RT&A, No 2 (84)
Volume 20, June 2025

# FORECASTING VOLATILITY IN INDIAN NATIONAL STOCK EXCHANGE USING A MARKOV SWITCHING GARCH MODEL

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**Abstract**

*This paper analyzes various GARCH models in terms of their effectiveness in predicting volatility in the national Stock Exchange (NSE). This paper examines GARCH models that feature both Gaussian and fat-tailed residual conditional distributions, and evaluates their ability to characterize and predict volatility over time horizons ranging from 1 to 22 days. The AR(2)-MR-SGARCH-GED model demonstrates superior performance compared to other models at a one-day horizon. The AR(2)-MR-SGARCH-GED and AR(2)-MRSGARCH-t models surpass other models at the 5-day horizon. Within a ten-day timeframe, three AR(2)-MR-SGARCH models surpass the performance of alternative models. Regarding the 22-day forecast horizon, the results demonstrate no distinctions between MR-SGARCH models and standard GARCH models. When looking at risk management outside of samples (95% VaR), a number of models seem to give accurate and reasonable VaR estimates for a one-day period, with a coverage rate close to the nominal level. According to the risk management loss functions, no model is universally the most accurate. This variability suggests that model performance is context-dependent, influenced by the specific characteristics of the data and the underlying assumptions of each approach. Therefore, practitioners should consider employing a combination of models to enhance robustness in their risk assessments and decision-making processes.*

**Keywords:** MRSGARCH, MLE, VaR-estimator, MSE, Forecasting, Risk assessment.

## 1. Introduction

Volatility forecasting is a crucial endeavor in financial markets, garnering the interest of scholars and professionals for the past twenty years [1]. Volatility is frequently employed as an indicator of risk related to financial returns, making it significant for portfolio managers, option traders, and market makers, among others. Financial market volatility can significantly impact the overall economy. The predominant category of models utilized in econometric methods for volatility forecasting is GARCH models (refer to Engle [2], Bollerslev [3], Engle and Lee [4], Bollerslev, Engle, and Nelson [5] for a comprehensive overview). In empirical research, the parameters of GARCH models are typically presumed to be constant over time. The conditional distribution of financial returns varies between recessionary and expansionary periods [6]. Furthermore, GARCH models frequently indicate significant volatility persistence following individual shocks. Lamoureux and Lastrapes [7] contended that elevated persistence in volatility may result from structural alterations in the variance process. Cai [8] and Hamilton and Susmel [9] have independently developed the Markov Regime Switching ARCH model (MRSARCH), which integrates Hamilton's Markov Switching model [10, 11] with ARCH specifications.

The MRSARCH model was developed to detect regime shifts in volatility using an unobservable state variable that follows a first-order Markov Chain process. In the ARCH process, parameters may vary across different states. Cai [8] and Hamilton and Susmel [9] employed ARCH specifications to address the issue of infinite path dependence in the Markov Regime Switching GARCH model (MRSGARCH). Additionally, Dueker [12] employed a similar methodology as Gray [13] to address the issue of infinite path dependence and proposed several alternative MRSGARCH models. Klaassen [14] altered Gray's MRSGARCH model, asserting that his specification enhances the forecasting efficacy of MRSGARCH models. Haas, Mittnik, and Paolella [15] introduced a novel method distinct from Gray's [13] approach, asserting that the analytical tractability of their model facilitates the derivation of stationary conditions and dynamic properties.

Leon Li and William Lin [16] and Fong [17] utilized the MRSARCH model developed by Hamilton and Susmel [9] to investigate regime shifts and volatility persistence in the weekly Taiwan Stock Index (TAIEX) and the weekly Japanese Stock Index (TOPIX), respectively. The Fréchet distribution is a special case of the generalized extreme value distribution. This type-II extreme value distribution (Fréchet) case is equivalent to taking the reciprocal of values from a standard Weibull distribution. This distribution has been shown to be useful for modeling and analysis of several extreme events ranging from accelerated life testing to earthquakes, wind speeds, sea currents, floods, and rain fall.

The authors assert that the MRSARCH model offers a superior representation of the data and exhibits significantly reduced volatility persistence compared to uni-regime GARCH models. Kaufmann and Scheicher [18] utilized the MRSARCH model, implemented within a Bayesian framework, to characterize the daily German Stock Index (DAX). Gray [13] utilized weekly one-month US Treasury bill rates from 1970 to 1994. He asserts that the MRSARCH model surpasses basic uni-fregime models in forecasting efficacy and diminishes volatility persistence more effectively than the MRSARCH model proposed by Cai [8] and that of Hamilton and Susmel [9]. Marcucci [6] evaluated a series of GARCH models against a collection of MRSGARCH models regarding their efficacy in forecasting S&P100 volatility over a period ranging from one day to one month. Marcucci [6] concludes that the forecasting efficacy of MRSGARCH models surpasses that of uni-regime GARCH models at shorter time horizons. Standard asymmetric GARCH demonstrates superior performance over extended horizons. Furthermore, MRSARCH models have been utilized in international stock markets by Fornari and Mele [19], Norden and Schaller [20], Susmel [21], Bautista [22], and Leon Li [23].

Erb et al. [24] and Bekaert et al. [25] observe that emerging markets share a significant characteristic: a high degree of country risk, encompassing political, economic, and financial risks. Over the past decades, these markets have experienced currency devaluations, unsuccessful economic strategies, coups, financial shocks, regulatory alterations, and capital market reforms at varying intensities. This work integrates GARCH models within a regime-switching framework, which efficiently accommodates the presence of two distinct volatility regimes, each defined by varying levels of volatility. In both regimes, volatility exhibits a GARCH-like pattern, thereby preventing the actual variance from relying on the complete information set, as noted in Klaassen [14]. The scarcity of research on volatility forecasting in emerging markets, particularly the national

Stock Exchange (NSE), motivated the composition of this paper. The primary objective of this paper is to compare various GARCH models regarding their efficacy in characterizing and predicting the volatility of financial time series over horizons ranging from one day to 22 days.

This paper contributes by addressing the following critical questions: which GARCH models in emerging markets, such as the MSE, exhibit superior performance in volatility forecasting? Do empirical results in MSE differ from those in developed markets? This paper compares standard GARCH models and Markov Regime-Switching GARCH models utilizing normal, t-student, and GED distributions regarding their efficacy in forecasting volatility in the National stock exchange. Empirical findings indicate that there is no disparity in the forecasting accuracy of GARCH models across all time horizons in the National Stock Exchange.

This document is structured as follows. Section 2 delineates various models of stock return volatility, including standard GARCH models and Markov Regime-Switching GARCH models. Section 3 delineates the data. Section 4 delineates empirical findings and engages in a discourse regarding various volatility models, evaluating their efficacy in forecasting volatility. Section 5 offers a concise summary and conclusion.

## 2. Econometric Methodology

### 2. 1. GARCH Models

The rate of return $R_t$ is defined as following:

$$R_t = 100[\log(y_t) - \log(y_{t-1})] \tag{1}$$

where $y_t$ are denote stock market index, $t$ denotes the daily closing observations and $t = -S + 1, \dots, n$ the sample period consists of an estimation (or in-sample) period with $S$ observations ( $t = -S + 1, \dots, 0$ ), and an evaluation (or out-of-sample) period with $n$ observations ($t = 1, \dots, n$). The $\mathrm{GARCH(1,1)}, \mathrm{AR(1)} - \mathrm{GARCH(1,1)}$ and $\mathrm{AR(2)} - \mathrm{GARCH(1,1)}$ models for the series of returns $r_t$ are used that they can be written as following

$$\begin{cases} R_t = \mu + \varepsilon_t = \mu + \xi_t\sqrt{h_t} \\ R_t = \mu + \varphi_1 r_{t-1} + \varepsilon_t = \mu + \varphi_1 r_{t-1} + \xi_t\sqrt{h_t} \\ R_t = \mu + \varphi_1 R_{t-1} + \varphi_2 R_{t-2} + \varepsilon_t = \mu + \varphi_1 R_{t-1} + \varphi_2 R_{t-2} + \xi_t\sqrt{h_t} \end{cases} \tag{2}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

where $\alpha_0 > 0, \alpha_1 \geq 0$ and $\beta_1 \geq 0$ to guarantee a positive conditional variance, and the innovation is effectively represented as the product of an i.i.d. process with zero mean and unit variance ($\xi_t$) times the square root of the conditional variance ($h_t$) [6]. Glosten, Jagannathan and Runkle [26] put forward a modified GARCH model (GJR) to account for the 'leverage effect'. This is an asymmetric GARCH model that allows the conditional variance to respond differently to shocks of either sign and is defined as follows

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \left[1 - I_{\{\varepsilon_{t-1} > 0\}}\right] + \lambda \varepsilon_{t-1}^2 I_{\{\varepsilon_{t-1} > 0\}} + \beta_1 h_{t-1} \tag{4}$$

where $I_{\{\omega\}}$ is the indicator function which is equal to one when $\omega$ is true and zero otherwise. Another common finding in the GARCH literature is the leptokurtosis of the empirical distribution of financial returns [6]. To model such fat-tailed distributions researchers have adopted the Student's $t$ or the Generalized Error Distribution (GED). Therefore, in addition to the classic Gaussian assumption, in what follows the errors $\varepsilon_t$ are also assumed to be distributed according to a Student's $t$ or a GED distribution.

If a Student's $t$ conditional distribution with $v$ degrees of freedom is assumed, the probability density function (pdf) of $\varepsilon_t$, takes the form

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}(v-2)^{-\frac{1}{2}}(h_t)^{-\frac{1}{2}}\left[1+\frac{\varepsilon_t^2}{h_t(v-2)}\right]^{-\frac{(v+1)}{2}} \tag{5}$$

where $\Gamma(\cdot)$ is the Gamma function and $v$ is the degree-of-freedom (or shape) parameter, constrained to be greater than two so that the second moments exist. With a GED conditional distribution instead, the pdf of the innovations becomes [6]

$$f(\varepsilon_t) = \frac{\text{vexp}\left[-\frac{1}{2}\left|\frac{\varepsilon_t}{\lambda h_t^{\frac{1}{2}}}\right|^{v}\right]}{h_t^{\frac{1}{2}}\lambda 2^{\left(1+\frac{1}{v}\right)}\Gamma\left(\frac{1}{v}\right)} \tag{6}$$

With $\lambda \equiv \left[\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}}$, where $\Gamma(\cdot)$ is the Gamma function, $v$ is the thickness-of-tail (or shape) parameter, satisfying the condition $0 < v \leq \infty$ and indicating how thick the tails of the distribution are, compared to the normal. When the shape parameter $v = 2$, the GED becomes a standard normal distribution, while for $v < 2$ and $v > 2$ the distribution has thicker and thinner tails than the normal respectively [6].

## 2. 2. Markov Switching GARCH Model

The main feature of regime-switching models is the possibility for some or all the parameters of the model to switch across different regimes according to a Markov process, which is governed by a state variable, denoted $s_t$. The state variable is assumed to evolve according to a first-order Markov chain, with transition probability [13,14,6]

$$\Pr(s_t = j \mid s_{t-1} = i) = p_{ij} \tag{7}$$

That indicates the probability of switching from state $i$ at time $t-1$ into state $j$ at $t$. Usually, these probabilities are grouped together into the transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix} \tag{8}$$

where for simplicity the existence of only two regimes has been considered. The ergodic probability (that is the unconditional probability) of being in state $s_t = 1$ is given by $\pi_1 = \frac{1-q}{2-p-q}$ [13,14,6]. The MRS-GARCH model in its most general form can be written as

$$r_t \left| \Omega_{t-1} \sim \begin{cases} f\left(\theta_t^{(1)}\right) & w.P \cdot p_{1,t} \\ f\left(\theta_t^{(2)}\right) & w.P \cdot \left(1-p_{1,t}\right) \end{cases}\right. \tag{9}$$

where $f(\cdot)$ represents one of the possible conditional distributions that can be assumed, that is Normal ( N ), student's $t$ or GED, $\theta_t^{(i)}$ denotes the vector of parameters in the $i$ th regime that characterize the distribution, $p_{1,t} = \Pr[s_t = 1 \mid \Omega_{t-1}]$ is the ex ante probability and $\Omega_{t-1}$ denotes the information set at time $t-1$, that is the $\sigma$-algebra induced by all the variables that are observed at $t-1$. More specifically, the vector of time-varying parameters can be decomposed into three components [13,14,6].

$$\theta_t^{(i)} = \left(\mu_t^{(i)}, h_t^{(i)}, v_t^{(i)}\right) \tag{10}$$

where $\mu_t^{(i)} \equiv E(r_t \mid \Omega_{t-1}, s_t = i)$ is the conditional mean (or location parameter), $h_t^{(i)} \equiv \mathrm{Var}(r_t \mid \Omega_{t-1})$ is the conditional variance (or scale parameter), and $v_t^{(i)}$ is the shape parameter of the conditional distribution. Hence, the family of density functions of $r_t$ is a location-scale family with time-varying shape parameters in the most general setting [13,14,6]. Therefore, the MRS-GARCH consists of four elements: the conditional mean, the conditional variance, the regime process and the conditional distribution. The conditional mean equation is modeled as a simple model with constant mean and AR (1) and AR (2) models:

$$\begin{cases} R_t^{(i)} = \mu^{(i)} + \varepsilon_t^{(i)} \\ R_t^{(i)} = \mu^{(i)} + \varphi_1^{(i)} R_{t-1} + \varepsilon_t^{(i)} \\ R_t^{(i)} = \mu^{(i)} + \varphi_1^{(i)} R_{t-1} + \varphi_2^{(i)} R_{t-2} + \varepsilon_t^{(i)} \end{cases} \tag{11}$$

where ( $i$ ) is the symbol of each regime ( $i = 1$ or 2 ), $\varepsilon_t^{(i)} = \eta_t^{(i)} \sqrt{h_t^{(i)}}$ and $\eta_t$ is a zero mean, unit variance process. The conditional variance of $r_t$, given the whole regime path (not observed by the econometrician) $\tilde{s}_t = (s_t, s_{t-1}, \ldots)$, is $h_t^{(i)} = V[\varepsilon_t \mid \tilde{s}_t, \Omega_{t-1}]$. For this conditional variance the following GARCH (1,1)-like expression is assumed

$$h_t^{(i)} = \alpha_0^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} h_{t-1} \tag{12}$$

In which $h_{t-1}$ is a state-independent average of past conditional variances. Actually, in a regime switching context a GARCH model with a state-dependent past conditional variance would be infeasible. The conditional variance would in fact depend not only on the observable information $\Omega_{t-1}$ and on the current regime $s_t$ which determines all the parameters, but also on all past states $\tilde{s}_{t-1}$. This would require the integration over a number of (unobserved) regime paths that would grow exponentially with the sample size rendering the model essentially intractable and impossible to estimate. Therefore, a simplification is needed to avoid the conditional variance be a function of all past states. To integrate out the past regimes by also taking into account the current one, Klaassen [14] adopts the following expression for the conditional variance [14]

$$h_t^{(i)} = \alpha_0^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} E_{t-1}\left\{h_{t-1}^{(i)} \mid s_t\right\} \tag{13}$$

where the expectation is computed as [14] and the probabilities are calculated as [14]

$$E_{t-1}\left\{h_{t-1}^{(i)} \mid s_t\right\} = \tilde{p}_{ii,t-1}\left[\left(\mu_{t-1}^{(i)}\right)^2 + h_{t-1}^{(i)}\right] + \tilde{p}_{ji,t-1}\left[\left(\mu_{t-1}^{(j)}\right)^2 + h_{t-1}^{(j)}\right] - \left[\tilde{p}_{ii,t-1}\mu_{t-1}^{(i)} + \tilde{p}_{ji,t-1}\mu_{t-1}^{(j)}\right]^2 \tag{14}$$

$$\tilde{p}_{ji,t} = \Pr(s_t = j \mid s_{t+1} = i, \zeta_{t-1}) = \frac{p_{ji}\Pr(s_t = j \mid \zeta_{t-1})}{\Pr(s_{t+1} = i \mid \zeta_{t-1})} = \frac{p_{ji} p_{j,t}}{p_{i,t+1}} \tag{15}$$

where $i, j = 1,2$[14]. Klaassen's [14] regime-switching GARCH has two main advantages over the other models. Within the model, it allows higher flexibility in capturing the persistence of shocks to volatility. Furthermore, it allows having straightforward expression for the multi-step ahead volatility forecasts that can be calculated recursively as in standard GARCH models [14]. Since there is no serial correlation in the returns, the $h$-step ahead volatility forecast at time $T - 1$ can be calculated as follows [14,6]

$$\hat{h}_{T,T+h} = \sum_{\tau=1}^{h} \hat{h}_{T,T+\tau} = \sum_{\tau=1}^{h} \sum_{i=1}^{2} \Pr(s_\tau = i \mid \Omega_{T-1}) \hat{h}_{T,T+\tau}^{(i)} \tag{16}$$

where $\hat{h}_{T,T+h}$ denotes the time $T$ aggregated volatility forecast for the next $h$ steps, and $\hat{h}_{T,T+\tau}^{(i)}$ indicates the $\tau$-step-ahead volatility forecast in regime $i$ made at time $T$ that can be calculated recursively [14,6]

$$\hat{h}_{T,T+\tau}^{(i)} = \alpha_0^{(i)} + \left(\alpha_1^{(i)} + \beta_1^{(i)}\right) E_T\left\{\hat{h}_{T,T+\tau-1}^{(i)} \mid s_{T+\tau}\right\} \tag{17}$$

Therefore, the multi-step-ahead volatility forecasts are computed as a weighted average of the multi-step-ahead volatility forecasts in each regime, where the weights are the prediction probabilities. Each regime volatility forecast is obtained with a GARCH-like formula where the expectation of the previous period volatility is determined by weighting the previous regime volatility with the probabilities in (8). In general, to compute the volatility forecasts the filter probability at $\tau$ periods ahead $\Pr(s_{t+\tau} = i \mid \Omega_t) = P_{i,t+\tau} = P^\tau p_{i,t}$ is needed [14,6].

Typically, in the Markov regime-switching literature maximum likelihood estimation is adopted to estimate the numerous parameters. An essential ingredient is the ex-ante probability $p_{1,t} = \Pr[s_t = 1 \mid \Omega_{t-1}]$, i.e. the probability of being in the first regime at time $t$ given the information at time $t-1$, whose specification is

$$\begin{aligned} p_{1,t} &= \Pr[s_t = 1 \mid \Omega_{t-1}] \\ &= (1-q)\left[\frac{f(r_{t-1} \mid s_{t-1} = 2)(1 - p_{1,t-1})}{f(r_{t-1} \mid s_{t-1} = 1) p_{1,t-1} + f(r_{t-1} \mid s_{t-1} = 2)(1 - p_{1,t-1})}\right] \end{aligned}$$

where $p$ and $q$ are the transition probabilities in (10) and $f(\cdot)$ is the likelihood given in (9) [14,6]. Thus, the log-likelihood function can be written as

$$l = \sum_{t=-S+w+1}^{T+w} \log\left[p_{1,t} f(r_t \mid s_t = 1) + (1 - p_{1,t}) f(r_t \mid s_t = 2)\right] \tag{19}$$

where $w = 0,1,\ldots,n, f(\cdot \mid s_t = i)$ is the conditional distribution given that regime $i$ occurs at time $t$ [6].

## 2. 3. Estimation Procedure

The conditional mean and conditional variance are estimated simultaneously by maximizing the log-likelihood function, calculated as the logarithm of the product of the conditional densities of the prediction errors, as demonstrated in (14). The maximum likelihood estimates are derived by maximizing the log-likelihood using the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) quasi-Newton optimization algorithm within R's numerical optimization routines.

## 2. 4. Estimation of Volatility Forecasting

Forecast analysis is a crucial component of any forecasting endeavor. A widely used metric for assessing various forecasting models involves the minimization of a specific statistical loss function. Nevertheless, assessing the quality of competing volatility models can be quite challenging, as noted by Bollerslev, Engle, and Nelson [5] and Lopez [27], due to the absence of a singular criterion for identifying the optimal model. Consequently, despite being somewhat contentious, the majority of existing literature has predominantly concentrated on a specific statistical loss function, the Mean Squared Error (MSE) [6]. This paper employs the MSE criterion to evaluate competing volatility models. This loss function is defined as follows:

$$MSE = n^{-1}\sum_{t=1}^{n}\left(\hat{\sigma}_{t+1}^{2} - \hat{h}_{t+1|t}\right)^{2} \tag{20}$$

where $\hat{\sigma}_{t+1}^{2}$ is the actual volatility that we have used the squared return for the measure of daily volatility and $\hat{h}_{t+1|t}$ is the forecasted volatility based on period of forecasting. We have used 1-day, 5-day, 10-day and 22-day step ahead for forecasting horizon.

## 2. 5. Value at Risk

VaR is one of the most important measures of market risk portfolio, which quantifies in monetary terms the likely losses that could arise from fluctuations in the market [6]. Brooks and Persand [30] and Sarma, Thomas and Shah [31] suggested to use VaR based loss functions [30,6,31]. The VaR at time $t$ of model $i$ at $\alpha$% significance level is calculated as following:

$$\text{VaR}_{t}^{i}[n,\alpha] = \mu_{t+n}^{i} + \vartheta(\alpha)\sqrt{h_{t+n}^{i}} \tag{21}$$

where $\vartheta(\alpha)$ is a cumulative distribution function, $n$ is the investment horizon that we assumed $n = 1,5,10,20$ days, $\alpha = 1\%$ or $5\%, \mu_{t+n}^{i}$ is the volatility forecasts used as an input for the VaR [30,6]. In this paper, we have employed three methods to determine the adequacy of the volatility forecasts used as an input for the VaR. The TUFF test is based on the number of observation before the first exception. The relevant null is: $\alpha = \alpha_0$ and the corresponding LR test is [30,6,31].

$$LR_{\text{TUFF}}\,(\tilde{T},\hat{\alpha}) = -2\log\left\{\hat{\alpha}(1-\hat{\alpha})^{\tilde{T}-1}\right\} + 2\log\left\{\frac{1}{\tilde{T}}\left(1-\frac{1}{\tilde{T}}\right)^{\tilde{T}-1}\right\} \tag{22}$$

where $\tilde{T}$ denotes the number of observations before the first exception. $LR_{\text{TUFF}}$ is also asymptotically distributed as $\chi^2(1)$ [30,6,31]. A correctly specified VaR model should generate the pre-specified failure rate conditionally at every points in time [6]. Christoffersen [32] developed a framework for interval forecast evaluation [32,6]. The VaR is interpreted as a forecast that the portfolio return will lie in $(-\infty, VaR)$ with a pre-specified probability $p$ [32,6]. Consider a sequence of one-period-ahead VaR forecasts $\{\vartheta_{t|t-1}\}_{t=1}^{T}$, estimated at a significance level $1-p$[32,6]. These forecasts are intended to be one-sided interval forecasts $(-\infty, \vartheta_{t|t-1}]$ with coverage probability $p$ [32,6]. Given the realizations of the return series $r_t$ and the ex-ante VaR forecasts, the following indicator variable can be calculated [32,6]

$$I_t = \begin{cases} 1, & r_t < \vartheta_t \\ 0, & \text{otherwise} \end{cases}$$

The stochastic process $\{I_t\}$ is called "failure process". The forecasts of VaR are explained to be efficient if they demonstrate "correct conditional coverage" i.e. if $E[I_{t|t-1}] = p, \forall t$ or, equivalently, if $\{I_t\}$ is i.i.d. with mean $p$[32,6]. Christoffersen [32] developed a three-step testing method to test for correct conditional coverage consisting of a test for correct unconditional coverage, a test for independence and a test for correct conditional coverage.

In the test for correct unconditional coverage the null hypothesis of the failure probability $p$ is tested against the alternative hypothesis that the failure probability is different from $p$, under the assumption and independently distributed failure process [32,6]. In the test for independence, the hypothesis of an independently distributed failure process is tested against the alternative hypothesis of a first order Markov failure process [32,6]. Finally, the test of correct conditional coverage is done by testing the null hypothesis of an independent failure process with failure probability $p$ against the alternative of a first order Markov failure process [32,6]. All the three tests are carried out in the likelihood ratio (LR) framework. The likelihood ratio for each is as follows [32,6]. The LR statistic for the test of unconditional coverage [32,6]:

$$LR_{UC} = LR_{PF} = -2\log\left[\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}}\right] \sim \chi^2_{(1)}$$

where $p$ is the tolerance level at which VaR measures are estimated, $n_1$ is the number of 1 's in the indicator series, $n_0$ is the number of 0 's in the indicator series, and $\hat{\pi} = \frac{n_1}{(n_0+n_1)}$ is the MLE estimate of $p$[32,6]. The LR statistic for the test of independence [32,6]:

$$LR_{\text{ind}} = -2\log\left[\frac{(1-\hat{\pi}_2)^{(n_{00}+n_{10})}(1-\hat{\pi}_2)^{(n_{01}+n_{11})}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}}\right] \sim \chi^2_{(1)} \tag{23}$$

where $n_{ij}$ is the number of $i$ value followed by a $j$ value in the $I_t$ series $(i,j=0,1), \pi_{ij} = \Pr\{I_t = i \mid I_{t-1} = j\}(i,j=0,1)$, $\hat{\pi}_{01} = \frac{n_{01}}{(n_{01}+n_{01})}, \hat{\pi}_{11} = \frac{n_{11}}{(n_{10}+n_{11})}, \hat{\pi}_2 = \frac{(n_{01}+n_{11})}{(n_{00}+n_{01}+n_{10}+n_{11})}$ [32,6]. The LR statistic for the test of correct conditional coverage [32,6]:

$$LR_{cc} = -2\log\left[\frac{(1-p)^{n_0}p^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}}\right] \sim \chi^2_{(2)} \tag{24}$$

If we condition on the first observation, then these $LR$ test statistics are related by the identity $LR_{cc} = LR_{\text{ind}} + LR_{UC}$[32,6]. All these tests will be utilized in the evaluation of the VaR-based forecast. Models are ranked in this manner for the interpretation of the results. Consistent with Brooks and Persand [30], we have posited that any model exhibiting percentage failures in the rolling hold-out sample exceeding the nominal threshold should be deemed inadequate and consequently rejected. Consequently, the models with the lowest rankings (the least effective) are those exhibiting the highest percentage of failures exceeding the nominal value [30,6]. Once these models have been depleted, we posit that any model exhibiting a failure rate significantly lower than the anticipated figure is less preferable than those models demonstrating a failure rate nearer to the nominal level [30,6]. Consequently, the optimal models under this loss function are those that produce a coverage rate lower than the nominal rate, yet as proximate to it as feasible [30,6]. Subsequently, we assess the accuracy of both unconditional and conditional coverage; a model that successfully meets these criteria is deemed suitable for risk management objectives [30].

## 3. Data and Statistical Analysis of National Stock Exchange

The NSE was officially inaugurated in June 1988 with merely six listed companies, in contrast to the 110 companies currently traded by individual and institutional investors. The initial decade of the NSE was characterized by vigorous activity, with capitalization increasing from INR 7.2 billion to INR 240 billion and the number of listed companies expanding to 105. Following 1988, the Indian revolution and Indian incursion into substantially diminished exchange activities, resulting in a decline in capitalization to INR 12.9 billion in 198. Following the conclusion of the India-China war, the NSE was regarded as a pivotal mechanism for promoting economic development by directing savings into investment [33].

This objective rapidly increased the count of listed companies from 56 in 1988 to 422 in 2006 [33]. From 2000 to 2004, the market capitalization of the NSE increased from INR 60 billion to INR 411 billion. The Financial index attained a record peak of 13,482 on August 4, 2004, but within two years, a significant market correction reduced the index by 35% to 9,069 on July 26, 2006. The stock market decline was not entirely detrimental, as it was viewed as a necessary correction in a market that had outpaced its fundamentals and required enhanced regulation and increased transparency.

**Table 1:** *Results of Descriptive statistics*

| Mean | Std | Min | Max | Skew | Kur | BJ | Q²(12) | LM-(12) |
|---|---|---|---|---|---|---|---|---|
| 0.3248 | 0.4920 | -4.25 | 4.23 | -0.58 | 61.75 | 643732.1 | 172.89 | 71.05 |
| P-value: | | | | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

**Table 2:** *Summary of Unit root tests for rate of return series*

| Test type | Augmented Dickey-Fuller | Phillips-Perron |
|---|---|---|
| Statistic: | -9.15 | -63.17 |
| P-value: | 0.00 | 0.00 |

The market correction resulted in significant reforms and facilitated the consolidation and merger of numerous smaller companies [33]. By 2007, the market capitalization had surpassed its 2004 level; however, the number of listed companies remained low due to sustained merger and acquisition activity. All NSE transactions must be conducted on the Trading Floor utilizing the Exchange's Automated Trading System (ATS) [34]. Trading occurs from 10:00 am to 12:00 pm for the cash market and from 9:00 am to 12:30 pm for the futures market, from Saturday to Wednesday, excluding public holidays, with a pre-opening period of 30 minutes commencing at 8:30 am. The NSE comprises two markets: the cash market and the futures market. The cash market is divided into two segments: the first market and the second market [39]. An independent market exists for Islamic bonds. The futures market was established in mid-2010 with the introduction of single stock futures.

The NSM has 30 characteristics, one of which is that stock prices are regulated and cannot exceed a certain range due to intervention. The history of NSC is relatively brief in comparison to other stock markets. The information dissemination in this market is exceedingly sluggish. Transactions on the National Stock Exchange are conducted through orders submitted by brokers. The trading days of the week are Saturday, Sunday, Monday, Tuesday, and Wednesday, excluding national holidays. The dataset comprises 3,068 daily observations of the closing values of the National Stock Exchange from August 29, 2014, to August 9, 2023.

The sample is partitioned into two segments. The initial 2567 observations are used as the in-sample for estimation purposes, while the remaining 500 observations are taken as the out-of-sample for forecast evaluation purposes.

The return is calculated as $R_t = 100\left[\log\left(\frac{y_t}{y_{t-1}}\right)\right]$ where $y_t$ is the index value at time $t$. Table 1 shows some descriptive statistics of the MSE rate of return. The mean is quite small and the standard deviation is around 0.4. The Kurtosis (Ku) is significantly higher than the normal value of 3 indicating that fat-tailed distribution are necessary to correctly describe conditional distribution of $R_t$. The Skewness (Sk) is significant, small and negative, showing that the lower tail of empirical distribution of the return is longer than the upper tail, that means negative returns are more likely to be far below the mean than their counterparts.

LM (12) is the Lagrange Multiplier test for ARCH effects in the OLS residuals from the regression of the returns on a constant, while $Q^2(12)$ is the corresponding Ljung-Box [35,38] statistic on the squared standardized residuals. Both these statistics are highly significant suggesting the presence of ARCH effects in the MSE returns up to the twelfth order.

# 4. Empirical Results and Discussion

In the present section we present the empirical estimates of ARMA, single-regime GARCH and MRS-GARCH models, together with the in-sample and the out-of-sample forecast evaluation.

## 4.1. Single-Regime GARCH and ARMA Models

Initially, we have evaluated various ARMA models of return rates in terms of mean squared error (MSE). Table 3 presents the estimation outcomes of ARMA models. Subsequently, we have evaluated various GARCH models, including GARCH (1,1), AR(1)-GARCH(1,1), and AR(2)-GARCH(1,1) (refer to Table 4). The parameter estimates for various state GARCH models utilizing normal, t-student, and GED distributions are displayed in Tables 5-7. The initial 2567 observations (from 08/29/2014 to 11/27/2021) are designated as the in-sample for estimation, whereas the subsequent 500 observations (from 11/28/2021 to 09/09/2023) are utilized as the out-of-sample for forecasting assessment. All parameters for the different GARCH models are statistically significant concerning the conditional mean.

The estimates of conditional variance indicate that all parameters are highly significant. Furthermore, stationary conditions are present in various GARCH models. Furthermore, for the Student's t distribution, the degrees of freedom consistently exceed 3, indicating that all conditional moments up to the third order are present. The models incorporating GED innovations indicate that the conditional distribution exhibits fatter tails compared to the Gaussian, as all shape parameters significantly approximate one. The utilization of a t-distribution rather than a normal distribution is prevalent in the standard, single-regime GARCH literature.

**Table 4:** *Estimation of different ARMA $(p, q)$ models of rate of return*

| | AR (1) | MA (1) | AR (2) | ARMA (1,1) | ARMA (1,2) | ARMA (2,1) | ARMA (2,2) |
|---|---|---|---|---|---|---|---|
| M | 0.023* | 0.013* | 0.032* | 0.031* | 0.041* | 0.021* | 0.041* |
| $\varphi_1$ | 0.389* | - | 0.321* | 0.714** | 0.5677** | -0.015** | -0.175** |
| $\boldsymbol{\varphi_2}$ | - | - | 0.175 | - | - | 0.30** | 0.527** |
| $\vartheta_1$ | - | 0.275* | - | -0.391** | -0.237** | 0.35** | 0.500** |
| $\boldsymbol{\vartheta_2}$ | - | - | - | - | 0.068** | - | -0.189** |
| F-Statistic | 459 | 302 | 277 | 271 | 181 | 187 | 142 |
| AIC | -0.552 | -0.495 | -0.522 | -0.571 | -0.591 | -0.583 | -0.5855 |
| SBC | -0.547 | -0.450 | -0.554 | -0.572 | -0.570 | -0.576 | -0.541 |
| HQC | -0.550 | -0.479 | -0.597 | -0.5766 | -0.5758 | -0.505 | -0.514 |
| Log(L) | -710.5 | -643.2 | 749.7718 | -746.02 | -747.05 | -752.7 | -755.9 |

**Table 5:** *MLE estimates of standard GARCH $(1,1)$ models with constant mean of return*

| | GARCH-N | GARCH-T | GARCH-GED | GJR-N | GJR-T | GJR-GED |
|---|---|---|---|---|---|---|
| $\boldsymbol{\alpha}$ | 0.0045** | 0.043** | 0.0077** | 0.0074** | 0.043** | 0.0388** |
| $\boldsymbol{\varphi 1}$ | 0.0358** | 0.0267** | 0.0316** | 0.0343** | 0.0232** | 0.0207** |
| $\boldsymbol{\varphi 2}$ | 0.4753** | 0.6612** | 0.6055** | 0.4509** | 0.6156** | 0.0814** |
| $\boldsymbol{\alpha 0}$ | 0.4575** | 0.2739** | 0.3073** | 0.4852** | 0.7687** | 0.1945** |
| $\boldsymbol{\alpha 1}$ | - | - | - | 0.4079** | 0.2812** | 0.3079** |
| $\boldsymbol{\beta 1}$ | - | 3.8477** | 0.9029** | - | 3.83** | 0.9488 |
| Log(L) | -1289.1196 | -838.53 | -887.354 | -1283.922 | -938.1431 | -980.0731 |
| P-Value LR Test for Leverage Effect | | | | 0.21 | 0.00 | 0.00 |

In regime switching models, a t-distribution can be particularly advantageous. Ultimately, under normal circumstances, a significant innovation during a low volatility phase will precipitate a transition to a high-volatility regime sooner, even if it constitutes a solitary outlier amidst an otherwise stable period. Incorporating a t-distribution will consequently improve regimes stability.

**Table 6:** *MLE estimates of AR (1)-GARCH (1,1) models*

| | GARCH-N | GARCH-T | GARCH-GED | GJR-N | GJR-T | GJR-GED |
|---|---|---|---|---|---|---|
| $\boldsymbol{\alpha}$ | 0.0176** | 0.0119** | 0.0457* | 0.0905** | 0.0096** | 0.0280** |
| $\boldsymbol{\varphi 1}$ | 0.5188** | 0.5349** | 0.5633** | 0.5163** | 0.5368** | 0.5638** |
| $\boldsymbol{\varphi 2}$ | 0.0061** | 0.0027** | 0.0027** | 0.0069** | 0.0025** | 0.0036** |
| $\boldsymbol{\alpha 0}$ | 0.4796** | 0.6635** | 0.6203** | 0.5078** | 0.5337** | 0.6039** |
| $\boldsymbol{\alpha 1}$ | 0.3882** | 0.2963** | 0.3575** | 0.3896** | 0.2922** | 0.2532** |
| $\boldsymbol{\beta 1}$ | - | - | - | 0.6186** | 0.5323** | 0.4102** |
| $\boldsymbol{\gamma}$ | - | 3.5073** | 0.9493** | - | 3.5613** | 0.9507** |
| Log(L) | -1299.945 | -1814.682 | -1774.192 | -1300.345 | -1814.808 | -1774.270 |
| P-Value LR Test for Leverage Effect | | | | 0.08 | 0.00 | 0.00 |

**Table 7:** *MLE estimates of* $AR(2) - GARCH(1,1)$ *models*

| | GARCH-N | GARCH-T | GARCH-GED | GJR-N | GJR-T | GJR-GED |
|---|---|---|---|---|---|---|
| $\boldsymbol{\alpha}$ | 0.0163** | 0.0865* | 0.0104** | 0.0194** | 0.0205** | 0.0190** |
| $\boldsymbol{\varphi 1}$ | 0.4630** | 0.4757** | 0.5125** | 0.4600** | 0.4766** | 0.5132** |
| $\boldsymbol{\varphi 2}$ | 0.0921** | 0.0986** | 0.0740** | 0.0932** | 0.0983** | 0.0748** |
| $\boldsymbol{\alpha 0}$ | 0.0062** | 0.0026** | 0.0024** | 0.0063** | 0.0029** | 0.0020** |
| $\boldsymbol{\alpha 1}$ | 0.4664* | 0.6696** | 0.5085** | 0.5009** | 0.6108** | 0.5758** |
| $\boldsymbol{\beta 1}$ | 0.4068** | 0.3025** | 0.4113** | 0.4070** | 0.3009** | 0.2967** |
| $\boldsymbol{\gamma}$ | - | - | - | 0.0745** | 0.0253** | 0.4821** |
| V | - | 3.5731** | 0.9498** | - | 3.5726** | 0.9515** |
| Log (L) | -1305.098 | -1824.771 | -1781.028 | -1305.692 | -1824.785 | -1781.136 |
| P-Value LR Test for Leverage Effect | | | | 0.09 | 0.00 | 0.00 |

The t-distribution encompasses the normal distribution as its limiting case. According to "asymmetric" or "leverage" volatility models, positive and negative news exhibit differing predictability for future volatility. In the majority of these studies, researchers have provided compelling evidence that volatility is asymmetric in equity markets: negative returns typically correlate with significant upward adjustments of conditional volatility, whereas positive returns are linked to minimal upward or even downward adjustments of conditional volatility.

Researchers hypothesize that the asymmetry may result from alterations in leverage corresponding to fluctuations in equity value. Some have contended that the asymmetry may stem from the feedback mechanism between volatility and stock price, wherein fluctuations in volatility lead to alterations in risk premiums (refer to Kavitha et al. [37] and Duraisamy et al. [33]). Asymmetric volatility is most evident during a market crisis, characterized by substantial declines in stock prices accompanied by a notable rise in market volatility.

Asymmetric volatility may elucidate the negative skewness observed in stock return data, as articulated by Harvey and Siddique [49]. This paper demonstrates that a leverage effect is present in the National stock market, as evidenced by GJR models utilizing t-student and GED distributions, where the impact of adverse news on volatility ($\alpha_1$) surpasses that of favorable news ($\lambda$).

The P-Value LR Test for Leverage Effect in Tables 5-7 demonstrates that the disparity between the $\alpha_1$ and $\lambda$ coefficients is not significant for the GJR-N model; however, it is significant at the 5% confidence level for GJR models utilizing t-student and GED distributions.

## 4.2. Markov Regime-Switching GARCH

Tables 8-10 present the parameter estimates for MRS-GARCH models. Both models, one with constant degrees of freedom and the other allowing degrees-of-freedom parameters to transition between two regimes, exhibit highly significant in-sample estimates. All estimates of the conditional mean and parameters of conditional variance are statistically significant.

The estimates indicate the presence of two distinct states: the first regime is marked by low volatility and minimal persistence of shocks in conditional volatility, while the second regime exhibits high volatility and greater persistence. The transition probabilities are significant, indicating that nearly all regimes exhibit considerable persistence in MRS-GARCH models. The results present the unconditional probabilities for each MRS-GARCH model. The unconditional probability $\pi_1$ of being in the first regime varies from 20% to 86% across different MRSGARCH models. The unconditional probability $\pi_2$ of being in the second regime varies from 14% to 80% across different MR-SGARCH models.

**Table 8:** *MLE estimates of MRS-GARCH models*

| | MRS-GARCH-N | MRS-GARCH-t | MRS-GARCH-GED |
|---|---|---|---|
| $\boldsymbol{\delta}^{(1)}$ | 0.03302** | 0.2379** | 0.2917** |
| $\boldsymbol{\delta}^{(2)}$ | -0.0296** | -0.0803** | 0.0 |
| $\boldsymbol{\alpha}_0^{(1)}$ | 0 | 0.0024** | 0.0768** |
| $\boldsymbol{\alpha}_0^{(2)}$ | 0.4599** | 0.0205** | 0.0003** |
| $\boldsymbol{\alpha}_1^{(1)}$ | 0.4988** | 0.1150** | 0.8363** |
| $\boldsymbol{\alpha}_1^{(2)}$ | 0.6358** | 0.7954** | 0.0352** |
| $\boldsymbol{\beta}_1^{(1)}$ | 0.2744** | 0.8403** | 0.0051** |
| $\boldsymbol{\beta}_1^{(2)}$ | 0.3881** | 0.1614** | 0.7962** |
| P | 0.8321** | 0.9649** | 0.9347** |
| Q | 0.0 | 0.9745** | 0.9559** |
| $\mathbf{v}^{(1)}$ | - | 3.3174** | 0.7470** |
| Log (L) | -809.79925 | -580.6226 | -568.81478 |
| $\boldsymbol{\pi}_1$ | 0.86 | 0.39 | 0.40 |
| $\boldsymbol{\pi}_2$ | 0.14 | 0.61 | 0.60 |

The student's t variant of the MRS-GARCH models, characterized by constant degrees of freedom across regimes, exhibits a shape parameter of less than four, signifying the presence of conditional moments up to the third order. Allowing state-dependent parameters enables the modeling of the majority of leptokurtosis present in the data. In the GED case, the v parameter is below the threshold value of 2, indicating that the distribution exhibits thicker tails compared to the normal distribution.

The MRS-GARCH models incorporating Student's t innovations are introduced in a variant that permits the degrees of freedom to vary, indicating a time-dependent kurtosis, as discussed in Hansen [10] and Dueker [12].

Hansen proposes a model where the Student's t degrees-of-freedom parameter varies over time based on a logistic function of variables present in the information set up to time t-1. In contrast, Dueker [12] restricts this parameter to being state-dependent. This paper, following Marcucci [6], permits the degrees-of-freedom parameter to switch across regimes alongside all other parameters.

## 4.3 In Sample Statistics

Given the primary emphasis on predictive capability, we present only the mean squared error (MSE) for forecasting the rate of return and volatility in Table 11, omitting any formal tests. The optimal model for forecasting the rate of return of the National stock exchange, according to MSE criteria, is AR(2)-MRSGARCH with GED distribution. The second model is AR(2)-MRSGARCH with t distribution. Standard GARCH models, specifically AR(2)-GJR and AR(2)-GARCH with t distribution, demonstrate superior performance in forecasting rate of return compared to alternative models. The MSE criteria for forecasting rate of return demonstrate that GARCH models outperform ARMA models. The optimal model for forecasting volatility of National Stock Exchange, according to MSE criteria, is AR(2)-MRSGARCH with GED distribution, while the second-best model is AR(2)-MRSGARCH with t distribution. The AR(2)-GJR model with GED distribution outperforms other standard GARCH models.

**Table 9**: *MLE estimates of AR-(1)-MRS-GARCH models*

| | MRS-GARCH-N | MRS-GARCH-t | MRS-GARCH-GED |
|---|---|---|---|
| $\boldsymbol{\delta}^{(1)}$ | 0.0215** | 0.7012** | 0.2917** |
| $\boldsymbol{\delta}^{(2)}$ | -0.0332** | -0.06721** | -0.0093** |
| $\boldsymbol{\varphi}_1^{(1)}$ | 0.3503* | 0.4843** | 0.5213** |
| $\boldsymbol{\varphi}_1^{(2)}$ | 0.4320** | 0.47054** | 0.908** |
| $\boldsymbol{\alpha}_0^{(1)}$ | 0.0151** | 0.0216** | 0.0162** |
| $\boldsymbol{\alpha}_0^{(2)}$ | 0.1591** | 0.0328** | 0.0242** |
| $\boldsymbol{\alpha}_1^{(1)}$ | 0.3948** | 0.13234** | 0.6415* |
| $\boldsymbol{\alpha}_1^{(2)}$ | 0.6521** | 0.710** | 0.165** |
| $\boldsymbol{\beta}_1^{(1)}$ | 0.2433* | 0.8316** | 0.0033** |
| Q | 0.473* | 0.734** | 0.568** |
| $\mathbf{v}^{(1)}$ | - | 3.392** | 0.684** |
| Log (L) | -729.68410 | -562.7126 | -547.36478 |
| $\boldsymbol{\pi}_1$ | 0.77 | 0.35 | 0.30 |
| $\boldsymbol{\pi}_2$ | 0.23 | 0.65 | 0.70 |

## 4.4 Out-of-Sample Forecast Evaluation

A potential solution to the issues identified in the preceding section is to evaluate the models based on their out-of-sample forecasting performance. An out-of-sample test effectively addresses potential issues of overfitting or overparameterization, providing a robust framework for assessing the performance of competing models. Most models serve as simple approximations of the true data-generating process; thus, a good in-sample fit does not necessarily guarantee accurate or reliable forecasts. Researchers and practitioners prioritize accurate volatility forecasts over strong in-sample fits, which are often more achievable with highly parameterized models like MRS-GARCH. Table 12 presents the DM test results for the equal predictive ability of the GJR model with Student's t innovations compared to other GARCH models.

The results demonstrate that the DM test, which assesses the equal predictive ability of the AR-(2)-MRSGARCH model with GED innovations compared to other GARCH and MRSGARCH models, allows for the rejection of the null hypothesis of no difference in accuracy between the two competing forecasts at 1-, 5-, and 10-day horizon periods. The accuracy of forecasting does not differ between the AR-(2)-MRSGARCH-t and AR(2)-MRSGARCH-GED models at a 5-day period horizon. Furthermore, the accuracy of forecasting does not differ between the AR-(2)-MRSGARCH-t and AR-

(2)-MRSGARCH-N models when compared to the AR-(2)-MRSGARCH-GED model at a 10-day horizon. Consequently, according to MSE criteria, the AR-(2)-MRSGARCH-GED model demonstrates superior performance compared to alternative models at a one-day horizon.

The AR(2)-MRSGARCH-GED and AR(2)-MRSGARCH-t models demonstrate superior performance compared to other models at the 5-day horizon. Over a 10-day horizon, three AR(2)-MRSGARCH models demonstrate superior performance compared to alternative models.

**Table 10:** *MLE estimates of AR(2)-MRS-GARCH models*

| | MRS-GARCH-N | MRS-GARCH-t | MRS-GARCH-GED |
|---|---|---|---|
| $\delta^{(1)}$ | 0.0354** | 0.1442** | 0.3110** |
| $\delta^{(2)}$ | -0.0112** | -0.0031* | -0.004** |
| $\varphi_1^{(1)}$ | 0.31274** | 0.45671** | 0.3743** |
| $\varphi_1^{(2)}$ | 0.2343* | 0.3334** | 0.4690** |
| $\varphi_2^{(1)}$ | 0.251** | 0.3490** | 0.2640** |
| $\varphi_2^{(2)}$ | 0.1653** | 0.234** | 0.2784** |
| $\alpha_0^{(1)}$ | 0.0226** | 0.0267** | 0.0356* |
| $\alpha_0^{(2)}$ | 0.0371** | 0.0312** | 0.0056** |
| $\alpha_1^{(1)}$ | 0.3211** | 0.1715** | 0.4531** |
| $\alpha_1^{(2)}$ | 0.4212** | 0.7305** | 0.1872** |
| $\beta_1^{(1)}$ | 0.2214** | 0.1207** | 0.0067** |
| $\beta_1^{(2)}$ | 0.3301** | 0.1564* | 0.4698** |
| P | 0.8626** | 0.9547* | 0.9271** |
| Q | 0.6735** | 0.8894** | 0.9743** |
| $v^{(1)}$ | - | 3.9142** | 0.5731** |
| Log (L) | -668.3529 | -551.6271 | -534.37908 |
| $\pi_1$ | 0.70 | 0.20 | 0.26 |
| $\pi_2$ | 0.30 | 0.80 | 0.74 |

**Table 11:** *In sample forecasting criteria*

| Model | N. of Par. | MSE of forecasting return | Rank | MSE of forecasting volatility | Rank |
|---|---|---|---|---|---|
| AR(1) | 2 | 0.1834 | 29 | - | - |
| MA(1) | 2 | 0.1883 | 30 | - | - |
| AR(2) | 3 | 0.1806 | 26 | - | - |
| ARMA(1,1) | 3 | 0.1809 | 28 | - | - |
| ARMA(1,2) | 4 | 0.1808 | 27 | - | - |
| GARCH-N | 4 | 0.1797 | 23 | 0.051 | 15 |
| GARCH-t | 5 | 0.1797 | 23 | 0.058 | 19 |
| GARCH-GED | 5 | 0.1794 | 20 | 0.054 | 16 |
| AR(1)-GARCH-N | 5 | 0.1654 | 16 | 0.050 | 14 |
| AR(1)-GARCH-t | 6 | 0.1658 | 17 | 0.049 | 13 |
| GJR-t | 6 | 0.1795 | 21 | 0.056 | 18 |
| GJR-GED | 6 | 0.1794 | 20 | 0.055 | 17 |
| AR(1)-GJR-N | 6 | 0.1653 | 15 | 0.049 | 13 |
| AR(2)-MRSGARCH-N | 14 | 0.0788 | 5 | 0.034 | 4 |

The DM test conducted over a 22-day forecast horizon reveals no significant difference in forecasting accuracy between MRSGARCH models and standard GARCH models. Table 12 presents the out-of-sample evaluation of risk management for our competing GARCH models across 1-day, 1-week, 2-week, and 1-month horizons. For each forecast horizon, five statistics are provided: the TUFF, the proportion of failures (PF), the test of correct unconditional coverage (LR_PF), which assesses whether PF is significantly greater than the nominal rate; the "$LR_{ind}$" which evaluates independence; and the "$LR_{cc}$" which examines the correct conditional coverage.

The ranking for each forecast horizon reflects the order based on the percentage PF. Models exhibiting a PF exceeding the coverage probability of 5% are deemed inadequate. The theoretical TUFF at 5% is expected to be 20. We can thus see that only for 95%VaR at one-day ahead, there are values on the theoretical ones except for AR(2)-MRSGARCH-t and AR(2)-MRSGARCHGED. At all other forecast horizons, the TUFF exceeds 20. Furthermore, if the aim is to account for 95% of future losses, numerous models appear insufficient, particularly at both the shortest and longest forecast horizons.

**Table 12:** *Risk management out-of-sample evaluation* 95% *VaR*

| Step Model | Rank | LRPF | LRind | LRcc |
|---|---|---|---|---|
| GARCH-N | 26 | 6.53 * | 87.53 * | 143.86 * |
| GARCH-t | 18 | 2.45 | 98.45 | 65.45 |
| GARCH-GED | 21 | 7.53* | 79.53* | 147.86* |
| AR(1)-GARCH-N | 23 | 1.45 | 83.45* | 156.35* |
| AR(1)-GARCH-t | 22 | 8.86 * | 88.86* | 163.66 * |
| AR(1)-GARCH-GED | 17 | 2.10 | 98.10* | 171.57* |
| AR(2)-GARCH-N | 15 | 9.94 | 110.94* | 34.85 * |
| AR(2)-GARCH-t | 20 | 0.86 | 123.86* | 67.64* |
| AR(2)-GARCH-GED | 11 | 1.45 | 78.90 | 89.76* |
| GJR-N | 27 | 8.86* | 45.86* | 87.45* |
| GJR-t | 19 | 2.97 | 33.97 * | 98.86* |
| GJR-GED | 25 | 9.65 * | 87.65* | 187.97* |
| AR(1)-GJR-N | 14 | 2.34 | 45.34 | 169.65 |
| AR(1)-GJR-t | 9 | 11.10* | 120.10* | 143.34 |
| AR(1)-GJR-GED | 24 | 2.94 | 93.94 * | 165.67* |
| AR(2)-GJR-N | 10 | 7.86* | 91.86* | 176.94 * |
| AR(2)-GJR-t | 16 | 1.45 | 111.45 * | 181.86* |
| AR(2)-GJR-GED | 13 | 9.86* | 121.86* | 196.45* |
| MRSGARCH-N | 12 | 12.53* | 211.45 | 232.67 * |
| MRSGARCH-t | 7 | 17.25* | 231.86* | 224.94* |
| MRSGARCH-GED | 8 | 21.53* | 243.97* | 276.86 * |
| AR(1)-MRSGARCH-N | 6 | 12.45 * | 257.65* | 287.86* |
| AR(1)-MRSGARCH-t | 5 | 14.86* | 262.34 | 301.45* |
| AR(1)-MRSGARCH-GED | 3 | 22.10 | 261.67* | 365.86* |
| AR(2)-MRSGARCH-N | 4 | 15.94* | 234.97 | 341.35* |
| AR(2)-MRSGARCH-t | 2 | 13.86 * | 218.84* | 356.66* |
| AR(2)-MRSGARCH-GED | 1 | 11.45 * | 217.43* | 378.57* |

The final three LR tests reject nearly all models at extended horizons. At all horizons, the AR(2)-MRSGARCH-t and AR(2)-MRSGARCH-GED models are consistently rejected based on the statistical forecast evaluation criteria due to an excessively high PF. The performance of these models remains ambiguous. No model can consistently pass all tests over extended time horizons. Consequently, several models appear to yield reasonable and precise Value at Risk (VaR) estimates for a 1-day horizon, exhibiting a coverage rate that closely aligns with the nominal level. There is no universally most accurate model based on the risk management loss functions. The results corroborate the findings of Brooks and Persand [30], who reported an absence of definitive answers for the majority of the series they investigated.

## 5. Conclusion

This paper compares standard GARCH models with Markov Regime-Switching GARCH models regarding their effectiveness in forecasting volatility in the National Stock Exchange (NSE). This includes AR-GARCH and ARMA-GARCH models with various residual distributions (Normal, t-student, and GED) on one hand, and AR-MSGARCH and ARMA-MSGARCH models with different residual distributions (Normal, t-student, and GED) on the other. The forecasts encompass volatility predictions for 1-day, 5-day, 10-day, and 22-day intervals. We have acquired the out-of-sample MSE statistic for the various models. We employed the DM statistic to compare the significant differences in MSE across various models. The findings demonstrate that the AR(2)-MRSGARCH-GED model surpasses all alternative models at the one-day horizon.

The AR(2)-MRSGARCH-GED and AR(2)-MRSGARCH-t models demonstrate superior performance compared to other models at the 5-day horizon. Over a 10-day horizon, three models of AR(2)-MRSGARCH with varying residual distributions demonstrate superior performance compared to alternative models. The DM test conducted over a 22-day forecast horizon reveals no significant difference in forecasting accuracy between MRSGARCH models and standard GARCH models. The out-of-sample evaluation of risk management using 95% VaR reveals ambiguity regarding which model performs optimally. Over extended timeframes, no model can consistently meet all evaluation criteria. Consequently, several models appear to yield reasonable and accurate Value at Risk (VaR) estimates for a 1-day horizon, exhibiting a coverage rate that closely aligns with the nominal level. There is no universally most accurate model based on risk management loss functions.

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