# MARSHALL – OLKIN TWO PARAMETER SUJATHA DISTRIBUTION AND ITS APPLICATIONS

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**Abstract**

*Sujatha distribution, often discussed in the context of reliability engineering and survival analysis, is a probability distribution that is useful for modeling lifetimes of objects and systems. In this paper, we introduce a three parameter distribution called the Marshall – Olkin two parameter Sujatha distribution. Marshall – Olkin generalization is a novel method of enlarging a known family of distributions. It leads to a rich class of distributions that can capture different shapes and behaviours and thus allows for better fitting to empirical data, thereby establishing its importance in reliability engineering. We derive many useful statistical properties such as the hazard rate function, reverse hazard rate function, order statistics, moments, measure of skewness and measure of kurtosis of the Marshall – Olkin two parameter Sujatha distribution. The stress – strength reliability of this distribution has also been derived. The application of this new distribution is established using real life data sets. We also compare the performance of this distribution with that of some existing distributions.*

**Keywords:** Lifetime Distributions, Sujatha Distribution, Reliability, Marshall – Olkin Distribution.

## 1. Introduction

Statistical analysis and modeling of lifetime data has now become a topic of relevance because of its widespread applicability in the fields of engineering, insurance and medical science. Numerous statistical methods which includes both parametric and non-parametric models have been evolved so far. Still the scope and importance of the topic are being expanded beyond limits. Exponential, gamma, lognormal, Weibull, Lindley, Akash, Shanker, Sujatha are some probability distributions which are very widely used in modeling lifetime data. Each of these distributions has its own merits and demerits while applying to a real life situation. For example, the survival functions of lognormal and gamma distributions cannot be written in closed forms. The exponential distribution has a constant hazard rate. At the same time, the hazard rate of Lindley, Shanker, Akash and Sujatha distributions are monotonically increasing.

Sujatha distribution has been introduced by [20]. It has been originally presented as a one parameter continuous distribution with probability density function

$$f(x,\theta) = \frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x};\ x>0,\ \theta>0 \tag{1}$$

and cumulative distribution function

$$F(x,\theta) = 1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x};\ x>0,\ \theta>0. \tag{2}$$

In fact, this distribution is a three-component mixture of an exponential distribution with scale parameter $\theta$, a gamma distribution with shape parameter 2 and scale parameter $\theta$, and a gamma distribution with shape parameter 3 and scale parameter $\theta$. A two parameter Sujatha distribution (TPSD) having parameters $\theta$ and $\alpha$ was proposed by [21]. The probability density function $f(x)$ and cumulative distribution function $F(x)$ of the two parameter Sujatha distribution are given by

$$f(x,\theta,\alpha) = \frac{\theta^3}{\alpha\theta^2+\theta+2}(\alpha+x+x^2)e^{-\theta x};\ x>0,\ \theta>0,\ \alpha\geq 0 \tag{3}$$

and

$$F(x,\theta) = 1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x};\ x>0,\ \theta>0,\ \alpha\geq 0. \tag{4}$$

Here $\theta$ is a scale parameter and $\alpha$ is a shape parameter. It is easy to verify that the two parameter Sujatha distribution reduces to the Sujatha distribution for $\alpha = 1$.

The hazard rate function, mean residual life function, stochastic ordering, Bonferroni and Lorenz curves, stress-strength reliability and many other important properties of Sujatha distribution as well as two parameter Sujatha distribution has been discussed in detail by [20] and [21] respectively.

In this manuscript, we present the Marshall – Olkin generalization of the two parameter Sujatha distribution. A new method of expanding a known family of distributions was put forward by [17]. This method consists of considering the survival function, say $\bar{F}(x)$ of an existing distribution and constructing a new function $\bar{G}(x)$ defined as

$$\bar{G}(x) = \frac{\bar{F}(x)}{\gamma+(1-\gamma)\bar{F}(x)}, \tag{5}$$

$\bar{G}(x)$ is a proper survival function and forms a new family of survival functions. This method of extending an existing distribution usually gives rise to distributions having useful hazard functions. As an immediate consequence, they can be employed for modeling real life situations in a better way than the already known distributions. Because of this wide applicability, the Marshall - Olkin extended family of distributions had been studied in detail by many researchers. Numerous examples can be seen in [12], [13], [1], [2], [3], [4], [5], [8], [6], [11], [7], [14], [8], [15],[16],[18], [19], [9]. The Marshall – Olkin Sujatha (MOS) distribution introduced by [10] has many useful statistical properties with respect to survival rate function, hazard rate function, reversed hazard rate function, cumulative hazard rate function, stochastic ordering, Shannon and Renyi entropies, order statistics etc.

## 2. Marshall-Olkin two parameter Sujatha distribution

The survival function $\bar{G}(x,\ \theta,\ \alpha,\ \gamma)$ of the Marshall-Olkin two parameter Sujatha distribution (MOTPSD) computed using (5) is given by

$$\bar{G}(x,\ \theta,\ \alpha,\ \gamma) = \frac{\bar{F}(x,\ \theta,\ \alpha)}{\gamma+(1-\gamma)\bar{F}(x,\ \theta,\ \alpha)} = \frac{\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}}{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}}.$$

The cumulative distribution function of MOTPSD is given by

$$\begin{aligned} G(x,\ \theta,\ \alpha,\ \gamma) &= 1-\bar{G}(x,\ \theta,\ \alpha,\ \gamma) \\ &= \frac{\gamma\left\{1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}}{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}}. \end{aligned} \tag{6}$$

The probability density function of $MOTPSD$ $g(x,\ \theta,\ \alpha,\ \gamma)$ is

$$g(x,\ \theta,\ \alpha,\ \gamma) = \frac{\gamma\theta^3(\alpha + x^2 + x)e^{-\theta x}}{(\alpha\theta^2 + \theta + 2)\left\{\gamma + (1-\gamma)\left[1 + \frac{\theta x(\theta x + \theta + 2)}{\alpha\theta^2 + \theta + 2}\right]e^{-\theta x}\right\}^2}; x > 0,\ \alpha \geq 0,\ 0 < \gamma < 1. \tag{7}$$

The plots of probability density function for different values of the parameters are given in figure 1.

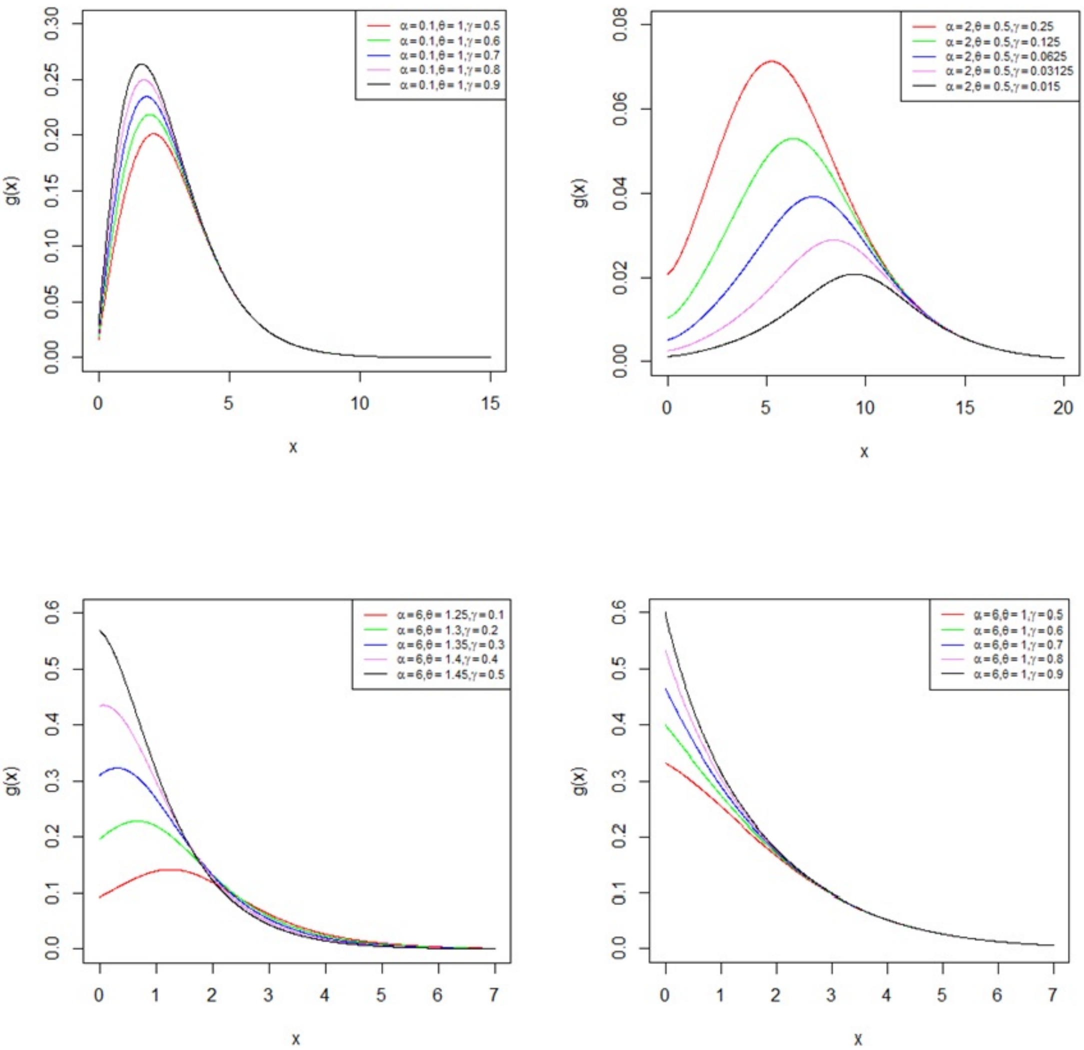


**Figure 1:** *Plots of probability density functions of MOTPSD for various parameter values.*

The hazard rate function is

$$h(x,\,\theta,\,\alpha,\,\gamma) = \frac{g(x,\,\theta,\,\alpha,\,\gamma)}{\bar{G}(x,\,\theta,\,\alpha,\,\gamma)} = \frac{\gamma\theta^3(\alpha + x^2 + x)e^{-\theta x}}{(\alpha\theta^2+\theta+2)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]} \times \frac{1}{\left\{\gamma + (1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}}. \quad (8)$$

The plot of the hazard rate function is given in figure 2.

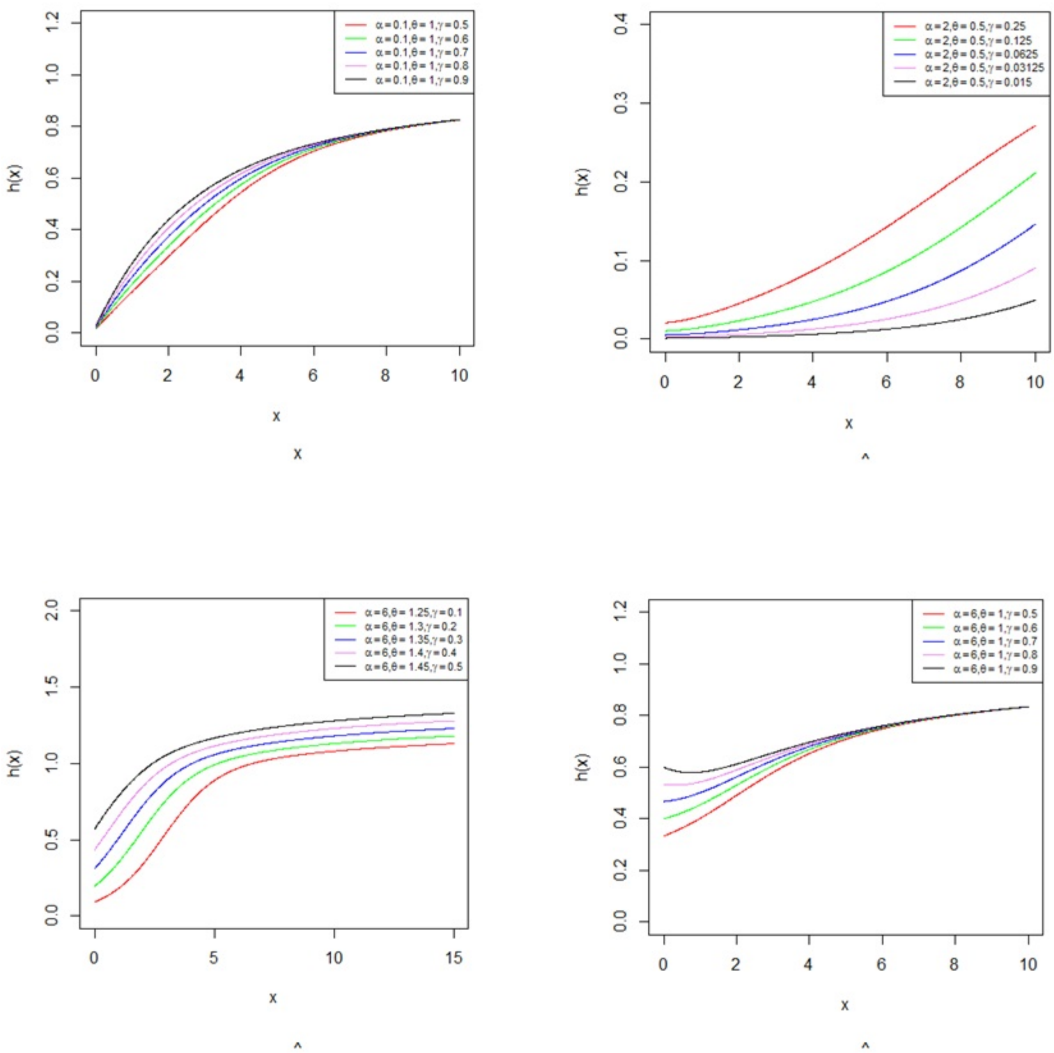


**Figure 2:** *Plots of hazard rate function of MOTPSD for various parameter values.*

The reverse hazard function is

$$h_r(x,\,\theta,\,\alpha,\,\gamma) = \frac{g(x,\,\theta,\,\alpha,\,\gamma)}{G(x,\,\theta,\,\alpha,\,\gamma)}$$

$$= \frac{\theta^3(\alpha + x^2 + x)e^{-\theta x}}{(\alpha\theta^2+\theta+2)\left\{1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}\left\{\gamma + (1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}}. \quad (9)$$

## 3. Order Statistics

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics based on a random sample $X_1, X_2, \ldots, X_n$ of size $n$ from the population having probability density function given by (7). Then the probability density function of the $r^{\text{th}}$ order statistic is given by

$$\begin{aligned} g_{x_{(r)}}(x,\theta,\alpha,\gamma) &= \frac{n!}{(r-1)!(n-r)!} g(x,\theta,\alpha,\gamma)[G(x,\theta,\alpha,\gamma)]^{r-1}[1-G(x,\theta,\alpha,\gamma)]^{n-r} \\ &= \frac{n!\gamma^r\theta^3(\alpha+x^2+x)e^{-\theta x}\left\{\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{n-r}}{(r-1)!(n-r)!(\alpha\theta^2+\theta+2)} \\ &\times \frac{\left\{1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{r-1}}{\left\{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{n+1}};\ x>0,\ \alpha\geq 0,\ 0<\gamma<1. \quad (10) \end{aligned}$$

Now, the probability density function of the first order statistic $X_{(1)}$ and the $n^{\text{th}}$ order statistic $X_{(n)}$ are

$$\begin{aligned} g_{x_{(1)}}(x,\theta,\alpha,\gamma) &= \frac{n\gamma\theta^3(\alpha+x^2+x)e^{-n\theta x}\left\{\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]\right\}^{n-1}}{(\alpha\theta^2+\theta+2)\left\{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{n+1}}; \\ & x>0,\ \alpha\geq 0,\ 0<\gamma<1. \quad (11) \end{aligned}$$

and

$$\begin{aligned} g_{x_{(n)}}(x,\theta,\alpha,\gamma) &= \frac{n\gamma^n\theta^3(\alpha+x^2+x)e^{-\theta x}\left\{1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]\right\}^{n-1}}{(\alpha\theta^2+\theta+2)\left\{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{n+1}}; \\ & x>0,\ \alpha\geq 0,\ 0<\gamma<1. \quad (12) \end{aligned}$$

## 4. Moments and related measures

We have

$$\begin{aligned} \left\{\gamma+(1-\gamma)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\}^{-2} &= \gamma^{-2}\left\{1+\left(\frac{1-\gamma}{\gamma}\right)\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x}\right\} \\ &= \gamma^{-2}\sum_{k=0}^{\infty}(-1)^k\binom{k+1}{k}\left(\frac{1-\gamma}{\gamma}\right)^k\left[1+\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]^k e^{-\theta kx} \\ &= \gamma^{-2}\sum_{k=0}^{\infty}\sum_{i=0}^{k}(-1)^k\left(\frac{1-\gamma}{\gamma}\right)^k\binom{k+1}{k}\binom{k}{i}\left[\frac{\theta x(\theta x+\theta+2)}{\alpha\theta^2+\theta+2}\right]^i e^{-\theta kx} \end{aligned}$$

$$\begin{aligned} &= \gamma^{-2}\sum_{k=0}^{\infty}\sum_{i=0}^{k}(-1)^k\left(\frac{1-\gamma}{\gamma}\right)^k\binom{k+1}{k}\binom{k}{i}\left[\frac{\theta x}{\alpha\theta^2+\theta+2}\right]^i \\ &\times e^{-\theta kx}\sum_{j=0}^{i}\binom{i}{j}(\theta+2)^{i-j}(\theta x)^j e^{-\theta kx} \\ &= \gamma^{-2}\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{j=0}^{i}(-1)^k\left(\frac{1-\gamma}{\gamma}\right)^k\binom{k+1}{k}\binom{k}{i}\binom{i}{j}(\theta)^{i+j} \\ &\times (\alpha\theta^2+\theta+2)^{-i}(\theta+2)^{i-j}x^{i+j}e^{-\theta kx}. \end{aligned}$$

Therefore

$$g(x,\ \theta,\ \alpha\ ,\gamma) = \frac{\theta^3}{\gamma(\alpha\theta^2+\theta+2)}\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{j=0}^{i}P_{kij}x^{i+j}e^{-\theta(k+1)x}(\alpha+x^2+x); \quad x>0,\ \alpha\geq 0;\ \theta>0,\ 0<\gamma<1, \tag{13}$$

where

$$P_{kij} = (-1)^k\left(\frac{1-\gamma}{\gamma}\right)^k\binom{k+1}{k}\binom{k}{i}\binom{i}{j}(\theta)^{i+j}(\alpha\theta^2+\theta+2)^{-i}(\theta+2)^{i-j}.$$

Thus, the $r$th moment about the origin is given by

$$\begin{aligned}\mu_r &= \frac{\theta^3}{\gamma(\alpha\theta^2+\theta+2)}\int_0^{\infty}\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{j=0}^{i}P_{kij}x^{i+j+r}e^{-\theta(k+1)x}(\alpha+x+x^2)\\ &= \sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{j=0}^{i}P_{kij}\frac{\theta^3}{\gamma(\alpha\theta^2+\theta+2)}\\ &\times \left\{\frac{\alpha\Gamma(i+j+r+1)}{[\theta(k+1)]^{i+j+r+1}}+\frac{\Gamma(i+j+r+2)}{[\theta(k+1)]^{i+j+r+2}}+\frac{\Gamma(i+j+r+3)}{[\theta(k+1)]^{i+j+r+3}}\right\}.\end{aligned} \tag{14}$$

To study the behaviour of the distribution regarding dispersion, skewness and kurtosis, the coefficient of variation ($CV$), the moment measure of skewness ($\sqrt{\beta_1}$) and the moment measure of kurtosis ($\beta_2$) have been computed for different values of the parameters and are shown in Table 1 below.

**Table 1:** *$CV$, $\sqrt{\beta_1}$ and $\beta_2$ of MOTPSD for different values of $\alpha$, $\theta$, $\gamma$.*

| **Parameters** | $\mu_1'$ | $\mu_2'$ | $\mu_3'$ | $\mu_4'$ | $CV$ | $\sqrt{\beta_1}$ | $\beta_2$ |
|---|---|---|---|---|---|---|---|
| $\alpha=1$, $\theta=2$, $\gamma=0.5$ | 1.188 | 2.1881 | 5.2428 | 15.2408 | 1.1653 | 4.7739 | 1.3580 |
| $\alpha=1.5$, $\theta=1$, $\gamma=0.95$ | 2.1574 | 7.5865 | 35.4228 | 203.672 | 1.2755 | 5.2179 | 1.6270 |
| $\alpha=2$, $\theta=1.8$, $\gamma=0.90$ | 0.9143 | 1.5475 | 3.6860 | 11.1401 | 0.922605 | 1.6160 | 6.5671 |
| $\alpha=6$, $\theta=1$, $\gamma=0.75$ | 1.7864 | 5.7403 | 25.5035 | 142.184 | 0.893743 | 1.509087 | 6.049135 |

## 5. Estimation of Parameters

In this section, we estimate the parameters of Marshall – Olkin two parameter Sujatha distribution using the method of maximum likelihood estimation. Let $X_1,\ X_2,\ \ldots,\ X_n$ be a random sample of size $n$ from the population having probability density function given by (7)

Then the likelihood function can be written as

$$L(\underline{x};\ \theta,\ \alpha,\ \gamma) = \left[\frac{\gamma\theta^3}{(\alpha\theta^2+\theta+2)}\right]^n e^{-\theta\sum_{1=1}^{n}x_i}\prod_{i=1}^{n}\left\{\frac{\alpha+x_i^2+x_i}{\left\{\gamma+(1-\gamma)\left[1+\frac{\theta x_i(\theta x_i+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x_i}\right\}^2}\right\}. \tag{15}$$

The log likelihood function is

$$\ln L(\underline{x};\ \theta,\ \alpha,\ \gamma) = n\ln(\gamma\theta^3) - n\ln(\alpha\theta^2+\theta+2) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\ln(\alpha + x_i^2 + x_i)$$
$$-2\sum_{1=1}^{n}\ln\left\{\gamma + (1-\gamma)\left[1+\frac{\theta x_i(\theta x_i+\theta+2)}{\alpha\theta^2+\theta+2}\right]e^{-\theta x_i}\right\}. \quad (16)$$

The log likelihood function does not have an explicit solution. Hence the likelihood equations are to be solved numerically with the help of computer software.

## 6. Stress Strength Reliability

Let $X$ and $Y$ be two independent random variables denoting the strength and stress of a component. If we assume that $X$ and $Y$ have the MOTPS distribution with parameters $(\theta_1, \alpha_1, \gamma_1)$ and $(\theta_2,\ \alpha_2,\ \gamma_2)$ respectively, then the stress – strength reliability $(R)$ can be obtained as

$$R = P(Y<X) = \int_0^\infty g(x,\ \theta_1, \alpha_1, \gamma_1) G(x, \theta_2,\ \alpha_2,\ \gamma_2)dx$$
$$= \int_0^\infty \left[\frac{\gamma_2\left[1-\left(1+\frac{\theta_2 x(\theta_2 x+\theta_2+2)}{\alpha_2\theta_2^2+\theta_2+2}\right)e^{-\theta_2 x}\right]}{\gamma_2+(1-\gamma_2)\left[1+\frac{\theta_2 x(\theta_2 x+\theta_2+2)}{\alpha_2\theta_2^2+\theta_2+2}\right]e^{-\theta_2 x}}\right]$$
$$\times \left[\frac{\gamma_1\theta_2^2(\alpha_1+x^2+x)e^{-\theta_1 x}}{(\alpha_1\theta_1^2+\theta_1+2)\left[\gamma_1+(1-\gamma_1)\left[1+\frac{\theta_1 x(\theta_1 x+\theta_1+2)}{\alpha_1\theta_1^2+\theta_1+2}\right]e^{-\theta_1 x}\right]}\right] dx.$$

Since this integral is difficult to compute, we employ some numerical methods for computation for some set of values of the parameters and the values of R so obtained is shown in table 2.

**Table 2:** *Values of Stress – Strength Reliability(R).*

| **Sl.No** | $\theta_1$ | $\alpha_1$ | $\gamma_1$ | $\theta_2$ | $\alpha_2$ | $\gamma_2$ | **R** |
|---|---|---|---|---|---|---|---|
| 1. | 0.5 | 2 | 0.25 | 0.5 | 2 | 0.125 | 0.386294 |
| 2. | 1 | 5 | 0.5 | 1.25 | 5 | 0.6 | 0.625242 |
| 3. | 1 | 6 | 0.5 | 1.25 | 5 | 0.5 | 0.583306 |
| 4. | 1 | 6 | 0.5 | 1.25 | 5 | 0.4 | 0.549194 |
| 5. | 1.25 | 6 | 0.1 | 1.3 | 6 | 0.2 | 0.635063 |

## 7. Application

In this section, we appl a real lifetime data set in the Marshall – Olkin two parameter Sujatha distribution to determine its goodness of fit and then comparison has been developed in order to show that the Marshall – Olkin two parameter Sujatha distribution provides a better fit over Sujatha distribution, two parameter Sujatha distribution and another two parameter Sujatha distribution. The real data set given in table 3 represents the remission time (in months) of 50 breast cancer women subjected to treatment using trastzuzumab as medication reported by cancer registry department, university of Benin teaching hospital, Benin, Edo state.

**Table 3:** *Data regarding the remission time (in months) of 50 breast cancer women reported by university cancer registry department of Benin teaching hospital.*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 74 | 35 | 39 | 21 | 37 | 27 | 35 | 30 | 35 |
| 26 | 28 | 34 | 34 | 26 | 41 | 61 | 33 | 33 | 26 |
| 25 | 41 | 35 | 34 | 34 | 33 | 60 | 61 | 42 | 30 |
| 80 | 31 | 24 | 49 | 26 | 31 | 28 | 41 | 37 | 41 |
| 61 | 33 | 26 | 34 | 50 | 73 | 45 | 80 | 39 | 21 |

To estimate the unknown parameters and for model comparison, R software is used. In order to compare the performance of MOTPSD with Length Biased Another two parameter Sujatha distribution, Another two parameter Sujatha distribution, two parameter Sujatha distribution and Sujatha distributions, we consider the criteria such as Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC) and Akaike Information Criterion Corrected (AICC). The distribution which shows lesser values of AIC, BIC, AICC is better. For computing the criteria values such as AIC, BIC, AICC and $-2\log L$ the following formulae are used:

$$AIC = 2k - 2\ln L,\ BIC = k\ln(n) - 2\ln L \text{ and } AICC = AIC + \frac{2k(k+1)}{n-k-1},$$

where $n$ is the sample size, $k$ is the number of parameters in the statistical model.

**Table 4:** *Model Comparison.*

| Distribution | MLE | AIC | BIC | AICC |
|---|---|---|---|---|
| Sujatha Distribution | $\hat{\theta} = 0.074719546$ | 422.4818 | 424.3938 | 422.5651 |
| Two parameter Sujatha Distribution | $\hat{\alpha} = 0.001$ $\hat{\theta} = 0.074847071$ | 424.2968 | 428.1208 | 424.5521 |
| Another Two parameter Sujatha Distribution | $\hat{\alpha} = 2.189802$ $\hat{\theta} = 7.7485049$ | 424.2974 | 428.1215 | 424.5527 |
| Marshall – Olkin Two parameter Sujatha Distribution | $\hat{\alpha} = 3.165$ $\hat{\theta} = 0.154855$ $\hat{\gamma} = 0.1$ | 416.0321 | 421.7682 | 416.5538 |

## 8. Simulation

Next we assess the performance of the maximum likelihood estimators of the parameters of $MOTPSD(\alpha,\ \theta\ \gamma)$. We simulated a sample of size $n = 20, 50, 100, 500, 1000$ and $10,000$ based on the following sets of parameters.

- $\alpha = 2,\ \theta = 1,\ \gamma = 0.5$
- $\alpha = 3,\ \theta = 0.15,\ \gamma = 0.1$
- $\alpha = 5,\ \theta = 3,\ \gamma = 0.75$

Then we computed the maximum likelihood estimates of these parameters and a comparison of the average bias and mean square of all the parameters have been carried out. The results obtained are summarized in table 5 shown below.

## 9. Conclusion

A new lifetime distribution known as Marshall – Olkin two parameter Sujatha distribution has been introduced. Its statistical properties such as hazard rate function, reverse hazard rate function, distribution of order statistics, skewness and kurtosis have been investigated. The method of maximum likelihood for estimation of parameters has been discussed. A comparison of this distribution with Sujatha, Two Parameter Sujatha and Another two parameter Sujatha

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**Table 5:** *Average Bias and Mean Squared Errors of MLEs of $MOTPSD(\alpha,\ \theta,\ ,\gamma)$.*

| Parameter | | $n = 20$ | $n = 50$ | $n = 100$ | $n = 500$ | $n = 1000$ | $n = 10000$ |
|---|---|---|---|---|---|---|---|
| $\alpha = 2$ | Avg. Bias | 1.970704 | 1.43478 | 1.15029 | 0.45921 | 0.36780 | 0.15733 |
| | MSE | 9.43113 | 4.89108 | 2.40008 | 1.79432 | 0.93313 | 0.39167 |
| $\theta = 1$ | Avg. Bias | 0.06633 | 0.04608 | 0.00139 | 0.00134 | -0.00106 | -0.00101 |
| | MSE | 0.06016 | 0.03186 | 0.10022 | 0.00213 | 0.00204 | 0.00022 |
| $\gamma = 0.5$ | Avg. Bias | -0.06676 | -0.06401 | -0.04781 | 0.00558 | -0.00029 | -0.00025 |
| | MSE | 0.17423 | 0.06407 | 0.02701 | 0.02857 | 0.01788 | 0.00500 |
| $\alpha = 3$ | Avg. Bias | 113.28788 | 111.71993 | 72.66700 | 41.30937 | 7.46501 | 0.00709 |
| | MSE | 17444.57814 | 14692.92071 | 8499.1245 | 4953.06184 | 750.05078 | 1.95245 |
| $\theta = 0.15$ | Avg. Bias | 0.00866 | 0.00464 | 0.00342 | 0.00218 | 0.00021 | -0.00020 |
| | MSE | 0.00071 | 0.00020 | 0.00015 | $4.39864 \times 10^{-5}$ | $1.75124 \times 10^{-5}$ | $1.563 \times 10^{-5}$ |
| $\gamma = 0.1$ | Avg. Bias | -0.04827 | 0.39261 | -0.03555 | -0.002109 | -0.00339 | 0.00102 |
| | MSE | 0.00377 | 3.97591 | 0.00272 | 0.00166 | 0.000039 | $1.524 \times 10^{-5}$ |
| $\alpha = 5$ | Avg. Bias | 2.45105 | 1.27315 | 0.87924 | 0.14098 | 0.04309 | 0.03736 |
| | MSE | 35.76077 | 15.92763 | 9.074422 | 8.41508 | 8.14613 | 3.27945 |
| $\theta = 3$ | Avg. Bias | -0.10186 | 0.07054 | 0.03867 | -0.01533 | 0.00552 | -0.00030 |
| | MSE | 1.89987 | 0.40468 | 0.05986 | 0.05369 | 0.05259 | 0.00678 |
| $\gamma = 0.75$ | Avg. Bias | 0.48633 | 0.06119 | 0.01544 | 0.00262 | $5.02 \times 10^{-5}$ | $4.255 \times 10^{-5}$ |
| | MSE | 1.11601 | 0.17848 | 0.03189 | 0.02028 | 0.00584 | 0.00056 |

distributions is also given with respect to a real lifetime data set. A simulation study has been done to check the consistency of estimators for different sets of values of parameters.

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