (GP) to adjust the hazard rate. A successful application of the model to a cassava grinding machine, reveals that while preventive maintenance temporarily improves operational time, its maintenance costs rise significantly with each phase of the machine's lifespan. The model, which utilizes the Weibull failure distribution, typical of systems with increasing hazard rate provides actionable insights into optimal maintenance scheduling and cost management. Its effectiveness in predicting maintenance needs and associated costs demonstrates its value, particularly in resource-constrained environment where managing machinery performance and expenses is crucial. The model's validation is demonstrated through its application to a real-world case of cassava grinding machine, where it successfully generates optimal maintenance schedules with its associated cost implications. The results substantiate the model's effectiveness in extending machine lifespan and managing maintenance costs with high cost and frequent preventive maintenance.

In summary, a geometric hazard-based preventive maintenance model for aging mechanically repairable machines was developed in this work. The utilization of geometric process as the hazard rate adjustment factor from [6] was successfully applied to the sequential hazard model of [4] in order to obtain a generalized hazard-based PM and replacement model. Hence, our proposed model is a generalization of the hazard model in [4]. It will be reduced to the hazard model of [4] in phase 1, when j = 2 for the maintenance of non-ageing and low degree deteriorating machines.

Finally, the proposed model was used to obtain the optimal PM/replacement schedule for a cassava grinding machine, characterized by ageing and prolong usage over many years. The results showed that for the three life phases of the machine, the operational times continue to decrease down to the replacement point. It is also observed that the higher the number of operational phases of the machine, the higher the associated costs of maintenance. Hence, an advisory to both design and maintenance engineers to consider this modification for products specification meant for developing countries because of their extended lifespan usage. Also, users of machines in the second and third life phases which is typical in developing countries should be conscious of the implications of the associated high maintenance cost. This is necessary to ensure efficient production management and profitability in business.

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