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EVALUATING MULTI SERVER QUEUING SYSTEM EFFICIENCY: A COMPARATIVE STUDY BETWEEN FUZZY QUEUING MODEL AND INTUITIONISTIC FUZZY QUEUING MODEL WITH INFINITE CAPACITY CONSTRAINTS

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Abstract

In this study, we suggest an innovative analytical approach employing triangular fuzzy and triangular intuitionistic fuzzy numbers to ascertain the membership functions governing significant service-execution proportions within multiserver queuing models. Departing from conventional methodologies, both the inter-entry and completion rates are characterized by fuzzy natures, introducing a novel dimension to the analysis. Through numerical validation, our model demonstrates viability in the multiserver queuing context, bolstering its credibility. Furthermore, we conduct a contextual investigation, juxtaposing individual fuzzy metrics, thereby revealing the superior categorization potential of intuitionistic fuzzy queuing models over their fuzzy counterparts. Expanding the horizons of traditional fuzzy queuing theory, our integration of intuitionistic fuzzy environments promises enhanced queuing model implementation efficacy. Our primary objective lies in evaluating the productivity of a multi-server queuing model with limitless capacity, leveraging both fuzzy queuing and intuitionistic fuzzy queuing theories. Embracing a framework where arrival and service rates are characterized by triangular and intuitionistic triangular fuzzy numbers, we undertake a thorough assessment to establish evaluation criteria, adhering to a design protocol that preserves fuzzy values without converting them into crisp values. Additionally, we address two statistical challenges to ascertain the method's validity.

Keywords: queuing model, fuzzy numbers, multi-server, execution proportions

I. Introduction

Queuing models have found extensive application in various domains, aiding in the formulation of routine procedures, workplace configurations, and labor regulations. However, real-world scenarios often present challenges where certain variables within these queuing models may be inaccurately known due to various factors. As a result, there arises a need to integrate fuzzy logic into queuing systems, enabling adaptability to diverse real-life situations. Nevertheless, in numerous practical instances, precise data is crucial for effective decision-making by system designers and researchers.

To address the inherent vagueness in such situations, many scholars have proposed methodologies. One such approach involves constructing a membership function for queue performance measures, thereby providing a means to navigate ambiguity. Additionally, the notion of intuitionistic fuzzy sets [2] has emerged as a viable method for defining fuzzy sets, particularly when data accessibility is limited, making it challenging to determine ambiguous conceptions accurately.

Acknowledging the quandary of operationalizing fuzzy set hypotheses, given that fuzzy numbers deviate from a traditional linear approach like real numbers, the paper propounds a methodology for resolving a multiple transmission queuing model in both fuzzy and intuitionistic fuzzy environments without undermining their essence. In comparison to antecedent methodologies, this approach is advantageous due to its succinctness, adaptability, and relevance. The analysis outcomes suggest that the performance metrics of the fuzzy queuing model align with the estimated spectrum of the intuitionistic fuzzy queuing model.

Building upon foundational research by Lie and Lee [10] and Buckley [4], subsequent progressions by Negi and Lee [12] and Chen [6] expanded upon these models, introducing fuzzified exponential time based on queuing theory. Srinivasan [17] introduced a fuzzy queuing model using the DSW algorithm, while Thamotharan [18] investigated a multi-server fuzzy queuing model using triangular and trapezoidal fuzzy numbers with α cuts. Shanmugasundaram et al. [15] developed a predictive fuzzy multi-server queuing model utilizing the DSW algorithm. Madhuri et al. (2017) conducted an analysis of fuzzy queues employing Zadeh's [19] augmentation rule, and Selvam et al. [14] proposed a ranking method for pentagonal fuzzy numbers applied to the incentre of centroids. Samouylov et al. [13] assessed a multi-server queuing system catering to two correlated flows of requests. Bin Sun et al. [3] investigated a multi-server queuing system with two input flows. Dudin et al. [8] interpreted a multi-server vacation queuing model. Kuaban et al. [9] scrutinized a multi-server queuing model with balking and correlated reneging, with implications for healthcare management. Anupama Kumar [1] examined a $M/M/3$ queuing system with a queue-dependent multi-server. Diletta Olliaro et al. [7] studied a multiserver queue where jobs request a variable number of servers for random service time. Selvam et al. [14] also explored a ranking technique for handling fuzzy numbers. S.P. Chen [5] used the α-cut approach to convert a fuzzy bulk-service queue into a series of traditional crisp bulk-service queues. Anggraito et al. [20] determined the stability condition for a multi-server queuing model with both large and small jobs, assuming an infinite number of servers and considering various service time distributions for the small jobs. Golovin et al. [21] studied about the multiresource queuing system with multiserver customers. Anupama [22] considered $M/M/3$ Queuing system with a Queue-dependent multi-server. Fuzzy set theory, an extension of traditional set theory, addresses the limitations of crisp numbers in handling uncertainty. Crisp numbers, commonly used in applications, may not be suitable for scenarios with uncertainties at multiple points. In such cases, intuitionistic fuzzy numbers prove to be a more robust tool.

The central objective of this work is to juxtapose fuzzy queuing models against intuitionistic fuzzy queuing models while also exploring methods to tackle fuzzy queuing challenges without fundamentally altering their core tenets. Existing endeavors typically involve translating fuzzy queuing problems into classical equivalents and then employing various techniques such as the α-cut approach, the L-R approach, the interval arithmetic approach, and the DSW algorithm for resolution. However, the outcomes derived from these methods produce definite results, thereby altering the intrinsic nature of the original problem. This manuscript, titled " Evaluating Multi-Server Queuing System Efficiency: A Comparative Study between Fuzzy Queuing Model and Intuitionistic Fuzzy Queuing Model with Infinite Capacity Constraints," endeavors to circumvent this issue. We propose a methodology to address queuing problems without the need for their conversion into classical equivalents, thus facilitating a direct comparison between fuzzy and intuitionistic fuzzy queuing models. The effectiveness of our proposed approach is demonstrated through the resolution of numerical problems and subsequent comparative analysis of outcomes.

The structural framework of the article can be delineated as follows: Module 2 elucidates essential definitions, while Module 3 delves into the intricate details of the system model. Manuals and preconceptions are scrutinized in Module 4, followed by the exploration of fundamental theorems in Module 5. Module 6 presents the envisaged queuing model. Module 7 offers a mathematical illustration, providing a tangible demonstration of the proposed methodology's efficacy. Module 8 serving as the culminating segment of the framework.

II. Preliminaries

The motive of this division is to give some basic definitions, annotations, and outcomes that are used in our subsequent calculations.

Definition 2.1. [11] A fuzzy number $\tilde{A}$ is defined on R, the set of real numbers is said to be a triangular fuzzy number (TFN) if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which satisfy the following conditions:

$$\mu_{\tilde{A}}(x)= \begin{cases} \frac{x-\tilde{\alpha}_1}{\tilde{\alpha}_2-\tilde{\alpha}_1} \text{ for } \tilde{\alpha}_1 \le x \le \tilde{\alpha}_2 \\ 1 \text{ for } x = \tilde{\alpha}_2 \\ \frac{\tilde{\alpha}_3-x}{\tilde{\alpha}_3-\tilde{\alpha}_2} \text{ for } \tilde{\alpha}_2 \le x \le \tilde{\alpha}_3 \\ 0 \text{ otherwise} \end{cases}$$

Definition 2.2. [11] Let the two triangular fuzzy numbers be $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2)$ and then the arithmetic operations on TFN be given as follows:

Addition

$$\tilde{P} + \tilde{Q} \approx (\tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{1}$$

Subtraction

$$\tilde{P} - \tilde{Q} \approx (\tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{2}$$

Multiplication

$$\tilde{P}.\tilde{Q} \approx (\tilde{m}_1.\tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \tag{3}$$

Division

$$\frac{\tilde{P}}{\tilde{Q}} \approx \left(\frac{\tilde{m}_1}{\tilde{m}_2}, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \tag{4}$$

Scalar Multiplication

$$\begin{aligned} k\tilde{P} &\approx (ka_2, \alpha_1, \beta_1), k \ge 0 \\ k\tilde{P} &\approx (-ka_2, \alpha_1, \beta_1), k < 0 \end{aligned} \tag{5}$$

A triangular fuzzy number $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) \in F(R)$ is known to be positive if $\Re(\tilde{P}) > 0$ and defined by $\tilde{P} \succ 0$.

Definition 2.3. [16] A fuzzy number $\tilde{A}'$ on R is said to be a triangular intuitionistic fuzzy number (TIFN) if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and non – membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ have the following conditions:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \dfrac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \le x \le \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \dfrac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \le x \le \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

and

$$\gamma_{\tilde{A}'}(x) = \begin{cases} 1 \text{ for } x < \tilde{a}'_1, x > \tilde{a}'_3 \\ \dfrac{\tilde{a}_2-x}{\tilde{a}_2-\tilde{a}'_1} \text{ for } \tilde{a}'_1 \le x \le \tilde{a}_2 \\ 0 \text{ for } x = \tilde{a}_2 \\ \dfrac{x-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \le x \le \tilde{a}'_3 \end{cases}$$

and is given by $\tilde{A}' = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ where $\tilde{a}'_1 \le \tilde{a}_1 \le \tilde{a}_2 \le \tilde{a}_3 \le \tilde{a}'_3$

Definition 2.4. The extension of fuzzy arithmetic operations of Ming Ma et al. [11] to the set of TIFN based upon both location indices and functions of fuzziness indices. The location indices number is taken in the regular arithmetic while the functions of fuzziness indices are assumed to follow the lattice rule, which is the least upper bound in the lattice $\tilde{I}'$. For any two arbitrary TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}'_1, \tilde{\beta}'_1)$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}'_2, \tilde{\beta}'_2)$ and $* \in \{+, -, \times, \div\}$, then the arithmetic operations on TIFN are defined by $\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}'_1 \vee \tilde{\alpha}'_2, \tilde{\beta}'_1 \vee \tilde{\beta}'_2)$.

In particular, for any two TIFNs $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}'_1, \tilde{\beta}'_1)$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}'_2, \tilde{\beta}'_2)$ the arithmetic operations are defined as

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}'_1, \tilde{\beta}'_1) * (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}'_2, \tilde{\beta}'_2)$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}'_1, \tilde{\alpha}'_2\}, max\{\tilde{\beta}'_1, \tilde{\beta}'_2\})$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}'_1 \vee \tilde{\alpha}'_2, \tilde{\beta}'_1 \vee \tilde{\beta}'_2)$$

In particular, for any two TIFN $\tilde{P}' \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}'_1, \tilde{a}'_2, \tilde{a}'_3) \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}'_1, \tilde{\beta}'_1)$, $\tilde{Q}' \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}'_1, \tilde{b}'_2, \tilde{b}'_3) \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}'_2, \tilde{\beta}'_2)$we define:

Addition

$$\tilde{P}' + \tilde{Q}' = (m_1 + m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 + m_2, max\{\alpha'_1, \alpha'_2\}, max\{\beta'_1, \beta'_2\}) \quad (6)$$

Subtraction

$$\tilde{P}' - \tilde{Q}' = (m_1 - m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 - m_2, max\{\alpha'_1, \alpha'_2\}, max\{\beta'_1, \beta'_2\}) \quad (7)$$

Multiplication

$$\tilde{P}' \times \tilde{Q}' = (m_1 \times m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 \times m_2, max\{\alpha'_1, \alpha'_2\}, max\{\beta'_1, \beta'_2\}) \quad (8)$$

Division

$$\tilde{P}' \div \tilde{Q}' = (m_1 \div m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 \div m_2, max\{\alpha'_1, \alpha'_2\}, max\{\beta'_1, \beta'_2\}) \quad (9)$$

Scalar Multiplication

$$k\tilde{P}' = (ka_2, \alpha_1, \beta_1; ka_2, \alpha'_1, \beta'_1), \text{ for } k \ge 0 \quad (10)$$

$$k\tilde{P}' = (-ka_2, \alpha_1, \beta_1; -ka_2, \alpha'_1, \beta'_1), \text{ for } k < 0$$

The triangular fuzzy number and triangular intuitionistic fuzzy number are illustrated in Figures 1 and 2.

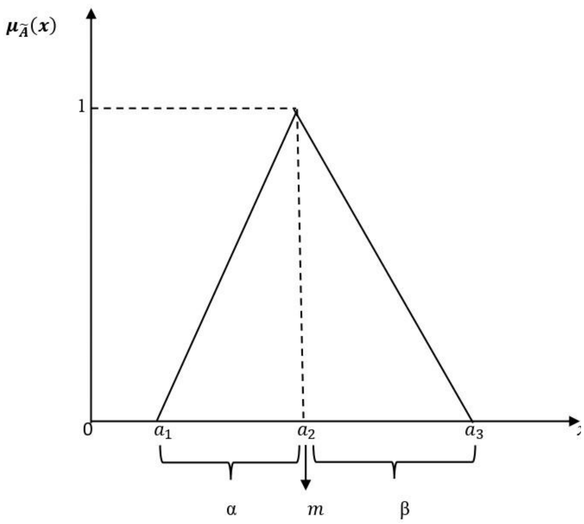

Figure 1: *Triangular fuzzy number*

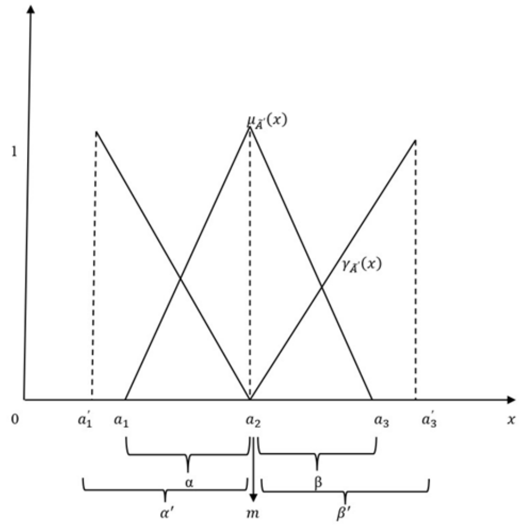

Figure 2: *Triangular intuitionistic fuzzy number*

III. Model Description

We suggest a queuing model with an infinite population and multiple parallel service facilities, following a First-Come-First-Served (FCFS) rule denoted as $(FM/FM/C)$: $(\infty/FCFS)$. In this model, entry times and service durations adhere to Poisson and Exponential distributions, respectively, with fuzzy parameters λ and μ.

Both TFN and TIFN are used for determining the entry rate and processing rate. By analyzing both triangular fuzzy numbers and intuitionistic fuzzy numbers, performance measures are determined, and models are compared according to the mean number of customers in the sequence and infrastructure as well as the duration of their time in the queue and system. As long as we maintain the fuzziness values until the end of the process, the problems can be solved without changing them from fuzzy to crisp. By doing so, it becomes easier to apply to real-life situations. Figure 3 depicts this situation.

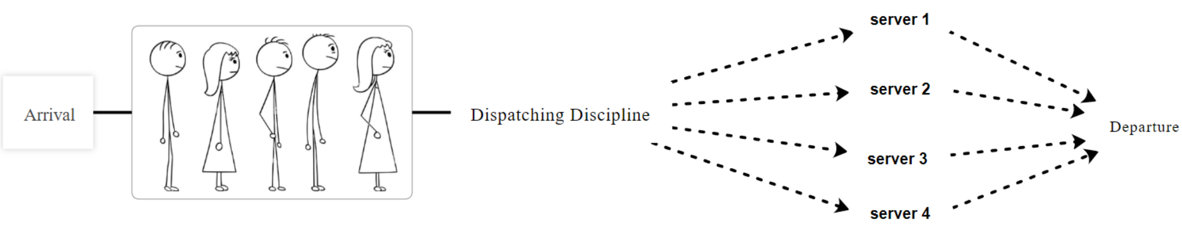

Figure 3: *Multiserver queuing system*

IV. Concerns and Engravings

I. Concerns

i) Consider the endless capacity $FM/FM/c$ fuzzy queuing model under the FCFS discipline.

ii) The system is served by a multiple server.

iii) The time between arrivals follows a Poisson distribution.

iv) Service time is distributed exponentially.

v) Both arrival rate and service rate are described using triangular fuzzy numbers and intuitionistic triangular fuzzy numbers.

II. Syntaxes

Here we are using the following notations:

$\tilde{\lambda}, \tilde{\lambda}'$ → The average number of patrons who arrive in a predetermined period of time.

$\tilde{\mu}, \tilde{\mu}'$ → The average number of patrons being serviced per unit of time.

$\tilde{\rho}$ →Traffic intensity.

$\tilde{N}_q, \tilde{N}_q'$ → The estimated number of patrons in the line.

$\tilde{N}_s, \tilde{N}_s'$ →The estimated number of patrons in the system.

$\tilde{T}_q, \tilde{T}_q'$ → The estimated sojourn time of the patrons in the queue.

$\tilde{T}_s, \tilde{T}_s'$ → The estimated sojourn time of the patrons in the system.

c → Number of servers

$\tilde{P}, \tilde{P}'$ → Interarrival rate.

$\tilde{Q}, \tilde{Q}'$ → Service rate.

X → Arrival time schedule.

Y → Service time schedule.

V. Multiserver queuing model with infinite capacity

The Probability on n units in the $(FM/FM/c)$: $(\infty/FIFO)$ is given as

The birth and death process in steady state,

$$\tilde{P}_n' = \left(\frac{\tilde{\lambda}_0'\tilde{\lambda}_1'\tilde{\lambda}_2'\tilde{\lambda}_3' \dots \tilde{\lambda}_{n-1}'}{\tilde{\mu}_1'\tilde{\mu}_2'\tilde{\mu}_3'\tilde{\mu}_4' \dots \tilde{\mu}_n'}\right)\tilde{P}_0' \quad (11)$$

For this model,

$$\tilde{\lambda}_n' = \tilde{\lambda}' \quad \text{for all } n$$

$$\tilde{\mu}_n' = \begin{cases} n\tilde{\mu}' & \text{for all } n < c \\ c\tilde{\mu}' & \text{for all } n \geq c \end{cases}$$

$$\tilde{P}_n' = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}_0' & \text{for all } n < c \\ \dfrac{\tilde{\lambda}'^n}{(1\tilde{\mu}')(2\tilde{\mu}')(3\tilde{\mu}')(4\tilde{\mu}') \cdots (c-1)\tilde{\mu}'(c\tilde{\mu}')(c\tilde{\mu}')(c\tilde{\mu}') \cdots (n-(c-1))times}\tilde{P}_0' & \text{for all } n \geq c \end{cases}$$

$$\tilde{P}'_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 & \text{for all } n < c \\ \dfrac{\tilde{\lambda}'^n}{(c-1)!\,\tilde{\mu}'^{c-1} \quad c^{n-c+1}\tilde{\mu}'^{n-c+1}} \tilde{P}'_0 & \text{for all } n \geq c \end{cases}$$

$$\tilde{P}'_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 & \text{for all } n < c \\ \dfrac{\tilde{\lambda}'^n}{c!\ c^{n-c}\tilde{\mu}'^n} \tilde{P}'_0 & \text{for all } n \geq c \end{cases}$$

$$\tilde{P}'_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 & \text{for all } n < c \\ \dfrac{1}{c!\ c^{n-c}}\left(\dfrac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 & \text{for all } n \geq c \end{cases}$$

From the property of discrete random variable, we have

$$\sum_{n=0}^{\infty} \tilde{P}'_n = 1$$

$$\Rightarrow \sum_{n=0}^{c-1} \tilde{P}'_n + \sum_{n=c}^{\infty} \tilde{P}'_n = 1$$

$$\Rightarrow \sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 + \sum_{n=c}^{\infty} \frac{1}{c!c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \tilde{P}'_0 = 1$$

$$\Rightarrow \tilde{P}'_0\left(\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n + \sum_{n=c}^{\infty} \frac{1}{c!c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n\right) = 1 \tag{12}$$

Consider

$$\sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n = \frac{1}{c!\,c^{-c}} \sum_{n=c}^{\infty} \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^n$$

$$\sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n = \frac{1}{c!\,c^{-c}}\left(\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^c + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^{c+1} + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^{c+2} + \cdots\right)$$

$$\sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n = \frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^c}{c!\,c^{-c}}\left(1 + \frac{\tilde{\lambda}'}{\tilde{\mu}'c} + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^2 + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^3 + \cdots\right)$$

$$\sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n = \frac{1}{c!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c\left(1 - \frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)^{-1}$$

$$\sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n = \frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{c!\left(1 - \frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)} \tag{13}$$

Substituting Equation (13) in Equation (12), we get

$$\tilde{P}'_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n+\frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{c!\left(1-\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)}\right]=1$$

$$\tilde{P}'_0=\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n+\frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{c!\left(1-\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)}\right]^{-1}$$

Or, we can write

$$\tilde{P}_0'^{-1}=\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n+\frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{c!\left(1-\frac{\tilde{\lambda}'}{\tilde{\mu}'c}\right)}$$

Theorem 5.1. In the $(FM/FM/c):(\infty/FIFO)$ queuing model, that there will be n patrons, where $n = 0$ (n is an integer) in the system at any time $(\tilde{t}' + \Delta\tilde{t}')$, then prove that the rate is $\tilde{\lambda}'\tilde{P}'_0 = \tilde{\mu}'\tilde{P}'_1$.

Proof. For $n = 0$,
The possibility that somehow there won't be any patrons in the system at $(\tilde{t}' + \Delta\tilde{t}')$ will be equal to the sum of the following two separate probabilities:

i) $\tilde{P}'_0(\tilde{t}').\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right)$ is the statistical likelihood that there would be no patrons in the system at $\tilde{t}'$ and no arrival at $\Delta\tilde{t}'$

ii) $\tilde{P}'_1(\tilde{t}').\tilde{\mu}'\Delta\tilde{t}'.\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right) \cong \tilde{P}'_1(\tilde{t}')\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}')$ is the statistical likelihood that one patron is present in the system at $\tilde{t}'$, one patron is serviced at $\Delta\tilde{t}'$ and no arrival at $\Delta\tilde{t}'$.

As a result, the probability of $n = 0$ is

$$\tilde{P}'_0(\tilde{t}'+\Delta\tilde{t}') = \tilde{P}'_0(\tilde{t}')\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right) + \tilde{P}'_1(\tilde{t}')\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}') \tag{14}$$

$$\tilde{P}'_0(\tilde{t}'+\Delta\tilde{t}') - \tilde{P}'_0(\tilde{t}') = -\tilde{\lambda}'\Delta\tilde{t}'\tilde{P}'_0(\tilde{t}') + \tilde{P}'_1(\tilde{t}')\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}')$$

Now divide the aforementioned Equation by $\Delta\tilde{t}'$ and assuming the limit as $\Delta\tilde{t}' \to 0$, hence it transforms into:

$$\Delta\tilde{t}' \to 0\left[\frac{\tilde{P}'_0(\tilde{t}'+\Delta\tilde{t}') - \tilde{P}'_0(\tilde{t}')}{\Delta\tilde{t}'}\right] = -\tilde{\lambda}'\tilde{P}'_0(\tilde{t}') + \tilde{\mu}'\tilde{P}'_1(\tilde{t}'); n = 0$$

$$\tilde{P}'_0(\tilde{t}') = -\tilde{\lambda}'\tilde{P}'_0(\tilde{t}') + \tilde{\mu}'\tilde{P}'_1(\tilde{t}') \text{ for } n = 0 \tag{15}$$

In the scenario of a steady state where $\tilde{t}' \to \infty$, $\tilde{P}'_n(\tilde{t}') \to \tilde{P}'_n$ (independent of $\tilde{t}'$) and consequently $\tilde{P}'_n(\tilde{t}') \to 0$. Eventually, the steady state difference equations of the system are provided by

$$0 = -\tilde{\lambda}'\tilde{P}'_0 + \tilde{\mu}'\tilde{P}'_1 \qquad \text{(From Equation (15))}$$

Furthermore, $\tilde{\lambda}'\tilde{P}'_0 = \tilde{\mu}'\tilde{P}'_1; n = 0$ (16)

Hence the proof.

Theorem 5.2. In the $(FM/FM/c):(\infty/FIFO)$ queuing model, at the state $1 \le n \le c-1$, where $0 \le n \le c$ prove that the rate is $\left(\tilde{\lambda}' + n\tilde{\mu}'\right)\tilde{P}'_n = \tilde{\lambda}'\tilde{P}'_{n-1} + (n+1)\tilde{\mu}'\tilde{P}'_{n+1}$.

Proof. Let,

$$\tilde{\lambda}'_n = \tilde{\lambda}' \quad \text{for all } n$$

$$\tilde{\mu}'_n = \begin{cases} n\tilde{\mu}' & \text{for all } n < c \\ c\tilde{\mu}' & \text{for all } n \ge c \end{cases}$$

For $n = 1,2,3,\dots c-1$,

When independent conditions are incorporated together, the probability will be shown as follows:

i) At any time $\tilde{t}'$, there are n patrons in the system are $\tilde{P}_n'(\tilde{t}')$. However, at $\Delta\tilde{t}'$, there are no arrivals and no services respectively, $(1-\tilde{\lambda}'\Delta\tilde{t}')$ and $(1-n\tilde{\mu}'\Delta\tilde{t}')$. As a consequence, the probability is provided as

$$\Rightarrow [\tilde{P}_n'(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}_n'(\tilde{t}').\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right).(1-n\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}_n'(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}_n'(\tilde{t}').\left(1-\tilde{\lambda}'\Delta\tilde{t}'-n\tilde{\mu}'\Delta\tilde{t}'\right)+o_1(\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}_n'(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}_n'(\tilde{t}')\left(1-\Delta\tilde{t}'\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\right)+o_1(\Delta\tilde{t}') \quad (17)$$

ii) At any time $\tilde{t}'$, there are $(n-1)$ patrons in the system are $\tilde{P}_{n-1}'(\tilde{t}')$; there is one entrance at $\Delta\tilde{t}'$ is $\tilde{\lambda}'\Delta\tilde{t}'$ and no assistance at $\Delta\tilde{t}'$ is $(1-n\tilde{\mu}'\Delta\tilde{t}')$. As a result, the probability is given as

$$\Rightarrow [\tilde{P}_{n-1}'(\tilde{t}')]_{\tilde{\lambda}'=1;\tilde{\mu}'=0} = \tilde{P}_{n-1}'(\tilde{t}').\tilde{\lambda}'\Delta\tilde{t}'.(1-n\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}_{n-1}'(\tilde{t}')]_{\tilde{\lambda}'=1;\tilde{\mu}'=0} = \tilde{\lambda}'\Delta\tilde{t}'\tilde{P}_{n-1}'(\tilde{t}')+o_2(\Delta\tilde{t}') \quad (18)$$

iii) At any time $\tilde{t}'$, there are $(n+1)$ patrons in the system are $\tilde{P}_{n+1}'(\tilde{t}')$; there is no inflow at $\Delta\tilde{t}'$ is $\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right)$ and one service at $\Delta\tilde{t}'$ is $(n+1)\tilde{\mu}'\Delta\tilde{t}'$. As an outcome, the probability is given as

$$\Rightarrow [\tilde{P}_{n+1}'(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=1} = \tilde{P}_{n+1}'(\tilde{t}').\left(1-\tilde{\lambda}'\Delta\tilde{t}'\right).(n+1)\tilde{\mu}'\Delta\tilde{t}'$$

$$\Rightarrow [\tilde{P}_{n+1}'(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=1} = (n+1)\tilde{\mu}'\Delta\tilde{t}'\tilde{P}_{n+1}'(\tilde{t}')+o_3(\Delta\tilde{t}') \quad (19)$$

Now adding the above three Equations (17), (18) and (19) independent compound probabilities, we obtain the probability of n patrons in the system at the time $(\tilde{t}'+\Delta\tilde{t}')$. We obtain

$$\tilde{P}_n'(\tilde{t}'+\Delta\tilde{t}') = \tilde{P}_n'(\tilde{t}')\left[1-\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\Delta\tilde{t}'\right]+\tilde{\lambda}'\Delta\tilde{t}'\tilde{P}_{n-1}'(\tilde{t}')+(n+1)\tilde{\mu}'\Delta\tilde{t}'\tilde{P}_{n+1}'(\tilde{t}')+o(\Delta\tilde{t}'); 1\le n\le c-1 \quad (20)$$

The Equation (20) now becomes

$$\tilde{P}_n'(\tilde{t}'+\Delta\tilde{t}')-\tilde{P}_n'(\tilde{t}') = -\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\tilde{P}_n'(\tilde{t}')\Delta\tilde{t}'+\tilde{\lambda}'\tilde{P}_{n-1}'(\tilde{t}')\Delta\tilde{t}'+(n+1)\tilde{\mu}'\tilde{P}_{n+1}'(\tilde{t}')\Delta\tilde{t}'+o(\Delta\tilde{t}'); 1\le n\le c-1 \quad (21)$$

Now, presuming that $\Delta\tilde{t}'\to 0$ is the limit and dividing it by $\Delta\tilde{t}'$ on both sides of the above equation, we obtain

$$\Delta\tilde{t}'\to 0\left[\frac{\tilde{P}_n'(\tilde{t}'+\Delta\tilde{t}')-\tilde{P}_n'(\tilde{t}')}{\Delta\tilde{t}'}\right] = -\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\tilde{P}_n'(\tilde{t}')+\tilde{\lambda}'\tilde{P}_{n-1}'(\tilde{t}')+(n+1)\tilde{\mu}'\tilde{P}_{n+1}'(\tilde{t}'); 1\le n\le c-1$$

By the definition of the first derivative, we get

$$\tilde{P}_n'(\tilde{t}') = -\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\tilde{P}_n'(\tilde{t}')+\tilde{\lambda}'\tilde{P}_{n-1}'(\tilde{t}')+(n+1)\tilde{\mu}'\tilde{P}_{n+1}'(\tilde{t}'); 1\le n\le c-1 \quad (22)$$

In the circumstance of a steady state where $\tilde{t}'\to\infty$, $\tilde{P}_n'(\tilde{t}')\to\tilde{P}_n'$ (independent of $\tilde{t}'$) and hence $\tilde{P}_n'(\tilde{t}')\to 0$. So, the system of steady-state difference equations is given by

$$0 = -\left(\tilde{\lambda}'+n\tilde{\mu}'\right)\tilde{P}_n'+\tilde{\lambda}'\tilde{P}_{n-1}'+(n+1)\tilde{\mu}'\tilde{P}_{n+1}'; 1\le n\le c-1 \quad \text{(From Equation (22))}$$

Hence,

$$\left(\tilde{\lambda}'+\tilde{\mu}'\right)\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-1}'+(n+1)\tilde{\mu}'\tilde{P}_{n+1}'; 1\le n\le c-1 \quad (23)$$

Hence the proof.

Theorem 5.3. In the $(FM/FM/c):(\infty/FIFO)$ queuing model, at the state $n\ge c$, (where n is an integer), proves that the rate is $\left(\tilde{\lambda}'+c\tilde{\mu}'\right)\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-1}'+c\tilde{\mu}'\tilde{P}_{n+1}'$.

Proof. Let,

$$\tilde{\lambda}'_n = \tilde{\lambda}' \qquad \text{for all } n$$

$$\tilde{\mu}'_n = \begin{cases} n\tilde{\mu}' & \text{for all } n < c \\ c\tilde{\mu}' & \text{for all } n \geq c \end{cases}$$

For $n \geq c$,

When independent conditions are incorporated together, the probability will be shown as follows:

i) At any time $\tilde{t}'$, there are n patrons in the system are $\tilde{P}'_n(\tilde{t}')$. However, at $\Delta\tilde{t}'$, there are no arrivals and no services respectively, $(1 - \tilde{\lambda}'\Delta\tilde{t}')$ and $(1 - c\tilde{\mu}'\Delta\tilde{t}')$. As a consequence, the probability is provided as

$$\Rightarrow [\tilde{P}'_n(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}'_n(\tilde{t}').(1 - \tilde{\lambda}'\Delta\tilde{t}').(1 - c\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}'_n(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}'_n(\tilde{t}').(1 - \tilde{\lambda}'\Delta\tilde{t}' - c\tilde{\mu}'\Delta\tilde{t}') + o_1(\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}'_n(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=0} = \tilde{P}'_n(\tilde{t}')\left(1 - \Delta\tilde{t}'(\tilde{\lambda}' + c\tilde{\mu}')\right) + o_1(\Delta\tilde{t}') \tag{24}$$

ii) At any time $\tilde{t}'$, there are $(n-1)$ patrons in the system are $\tilde{P}'_{n-1}(\tilde{t}')$; there is one entrance at $\Delta\tilde{t}'$ is $\tilde{\lambda}'\Delta\tilde{t}'$ and no assistance at $\Delta\tilde{t}'$ is $(1 - c\tilde{\mu}'\Delta\tilde{t}')$. As a result, the probability is given as

$$\Rightarrow [\tilde{P}'_{n-1}(\tilde{t}')]_{\tilde{\lambda}'=1;\tilde{\mu}'=0} = \tilde{P}'_{n-1}(\tilde{t}').\tilde{\lambda}'\Delta\tilde{t}'.(1 - c\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow [\tilde{P}'_{n-1}(\tilde{t}')]_{\tilde{\lambda}'=1;\tilde{\mu}'=0} = \tilde{\lambda}'\Delta\tilde{t}'\tilde{P}'_{n-1}(\tilde{t}') + o_2(\Delta\tilde{t}') \tag{25}$$

iii) At any time $\tilde{t}'$, there are $(n+1)$ patrons in the system are $\tilde{P}'_{n+1}(\tilde{t}')$; there is no inflow at $\Delta\tilde{t}'$ is $(1 - \tilde{\lambda}'\Delta\tilde{t}')$ and one service at $\Delta\tilde{t}'$ is $c\tilde{\mu}'\Delta\tilde{t}'$. As an outcome, the probability is given as

$$\Rightarrow [\tilde{P}'_{n+1}(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=1} = \tilde{P}'_{n+1}(\tilde{t}').(1 - \tilde{\lambda}'\Delta\tilde{t}').c\tilde{\mu}'\Delta\tilde{t}'$$

$$\Rightarrow [\tilde{P}'_{n+1}(\tilde{t}')]_{\tilde{\lambda}'=0;\tilde{\mu}'=1} = c\tilde{\mu}'\Delta\tilde{t}'\tilde{P}'_{n+1}(\tilde{t}') + o_3(\Delta\tilde{t}') \tag{26}$$

Now adding the above three Equations (24), (25) and (26) independent compound probabilities, we obtain the probability of n patrons in the system at the time $(\tilde{t}' + \Delta\tilde{t}')$. We obtain

$$\tilde{P}'_n(\tilde{t}' + \Delta\tilde{t}') = \tilde{P}'_n(\tilde{t}')\left[1 - (\tilde{\lambda}' + c\tilde{\mu}')\Delta\tilde{t}'\right] + \tilde{\lambda}'\Delta\tilde{t}'\tilde{P}'_{n-1}(\tilde{t}') + c\tilde{\mu}'\Delta\tilde{t}'\tilde{P}'_{n+1}(\tilde{t}') + o(\Delta\tilde{t}'); n \geq c \tag{27}$$

The Equation (27) now becomes

$$\tilde{P}'_n(\tilde{t}' + \Delta\tilde{t}') - \tilde{P}'_n(\tilde{t}') = -(\tilde{\lambda}' + c\tilde{\mu}')\tilde{P}'_n(\tilde{t}')\Delta\tilde{t}' + \tilde{\lambda}'\tilde{P}'_{n-1}(\tilde{t}')\Delta\tilde{t}' + c\tilde{\mu}'\tilde{P}'_{n+1}(\tilde{t}')\Delta\tilde{t}' + o(\Delta\tilde{t}'); n \geq c \tag{28}$$

Now, presuming that $\Delta\tilde{t}' \to 0$ is the limit and dividing it by $\Delta\tilde{t}'$ on both sides of the above equation, we obtain

$$\Delta\tilde{t}' \to 0\left[\frac{\tilde{P}'_n(\tilde{t}' + \Delta\tilde{t}') - \tilde{P}'_n(\tilde{t}')}{\Delta\tilde{t}'}\right] = -(\tilde{\lambda}' + c\tilde{\mu}')\tilde{P}'_n(\tilde{t}') + \tilde{\lambda}'\tilde{P}'_{n-1}(\tilde{t}') + c\tilde{\mu}'\tilde{P}'_{n+1}(\tilde{t}'); n \geq c$$

By the definition of the first derivative, we get

$$\tilde{P}'_n(\tilde{t}') = -(\tilde{\lambda}' + c\tilde{\mu}')\tilde{P}'_n(\tilde{t}') + \tilde{\lambda}'\tilde{P}'_{n-1}(\tilde{t}') + c\tilde{\mu}'\tilde{P}'_{n+1}(\tilde{t}'); n \geq c \tag{29}$$

In the circumstance of a steady state where $\tilde{t}' \to \infty$, $\tilde{P}'_n(\tilde{t}') \to \tilde{P}'_n$ (independent of $\tilde{t}'$) and hence $\tilde{P}'_n(\tilde{t}') \to 0$. So, the system of steady-state difference equations is given by

$$0 = -(\tilde{\lambda}' + c\tilde{\mu}')\tilde{P}'_n + \tilde{\lambda}'\tilde{P}'_{n-1} + c\tilde{\mu}'\tilde{P}'_{n+1}; n \geq c \qquad \text{(From Equation (29))}$$

Hence,

$$(\tilde{\lambda}' + c\tilde{\mu}')\tilde{P}'_n = \tilde{\lambda}'\tilde{P}'_{n-1} + c\tilde{\mu}'\tilde{P}'_{n+1}; n \geq c \tag{30}$$

Hence the proof.

VI. Suggested Queuing Framework and Its Execution

Let $\tilde{\lambda}$ and $\tilde{\lambda}'$ be the fuzzy and intuitionistic fuzzy arrival rates respectively. Let $\tilde{\mu}$ and $\tilde{\mu}'$ be the fuzzy and intuitionistic fuzzy service rates, respectively. At the steady-state, the FCFS discipline is upheld, and the capacity is unlimited.

The operational characteristics of the multi-server model are provided below:

i) Anticipated number of patrons in the system

$$\tilde{N}_s' = \frac{\tilde{\lambda}'\tilde{\mu}'\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{(c-1)!\left(c\tilde{\mu}'-\tilde{\lambda}'\right)^2}\tilde{P}_o + \frac{\tilde{\lambda}'}{\tilde{\mu}'} \tag{31}$$

ii) Anticipated number of patrons standing up in the queue

$$\tilde{N}_q' = \frac{\tilde{\lambda}'\tilde{\mu}'\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{(c-1)!\left(c\tilde{\mu}'-\tilde{\lambda}'\right)^2}\tilde{P}_o \tag{32}$$

iii) Mean waiting time a patron spends in the system

$$\tilde{T}_s' = \frac{\tilde{\mu}'\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{(c-1)!\left(c\tilde{\mu}'-\tilde{\lambda}'\right)^2}\tilde{P}_o + \frac{1}{\tilde{\mu}'} \tag{33}$$

iv) Mean standing up time of a patron in the queue

$$\tilde{T}_q' = \frac{\tilde{\mu}'\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^c}{(c-1)!\left(c\tilde{\mu}'-\tilde{\lambda}'\right)^2}\tilde{P}_o \tag{34}$$

Where, $\tilde{P}_o = \frac{1}{\sum_{n=0}^{c-1}\frac{\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n}{n!} + \frac{\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^c}{c!}\cdot\frac{c\tilde{\mu}}{c\tilde{\mu}-\tilde{\lambda}}}$

VII. Mathematical description

Imagine a bustling supermarket where customers approach the checkout counters at unpredictable intervals. In this model, there are four available checkout counters (servers), each operating at a unique service rate. Interpret the entry rate and the departure rate as both TFNs and TIFNs are symbolized by $\tilde{\lambda}, \tilde{\lambda}'$ and $\tilde{\mu}, \tilde{\mu}'$ respectively.

I. Single server fuzzy queuing model with infinite capacity

Let $\tilde{\lambda} = (12,13,14)$ is the arrival rate and $\tilde{\mu} = (22,23,24)$ is the service rate of the queuing model. Determine the TFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta})$ as $\tilde{\lambda} = (13,1,1)$ and $\tilde{\mu} = (23,1,1)$.

To determine the values of the number of patrons and their sojourn time in the queue as well as a system using suitable formulas among (31), (32), (33), & (34). It is necessary to use the appropriate arithmetic operations described in (1), (2), (3), (4), and (5) for add, sub, multiply, and divide, respectively.

The metrics of performance are calculated and tabulated in Table 1.

Table 1: *Performance Measures Using Triangular Fuzzy Numbers*

S. No	Parameters	Quantifiable Metrics Using TFN
1	$\tilde{N}_s$	$(-0.4345, 0.5655, 1.5655)$
2	$\tilde{N}_q$	$(-0.9997, 0.0003, 1.0003)$
3	$\tilde{T}_s$	$(-0.9566, 0.0434, 1.0434)$
4	$\tilde{T}_q$	$(-0.99997, 0.00003, 1.00003)$

The following Fig. 4 – 7 depict the visualizations of Table 1.

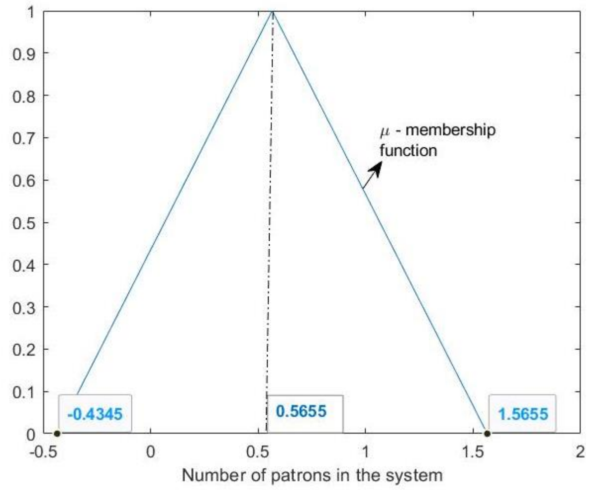

Figure 4: *Number of patrons in the system* $\tilde{N}_s$

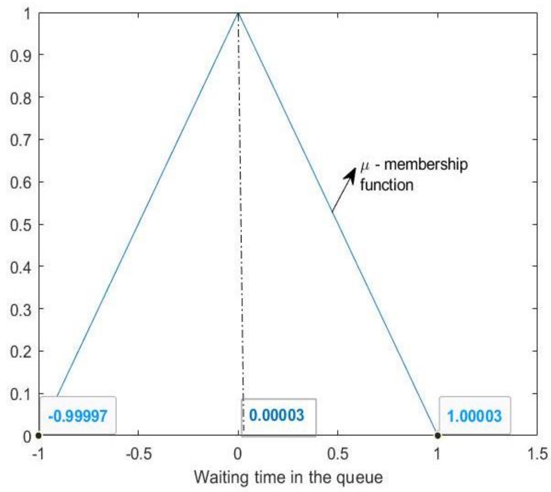

Figure 6: *Waiting time of patrons in the queue* $\tilde{T}_q$

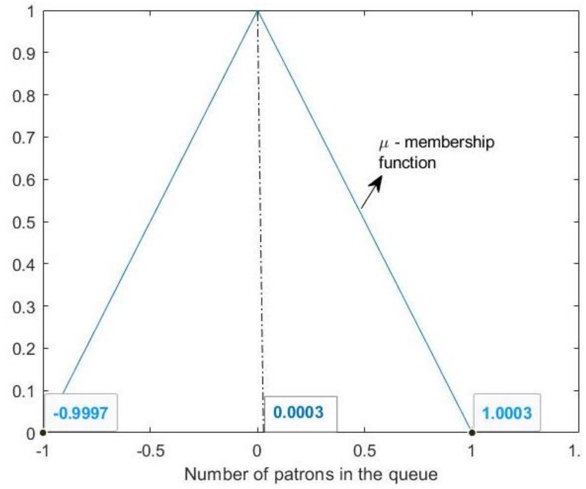

Figure 5: *Number of patrons in the queue* $\tilde{N}_q$

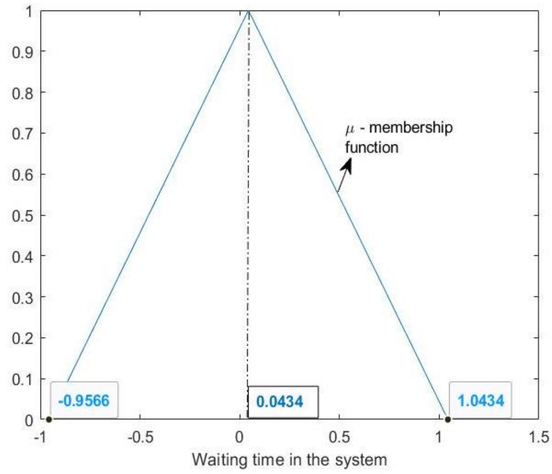

Figure 7: *Waiting time of patrons in the system* $\tilde{T}_s$

II. Single server intuitionistic fuzzy queuing model with infinite capacity

Let $\tilde{\lambda}' = (12,13,14; 11,13,15)$ is the arrival rate and $\tilde{\mu}' = (22,23,24; 21,23,25)$ is the service rate of the queuing model. Determine the TIFN in the guise of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta}; \tilde{m}, \tilde{\alpha}', \tilde{\beta}')$ as $\tilde{\lambda}' = (13,1,1; 13,2,2)$ and $\tilde{\mu}' = (23,1,1; 23,2,2)$.

To determine the values of the number of patrons and their sojourn time in the queue as well as a system using suitable formulas among (31), (32), (33), & (34). It is necessary to use the appropriate arithmetic operations described in (6), (7), (8), (9), and (10) for addition, subtraction, multiplication, and division, respectively.

The metrics of performance are gauged and tabulated in Table 2.

Table 2: *Performance Measures using triangular intuitionistic fuzzy numbers*

S. No	Parameters	Quantifiable Metrics Using TIFN
1	$\tilde{N}_s'$	$(-0.4345, 0.5655, 1.5655; -1.4345, 0.5655, 2.5655)$
2	$\tilde{N}_q'$	$(-0.9997, 0.0003, 1.0003; -1.999, 0.0003, 2.0003)$
3	$\tilde{T}_s'$	$(-0.9566, 0.0434, 1.0434; -1.9566, 0.0434, 2.0434)$
4	$\tilde{T}_q'$	$(-0.9999, 0.00003, 1.00003; -1.99997, 0.00003, 2.00003)$

The following Fig. 8 – 11 depict the visualizations of Table 2.

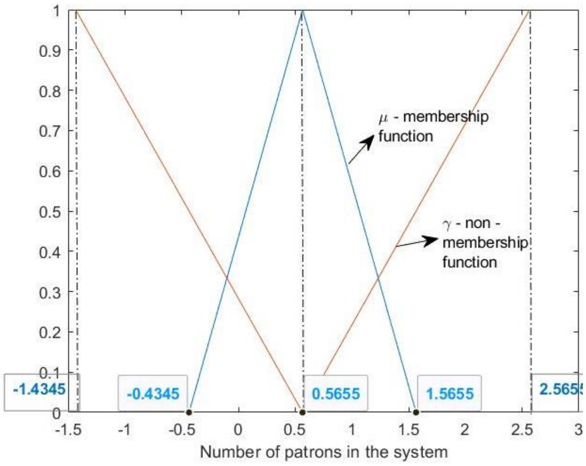

Figure 8: *The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the number of patrons in the system $\tilde{N}_s'$*

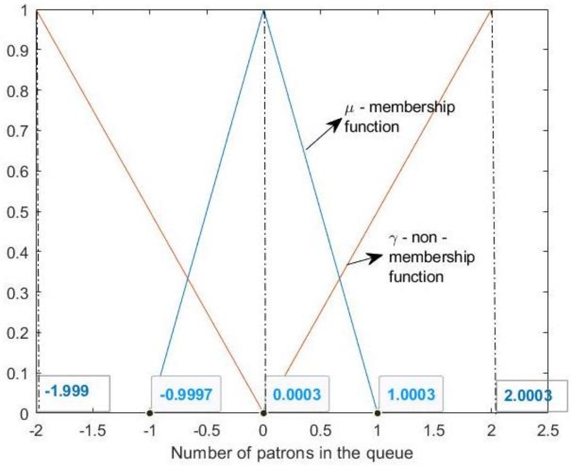

Figure 10: *The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the number of patrons in the queue $\tilde{N}_q'$*

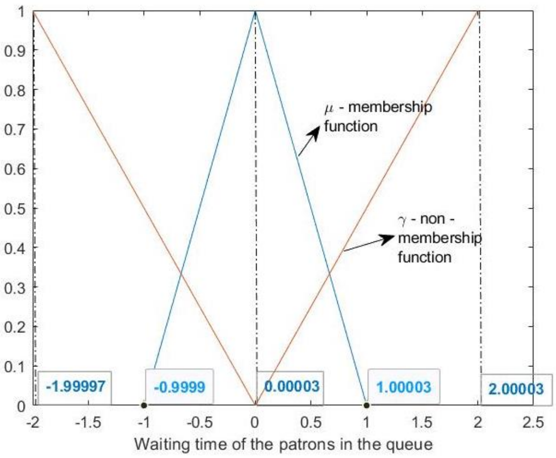

Figure 9: *The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the waiting time of patrons in the queue $\tilde{T}_q'$*

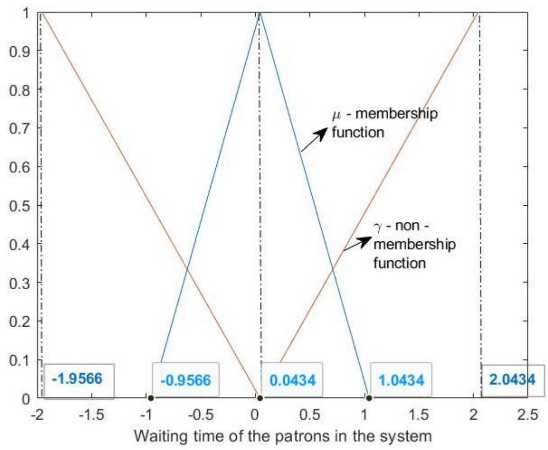

Figure 11: *The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the waiting time of patrons in the system $\tilde{T}_s'$*

VIII. Conclusion

IFS show a predisposition toward addressing decision-making challenges. Oftentimes, decision-makers struggle to articulate their viewpoints due to insufficient, dependable, or precise data or their hesitancy to explicitly delineate the extent of preference for one option over others. While decision-makers may lean toward certain alternatives, they may not be entirely convinced. Many researchers have focused on IFS theory to tackle optimization problems characterized by ambiguity and inconsistency.

In this study, we delve into the investigation of the $(FM/FM/C)$: $(\infty/FCFS)$ queuing model using algebraic operations techniques. We presume that the entry and processing times are indeterminate. The outcomes of this procedure appear somewhat unclear. However, numerical demonstrations prove the efficacy of this technique. It has been observed that augmenting the convergence of variables can bolster the success of the queuing model. The proposed model is anticipated to assist businesses, distributors, and merchants in effectively identifying optimal performance metrics for queuing systems. There are numerous avenues for expanding the scope of this report, one of which involves interpreting arrival and service rates as fluctuating random variables or fuzzy random variables. Moreover, it is widely acknowledged in the realm of fuzzy logic literature that analytical findings derived within an intuitionistic fuzzy context offer more practical and enlightening insights for software developers and programmers compared to those obtained from a fuzzy model. The fuzzy and intuitionistic fuzzy queues under multiple servers are delineated with greater precision, employing prediction models to yield scientific outcomes. TFN and TIFN numerical descriptors are utilized to evaluate the robustness of the proposed queuing system.

Looking ahead, the proposed queuing model may eventually be expanded to encompass multiple objectives in Erlang. Furthermore, it can serve as a platform for exploring novel facets of intuitionistic set extensions, such as neutrosophic sets.

References

[1] Kumar, C. A. (2023). Analysis of a multiserver system of queue-dependent channel using genetic algorithm. *Mathematics and Computer Science*, 2(23):337–348. Doi:10.1002/9781119896715.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20:87–96. Doi: https://doi.org/10.1016/S0165-0114(86)80034-3.

[3] Sun, B., Lee, M. H., Dudin, S. A. and Dudin, N. A. (2014). Analysis of multiserver queuing system with opportunistic occupation and reservation of servers. *Mathematical Problems in Engineering*, 178108. Doi: https://doi.org/10.1155/2014/178108.

[4] Buckley, J. J. (1990). Elementary queuing theory based on possibility theory. *Fuzzy Sets and Systems*, 35:43–52.

[5] Chen, S. P. (2005). Parametric nonlinear programming approach to fuzzy queues with bulk service. *European Journal of Operations Research*, 163:434–444.

[6] Chen, S. P. (2006). A mathematics programming approach to the machine interference problem with fuzzy parameters. *Applied Mathematics and Computation*, 174:374–387.

[7] Olliaro, D., Marsan, M. A., Balsamo, S. and Marin, A. (2023). The saturated multiserver job queuing model with two classes of jobs: exact and approximate results. *Performance Evaluation*, 162:102370. Doi: https://doi.org/10.1016/j.peva.2023.102370.

[8] Dudin, A., Dudina, O., Dudin, S. and Samouylov, K. (2021). Analysis of multi-server queue with self-sustained servers. *Mathematics*, 9(17): Article ID 2134. Doi: https://doi.org/10.3390/math9172134.

[9] Kuaban, G. S., Kumar, R., Soodan, B. S. and Czekalski, P. (2020). A multi-server queuing model with balking and correlated reneging with application in health care management. *IEEE Access*, 8:169623–169639. Doi: 10.1109/ACCESS.2020.3024259.

[10] Li, R. J. and Lee, E. S. (1989). Analysis of fuzzy queues. *Computers and Mathematics with Applications*, 17(7):1143–1147. Doi: https://doi.org/10.1016/0898-1221(89)90044-8.

[11] Ma, M., Friedman, M. and Kandel, A. (1999). A new fuzzy arithmetic. *Fuzzy Sets and Systems*, 108(1):83–90.

[12] Negi, D. S. and Lee, E. S. (1992). Analysis and simulation of fuzzy queue. *Fuzzy Sets and Systems*, 46:321–330.

[13] Samouylov, K., Dudina, O. and Dudin, A. (2023). Analysis of multi-server queuing system with flexible priorities. *Mathematics*, 11(4): Article ID 1040. Doi: https://doi.org/10.3390/math11041040.

[14] Selvam, P., Rajkumar, A. and Easwari, J. S. (2017). Ranking of pentagonal fuzzy numbers applying incentre of centroids. *International Journal of Pure and Applied Mathematics*, 117(13):165–174.

[15] Shanmugasundaram, S. and Venkatesh, B. B. (2015). Fuzzy multi-server queuing model through DSW algorithm. *International Journal of Latest Trends in Engineering and Technology*, 5(3):452–457.

[16] Shaw, A. K. and Roy, T. K. (2012). Some arithmetic operations on triangular intuitionistic fuzzy number and its application on reliability evaluation. *International Journal of Fuzzy Mathematics and Systems*, 2(4):363–382.

[17] Srinivasan, V. (2014). Fuzzy queuing model using DSW algorithm. *International Journal of Advanced Research in Mathematics and Applications*, 1(1):57–62.

[18] Thamotharan, S. (2016). A study on multi-server fuzzy queuing model in triangular and trapezoidal fuzzy numbers using α cuts. *International Journal of Science and Research*, 5(1):226–230.

[19] Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control*, 8:338–353. Doi: https://doi.org/10.1016/S0019-9958(65)90241-X.

[20] Anggraito, A., Olliaro, D., Ajmone Marsan, M. and Marin, A. (2024). Stability Condition for the Multi-server Job Queuing Model: Sensitivity Analysis. In *International Conference on Analytical and Stochastic Modeling Techniques and Applications*, Cham: Springer Nature Switzerland, 148–163. Doi: https://doi.org/10.1007/978-3-031-70753-7_10.

[21] Golovin, A. and Rumyantsev, A. (2023). On Multiresource Queues with Multiserver Customers. In *2023 5th International Conference on Problems of Cybernetics and Informatics (PCI)*, IEEE, 1–3. Doi: https://doi.org/10.1109/PCI60110.2023.10325941.

[22] Anupama and Kumar, C. (2023). Analysis of a Multiserver System of Queue-Dependent Channel Using Genetic Algorithm. *Mathematics and Computer Science*, 2:337–348. Doi: https://doi.org/10.1002/9781119896715.ch23.