

ANALYSIS OF THE PERFORMANCE ON SINGLE SERVER BATCH SERVICE MULTIPLE WORKING VACATION QUEUING MODEL WITH COMPULSORY AND EXTENDED REPAIR

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Abstract

In this paper, the concept of single-server multiple-working vacations queuing model with compulsory and extended repair is analyzed. In this model, customers arrive at a service facility and form a queue to be served by a single server. The arrival follows the Poisson distribution, and the service follows the exponential distribution. Batches of customers are served under the General Bulk Service Rule. In GBSR, rather than the individual customer arriving in a queue one by one, the customers arrive in groups or batches. Thus, each batch of services contains a minimum of 'a' units and a maximum of 'b' units of customers. In this study, two types of repairs, that is, compulsory and extended repair, are considered. The steady-state equation, the performance of measures for the system, and particular cases of described model are derived. Finally, in the form of tables and graphs, numerical results have been analyzed.

Keywords: Repair, Batch Service, performance measures, Multiple Working vacation (MWV), queue length

1. INTRODUCTION

Erlang developed queuing theory while working for the Telephone Company to analyze the behavior of telephone traffic and optimize the capacity of telephone exchanges. His work laid the foundation for the study of waiting lines and has since been widely applied in various fields to improve system performance and efficiency. The main objective of queuing theory is to understand and optimize the performance of systems that involve waiting lines. By studying factors such as arrival rates, service rates, queue lengths, and waiting times, queuing theory provides insights into how to improve efficiency and reduce waiting times.

In queuing theory, a vacation queuing model is a type of queuing system where the server may take breaks or go on vacation, when service is not available. This type of model is often used in scenarios where service providers have scheduled breaks, such as in customer service centers, healthcare facilities, or manufacturing processes. Analyzing and optimizing vacation queuing models involves factors of the duration and frequency of vacations, the impact on service during vacation periods, and strategies to minimize the effects of downtime on customer satisfaction. The concept of "multiple working vacations" refers to a scenario where the server in a queuing system takes several breaks or vacations during their work.

The concept of the GBS rule was indeed introduced by Neuts. The GBS rule, which he introduced, is used to analyze queuing systems where customers arrive in batches and are served

as a single entity with a fixed service time for the entire batch. In a batch arrival process, rather than individual customers arriving at a queue one by one, customers arrive in groups or batches.

The breakdown refers to a situation where a service facility or a server becomes unavailable or inoperable, leading to a disruption in the service process. Breakdowns can occur due to various reasons, such as equipment failure, system malfunction, or scheduled maintenance. When a breakdown occurs, it can lead to increase waiting times for entities in the queue and can impact the overall performance of the system. Understanding the behavior of a queuing system under breakdown conditions help to improve system performance.

The repair process refers to the activities involved in restoring a failed or broken component back to a functional state. When a breakdown occurs in a system, the repair process is triggered to address the issue and restore the system to normal operation. Planning and scheduling the repair activities based on factors such as repair priorities, resource availability, and system constraints. The compulsory and extended repair model is a concept within queuing theory that is used to analyze and optimize repair processes in systems where maintenance and repair activities are essential. This model considers two types of repair scenarios: compulsory repairs, which are necessary to restore a failed component to working condition, and extended repairs, which involve more complex or time-consuming maintenance tasks. Overall, the compulsory and extended repair model provides a framework for understanding and managing repair processes in various systems and industries.

This paper analyzes the queuing system that combines multiple working vacations, batch service, a single server, compulsory and extended repair. For this model, we obtained steady state equations, measures of performances, and analyzed the particular cases.

2. REVIEW OF LITERATURE

A single server feedback retrial queuing system with multiple working vacations, balking customer, breakdown and vacation interruption studied by [24]. Using the supplementary variable method they obtained steady state probability generating function for the system size. The queue caused by a lack of servers, (M / G / I) with single working vacation and vacation interruption presented by [14]. The results of that model analysis show that the server is not busy or does not need to add a server. The multiple working vacations queuing-inventory model with compliment analyzed by [19]. Also studied steady state, expected total cost rate and measures of system performance. A discrete-time queuing system with multiple working vacations and reneging analyzed by [22]. For that steady-state probabilities are obtained recursively from the difference equations that represent the model. In addition, they obtain some other performance measures and a cost model is formulated to determine the optimal service rate during working vacation. $M^{[X]}/G(a, b)/1$ queuing system with additional optional service, multiple vacation and setup time analyzed by [2]. Using supplementary variable technique, the probability generating function of the performance measures is developed. The non-Markovian single server batch arrival queuing system of compulsory three stages of services with fourth optional stage service, service interruptions and deterministic server vacations developed by [29]. A non-Markovian batch arrival general bulk service single server queuing system with server breakdown and repair, single vacation and stand-by server discussed by [4]. The probability generating function of queue size at an arbitrary time and some performance measures of the system are derived.

The backup server, breakdown, repair, and vacation queuing system derived by [7]. They also prove decomposition results for the rate matrix and the mean number in the system under some special cases. A renewal input single server batch service queue with impatient customers and multiple working vacations presented by [11]. Applying the embedded Markov chain approach and the displacement operator method, they find the probability distribution of the queue-length. $M^{[X]}/G(a, b)/1$ queuing model with multiple vacation controllable arrival during multiple vacation and two phases of repair with delay analyzed by [3]. Using supplementary variable technique, the probability generating function of the queue size. the modeling the steady-state behavior of a retrial queuing system with phases of service analyzed by [1]. The first

phase of service is essential for all customers, any customer has three options after the completion of the phase, after each phase, the probabilistic failure, delay, repair and vacation are considered. A single server queuing model with setup process, close-down process, and standby server are analyzed by using the Matrix-Analytic method helped to obtain steady-state probability are investigated by [5]. $M^X/G/1$ retrial queue with impatient customers, modified vacation policy and Bernoulli feedback analyzed by [28]. Supplementary variable technique and probability generating function method is used to derive the system size distribution. An algorithm for determining optimal parameters of the queuing system that allows server vacation proposed by [18]. Also they first propose an efficient algorithm to determine steady-state probabilities of the system, and second, a non-linear integer programming problem is formulated to determine optimal system parameters.

M/M/1 retrial queuing model with a waiting server subject to breakdown and repair under working vacation, vacation interruption presented by [12]. Steady-state solutions are obtained using the probability generating function technique. A mathematical queuing model for multi-phase essential repair (MPER) systems with unreliable servers developed by [26]. Moreover, the service of the customer is interjected due to unavailability of the server. Also time-dependent probabilities are calculated by Runge-Kutta. The M/M/2 machine repair problem with operating machines and two removable servers are investigated described by [17]. The failed machines are repaired under the multiple working vacations and triadic policy. A retrial queuing system with working vacation, interruption, feedback, and setup time with the perfect repair studied by [13]. Also to save power, the concept of setup time is included in that model. The Bernoulli vacation on each service includes additional service after breakdowns and repairs by the method of supplementary variable techniques analyzed by [25]. To implement these concepts we build a mathematical model using the complementary variable method and obtained a function that gives the probabilities for the number of customers in the system when the server is free or on repair or on vacation.

Open network queuing system problems with breakdowns and its alternative repair strategies proposed by [9]. Queuing model with working vacation, balking, unreliable server with multi-phase repair discussed by [16]. The steady state queue size distribution and mean operational system metrics have been obtained by using probability generating function and supplementary variable methodology. Also cost optimization using particle swarm optimization technique have been drawn to explore the practical utility of the model. An unreliable bulk arrival single server queue rendering two-heterogeneous optional repeated service with delayed repair, under Bernoulli Vacation Schedule and N-policy analyzed by [6].

The queuing system with phase type vacations, breakdowns, and repairs developed by [8]. In the phase type they consider, first the service is restored to normalcy immediately after the server becomes idle or a fixed number of services is offered. In the second one, the restoration takes a random time that is modeled using a phase type distribution. The single server queuing model, working vacation, working breakdown and two-phase repair examined by [27]. They considered the repair process in two phases to recover the server from breakdown to working state with improved service rate phase-wise. Matrix geometric solution method is used to obtain stationary probability vectors. Single server multiple working vacation queuing system under breakdown, balking and encouraged arrival, and heterogeneous arrival analyzed by [20, 23, 21]. Batch arrival queuing system with a random vacation policy and optional second service. They consider the servers are unreliable and can be repaired immediately demonstrated by [15]. The queuing model for machine repair problem with two failure modes and working vacation analyzed by [30]. The steady-state system-size distribution is derived via the matrix analysis method and matrix decomposition. The repair personnel of the enterprise using the theory of queuing was developed by [10]. The proposed methodology can be applied to evaluate the effectiveness of production equipment repair operations.

3. METHODOLOGY

In this paper, $M/M(a, b)/1/MWV$ queuing model with compulsory and extended repair analyzed. Instead of the server being fully idle during the vacation period, server serves at a different rate during multiple working vacations. The service rate varies depending on the arrival state. Customers arrive to the system with parameter λ_v follows Poisson distribution where the server provides service in regular busy period with parameter μ_{rb} and under the multiple working vacations server provides service with parameter μ_{wv} with exponential distribution. In this model batches of customers are served under General Bulk Service Rule. Thus each batch of service contains minimum ' a ' units and maximum ' b ' units. Suppose the customers waiting in the queue is less than ' a ' server begins a vacation random variable V with parameter ξ .

In this model, we consider two types of repairs-compulsory repair ψ and extended repair ϕ . It is also assumed that breakdown happens only in busy period and denoted with the parameter β_v . When a breakdown occurs during busy period, a compulsory repair process begins instantly as aimed to quickly restore the breakdown condition. The immediate repair is essential to minimize disruptions in service and maintain system's performance during peak demand times. Also, in this model the extended repair process is considered in working vacations periods. The repair process during working vacations may be extended for various reasons. This could be due to limited availability of repair resources, or extended maintenance schedules.

Consider the customers in the queue at time t denote as $N_c(t)$, and $L(t)=0, 1$ or 2 according to the server is idle, busy or regular busy on vacation state respectively. Let

$$\begin{aligned} R_n^I(t) &= Pr\{N_c(t) = n, L(t) = 0\} & 0 \leq n \leq a - 1 \\ Q_n^V(t) &= Pr\{N_c(t) = n, L(t) = 1\} & n \geq 0 \\ P_n^B(t) &= Pr\{N_c(t) = n, L(t) = 2\} & n \geq 0 \end{aligned}$$

When the server is idle: $L(t) = 0$, the size of the queue and system are the same.

When the server is busy or in working start during the vacation: $L(t) = 1$ or 2 , denote the total number of customers in the system is the sum of the number of customers in queue and the size of the service batches that contains particular $a \leq x \leq b$ customers.

Probabilities of the steady state are ,

$$Q_n^V = \lim_{t \rightarrow \infty} Q_n^V(t); \quad R_n^I = \lim_{t \rightarrow \infty} R_n^I(t); \quad P_n^B = \lim_{t \rightarrow \infty} P_n^B(t);$$

exist and the Chapman Kolmogrove equations satisfied by them in the steady state are given by,

$$\lambda_v R_0^I = \mu_{rb} P_0^B + \mu_{wv} Q_0^V \tag{1}$$

$$\lambda_v R_n^I = \lambda_v R_{n-1}^I + \mu_{rb} P_n^B + \mu_{wv} Q_n^V; \quad 1 \leq n \leq a - 1 \tag{2}$$

$$(\lambda_v + \mu_{wv} + \phi + \xi) Q_0^V = \lambda_v R_{a-1}^I + \mu_{wv} \sum_{n=a}^b Q_n^V \tag{3}$$

$$(\lambda_v + \mu_{wv} + \phi + \xi) Q_n^V = (\lambda_v + \phi) Q_{n-1}^V + \mu_{wv} Q_{n+b}^V; \quad n \geq 1 \tag{4}$$

$$(\lambda_v + \mu_{rb} + \psi + \beta_v) P_0^B = \mu_{rb} \sum_{n=a}^b P_n^B + \xi Q_0^V \tag{5}$$

$$(\lambda_v + \mu_{rb} + \psi + \beta_v) P_n^B = (\lambda_v + \psi + \beta_v) P_{n-1}^B + \mu_{rb} P_{n+b}^B + \xi Q_n^V; \quad n \geq 1 \tag{6}$$

4. STEADY STATE SOLUTION

To solve the steady state equation, the forward shifting operator E on P_n^B and Q_n^V are introduced then,

$$E(P_n^B) = P_{n+1}^B; \quad E(Q_n^V) = Q_{n+1}^V \quad \text{for } n \geq 0$$

Thus the equation (4) gives homogeneous difference equation as,

$$[\lambda_v + \phi + \mu_{wv}E^{b+1} - (\lambda_v + \phi + \xi + \mu_{wv})E]Q_n^V = 0 \tag{7}$$

The characteristics equation of (7) is obtained as,

$$z(u) = \lambda_v + \phi + \mu_{wv}u^{b+1} - (\lambda_v + \phi + \xi + \mu_{wv})u = 0 \tag{8}$$

by taking $x(u) = (\lambda_v + \phi + \xi + \mu_{wv})u$ and $y(u) = \lambda_v + \phi + \mu_{wv}u^{b+1}$, it is found that $|y(u)| < |x(u)|$ on $|u| = 1$. By Rouché's theorem $z(u)$ has unique root r_v inside the contour $|u| = 1$. Equation (7) has a homogeneous solution as,

$$Q_n^V = r_v^n Q_0^V \tag{9}$$

From equation (6) we get,

$$[(\lambda_v + \psi + \beta_v) + \mu_{rb}E^{b+1} - (\lambda_v + \mu_{rb} + \psi + \beta_v)E]P_n^B = -\xi r_v^{n+1} Q_0^V \tag{10}$$

By applying Rouches theorem to equation (10) as,

$$[(\lambda_v + \psi + \beta_v) + \mu_{rb}E^{b+1} - (\lambda_v + \mu_{rb} + \psi + \beta_v)E]P_n^B = 0$$

The above equation has unique root r with $|r| < 1$. Also equation (10) gives a non-homogeneous solution as,

$$P_n^B = \left[Zr^n - \frac{\xi r_v^{n+1}}{[(\lambda_v + \psi + \beta_v) + \mu_{rb}r_v^{b+1} - (\lambda_v + \mu_{rb} + \psi + \beta_v)r_v]} \right] Q_0^V \tag{11}$$

$$P_n^B = (Zr^n + Z^*r_v^n)Q_0^V \tag{12}$$

where

$$Z^* = \frac{\xi r_v}{[\lambda_v(r_v - 1) + \psi(r_v - 1) + \beta_v(r_v - 1) + \mu_{rb}r_v(1 - r_v^b)]} \tag{13}$$

The expression for R_n^I is obtained by adding equations (1) & (2) and substitute P_n^B and Q_n^V values,

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{Z(1 - r^{n+1})}{(1 - r)} + \frac{Z^*(1 - r_v^{n+1})}{(1 - r_v)} \right) + \frac{\mu_{wv}}{\lambda_v} \frac{(1 - r_v^{n+1})}{(1 - r_v)} \right] Q_0^V$$

Now to calculate Z , consider equation (5) and substitute P_n^B and Q_n^V value,

$$Z \left[(\lambda_v + \mu_{rb} + \psi + \beta_v) - \frac{\mu_{rb}(r^a - r^{b+1})}{(1 - r)} \right] = \xi - Z^* \left((\lambda_v + \mu_{rb} + \psi + \beta_v) - \frac{\mu_{rb}(r_v^a - r_v^{b+1})}{(1 - r_v)} \right) \tag{14}$$

the above expression can be simplified as,

$$\frac{Z\mu_{rb}(1 - r^a)}{(1 - r)} = \frac{\xi}{(1 - r_v)} - \frac{Z^*\mu_{rb}(1 - r_v^a)}{(1 - r_v)} \tag{15}$$

Hence the probability queue size of the steady-state equation in terms of Q_0^V are obtained,

$$Q_n^V = (r_v^n)Q_0^V \quad n \geq 0 \tag{16}$$

$$P_n^B = (Zr^n + Z^*r_v^n)Q_0^V \quad n \geq 0 \tag{17}$$

where

$$Z = \frac{(1 - r)}{\mu_{rb}(1 - r^a)} \left[\frac{\xi}{(1 - r_v)} - \frac{Z^*\mu_{rb}(1 - r_v^a)}{(1 - r_v)} \right] \tag{18}$$

$$Z^* = \frac{\xi r_v}{[\lambda_v(r_v - 1) + \psi(r_v - 1) + \beta_v(r_v - 1) + \mu_{rb}r_v(1 - r_v^b)]} \tag{19}$$

and

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{Z(1 - r^{n+1})}{(1 - r)} + \frac{Z^*(1 - r_v^{n+1})}{(1 - r_v)} \right) + \frac{\mu_{wv}}{\lambda_v} \frac{(1 - r_v^{n+1})}{(1 - r_v)} \right] Q_0^V \tag{20}$$

by using normalizing condition and calculated the value of Q_0^V

$$\sum_{n=0}^{\infty} Q_n^V + \sum_{n=0}^{\infty} P_n^B + \sum_{n=0}^{a-1} R_n^I = 1$$

By substituting P_n^B , Q_n^V and R_n^I is observe that,

$$\sum_{n=0}^{\infty} r_v^n Q_0^V + \sum_{n=0}^{\infty} (Zr^n + Z^*r_v^n)Q_0^V + \sum_{n=0}^{a-1} \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{Z(1 - r^{n+1})}{(1 - r)} + \frac{Z^*(1 - r_v^{n+1})}{(1 - r_v)} \right) + \frac{\mu_{wv}}{\lambda_v} \frac{(1 - r_v^{n+1})}{(1 - r_v)} \right] Q_0^V = 1$$

Hence $(Q_0^V)^{-1}$, is given by,

$$(Q_0^V)^{-1} = \omega(r_v, \mu_{wv}) + Z\omega(r, \mu_{rb}) + Z^*\omega(r_v, \mu_{rb}) \tag{21}$$

where

$$\omega(x, y) = \frac{1}{(1 - x)} \left(1 + \frac{y}{\lambda_v} \left(c - \frac{x(1 - x^a)}{(1 - x)} \right) \right)$$

5. PERFORMANCE MEASURES

In this section, we derive measures of performances for multiple working vacations model with compulsory and extended repair, the expected queue length, expected waiting time of the queue, expected system length and expected waiting time of the queue.

5.1. Mean queue length

The expected queue length of multiple working vacations model with reverse balking is given by;

$$L_q = \sum_{n=1}^{\infty} n(Q_n^V + P_n^B) + \sum_{n=1}^{a-1} nR_n^I$$

By substituting P_n^B , Q_n^V and R_n^I we observe that,

$$L_q = \sum_{n=1}^{\infty} n(r_v^n Q_0^V) + \sum_{n=1}^{\infty} n(Zr^n + Z^*r_v^n)Q_0^V + \sum_{n=1}^{a-1} n \left[\frac{\mu_{rb}}{\lambda_v} \left(\frac{Z(1 - r^{n+1})}{(1 - r)} + \frac{Z^*(1 - r_v^{n+1})}{(1 - r_v)} \right) + \frac{\mu_{wv}}{\lambda_v} \frac{(1 - r_v^{n+1})}{(1 - r_v)} \right] Q_0^V$$

$$L_q = Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) \tag{22}$$

where

$$\omega^*(x, y) = \frac{x}{(1 - x)^2} + \frac{y}{(\lambda_v)(1 - x)} \left[\frac{a(a - 1)}{2} + \frac{ax^{a+1}(1 - x) - x^2(1 - x^a)}{(1 - x)^2} \right]$$

and Z & Z^* are given by equations (18) & (19).

5.2. Mean System Length

The expected system length for the model,

$$L_s = L_q + \rho$$

$$L_s = \{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv})\} + \rho$$

where

$$\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{(\lambda_v)(1-x)} \left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right]$$

5.3. Mean Waiting Time of the Queue

The expected waiting time of the queue of our model is given by,

$$W_q = \frac{L_q}{\lambda_v}$$

$$\text{i.e., } W_q = \frac{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv})}{\lambda_v}$$

where

$$\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{(\lambda_v)(1-x)} \left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right]$$

5.4. Mean Waiting Time of the System

The expected waiting time of the system of above specified model,

$$W_s = \frac{L_s}{\lambda_v}$$

$$W_s = \frac{\{Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv})\} + \rho}{\lambda_v}$$

where

$$\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{(\lambda_v)(1-x)} \left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right]$$

If $Pr_{(wv)}$, $Pr_{(busy)}$ and $Pr_{(idle)}$ denote the probability that the server in idle, regular busy and busy vacation period then

$$Pr_{(idle)} = \sum_{n=0}^{a-1} R_n^I \tag{23}$$

where the R_n^I is given by equation (19).

$$Pr_{(busy)} = \sum_{n=0}^{\infty} P_n^B = \left(\frac{Z}{(1-r)} + \frac{Z^*}{(1-r_v)} \right) Q_0^V \tag{24}$$

$$Pr_{(wv)} = \sum_{n=0}^{\infty} Q_n^V = \frac{Q_0^V}{(1-r_v)} \tag{25}$$

6. PARTICULAR CASES

6.1. Classical $M/M(a, b)/1/MWV$ model

By letting $\psi = \phi = 0$ in equations (16) to (22) and we obtain,

$$Q_n^V = (r_v^n)Q_0^V \quad n \geq 0$$

$$P_n^B = (Zr^n + Z^*r_v^n)Q_0^V \quad n \geq 0$$

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v} (Zg_n(r) + Z^*g_n(r_v) + g_n(r_v)) \right] Q_0^V \quad 0 \leq n \leq a - 1$$

where

$$Z = \frac{(1-r)}{\mu_{rb}(1-r^a)} \left[\frac{\xi}{(1-r_v)} - \frac{Z^*\mu_{rb}(1-r_v^a)}{(1-r_v)} \right]$$

$$Z^* = \frac{\xi r_v}{\lambda_v(r_v - 1) + \mu_{rb}r_v(1-r_v^b)}$$

Further we observe that L_q of multiple working vacations queuing model with compulsory and extended repair by $\psi = \phi = 0$ we have;

$$L_q = Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv})$$

where

$$\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)} \left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right]$$

Thus, observed that our specified model coincides with the $M/M(a, b)/1/MWV$ queuing model analyzed by Julia Rose Mary and Aftab Begum (2011).

7. NUMERICAL ANALYSIS

In the numerical section, we consider various parameters like the the arrival rate (λ_v), regular service rate (μ_{rb}), the service rate during multiple working vacations (μ_{wv}), vacation parameter (ξ) & the compulsory and extended repair rate (ψ & ϕ). With the aid of the above parameters numerical analysis is carried out to analyze the performance measures of the specified model. For that purpose, consider the (7) & (10), where r_v and r are the roots of the following characteristic equations,

$$[\lambda_v + \phi + \mu_{wv}E^{b+1} - (\lambda_v + \phi + \xi + \mu_{wv})E] = 0$$

$$[(\lambda_v + \psi + \beta_v) + \mu_{rb}E^{b+1} - (\lambda_v + \mu_{rb} + \psi + \beta_v)E] = 0$$

respectively, and that lie in the interval $[0,1]$.

The system performance measures and probabilities of each state is calculated and tabulated below. For the different values for $\lambda_v, \mu_{rb}, \mu_{wv}, \xi, \psi$ and ϕ . Table 1 to Table 3 show various performance measures and Table 4 shows the probabilities of the idle state, the working vacation state and busy state.

Table 1: $L_q, L_s, W_q,$ and W_s with respect to ξ and μ_{wv}

μ_{wv}	r_v	ξ	L_q	L_s	W_q	W_s
0.05	0.9985	0.01	664.9178	665.2881	132.9836	133.0576
	0.9961	0.03	254.9934	255.3638	50.9987	51.0728
	0.9938	0.05	160.2198	160.5902	32.0440	32.1180
	0.9915	0.07	116.9284	117.2988	23.3857	23.4598
	0.9892	0.09	92.2283	92.5987	18.4457	18.5197
0.10	0.9981	0.01	524.6106	524.9806	104.9220	104.9961
	0.9956	0.03	225.8886	226.2590	45.1777	45.2518
	0.9934	0.05	150.4587	150.8290	30.0917	30.1658
	0.9908	0.07	108.0308	108.4011	21.6062	21.6802
	0.9881	0.09	83.7757	84.1461	16.7551	16.8292
0.15	0.9978	0.01	452.8352	453.2055	90.5670	90.6411
	0.9951	0.03	202.7179	203.0883	40.5436	40.6177
	0.9927	0.05	135.9779	136.3482	27.1956	27.2696
	0.9894	0.07	93.8078	94.1781	18.7616	18.8356
	0.9871	0.09	77.3475	77.7179	15.4695	15.5436
0.20	0.9967	0.01	301.4009	301.7713	60.2802	60.3543
	0.9945	0.03	180.4764	180.8467	36.0953	36.1693
	0.9919	0.05	122.4987	122.8690	24.4997	24.5738
	0.9886	0.07	87.2368	87.6072	17.4474	17.5214
	0.9856	0.09	69.4125	69.7829	13.8825	13.9566

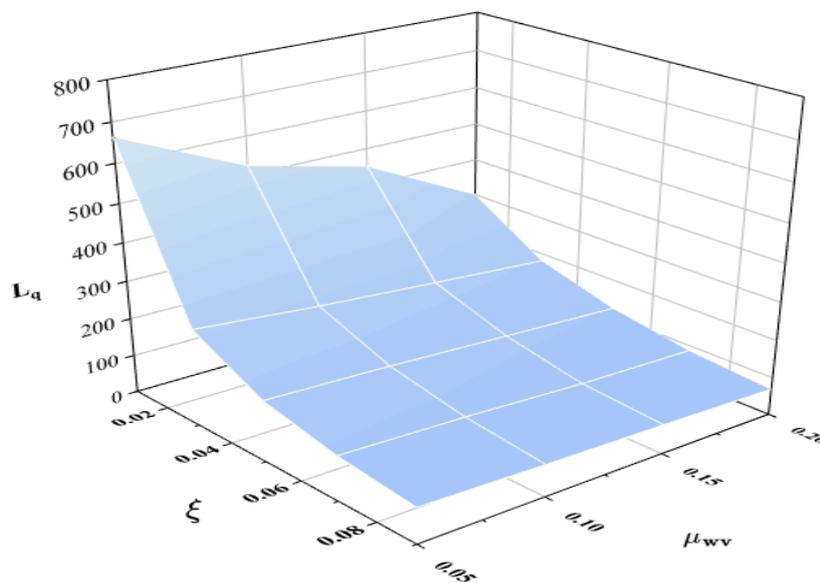


Figure 1: Expected length of the queue with respect to ξ and μ_{wv}

From Table (1) and Figure (1), we consider the parameters for arrival rate as $\lambda_v = 5$, service rate during regular busy period $\mu_{rb} = 0.9$, compulsory and extended repair rate $\psi = 4$ & $\phi = 4.5$, and then by varying vacation parameter ξ & service rate during vacation period μ_{wv} . Using these parameters, we calculated various measures of performances like expected queue length, & system length and their waiting time also. For a certain value for μ_{wv} (i.e $\mu_{wv} = 0.05$) noted that as vacation parameter (ξ) increases, L_s, L_q, W_s & W_q decrease gradually shown in Table (1).

From Figure (1) we observe that, the service rate during vacation period (μ_{wv}) and vacation

parameter (ξ) increases the length of the queue decreases, i.e., $\mu_{wv} = 0.05$ and $\xi = 0.01$ the length of the queue reaches the maximum value (664.9178) and $\mu_{wv} = 0.20$ & $\xi = 0.09$ length of the queue reaches the minimum value (69.4125).

Table 2: L_q, L_s, W_q, W_s with respect to λ_v and μ_{wv}

μ_{wv}	r_v	λ_v	L_q	L_s	W_q	W_s
0.05	0.9985	6.0	665.9296	666.3741	110.9883	111.0623
	0.9961	5.8	255.5344	255.9640	44.0577	44.1317
	0.9938	5.6	160.3357	160.7505	28.6314	28.7054
	0.9915	5.4	116.6397	117.0397	21.6000	21.6740
	0.9892	5.2	91.5454	91.9306	17.6049	17.6790
0.10	0.9981	6.0	525.5545	525.9989	87.5924	87.6665
	0.9956	5.8	226.3512	226.7808	39.0261	39.1001
	0.9934	5.6	150.4929	150.9077	26.8737	26.9478
	0.9908	5.4	107.6203	108.0203	19.9297	20.0038
	0.9881	5.2	82.9210	83.3062	15.9464	16.0204
0.15	0.9978	6.0	453.7440	454.1884	75.6240	75.6981
	0.9951	5.8	203.1062	203.5358	35.0183	35.0924
	0.9927	5.6	135.9050	136.3198	24.2688	24.3428
	0.9894	5.4	93.2164	93.6164	17.2623	17.3364
	0.9871	5.2	76.3332	76.7183	14.6795	14.7535
0.20	0.9967	6.0	302.1914	302.6359	50.3652	50.4393
	0.9945	5.8	180.7857	181.2153	31.1699	31.2440
	0.9919	5.6	122.3141	122.7289	21.8418	21.9159
	0.9886	5.4	86.5216	86.9216	16.0225	16.0966
	0.9856	5.2	68.1961	68.5813	13.1146	13.1887

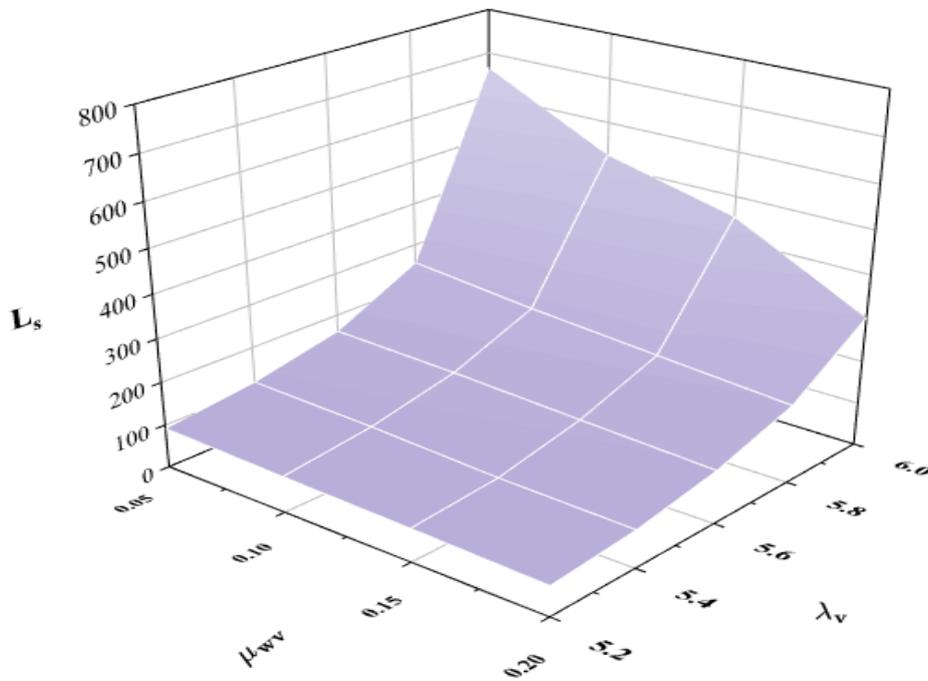


Figure 2: Expected length of the system with respect to λ_v and μ_{wv}

For the fixed values for the parameters $\mu_{rb} = 0.9$, vacation parameter $\zeta = 0.01$, $\psi = 4$ & $\phi = 4.5$, and by varying service rate during vacation period (μ_{wv}) with respect to arrival rate (λ_v) we calculated other characteristics of the model shown in Table (2) & Figure (2). We consider a fixed value for μ_{wv} (i.e. $\mu_{wv} = 0.05$) and by varying the arrival rate (λ_v), we notice that as the arrival rate increase, then the various operating characteristics also increase. Also, for a fixed value for arrival rate λ_v (i.e. $\lambda_v = 6$) and by varying service rate of working vacation (μ_{wv}), we observe that μ_{wv} increases, the operating characteristics are decreasing. For example, $\lambda_v = 6$, $\mu_{wv} = 0.05$ the characteristic values reached their highest point ($L_q = 665.93$, $L_s = 666.37$, $W_q = 110.99$ & $W_s = 111.06$) and $\lambda_v = 6$, $\mu_{wv} = 0.20$ the characteristic values reached lowest point ($L_q = 302.19$, $L_s = 302.64$, $W_q = 50.37$ & $W_s = 50.44$)

Table 3: L_q, L_s, W_q, W_s with respect to ψ and μ_{wv}

μ_{wv}	r_v	ψ	L_q	L_s	W_q	W_s
0.05	0.9975	4.1	398.0205	398.3909	79.6041	79.6782
	0.9949	4.3	194.7185	195.0889	38.9437	39.0178
	0.9927	4.5	135.9419	136.3123	27.1884	27.2625
	0.9903	4.7	102.2985	102.6689	20.4597	20.5338
	0.9879	4.9	82.0785	82.4488	16.4157	16.4898
0.10	0.9972	4.1	355.1622	355.5326	71.0324	71.1065
	0.9943	4.3	174.0373	174.4077	34.8075	34.8815
	0.9920	4.5	123.8886	124.2590	24.7777	24.8518
	0.9895	4.7	94.3656	94.7360	18.8731	18.9472
	0.9870	4.9	76.2777	76.6480	15.2555	15.3296
0.15	0.9969	4.1	320.5837	320.9541	64.1167	64.1908
	0.9938	4.3	159.8289	160.1992	31.9658	32.0398
	0.9912	4.5	112.4559	112.8263	22.4912	22.5653
	0.9882	4.7	83.8072	84.1776	16.7614	16.8355
	0.9855	4.9	68.2585	68.6289	13.6517	13.7258
0.20	0.9959	4.1	241.9815	242.3518	48.3963	48.4704
	0.9934	4.3	149.9802	150.3506	29.9960	30.0701
	0.9901	4.5	99.7657	100.1360	19.9531	20.0272
	0.9871	4.7	76.5057	76.8761	15.3011	15.3752
	0.9835	4.9	59.8556	60.2260	11.9711	12.0452

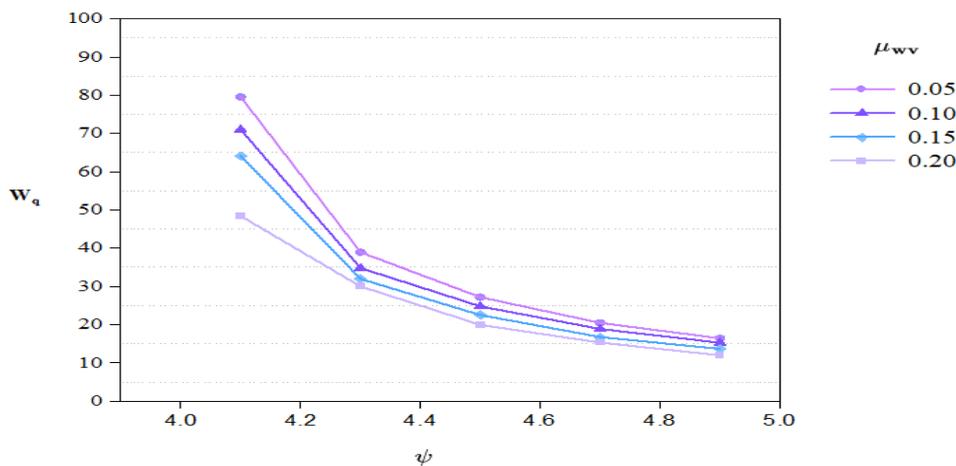


Figure 3: Expected waiting time of the queue with respect to ψ and μ_{wv}

Various performance measures of the compulsory and extended repair model are calculated and tabulated in Table (3). For that, we consider the values for $\lambda_v = 5$, $\mu_{rb} = 0.9$, $\zeta = 0.01$, & $\phi = 4.5$ and different values for μ_{wv} & ψ .

Table (3), shows that for a desired value for μ_{wv} (i.e. $\mu_{wv} = 0.05$) and by different values for (increasing value) compulsory repair rate (ψ), the performance measures of the model L_s , L_q , W_s & W_q are decreasing. As μ_{wv} increases, the system operators are decreasing for a particular values for ψ . Figure (3) explains that as the expected waiting time of the queue with parameters like μ_{wv} & ψ values increase, the waiting time of the queue is decreasing step by step. That shows as the compulsory repair rate increases then waiting time of the queue decreases significantly.

Table 4: $Pr(busy)$, $Pr(wv)$, and $Pr(idle)$ with respect to ζ and μ_{wv}

μ_{wv}	r_v	ζ	$Pr(busy)$	$Pr(wv)$	$Pr(idle)$
0.05	0.9985	0.01	0.6325	0.3653	0.0022
	0.9975	0.02	0.6780	0.3182	0.0039
	0.9961	0.03	0.6760	0.3183	0.0058
	0.9949	0.04	0.6857	0.3069	0.0074
	0.9938	0.05	0.6966	0.2946	0.0088
0.10	0.9981	0.01	0.5784	0.4189	0.0027
	0.9972	0.02	0.6544	0.3413	0.0043
	0.9956	0.03	0.6518	0.3418	0.0064
	0.9943	0.04	0.6646	0.3273	0.0082
	0.9934	0.05	0.6853	0.3053	0.0094
0.15	0.9978	0.01	0.5441	0.4528	0.0031
	0.9969	0.02	0.6327	0.3625	0.0047
	0.9951	0.03	0.6298	0.3630	0.0071
	0.9938	0.04	0.6484	0.3427	0.0089
	0.9927	0.05	0.6671	0.3226	0.0104
0.20	0.9967	0.01	0.4498	0.5459	0.0043
	0.9959	0.02	0.5717	0.4223	0.0060
	0.9945	0.03	0.6060	0.3860	0.0080
	0.9934	0.04	0.6362	0.3542	0.0096
	0.9919	0.05	0.6482	0.3404	0.0114

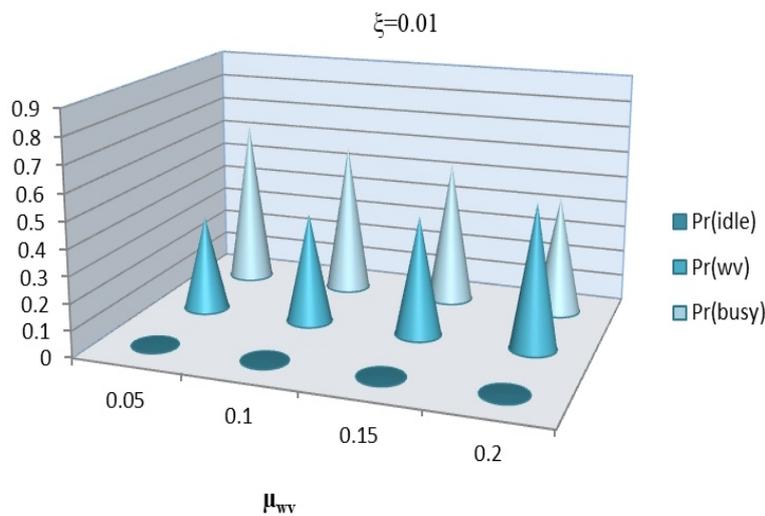


Figure 4: $Pr(busy)$, $Pr(wv)$, and $Pr(idle)$ with respect to ζ and μ_{wv}

The probability for various states, $Pr(idle)$, $Pr(wv)$ and $Pr(busy)$ are analyzed and the characteristic values are calculated and tabulated in (Table 4) by using the equations (23) to (25). The chosen parameters are $\lambda_v = 5$, $\mu_{rb} = 0.9$, $\psi = 4$ & $\phi = 4.5$, and by varying Service rate during working vacation (μ_{wv}) & vacation parameter (ζ), from Figure (4) we observe that as particular value for ζ (i.e., $\zeta = 0.01$) and μ_{wv} values increase, $Pr(busy)$ decreases significantly and $Pr(idle)$ & $Pr(wv)$ increase gradually.

CONCLUSION

In this study, $M/M(a,b)/1/MWV$ queuing model for compulsory and extended repair are analyzed. In this model, GBSR is followed. The steady-state solution, the various performance measures for the system and particular cases are calculated. Finally, in the form of tables and graphs, numerical results have been analyzed.

Analyzing a batch arrival multiple working vacations model with compulsory and extended repair processes helps in understanding the complex system. In the compulsory repair model, failed components or entities are required to undergo repair immediately to ensure the system's continued operation. These repairs are essential to prevent system failures. An extended repair process refers to a situation where a failed entity or component undergoes a repair process that takes longer than usual repairs. While extended repairs may result in longer downtime. By combining compulsory and extended repair processes, organizations can optimize maintenance schedules, allocate resources effectively, and enhance system performance. The model helps in analysing repair queues, and determining optimal repair strategies to minimize downtime, and improve service quality. Overall, the compulsory and extended repair model serves as a valuable tool for organizations to enhance system performance. In future the model maybe extended to concept of breakdown situation.

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