OPTIMIZED SCHEDULING FOR THREE-STAGE FLOW SHOP WITH PARALLEL MACHINES: APPLICATION IN WASTE MANAGEMENT AND RECYCLING FACILITIES

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Abstract

Scheduling is one of the most important aspects of manufacturing sectors. Various factors in terms of performance evaluation, make business more competitive. Scheduling is the process of allocating scarce resources to tasks throughout time in order to accomplish one or more optimization goals. In fact, the methodical process of organizing, managing and maximizing work while ensuring the greatest possible use of both time and resources is the main aim of scheduling. This paper indicates the desirable and necessary steps to discover the optimum solution by efficient scheduling in waste recycling facilities which is essential for optimizing resources utilization and reducing operational delay. In this study, we have taken three stage flow shop scheduling with multiple machines at first two stages and single machine at last stage. This type of model has a number of applications not only in several industries but also in saving natural resources like water, electricity and reducing waste, idle time and unnecessary energy consumption. As, this research specially focused on the scenario of waste management and recycling facility where incoming waste undergoes three primary stages sorting, cleaning, and recycling. Here the scheduling evaluation is done by comparing the Branch and Bound method with Palmer's and Dannenbring method to determine an optimal job sequence that minimize the total processing time. The processing time is represented as a trapezoidal fuzzy number making an impact. The goal of this activity is to determine the optimal job plan in order to reduce the total amount of time that has processed and allocate the lead in the best feasible way to reduce the machine idle time, minimizes the water usage and energy consumption. This comparison demonstrated how well our suggested approach handled this challenging scheduling issue the findings contribute to the efficient management of recycling facilities, ensuring improved throughput. Because better scheduling strategies must be implemented in order to the growing demand for waste processing and recycling. So, the proposed approach aids in streamlining waste management processes, ultimately supporting environmental conservation efforts. Here, Branch and Bound is the best approach as compared to Palmer and Dannenbring approach. Future work can be extending this methodology to the three-phase flow shop scheduling with parallel machines at all the stages.

Keywords: Scheduling, Equipotential machines, trapezoidal fuzzy numbers

I. Introduction

Customer satisfaction in a global supply chain is crucial to the long-term viability of the company. Given the abundance of items on the market, the goal of any manufacturing sector is to satisfy customer demands on schedule, with the appropriate quality, and most important at the lowest possible cost. One of the most important choices for maximizing the use of tools and resources is scheduling.

It is intimately related to how well the company performs in terms of cost, quality, speed, dependability, flexibility, and time. Johnson [1] conducted the crucial research on flow shop scheduling for the first time for two and three stages. Ignall and Schrage [2] worked on the B&B method to systematically explore possible job sequences. Maggu et.al [4] extended the classical flow shop scheduling by transportation and job block concepts. Chung Yee Lee [5] investigates the scheduling problems with the concept of parallel machines and proposed Polynomial time algorithm. MasriKhan et.al [11] explore the hybrid scheduling algorithms palmer-NEH, Gupta-NEH and Dannenbring-NEH to reduce energy costs in production environment. Narain [6] addressed the scheduling problems under no idle constraints and demonstrate the application and effectiveness of the proposed algorithm. Sharma et.al [8] studied probabilistic processing times and machine breakdown intervals into the scheduling model and included breakdowns where machine might stop working. Palmer [3] provided priority index to rank the jobs served as a foundation for later algorithms for flow shop scheduling. T.P. Singh [7] focused on 3-stage flow shop scheduling problems with parallel machines. Gupta [9] studied on heuristic optimization technique for total flow time of three sequential processing stages of equipotential flow shop scheduling. Gupta et.al [12] presented an exact branch and bound algorithm for two stage flow shop environment and offered practical value for industries. Utama et.al [14] provided a comprehensive review of energy efficient scheduling approaches in hybrid flow shop environment. Malhotra et.al [13] worked on the scheduling problem with three identical parallel machines based on branch and bound. Rahmani et.al [10] worked on Dannenbring method to find the optimal schedule including work weights and transportation time. Kaushik et.al [15] investigated scheduling strategies in a bi-stage flow shop by comparing B&B with two heuristic algorithms HEH and CDS.

II. Statistical Development

In this study we are presenting the ideas of multiple machines at level 1st and 2nd with single machine at level 3rd . This kind of model may produce many things on a huge scale in a variety of industrial businesses. There was another approach to solving the challenges provided by the idea of trapezoidal fuzzy numbers accepted as time. The description of a mathematical problem is assuming three processors, $\mathcal{H}$, $\mathcal{I}$ and $\mathcal{R}$ to complete n jobs (i = 1, 2, 3, 4, ... n) referred to as the first, second, and third stages respectively. At phase one and two, there are parallel machines of type $\mathcal{H}_j$ and $\mathcal{I}_k$ (j = 1, 2, 3, 4, ..., m_1) (k= 1, 2, 3, 4, ..., m_2) respectively, where as in the third stages, there is only one machine. In the initial step, each parallel machine's unit operational cost for ith job on jth machine is also provided in the form of $\mathcal{H}_{ij}$ and $\mathcal{I}_{ij}$ respectively. Here the processing time is represented as trapezoidal fuzzy numbers of processors $\mathcal{H}$, $\mathcal{I}$ and $\mathcal{R}$ in the form of ($\hat{a}_i$, $\hat{e}_i$, $\hat{\imath}_i$, $\hat{o}_i$), ($\hat{u}_i$, $\hat{r}_i$, ṣ$_i$, ţ$_i$) and (ɭ$_i$, ɳ$_i$, ƫ$_i$, ȷ$_i$) respectively. The total available time is denoted by t_{rs} of like machines of type $\mathcal{H}_j$ and $\mathcal{I}_k$ (j = 1, 2, 3, 4, ..., m_1), (k = 1, 2, 3, 4, ..., m_2) respectively. The statistical form of model is shown as follow,

Table1: *Statistical form of model*

Jobs (i)	Stage1($\mathcal{H}$)					Stage2($\mathcal{J}$)					Stage3($\mathcal{R}$)
	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_{m1}$	Processing time of $\mathcal{H}$ α_i = (â$_i$, ê$_i$, î$_i$, ô$_i$)	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\mathcal{J}_{m2}$	Processing time of $\mathcal{J}$ β_i = (û$_i$, r̂$_i$, ș$_i$, ț$_i$)	Processing time of $\mathcal{R}$ γ_i = (ɭ$_i$, ɳ$_i$, ƫ$_i$, ȷ$_i$)
1	$\mathcal{H}_{11}$	$\mathcal{H}_{12}$	$\mathcal{H}_{13}$	$\mathcal{H}_{1m1}$	(â$_1$, ê$_1$, î$_1$, ô$_1$)	$\mathcal{J}_{11}$	$\mathcal{J}_{12}$	$\mathcal{J}_{13}$	$\mathcal{J}_{1m2}$	(û$_1$, r̂$_1$, ș$_1$, ț$_1$)	(ɭ$_1$, ɳ$_1$, ƫ$_1$, ȷ$_1$)
2	$\mathcal{H}_{21}$	$\mathcal{H}_{22}$	$\mathcal{H}_{23}$	$\mathcal{H}_{2m1}$	(â$_2$, ê$_2$, î$_2$, ô$_2$)	$\mathcal{J}_{21}$	$\mathcal{J}_{22}$	$\mathcal{J}_{23}$	$\mathcal{J}_{2m2}$	(û$_2$, r̂$_2$, ș$_2$, ț$_2$)	(ɭ$_2$, ɳ$_2$, ƫ$_2$, ȷ$_2$)
3	$\mathcal{H}_{31}$	$\mathcal{H}_{32}$	$\mathcal{H}_{33}$	$\mathcal{H}_{3m1}$	(â$_3$, ê$_3$, î$_3$, ô$_3$)	$\mathcal{J}_{31}$	$\mathcal{J}_{32}$	$\mathcal{J}_{33}$	$\mathcal{J}_{3m2}$	(û$_3$, r̂$_3$, ș$_3$, ț$_3$)	(ɭ$_3$, ɳ$_3$, ƫ$_3$, ȷ$_3$)
4	$\mathcal{H}_{41}$	$\mathcal{H}_{42}$	$\mathcal{H}_{43}$	$\mathcal{H}_{4m1}$	(â$_4$, ê$_4$, î$_4$, ô$_4$)	$\mathcal{J}_{41}$	$\mathcal{J}_{42}$	$\mathcal{J}_{43}$	$\mathcal{J}_{4m2}$	(û$_4$, r̂$_4$, ș$_4$, ț$_4$)	(ɭ$_4$, ɳ$_4$, ƫ$_4$, ȷ$_4$)
n	$\mathcal{H}_{n1}$	$\mathcal{H}_{n2}$	$\mathcal{H}_{n3}$	$\mathcal{H}_{nm1}$	(â$_n$, ê$_n$, î$_n$, ô$_n$)	$\mathcal{J}_{n1}$	$\mathcal{J}_{n2}$	$\mathcal{J}_{n3}$	$\mathcal{J}_{nm2}$	(û$_n$, r̂$_n$, ș$_n$, ț$_n$)	(ɭ$_n$, ɳ$_n$, ƫ$_n$, ȷ$_n$)
t_{rs}	t_{11}	t_{12}	t_{13}	t_{1m1}		t_{21}	t_{22}	t_{23}	t_{2m2}		

I. Assumptions

- Each task has its own autonomy.
- The setup time is not taken into account.
- It is not necessary to process every task on every possible machine.
- Each task has a unique operational cost.
- All of the equipotential parallel processors may begin working at the same time.
- The availability of processors is constant.
- Here, pre-emption is not taken into consideration.

II. Methodology to solve problem

Step (1): Initially, we will use crisp(A)= 1/2[(b + c)- 4/5(b-a) +2/3(d-c)] to apply Yager's Formula of fuzzy dispensation time of different jobs, where (a, b, c, d) represents the fuzzy processing time of ith job of corresponding machine. The processing time as trapezoidal fuzzy will be reduced to a single number using this procedure.

Step (2): Check the condition $\sum_{r=1}^{m1} t_{1r} = \sum_{i=1}^{n} \alpha_i$ for first stage and $\sum_{r=1}^{m2} t_{1r} = \sum_{i=1}^{n} \beta_i$ for second stage. Then apply Vogal Approximation Method (VAM) and Modified Distribution Method. If condition is not satisfied then apply Modified Distribution Method for unbalanced problem.

Step (3): Use the Branch and Bound approach. The procedure is summarized in the following steps.

- Apply the formula
  - $g' = \max\left(\sum_{i=1}^{n} \mathcal{H}_{ij}\right) + \min_{i \epsilon j_{r'}}(\max \mathcal{J}_{ij} + \mathcal{R}_{ij})$
  - $g'' = \max_{i \epsilon j_r}(\mathcal{H}_{ij}) + \max\left(\sum_{i=1}^{n} \mathcal{J}_{ij}\right) + \min_{i \epsilon j_{r'}} \mathcal{R}_{ij}$

  - $g''' = \max_{i\epsilon j_r}(\mathcal{H}_{ij}) + \max_{i\epsilon j_r}(\mathcal{J}_{ij}) + \sum_{i\epsilon j_r} \mathcal{R}_{ij}$

- Calculate $\mathcal{M}$= max (g′, g″, g‴)
- Calculate $\mathcal{M}$ for all corresponding jobs.
- Determine the lowest value of $\mathcal{M}$ obtained in step(c).
- Then begin the optimal sequence by the job corresponding to minimum value $\mathcal{M}$ obtained in step (d).
- Now repeat the above process for (n-1) subsequence, then for (n-2) and so on. Finally, the process gives the optimal or near optimal schedule.
- Formulate in-out table for the optimal sequence obtained in step (f).

III. Practical Situation

In our model the first two stages challenge with parallel machines and third stage with single machines. This kind of scheduling can be effectively used in waste management which involve multiple steps in different stages. Here the case study involved the three stages.

The first stage is sorting stage. In this stage the waste is sorted into different categories like plastics, metals and papers using multiple parallel machines i.e. $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_3$. The second stage is cleaning stage with multiple parallel machines $\mathcal{J}_1$, $\mathcal{J}_2$ and $\mathcal{J}_3$. where the sorted recyclables are washed and cleaned to remove all kind of impurities. The third stage is processing or recycling stage with a single machine, where the cleaned sorted recyclables are melted and compacted into reusable materials. The difficulties lie in using the machineries as efficiently as possible while reducing the makespan i.e. by scheduling we can minimize the sorting time, the water and electricity usage by balancing the load on cleaning machines. Overall, we can save money by reducing the cost of waste disposal and protect the environment. So, the scheduling has a crucial role in this type of scenario also.

I. Process Stages

Let us assume we want to process 5 jobs i.e. five types of waste, that can be plastic waste, metal waste, glass waste, paper waste and electronic waste etc. We want to schedule all these jobs across three stages. This model worked in the following way,

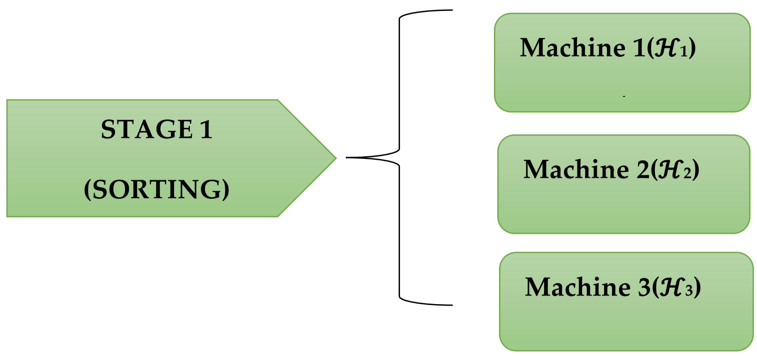

Figure I(a): *Three parallel machines at 1st stage*

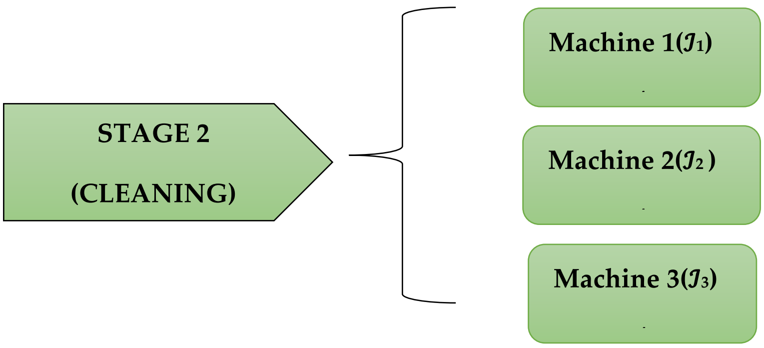

Figure I(b): *Three parallel machines at 2nd stage*

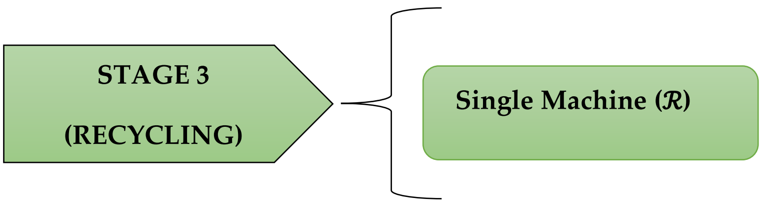

Figure I(c): *Single machines at 3rd stage*

IV. Numerical Example

Consider the three phase Flow Shop Scheduling with three parallel machines at first two stages each and single machine at last stage. The statement of numerical problem is given in table 2.

Table2: *Numerical Example*

Jobs	Stage1($\mathcal{H}$)				Stage2($\mathcal{J}$)				Stage3($\mathcal{R}$)
(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	Processing time of $\mathcal{H}$	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	Processing time of $\mathcal{J}$	Processing time of $\mathcal{R}$
1	1	5	4	(1,3,4,5)	7	2	10	(3,5,6,8)	(2,3,5,7)
2	7	1	9	(2,4,5,7)	8	3	4	(4,7,8,9)	(1,2,3,4)
3	6	7	3	(6,7,8,9)	5	1	6	(3,5,4,7)	(1,2,4,9)
4	4	8	5	(3,5,7,8)	3	7	2	(4,5,6,8)	(3,5,7,8)
5	10	2	8	(2,3,4,5)	1	6	8	(2,5,8,9)	(4,5,6,8)
t_{rs}	10	4.3	9		10	10	7.95		

Solution:

Step 1: Apply Yager's ranking formula, the simplified form of problem is represented in table3.

Table3: *Yager's ranking formula of fuzzy dispensation time*

Jobs	Stage1($\mathcal{H}$)				Stage2($\mathcal{J}$)				Stage3($\mathcal{R}$)
(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	Processing time of $\mathcal{H}$	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	Processing time of $\mathcal{J}$	Processing time of $\mathcal{R}$
1	1	5	4	3.0	7	2	10	5.35	4.25
2	7	1	9	4.0	8	3	4	6.6	2.4
3	6	7	3	7.4	5	1	6	4.7	4.25
4	4	8	5	5.5	3	7	2	5.7	5.5
5	10	2	8	3.4	1	6	8	5.6	5.75
t_{rs}	10	4.3	9		10	10	7.95		

Step 2: Since our problem is balanced as it satisfies the condition of step 2 of algorithm. Now, to find the optimal allocations of processing time on machine $\mathcal{H}$, $\mathcal{J}$ and $\mathcal{R}$ by using VAM method is given in table 4 below;

Table4: *Optimal allocations of processing time*

Jobs	Stage1($\mathcal{H}$)			Stage2($\mathcal{J}$)			Stage3($\mathcal{R}$)
(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\mathcal{R}_{ij}$
1	3.0	0	0	0	5.35	0	4.25
2	0	4	0	0	0	6.6	2.4
3	0	0	7.4	0.05	4.65	0	4.25
4	5.5	0	0	4.35	0	1.35	5.5
5	1.5	0.3	1.6	5.6	0	0	5.75

Step 3: Now to apply B&B algorithm to find out the lower bound of jobs for the optimal sequence. The calculations are shown in table 5.

Table5: *Branch and Bound procedure*

Task (i)	g′= max ($\sum_{i=1}^{n}\mathcal{H}_{ij}$) + $\min_{i\epsilon j_{r\prime}}(\max \mathcal{J}_{ij} + \mathcal{R}_{ij})$	g″ = $\max_{i\epsilon j_r}(\mathcal{H}_{ij})$ + max ($\sum_{i=1}^{n}\mathcal{J}_{ij}$) + $\min_{i\epsilon j_{r\prime}} \mathcal{R}_{ij}$	g‴= $\max_{i\epsilon j_r}(\mathcal{H}_{ij})$ + $\max_{i\epsilon j_r}(\mathcal{J}_{ij})$ + $\sum_{i\epsilon j_r}\mathcal{R}_{ij}$	$\mathcal{M}$= max (g′, g″, g‴)
1	10 + 8.9 = 18.9	3.0 +10 +2.4 = 15.4	3.0 + 5.35 + 22.15 = 30.5	30.5
2	10 + 8.9 = 18.9	4+10+4.25 = 18.25	4 + 6.6+22.15 = 32.75	32.75
3	10 + 9 = 19	7.4+10+2.4 = 19.8	7.4 + 4.65 + 22.15 = 34.2	34.2
4	10 + 8.9 = 18.9	5.5 + 10 +2.4 = 17.9	5.5+4.35+22.15 = 32	32
5	10 + 8.9 = 18.9	1.6 +10 +2.4 = 14	1.6 + 5.6 + 22.15 = 29.35	29.35

In table 5, min {30.5, 32.75, 34.2, 32, 29.35} = 29.35, which corresponds to job5. So, job5 will be at the first position of the optimal sequence. Continue with the same process of branch and bound for the subsequence {5, 1}, {5,2}, {5,3} and {5, 4}. The results shown in table 6.

Table6: *Branch and Bound procedure for 2nd position in optimal schedule*

Task (i)	g′= max ($\sum_{i=1}^{n}\mathcal{H}_{ij}$) + $\min_{i\epsilon j_{r\prime}}(\max \mathcal{J}_{ij} + \mathcal{R}_{ij})$	g″ = $\max_{i\epsilon j_r}(\mathcal{H}_{ij})$ + max ($\sum_{i=1}^{n}\mathcal{J}_{ij}$) + $\min_{i\epsilon j_{r\prime}} \mathcal{R}_{ij}$	g‴= $\max_{i\epsilon j_r}(\mathcal{H}_{ij})$ + $\max_{i\epsilon j_r}(\mathcal{J}_{ij})$ + $\sum_{i\epsilon j_r}\mathcal{R}_{ij}$	$\mathcal{M}$= max (g′, g″, g‴)
{5,1}	10 + 8.9 = 18.9	9.85 +7.95 +2.4 =20.2	9.85 + 22.15 = 32	32
{5,2}	10 + 8.9 = 18.9	10.9 + 4.65 +4.25 =19.8	10.9 + 22.15 =33.05	33.05
{5,3}	10 + 9 = 19	9.05 +7.95+2.4 = 19.4	9.05 + 22.15 = 31.2	31.2
{5,4}	10 + 8.9 = 18.9	12.25 + 10 +2.4 = 24.65	12.25 + 22.15 = 34.4	34.4

From table 6, we observe that the min {32, 33.05, 31.2, 34.4} =31.2, which corresponds to the subsequence {5,3}. Therefore job 3 will be at second position. Continue the process for the 3rd and 4th position in the optimal schedule. Table 7 and table 8 show the outcomes.

Table7: *Branch and Bound procedure for the 3rd position in optimal schedule*

Task (i)	$g' = \max(\sum_{i=1}^{n} \mathcal{H}_{ij}) + \min_{i\epsilon j_{r'}}(\max \mathcal{I}_{ij} + \mathcal{R}_{ij})$	$g'' = \max_{i\epsilon j_r}(\mathcal{H}_{ij}) + \max(\sum_{i=1}^{n} \mathcal{I}_{ij}) + \min_{i\epsilon j_{r'}} \mathcal{R}_{ij}$	$g''' = \max_{i\epsilon j_r}(\mathcal{H}_{ij}) + \max_{i\epsilon j_r}(\mathcal{I}_{ij}) + \sum_{i\epsilon j_r} \mathcal{R}_{ij}$	$\mathcal{M}$= max (g′, g″, g‴)
{5,3,1}	10 + 9 =19	10 + 7.95 +2.4 = 20.35	10 + 22.15 =32.15	32.15
{5,3,2}	10 + 9.6 =19.6	6.6 + 5.35 + 4.25 =16.2	6.6 + 22.15 = 28.75	28.75
{5,3,4}	10 + 9 = 19	13.4 + 6.6 + 2.4 = 22.4	13.4 +22.15 =35.55	35.55

Here min {32.15, 28.75, 35.55} = 28.75, which corresponds to the subsequence {5,3,2}. Which show that job 2 will be at 3rd position.

Table8: *Branch and Bound procedure for the 4th position in optimal schedule*

Task (i)	$g' = \max(\sum_{i=1}^{n} \mathcal{H}_{ij}) + \min_{i\epsilon j_{r'}}(\max \mathcal{I}_{ij} + \mathcal{R}_{ij})$	$g'' = \max_{i\epsilon j_r}(\mathcal{H}_{ij}) + \max(\sum_{i=1}^{n} \mathcal{I}_{ij}) + \min_{i\epsilon j_{r'}} \mathcal{R}_{ij}$	$g''' = \max_{i\epsilon j_r}(\mathcal{H}_{ij}) + \max_{i\epsilon j_r}(\mathcal{I}_{ij}) + \sum_{i\epsilon j_r} \mathcal{R}_{ij}$	$\mathcal{M}$= max (g′, g″, g‴)
{5,3,2,1}	10 + 9.85 = 19.85	10 + 4.35 +5.5 =19.85	10 + 22.15 = 32.15	32.15
{5,3,2,4}	10 + 9.6 =19.6	13.4 + 5.35 +4.25 = 23	13.4 + 22.15 =35.55	35.55

Clearly, min {32.15, 35.55} = 32.15, which corresponds to {5,3,2,1}. So, job 1 will be at the 4th place and job 4 at the 5th place in the optimal schedule.
Finally, {5, 3, 2, 1,4} is the optimal schedule by B&B and table 9 show the minimum makespan.

Table 9: *In/Out table for the minimum makespan*

Jobs(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{R}$
5	0 - 1.5	0 - 0.3	0 – 1.6	1.6 – 7.2	-	-	7.2 – 12.95
3	-	-	1.6 – 9	9 – 9.05	0 – 4.65	-	12.95 – 17.2
2	-	0.3 - 4.3	-	-	-	0 – 6.6	17.2 – 19.6
1	1.5 - 4.5	-	-	-	4.65 – 10	-	19.6 – 23.85
4	4.5 - 10	-	-	10-14.35	-	6.6 –7.95	23.85 - **29.35**

The minimum makespan by B&B is 29.35

V. Solution by Dannenbring Algorithm

Dannenbring Method is a heuristic approach designed for solving the scheduling issues involving more than two machines. The Johnson method served as the basis for the Dannenbring approach. Here the complete work with the nth machine processing time is transformed into a recapitulation table with two values says V_{i1} and V_{i2}. This method ranks jobs using a composite priority function based on the sum of processing time. It improved the Johnsons rule by considering more than first two machines.

I. Working procedure

Step1:

Calculate aggregate processing time of machines at level first and two i.e. $\frac{\sum pij}{m1}$ where m_1 is number of parallel machines and p_{ij} is the processing time of i^{th} job on j^{th} machine at first stage. Similarly calculate the same for the second stage also.

Step2:

Find the processing time for first machine by using the formula $T_{j1} = \sum_{i=1}^{m}(m-i+1)^{*}\ t_{ij}$ and the processing time for the second machine will be calculated by $T_{j2} = \sum_{i=1}^{m} i^{*}\ t_{ij}$.

Where;

m : Total number stages

i : The stage index

t_{ij} : Processing time for job 'i' on machine 'j'

Step3: Calculate the value V_{i1} and V_{i2} by using the formula in step 2.

Step4: Find out the optimal schedule by ordering the either of the value V_{i1} and V_{i2} in descending order.

Step5: find the in – out schedule for calculating makespan.

Solution:

Find the aggregate processing time of machine at first and second stage using table 4.

We get the result in table 10.

Table10: *Aggregate processing time*

Jobs(i)	$\mathcal{H}$	$\mathcal{I}$	$\mathcal{R}$
1	1	1.78	4.25
2	1.33	2.2	2.4
3	2.46	1.56	4.25
4	1.8	1.9	5.5
5	1.13	1.86	5.75

Now, Calculate the value V_{i1} and V_{i2} by using the formula in step 2. Table 11 demonstrate the results below;

Table 11: *The value of V_{i1} and V_{i2}*

Jobs	V_{i1}	V_{i2}
1	3(1 + 1.78 + 4.25) = 21.09	1 + 3.56 + 12.75 =17.31
2	3(1.33 + 2.2 + 2.4) = 17.79	1.33 + 4.4 + 7.2 = 12.93
3	3(2.46 + 1.56 + 4.25) = 24.81	2.46 + 3.12 + 12.75 = 18.33
4	3(1.8 + 1.9 +5.5) = 27.6	1.8 + 3.8 + 16.5 = 22.1
5	3(1.13 +1.86 +5.75) 26.22	1.13 + 3.72 + 17.25 =22.1

From step 4 the optimal order is {4, 5, 3, 1,2}. Now, find the makespan corresponding to the obtained schedule. Result is shown in table 12 below.

Table 12: *In-out table for makespan*

Jobs(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\mathcal{R}$
4	0 – 5.5	-	-	5.5 – 9.85	-	5.5 – 6.85	9.85 –15.35
5	5.5 – 7	0 – 0.3	0 – 1.6	9.85 – 15.45	-	-	15.45 – 21.2
3	-	-	1.6 – 9	15.45 – 15.5	9 – 13.65	-	21.2 – 25.45
1	7 – 10	-	-	-	13.65 – 19	-	25.45 – 29.7
2	-	0.3 – 4.3	-	-	-	6.85 -13.45	29.7 – **32.1**

VI. Solution with the help of Palmer's Heuristic Algorithm

Palmer Heuristic [3] is well-known guideline for solving scheduling issues in flow shop. This is quick solving approach specifically designed for flow shop scheduling to find near optimal solution. Palmer's heuristic reduces the total makespan by implicitly attempting to minimize the peak load on any one machine by giving the middle machines larger weights. This method gives a job index to every job according to its processing time after that jobs are ordered according to given index. The algorithm of Palmer's heuristic for the three-stage flow shop scheduling with parallel machines at first two stages is given below;

Step1 Calculate aggregate processing time at level 1 and level 2:

For the step 1, find the aggregate processing time of machines at the first and second stage i.e. $\frac{\sum \text{pij}}{\text{m1}}$ where m_1 is number of parallel machines and p_{ij} is the processing time of i^{th} job on j^{th} machine at first two stages.

Step2 Calculate weighted Slop Index (WSI):

First of all, give some specific weights to each machine by using the formula

S(i) = [$\sum_{i=2}^{m}(m-2i+1)$]* T_{ij} where

T_{ij} = processing time of job 'i' on machine 'j 'at all stages

m = number of stages

Step3 Job Rank

Sort the job in descending order according to the job indices S(i) calculated in step 2.

Step4 Makespan

Calculate the total completion time of job for the job sequence obtained in step 3.

Solution: To find the aggregate of processing time at stage first and two. We proceed from table 10, where the results already shown. Now, Calculate weighted Slop Index (WSI) as follow;

S(1) = (2*1) + (0*1.78) + (-2*4.25) = 2+0-8.5 = -6.5

S(2) = (2*1.3) + (0*2.2) + (-2*2.4) = 2.6+0-4.8= -2.2

S(3) = (2*2.4) + (0*1.56) + (-2*4.25) = 4.8+0- 8.5= -3.7

S(4) = (2*1.8) + (0*1.9) + (-2*5.5) = 3.6+0-11=-7.4

S(5) = (2*1.13) + (0*1.86) + (-2*5.7) = 2.26+0-11.4=-9.24

As mention above, sort the job in descending order according to the job indices S(i), we get the job sequence {J_2, J_3, J_1, J_4, J_5}. The makespan for the obtained schedule is shown in table13.

Table13: *Makespan for the optimal sequence by Palmer's heuristics*

Jobs(i)	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{R}$
2	-	0 - 4	-	-	-	4 – 10.6	10.6 - 13
3	-	-	0 - 7.4	7.4 – 7.45	7.4 -12.05	-	13 – 17.25
1	0 -3.0	-	-	-	12.05-17.4	-	17.25-21.5
4	3.0-8.5	-	-	8.5-12.85	-	12.85-14.2	21.5-28
5	8.5-10	4- 4.3	7.4-9	12.85-18.45	-	-	28 - **33.75**

Conclusion

We observe from the solution using all the three methods, we found that the B&B technique yield a lower minimum makespan as compared to Palmer and Dannenbring heuristic. It systematically evaluates all possible sequences, ensuring the best possible schedule. Dannenbring heuristic improves upon Palmer's heuristic by introducing a weighted priority index. It provides better approximation than Palmer but does not guarantee an optimal solution. Thus, we may conclude that B&B is the precise approach when compared to Dannenbring and Palmer approach.

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