PARABOLIC DENSE CLOUDY FUZZY MODEL WITH DEMAND DEPENDENT PRODUCTION RATE IN AN IMPRECISE PRODUCTION PROCESS

Upasana Rana[1], Tanuj Kumar[1,*]

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[1]Department of Mathematics, SRM Institute of Science and Technology,
Delhi-NCR Campus, Ghaziabad, India, 201204
ur2288@srmist.edu.in, tanujkuo@srmist.edu.in
*Corresponding author

Abstract

In this paper we develop a classical economic production lot size model of a manufactured item in a inadequate production process in which the ordering cost is fixed and shortage is not allowed. In this model the valuation of an item is considered as a parabolic cloudy fuzzy number and production rate is depends on demand. In the previous studies, fuzziness of parameters of the model is presented in the form of fuzzy numbers using the membership function only. Parabolic dense cloudy intuitionistic fuzzy numbers are found to be more generalized than fuzzy numbers and have the capability to express the fuzziness of parameters of the model in a more generalized way using both membership and non-membership functions. The presented model is solved in crisp, fuzzy and Intuitionistic fuzzy setup using proposed generalize and defuzzification method with minimum loss of data. The present model is solved by DBPSO algorithm to determine optimal values. The validity of the proposed model and parameter sensitivity analysis is presented to justify the idea by the numerical analysis.

Keywords: Classical Economic Model, cloudy fuzzy number, Parabolic dense cloudy Fuzzy Number, Score function, Defuzzification Method, Swarm Algorithm.

1. Introduction

According to a general scenario, when developing an economic production lot-size model, the demand rate for inventory products is usually assumed to be constant. However, in the actual world, inventory quantities will differ slightly from accurate figures. As a result, in practical settings, the demand variable should be viewed as ambiguous. The fuzzy notion has recently been used to production and inventory difficulties. The fuzzy set theory was first developed by Zadeh [4]. Bellman and Zadeh [5] used the fuzzy set theoretic technique to solve difficulties later. This concept is reflected in the gloomy, hazy surroundings. Following Yager [6], other scholars such as Ezzati, Allahviranloo, Khezerloo, and Khezerloo [7], Deng [8], Zhang, Ignatius, Lim, and Zhao [9], and others supported the method for sorting ambiguous numbers using the centre of gravity. In addition, De and Beg [10], Maity, Suman, et al. [2], Garg [3] and De and Mahata [11], [12] devised new defuzzification methods for triangular dense and triangular cloudy fuzzy sets, respectively. Until recently, no one has tackled a realistic production inventory model with a foggy, ambiguous demand rate. The production rate of a single item or many items is believed to be rigid and set in the traditional economic production lot-size (EPL) paradigm.

In truth, demand for every good has an impact on the manufacturing process. When demand progressively increases, client consumption increases, and manufacturers are forced to produce more things to match the increased need. In the opposite circumstance, the opposite is true. Several academics have developed EPL models for single/multiple items that take into account either a constant or variable production rate (depending on time, demand, and/or the amount of inventory on hand). Bhunia and Maiti [13], Balkhi and Benkherouf [14], Abad [15], Mandal and Maiti [16], Das, Roy, and Kar [18], [19], [20], [21], and others developed inventory models with uniform or variable production rates in mind. Manufacturing flexibility, on the other hand, has become a more significant issue in inventory management. Different types of flexibility have been assessed in the manufacturing system, with volume flexibility being the most relevant. Volume flexibility refers to the ability to adjust production volume in a manufacturing system. In the EPQ model, Cheng [22] was the first to establish the demand-dependent production unit cost, while Khouja [23] included volume flexibility and reliability considerations. Under inflationary conditions and reliability, Shah and Shah [24] established the EPQ model for time falling demand with imperfect production processes. Items are manufactured with a high level of consistency utilising a traditional manufacturing technique. With high expectations, more reliability boosts the efficiency of the production process. Any manufacturing company strives to improve its production efficiency and ability to operate at peak levels by lowering the cost of scraps, rework of poor goods, wasted materials, and worker hours, among other things. Rosenblatt and Lee [25], Ben-Daya and Hariga [26], Goyal, Hung, and Chen [27], Maiti, Bhunia, and Maiti [28], Sana, Goyal, and Chaudhuri [29], Das et al. [19], [20], Manna, Dey, and Mondal [30], Pal, Sana, and Chaudhuri [31], Yadav et al. [40], Yadav et al. [41], [42], [43], Rana and Kumar [45], Negi and Singh [47], Dutta, Anurag, et al. [48] and others have published numerous research papers on imperfect production. Manna, Das, Dey, and Mondal [32] looked at a multi-item EPQ model with learning effect on imperfect production over a fuzzy random planning horizon in a recent paper. Khara, Dey, and Mondal [33] created an inventory model with imperfect production and demand that is based on reliability. Soft computing approaches have been used in a number of research articles on inventory control difficulties. Several authors have used Genetic Algorithms (GA) in various forms to solve marketing difficulties. In an unclear planning horizon, Pal, Maiti, and Maiti [34] use GA to solve an EPQ model with price discounted promotional demand. Bera and Maiti [35] employed GA to solve a discount-based multi-item inventory model. Maiti, Maiti, and Maiti [36] employed GA to solve a stochastic lead time inventory model with price-dependent demand that included advance payment. Many additional scholars have used GA in inventory control problems, including Mondal and Maiti [37], Maiti and Maiti [38], [39], Jiang, Xu, Wang, and Wang [44], and Maiti et al. [46]. PSO was also employed by Bhunia and Shaikh [49] to solve a two-warehouse inventory model for decaying items with an allowed payment delay. To solve this fuzzy inventory model, dominance-based particle swarm optimization was created. The fuzzy inventory model is defined here under an imperfect production process with a hazy fuzzy demand rate, with the production rate being demand dependent. The model is solved using Yager's index approach and De and Beg's ranking index method for defuzzification in a crisp, general fuzzy, and cloudy fuzzy environment, and the outcomes in the crisp, fuzzy, and cloudy fuzzy environments are compared.

The goal of this research is to use a dominance-based Particle Swarm Optimization (PSO) algorithm to identify the best decision for the decision maker by minimising average total cost to acquire the best order quantity and cycle time (DM). A number of numerical examples are used to support the model, as well as some sensitivity studies.

2. Definitions and preliminaries

2.1. Normalized General Triangular Cloudy Fuzzy Number (NGTCFN)

A Normalized General Triangular Cloudy Fuzzy Number (NGTCFN) having the form $(\widetilde{A}) = \left(a\left(1-\frac{\rho_1}{1+t}\right), a, a\left(1+\frac{\sigma_1}{1+t}\right)\right)$ based on two parameters ρ_1 and σ_1 where $a\left(1-\frac{\rho_1}{1+t}\right) < a < a\left(1+\frac{\sigma_1}{1+t}\right)$

and is characterized by its continuous membership function(MF) $\zeta_{\widetilde{A}}(x) : X \to [0,1]$, defined as

$$\zeta_{\widetilde{A}}(x) = \begin{cases} \frac{x-a\left(1-\frac{\rho_1}{1+t}\right)}{a-a\left(1-\frac{\rho_1}{1+t}\right)}, & a\left(1-\frac{\rho_1}{1+t}\right) \le x \le a \\ \frac{a\left(1+\frac{\sigma_1}{1+t}\right)-x}{a\left(1+\frac{\sigma_1}{1+t}\right)-a}, & a \le x \le a\left(1+\frac{\sigma_1}{1+t}\right) \\ 0, & Otherwise \end{cases}$$

$$\eta_{\widetilde{A}}(x) = \begin{cases} \frac{(a-x)+\left(x-a\left(1-\frac{\rho_2}{1+t}\right)\right)}{a-a\left(1-\frac{\rho_1}{1+t}\right)}, & a\left(1-\frac{\rho_2}{1+t}\right) \le x \le a \\ \frac{(x-a)+\left(a\left(1+\frac{\sigma_2}{1+t}\right)-x\right)}{a\left(1+\frac{\sigma_2}{1+t}\right)-a}, & a \le x \le a\left(1+\frac{\sigma_2}{1+t}\right) \\ 0, & Otherwise \end{cases}$$

with $0 \le \zeta, \eta \le 1$ and $0 \le \zeta + \eta \le 1$.

2.2. Generalized Parabolic Fuzzy Numbers (GPFN)

A Generalized Parabolic Fuzzy number(GPFN) $\tilde{A} = (a_l, a_m, a_u)$ is called generalized parabolic fuzzy number if its membership function(MF) is defined as

$$\zeta_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a_l}{a_m-a_l}\right)^2 \theta, & if \;\; a_l \le x \le a_m \\ \theta, & x = a_m \\ \left(\frac{a_u-x}{a_u-a_m}\right)^2 \theta, & if \;\; a_m \le x \le a_u \end{cases}$$

2.3. Generalized Parabolic Dense Cloudy Fuzzy Number (GPDCFN)

A Generalized Parabolic Dense Cloudy Fuzzy Number (GPDCFN) $\tilde{A} = \langle [a(1-\frac{\rho_1}{1+t}), a, a(1+\frac{\sigma_1}{1+t}); \zeta], [a(1-\frac{\rho_2}{1+t}), a, a(1+\frac{\sigma_2}{1+t}); \eta] \rangle$ by adding membership function(MF) and non-membership function(NMF) is called generalized parabolic dense cloudy fuzzy number if its membership function(MF) and non-membership function(NMF) is defined as

$$\zeta_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a\left(1-\frac{\rho_1}{1+t}\right)}{a-a\left(1-\frac{\rho_1}{1+t}\right)}\right)^2 \theta, & if \;\; a\left(1-\frac{\rho_1}{1+t}\right) \le x \le a \\ \theta, & x = a \\ \left(\frac{a\left(1+\frac{\sigma_1}{1+t}\right)-x}{a\left(1+\frac{\sigma_1}{1+t}\right)-a}\right)^2 \theta & if \;\; a \le x \le a\left(1+\frac{\sigma_1}{1+t}\right) \end{cases}$$

$$\eta_{\tilde{A}}(x)=\begin{cases}\left(\frac{(a-x)+\sqrt{\eta}\left(x-a\left(1-\frac{\rho_2}{1+t}\right)\right)}{\left(a-a\left(1-\frac{\rho_2}{1+t}\right)\right)}\right)^2, & if\ \ a\left(1-\frac{\rho_2}{1+t}\right)\leq x\leq a\\ \eta, & x=a\\ \left(\frac{(x-a)+\sqrt{\eta}\left(a\left(1+\frac{\sigma_2}{1+t}\right)-x\right)}{\left(a\left(1+\frac{\sigma_2}{1+t}\right)-a\right)}\right)^2, & if\ \ a\leq x\leq a\left(1+\frac{\sigma_2}{1+t}\right)\end{cases}$$

Where $a\left(1-\frac{\rho_2}{1+t}\right)\leq a\left(1-\frac{\rho_1}{1+t}\right)\leq a\leq a\left(1+\frac{\sigma_1}{1+t}\right)\leq a\left(1+\frac{\sigma_2}{1+t}\right),\ 0\leq\zeta,\eta\leq 1$ and $0\leq\zeta+\eta\leq 1$.

2.4. (α,β) - cut of a cloudy fuzzy number

Let $\alpha,\beta\in[0,1]$ be a fixed numbers such that $0\leq\alpha\leq\zeta,\ \eta\leq\beta\leq 1$ and $0\leq\sqrt{\alpha}+\sqrt{\beta}\leq 1$.
A (α,β) cuts generated by Generalized Parabolic Dense Cloudy Fuzzy Number (GPDCFN) A is defined as

$$A_{\alpha,\beta}=\{\langle x,\zeta_{\tilde{A}}(x)\geq\alpha,\eta_{\tilde{A}}(x)\leq\beta\rangle\quad : x\in X\}$$

The α cut of GPDCFN $\tilde{A}$ is defined as

$$\tilde{A}^{\zeta}_{\alpha}=\{\langle x,\zeta_{\tilde{A}}(x)\geq\alpha\rangle\quad : x\in X\},\ \ 0\leq\alpha\leq\zeta$$

The β cut of GPDCFN $\tilde{A}$ is defined as

$$\tilde{A}^{\eta}_{\beta}=\{\langle x,\eta_{\tilde{A}}(x)\leq\beta\rangle\quad : x\in X\},\ \ \eta\leq\beta\leq 1$$

2.5. Arithmetic Operations on Generalized Triangular Parabolic Dense Cloudy Fuzzy Numbers (GTPDCFN)

$$\mu_{A+B}(x)=\langle(a_1+a_2,\ b_1+b_2,\ c_1+c_2);\mu_1\rangle$$
$$\nu_{A+B}(x)=\langle(a'_1+a'_2,\ b'_1+b'_2,\ c'_1+c'_2);\nu_2\rangle$$
$$\mu_{A-B}(x)=\langle(a_1-c_2,\ b_1-b_2,\ c_1-a_2);\mu_1\rangle$$
$$\nu_{A-B}(x)=\langle(a'_1-c'_2,\ b'_1-b'_2,\ c'_1-a'_2);\nu_2\rangle$$
$$\mu_{A\cdot B}(x)=\langle(a_1\cdot a_2,\ b_1\cdot b_2,\ c_1\cdot c_2);\mu_1\rangle$$
$$\nu_{A\cdot B}(x)=\langle(a'_1\cdot a'_2,\ b'_1\cdot b'_2,\ c'_1\cdot c'_2);\nu_2\rangle$$
$$\mu_{\frac{A}{B}}(x)=\left\langle\left(\frac{a_1}{c_2},\ \frac{b_1}{b_2},\ \frac{c_1}{a_2}\right);\mu_1\right\rangle$$
$$\nu_{\frac{A}{B}}(x)=\left\langle\left(\frac{a'_1}{c'_2},\ \frac{b'_1}{b'_2},\ \frac{c'_1}{a'_2}\right);\nu_2\right\rangle$$

3. Defuzzification of parabolic dense cloudy fuzzy number

Let $\tilde{A}=\langle[a\left(1-\frac{\rho_1}{1+t}\right),a,a\left(1+\frac{\sigma_1}{1+t}\right);\mu_1],\ [a\left(1-\frac{\rho_2}{1+t}\right),a,a\left(1+\frac{\sigma_2}{1+t}\right);\nu_1]\rangle$ be a generalized triangular parabolic dense cloudy fuzzy number with the following membership(MF) and non-membership functions(NMF).

$$\zeta_{\tilde{A}}(x)=\begin{cases}\zeta\left(\frac{x-a\left(1-\frac{\rho_1}{1+t}\right)}{a-a\left(1-\frac{\rho_1}{1+t}\right)}\right)^2, & if\ \ a\left(1-\frac{\rho_1}{1+t}\right)\leq x\leq a\\ \zeta, & x=a\\ \zeta\left(\frac{a\left(1+\frac{\sigma_1}{1+t}\right)-x}{a\left(1+\frac{\sigma_1}{1+t}\right)-a}\right)^2, & if\ \ a\leq x\leq a\left(1+\frac{\sigma_1}{1+t}\right)\end{cases}$$

$$\eta_{\tilde{A}}(x) = \begin{cases} \left(\frac{(a-x)+\sqrt{\eta}(x-a\left(1-\frac{\rho_2}{1+t}\right)}{(a-a\left(1-\frac{\rho_2}{1+t}\right)}\right)^2, & if \;\; a\left(1-\frac{\rho_2}{1+t}\right) \leq x \leq a \\ \eta, & x = a \\ \left(\frac{(x-a)+\sqrt{\eta}(a\left(1+\frac{\sigma_2}{1+t}-x\right)}{(a\left(1+\frac{\sigma_2}{1+t}-a\right)}\right)^2, & if \;\; a \leq x \leq a\left(1+\frac{\sigma_2}{1+t}\right) \end{cases}$$

respectively. The score functions is defined as the difference between membership and non-membership functions.

$$S(\tilde{A}) = \zeta(\tilde{A}) - \eta(\tilde{A})$$

$$S(\tilde{A}) = \begin{cases} \phi(x-\Psi); & \Psi \leq x \leq a \\ \sqrt{\zeta} - \sqrt{\eta}; & x = a \\ \rho(\tau - x); & a \leq x \leq \tau \end{cases}$$

$$\phi = \frac{\sqrt{\mu}(a - a\left(1-\frac{\rho_1}{1+t}\right)) + (a - a\left(1-\frac{\rho_1}{1+t}\right))(1-\sqrt{\nu})}{(a - a\left(1-\frac{\rho_1}{1+t}\right))(a - a\left(1-\frac{\rho_2}{1+t}\right))}$$

$$\Psi = \frac{\sqrt{\mu}a\left(1-\frac{\rho_1}{1+t}\right)(a - a\left(1-\frac{\rho_2}{1+t}\right)) + (a - a\left(1-\frac{\rho_1}{1+t}\right))(a - \sqrt{\nu}a\left(1-\frac{\rho_2}{1+t}\right))}{\sqrt{\mu}(a - a\left(1-\frac{\rho_2}{1+t}\right)) + (a - a\left(1-\frac{\rho_1}{1+t}\right))(1-\sqrt{\nu})}$$

$$\rho = \frac{\sqrt{\mu}(a\left(1+\frac{\sigma_2}{1+t}\right) - a) + (a\left(1+\frac{\sigma_1}{1+t}\right) - a)(1-\sqrt{\nu})}{(a\left(1+\frac{\sigma_1}{1+t}\right) - a)(a\left(1+\frac{\sigma_2}{1+t}\right) - a)}$$

$$\tau = \frac{\sqrt{\mu}a\left(1+\frac{\sigma_1}{1+t}\right)(a\left(1+\frac{\sigma_2}{1+t}\right) - a) + (a\left(1+\frac{\sigma_1}{1+t}\right) - a)(a - \sqrt{\nu}(a\left(1+\frac{\sigma_2}{1+t}\right))}{\sqrt{\mu}(a\left(1+\frac{\sigma_2}{1+t}\right) - a) + (a\left(1+\frac{\sigma_1}{1+t}\right) - a)(1-\sqrt{\nu})}$$

with $0 \leq \zeta,\ \eta \leq 1$ and $0 \leq \zeta + \eta \leq 1$. The α cut of $\bar{S} = \{\langle(\psi, a, \tau);\ \mu, \nu\rangle$ is defined as

$$\widetilde{A}(\alpha) = \left[\widetilde{A}_L(\alpha), \widetilde{A}_R(\alpha)\right] = \left[\psi + \frac{\alpha}{\phi}, \tau - \frac{\alpha}{\rho}\right]$$

The defuzzification formula for GPDCFN is defined as follows

$$D_f(\widetilde{A}) = \int_0^1 \left(\lambda\ L^{-1}(\alpha) + (1-\lambda)R^{-1}(\alpha)\right) d\alpha$$

$$D_f(\widetilde{A}) = \left[\frac{\lambda}{\phi} + \frac{(1-\lambda)}{\rho}\right]\left(\frac{\sqrt{\zeta} - \sqrt{\eta}}{2}\right)^2 + \left[\lambda\psi + (1-\lambda)\tau\right](\sqrt{\zeta} - \sqrt{\eta}).$$

4. Notations and assumptions

4.1. Notations

The mathematical model was formulated based on the following notations.

- K : Production rate per cycle.
- D : Demand rate per cycle, $(D < K)$.
- R : Production process reliability.

$P(T)$: Instantaneous inventory level.

P : Maximum inventory level (decision variable).

T_2 : Cycle length (decision variable).

T_1 : Production period (decision variable).

C : Production cost per unit.

C_1 : Setup cost per cycle.

H : Inventory carrying cost per unit quantity per unit time.

Z : Average total inventory cost.

P^* : Optimum value of P.

T_2^* : Optimum value of T_2.

Z^* : Optimum value of Z.

T_1^* : Optimum value of T_1.

4.2. Assumptions

The mathematical model was formulated based on the following assumptions.

1. Replenishment occurs instantaneously on placing of order quantity, so lead time is zero.
2. The inventory is developed for a single item in an imperfect process.
3. Shortage are not allowed.
4. The time horizon of the inventory system is infinite.
5. The production rate K is demand dependent and is of the form

$$K = A + B \cdot D,$$

where A and B are positive constants.

5. Model development and analysis

The process reliability r means that among the items product in a production, only $R\%$ is acceptable that can be used to meet the customer's demand. Initially the production process starts with zero inventories with production rate K and demand rate D. Figure 1 illustrates the variation between inventory levels and demand rate for the proposed model.

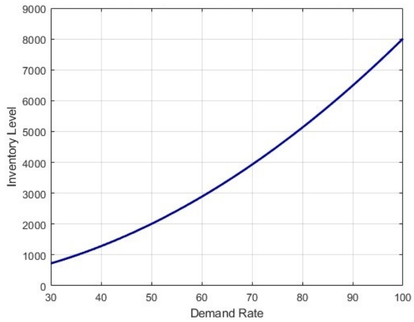

Figure 1: *Variation between inventory level and demand rate*

During the interval $[0, T_1]$ inventory level gradually built up at rate $(RK - D)$ and reached at its maximum level P at the end of production process. The inventory level gradually depleted during the period $[T_1, T_2]$ due to customer's demand and ultimately became zero at $[T_1, T_2]$. The instantaneous state of $P(t)$ desuring the differential equation in the interval $[0, T_2]$ of that item is given by

$$\frac{dP(T)}{dT} = \begin{cases} RK - D, & 0 \leq T \leq T_1 \\ \\ -D, & T_1 \leq T \leq T_2 \end{cases} \tag{1}$$

where $RK - D > 0$. With boundary condition $P(0) = 0,\ P(T_1) = P,\ P(T_2) = 0$. The solution of the differential equation using the boundary condition is given by

$$P(T) = \begin{cases} (RK - D)T, & 0 \leq T \leq T_1 \\ \\ D(T_2 - T), & T_1 \leq T \leq T_2 \end{cases} \tag{2}$$

Hence our problem is given by

$$Minimizing\ \ Z = \frac{CD}{R} + \frac{C_1}{T_2} + \frac{HDT_2(AR + (BR - 1)D)}{2(A + BD)R}, \cdots \cdots (A)$$

$$Subject\ to\ \ d(T_2 - T_1) = (RK - D)T_1,$$

$$i.e.,\ \ RKT_1 = DT_2$$

$$P = D(T_2 - T_1).$$

Now, the problem is reduced to minimize the average cost Z and to find the optimum value of P and T_2 for which $Z(P, T_2)$ is minimum and the corresponding value of T_1. The average cost is minimized by DBPSO.

6. Parabolic Cloudy Fuzzy mathematical model

Initially, when the production process starts, the demand rate R item is ambiguous. Naturally, demand rate is assumed to be general fuzzy over the cycle length. Then fuzzy demand rate $\tilde{D}$ is as follows, $\tilde{D} = \left(D\left(1 - \frac{\rho_2}{1+t}\right), D\left(1 - \frac{\rho_1}{1+t}\right), D, D\left(1 + \frac{\sigma_1}{1+t}\right), D\left(1 + \frac{\sigma_2}{1+t}\right)\right)$ for GTPDFN. Therefore, Maiti [1] transform the problem (A) to a fuzzy problem as folows.

$$\begin{cases} Minimizing\ \tilde{Z} & = \frac{C\tilde{D}}{R} + \frac{C_1}{T_2} + \frac{H\tilde{D}T_2(AR+(BR-1)\tilde{D})}{2(A+B\tilde{D})R}, \\ \\ Subject\ to\ \ R\tilde{K}T_1 = \tilde{D}T_2, \\ \\ \qquad \tilde{P} = \tilde{D}(T_2 - T_1). \end{cases}$$

$$\mu_1(Z) = \begin{cases} \left(\frac{Z - Z_1}{Z_2 - Z_1}\right)^2 \mu, & Z_1 \leq Z \leq Z_2 \\ \\ \mu, & Z = Z_2 \\ \\ \left(\frac{Z_3 - Z}{Z_3 - Z_2}\right)^2 \mu, & Z_2 \leq Z \leq Z_3 \end{cases}$$

$$\nu_1(Z)=\begin{cases}\left(\frac{(Z_2-Z)+(Z-Z_1')\sqrt{\nu}}{Z_2-Z_1'}\right)^2, & Z_1'\le Z\le Z_2\\ \nu, & Z=Z_2\\ \left(\frac{(Z-Z_2)+(Z_3'-Z)\sqrt{\nu}}{Z_3-Z_2'}\right)^2, & Z_2\le Z\le Z_3'\end{cases}$$

If we take parabolic dense intuitionistic fuzzy number $\left(D\left(1-\frac{\rho_2}{1+t}\right),D\left(1-\frac{\rho_1}{1+t}\right),D,D\left(1+\frac{\sigma_1}{1+t}\right),D\left(1+\frac{\sigma_2}{1+t}\right)\right)$.

$$Z_1'=\frac{CD\left(1-\frac{\rho_2}{1+t}\right)}{R}+\frac{C_1}{T_2}+\frac{HT_2D\left(1-\frac{\rho_2}{1+t}\right)\left(AR+(BR-1)D\left(1-\frac{\rho_2}{1+t}\right)\right)}{2R\left(A+BD\left(1+\frac{\sigma_2}{1+t}\right)\right)}$$

$$Z_1=\frac{CD\left(1-\frac{\rho_1}{1+t}\right)}{R}+\frac{C_1}{T_2}+\frac{HT_2D\left(1-\frac{\rho_1}{1+t}\right)\left(AR+(BR-1)D\left(1-\frac{\rho_1}{1+t}\right)\right)}{2R\left(A+BD\left(1+\frac{\sigma_1}{1+t}\right)\right)}$$

$$Z_2=\frac{CD}{R}+\frac{C_1}{T_2}+\frac{HT_2D(AR+(BR-1)D)}{2R(A+BD)}$$

$$Z_3=\frac{CD\left(1+\frac{\sigma_1}{1+t}\right)}{R}+\frac{C_1}{T_2}+\frac{HT_2D\left(1+\frac{\sigma_1}{1+t}\right)\left(AR+(BR-1)D\left(1+\frac{\sigma_1}{1+t}\right)\right)}{2R\left(A+BD\left(1-\frac{\rho_1}{1+t}\right)\right)}$$

$$Z_3'=\frac{CD\left(1+\frac{\sigma_2}{1+t}\right)}{R}+\frac{C_1}{T_2}+\frac{HT_2D\left(1+\frac{\sigma_2}{1+t}\right)\left(AR+(BR-1)D\left(1+\frac{\sigma_2}{1+t}\right)\right)}{2R\left(A+BD\left(1-\frac{\rho_2}{1+t}\right)\right)}$$

$$D_f(Z)=\left[\frac{\lambda}{\phi}+\frac{(1-\lambda)}{\rho}\right]\left(\frac{\sqrt{\mu}-\sqrt{\nu}}{2}\right)^2+\Big[\lambda\psi+(1-\lambda)\tau\Big](\sqrt{\mu}-\sqrt{\nu}).$$

$$\phi=\frac{\sqrt{\mu}(Z_2-Z_1')+(Z_2-Z_1)(1-\sqrt{\nu})}{(Z_2-Z_1)(Z_2-Z_1')},\ \ \Psi=\frac{\sqrt{\mu}Z_1(Z_2-Z_1')+(Z_2-Z_1)(Z_2-\sqrt{\nu}Z_1')}{\sqrt{\mu}(Z_2-Z_1')+(Z_2-Z_1)(1-\sqrt{\nu})}$$

$$\rho=\frac{\sqrt{\mu}(Z_3'-Z_2)+(Z_3-Z_2)(1-\sqrt{\nu})}{(Z_3'-Z_2)(Z_3-Z_2)},\ \ \tau=\frac{\sqrt{\mu}Z_3(Z_3'-Z_2)+(Z_3-Z_2)(Z_2-\sqrt{\nu}Z_3')}{\sqrt{\mu}(Z_3'-Z_2)+(Z_3-Z_2)(1-\sqrt{\nu})}$$

$$\mu_2(P)=\begin{cases}\left(\frac{P-P_1}{P_2-P_1}\right)^2\mu, & P_1\le P\le P_2\\ \mu, & P=P_2\\ \left(\frac{P_3-P}{P_3-P_2}\right)^2\mu, & P_2\le P\le P_3\end{cases}$$

$$\nu_2(P)=\begin{cases}\left(\frac{(P_2-P)+(P-P_1')\sqrt{\nu}}{P_2-P_1'}\right)^2, & P_1'\le P\le P_2\\ \nu, & P=P_2\\ \left(\frac{(P-P_2)+(P_3'-P)\sqrt{\nu}}{P_3-P_2'}\right)^2, & P_2\le z\le P_3'\end{cases}$$

$$P_1' = D\big(1 - \frac{\rho_2}{1+t}\big)(T_2 - T_1), \;\; P_1 = D\big(1 - \frac{\rho_1}{1+t}\big)(T_2 - T_1)$$

$$P_2 = D(T_2 - T_1), \;\; P_3 = D\big(1 + \frac{\sigma_1}{1+t}\big)(T_2 - T_1), \;\; P_3' = D\big(1 + \frac{\sigma_2}{1+t}\big)(T_2 - T_1)$$

$$D_f(P) = \left[\frac{\lambda}{\phi} + \frac{(1-\lambda)}{\rho}\right]\left(\frac{\sqrt{\mu}-\sqrt{\nu}}{2}\right)^2 + \left[\lambda\psi + (1-\lambda)\tau\right](\sqrt{\mu}-\sqrt{\nu}).$$

$$\phi = \frac{\sqrt{\mu}(P_2 - P_1') + (P_2 - P_1)(1-\sqrt{\nu})}{(P_2 - P_1)(P_2 - P_1')}, \;\; \Psi = \frac{\sqrt{\mu}P_1(P_2 - P_1') + (P_2 - P_1)(P_2 - \sqrt{\nu}P_1')}{\sqrt{\mu}(P_2 - P_1') + (P_2 - P_1)(1-\sqrt{\nu})}$$

$$\rho = \frac{\sqrt{\mu}(P_3' - P_2) + (P_3 - P_2)(1-\sqrt{\nu})}{(P_3' - P_2)(P_3 - P_2)}, \;\; \tau = \frac{\sqrt{\mu}P_3(P_3' - P_2) + (P_3 - P_2)(P_2 - \sqrt{\nu}P_3')}{\sqrt{\mu}(P_3' - P_2) + (P_3 - P_2)(1-\sqrt{\nu})}$$

$$\mu_3(K) = \begin{cases} \left(\frac{K-K_1}{K_2-K_1}\right)^2\mu, & K_1' \le K \le K_2 \\ \mu, & K = K_2 \\ \left(\frac{K_3-K}{K_3-K_2}\right)^2\mu, & K_2 \le K \le K_3' \end{cases}$$

$$\nu_3(K) = \begin{cases} \left(\frac{(K_2-K)+(K-K_1')\sqrt{\nu}}{K_2-K_1'}\right)^2, & K_1' \le K \le K_2 \\ \nu, & K = K_2 \\ \left(\frac{(K-K_2)+(K_3'-K)\sqrt{\nu}}{K_3-K_2'}\right)^2, & K_2 \le K \le K_3' \end{cases}$$

$$RK_1'T_1 = D\big(1 - \frac{\rho_2}{1+t}\big)T_2, \;\; RK_1T_1 = D\big(1 - \frac{\rho_1}{1+t}\big)T_2$$

$$RK_2T_1 = DT_2, \;\; RK_3T_1 = D\big(1 + \frac{\sigma_1}{1+t}\big)T_2, \;\; RK_3'T_1 = D\big(1 + \frac{\sigma_2}{1+t}\big)T_2$$

$$D_f(K) = \left[\frac{\lambda}{\phi} + \frac{(1-\lambda)}{\rho}\right]\left(\frac{\sqrt{\mu}-\sqrt{\nu}}{2}\right)^2 + \left[\lambda\psi + (1-\lambda)\tau\right](\sqrt{\mu}-\sqrt{\nu}).$$

$$\phi = \frac{\sqrt{\mu}(K_2 - K_1') + (K_2 - K_1)(1-\sqrt{\nu})}{(K_2 - K_1)(K_2 - K_1')}, \;\; \Psi = \frac{\sqrt{\mu}K_1(K_2 - K_1') + (K_2 - K_1)(K_2 - \sqrt{\nu}K_1')}{\sqrt{\mu}(K_2 - K_1') + (K_2 - K_1)(1-\sqrt{\nu})}$$

$$\rho = \frac{\sqrt{\mu}(K_3' - K_2) + (K_3 - K_2)(1-\sqrt{\nu})}{(K_3' - K_2)(K_3 - K_2)}, \;\; \tau = \frac{\sqrt{\mu}K_3(K_3' - K_2) + (K_3 - K_2)(K_2 - \sqrt{\nu}K_3')}{\sqrt{\mu}(K_3' - K_2) + (K_3 - K_2)(1-\sqrt{\nu})}$$

7. Numerical Example

Now we consider the following values of parameters in inventory models:

1. **Crisp model:** Let us consider $A = 850\,(\$)$ per cycle, $C_b = 1.5\,(\$)$ per unit, $p = 15\,(\$)$ per unit, $C_h = 5\,(\$)$. Per unit, $D = 500 units$. We get the optimal solution of our model as: the optimum average inventory cost is $X^* = 8501.92(\$)$ with shortage quantity $S^* = 813$ *units* and ordering quantity $Q^* = 1045$ *units* for the cycle time $T^* = 2.9$ $(weeks)$ and the shortages begin from 0.47 $(weeks)$, as Table 1. Figure 2 illustrates the graphical representation of convexity. This visual depiction highlights the key aspects of convexity.

2. **Cloudy intuitionistic fuzzy model:** Let us consider $\omega = 0.6$, $v = 0.2$, $\delta = 0.15$ $\epsilon = 0.12, \rho = 0.05$, and the other parameters will be same as the crisp model, as Table 1. Figure 2 provides a graphical representation of convexity, visually demonstrating its key features. This illustration helps to clarify the concept of convexity.

Table 1: *Optimum values of Crisp and Cloudy intuitionistic fuzzy model*

Model	$T^*(weeks)$	$(t_1)^*(weeks)$	$S^*(units)$	$Q^*(units)$	$X^*(\$)$
Crisp model	2.09	0.47	809	1045	13313.09
Cloudy intuitionistic fuzzy model	4.47	1.01	845.29	1009.69	14535.21

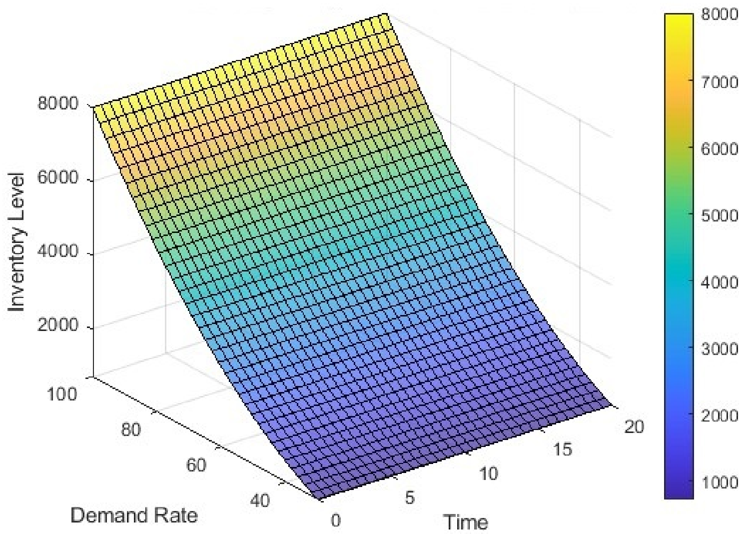

Figure 2: *Convexity with respect to D and T*

8. Conclusion and Future Directions

This study examined the impact of product reliability and time-dependent demand on an inventory model for perishable items within a pentagonal-fuzzy framework. Using the Graded Mean Integration Representation (GMIR) method for defuzzification, the research delivered an in-depth analysis of the Total Inventory Cost function and its optimal values. The findings show that integrating product reliability and time-dependent demand into the model has a substantial effect on inventory management strategies and cost optimization for perishable products. The numerical examples and sensitivity analysis provide important insights into how changes in parameters influence optimal inventory decisions, thereby increasing the model's practical relevance. This optimization technique is particularly relevant in the food and beverage industry, where managing perishable inventory is crucial to cutting waste, ensuring product quality, and meeting shifting consumer demand. It can also be applied in the pharmaceutical sector, where product shelf life and dependability have a direct bearing on both patient safety and regulatory compliance.

Future research could build on this study by investigating several new areas. For instance, including additional factors like multi-echelon supply chains or different types of perishable products could offer a deeper understanding of inventory dynamics. Applying various defuzzification methods or advanced optimization algorithms might provide more accurate results and insights. Moreover, validating the model with real-world data could strengthen its robustness and applicability. Lastly, adapting the model to consider external factors such as market fluctuations or regulatory changes could increase its relevance and applicability across different contexts.

References

[1] Maiti, Ajoy Kumar. "Cloudy fuzzy inventory model under imperfect production process with demand dependent production rate." Journal of Management Analytics 8.4 (2021): 741-763.

[2] Maity, Suman, et al. "A Study of an EOQ Model of Growing Items with Parabolic Dense Fuzzy Lock Demand Rate." Applied System Innovation 4.4 (2021): 81.

[3] Garg, Harish. "Arithmetic operations on generalized parabolic fuzzy numbers and its application." Proceedings of the national academy of sciences, India section A: Physical sciences 88.1 (2018): 15-26.

[4] Zadeh, Lotfi A. "Fuzzy sets." Information and Control (1965).

[5] Bellman, Richard E., and Lotfi Asker Zadeh. "Decision-making in a fuzzy environment." Management science 17.4 (1970): B-141.

[6] Yager, Ronald R. "A procedure for ordering fuzzy subsets of the unit interval." Information sciences 24.2 (1981): 143-161.

[7] Ezzati, Reza, et al. "An approach for ranking of fuzzy numbers." Expert Systems with applications 39.1 (2012): 690-695.

[8] Deng, Jun, et al. "Cenozoic tectono-magmatic and metallogenic processes in the Sanjiang region, southwestern China." Earth-Science Reviews 138 (2014): 268-299.

[9] Zhang, Feng, et al. "A new method for ranking fuzzy numbers and its application to group decision making." Applied mathematical modelling 38.4 (2014): 1563-1582.

[10] De, Sujit Kumar, and Ismat Beg. "Triangular dense fuzzy sets and new defuzzification methods." Journal of Intelligent & Fuzzy systems 31.1 (2016): 469-477.

[11] De, Sujit Kumar, and Gour Chandra Mahata. "Decision of a fuzzy inventory with fuzzy backorder model under cloudy fuzzy demand rate." International Journal of Applied and computational mathematics 3 (2017): 2593-2609.

[12] De, Sujit Kumar, and Gour Chandra Mahata. "A cloudy fuzzy economic order quantity model for imperfect-quality items with allowable proportionate discounts." Journal of Industrial Engineering International 15.4 (2019): 571-583.

[13] Bhunia, A. K., and M. Maiti. "Deterministic inventory models for variable production." Journal of the Operational Research Society 48.2 (1997): 221-224.

[14] Balkhi, Z. T. "On the global optimality of a general deterministic production lot size inventory model for detoriating items." JORBEL-Belgian Journal of Operations Research, Statistics, and Computer Science 38.4 (1998): 33-44.

[15] Abad, Cristina, et al. "An evaluation of the value relevance of consolidated versus unconsolidated accounting information: evidence from quoted Spanish firms." Journal of International Financial Management & Accounting 11.3 (2000): 156-177.

[16] Mandal, M., and M. Maiti. "INVENTORY OF DAMAGABLE ITEMS WITH VARIABLE REPLENISHMENT AND STOCK-DEPENDENT DEMAND." Asia-Pacific Journal of Operational Research 17.1 (2000).

[17] Saha, Anirban, et al. "Inventory models for breakable items with stock dependent demand and imprecise constraints." Mathematical and Computer Modelling 52.9-10 (2010): 1771-1782.
[18] Das, Debasis, Arindam Roy, and Samarjit Kar. "A ProductionInventory Model for a Deteriorating Item Incorporating Learning Effect Using Genetic Algorithm." Advances in Operations Research 2010.1 (2010): 146042.
[19] Das, Debasis, Arindam Roy, and Samarjit Kar. "A volume flexible economic production lot-sizing problem with imperfect quality and random machine failure in fuzzy-stochastic environment." Computers & Mathematics with Applications 61.9 (2011): 2388-2400.
[20] Das, Debasis, Arindam Roy, and Samarjit Kar. "Optimal payment time for a retailer under permitted delay of payment by the wholesaler with dynamic demand and hybrid number cost parameters." Opsearch 48 (2011): 171-196.
[21] Das, Debasis, Arindam Roy, and Samarjit Kar. "A multi-item inventory model for two-stage production system with imperfect processes using differential evolution and credibility measure." International Journal of Operations Research 9.2 (2012): 87-99.
[22] Cheng, T. C. E. "An economic order quantity model with demand-dependent unit cost." European Journal of Operational Research 40.2 (1989): 252-256.
[23] Khouja, Moutaz. "The use of data envelopment analysis for technology selection." Computers & Industrial Engineering 28.1 (1995): 123-132.
[24] Shah, Nita H., and Bhavin J. Shah. "EPQ model for time-declining demand with imperfect production process under inflationary conditions and reliability." International Journal of Operations Research 11.3 (2014): 91-99.
[25] Rosenblatt, Meir J., and Hau L. Lee. "Economic production cycles with imperfect production processes." IIE transactions 18.1 (1986): 48-55.
[26] Ben-Daya, Mohamed, and Moncer Hariga. "Economic lot scheduling problem with imperfect production processes." Journal of the Operational Research Society 51.7 (2000): 875-881.
[27] Goyal, S. K., Hung, J. H., & Chen, L. H. (2003). An imperfect production model with variable production rate and inspection. European Journal of Operational Research, 147(3), 568-578.
[28] Maiti, Ajoy Kumar, Asoke Kumar Bhunia, and Manoranjan Maiti. "An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy." Applied Mathematics and computation 183.2 (2006): 903-915.
[29] Sana, Shib Sankar, Suresh Kumar Goyal, and Kripasindhu Chaudhuri. "An imperfect production process in a volume flexible inventory model." International Journal of Production Economics 105.2 (2007): 548-559.
[30] Manna, Amalesh Kumar, Jayanta Kumar Dey, and Shyamal Kumar Mondal. "Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment." Journal of Uncertainty Analysis and Applications 2 (2014): 1-31.
[31] Pal, Brojeswar, Shib Sankar Sana, and Kripasindhu Chaudhuri. "Two-echelon competitive integrated supply chain model with price and credit period dependent demand." International Journal of Systems Science 47.5 (2016): 995-1007.
[32] Manna, Amalesh Kumar, et al. "Multi-item EPQ model with learning effect on imperfect production over fuzzy-random planning horizon." Journal of Management Analytics 4.1 (2017): 80-110.
[33] Khara, Barun, Jayanta Kumar Dey, and Shyamal Kumar Mondal. "An inventory model under development cost-dependent imperfect production and reliability-dependent demand." Journal of Management Analytics 4.3 (2017): 258-275.
[34] Pal, Sova, Manas Kumar Maiti, and Manoranjan Maiti. "An EPQ model with price discounted promotional demand in an imprecise planning horizon via Genetic Algorithm." Computers & Industrial Engineering 57.1 (2009): 181-187.
[35] Bera, Uttam Kumar, and Ajoy Kumar Maiti. "A soft-computing approach to multi-item fuzzy EOQ model incorporating discount." International Journal of Information and Decision Sciences 4.4 (2012): 313-328.

[36] Maiti, A. K., M. K. Maiti, and M. Maiti. "Inventory model with stochastic lead-time and price dependent demand incorporating advance payment." Applied Mathematical Modelling 33.5 (2009): 2433-2443.
[37] Mondal, S., and M. Maiti. "Multi-item fuzzy EOQ models using genetic algorithm." Computers & Industrial Engineering 44.1 (2003): 105-117.
[38] Maiti, Manas Kumar, and Manoranjan Maiti. "Fuzzy inventory model with two warehouses under possibility constraints." Fuzzy Sets and systems 157.1 (2006): 52-73.
[39] Maiti, Manas Kumar, and Manoranjan Maiti. "Two-storage inventory model with lot-size dependent fuzzy lead-time under possibility constraints via genetic algorithm." European Journal of Operational Research 179.2 (2007): 352-371.
[40] Yadav, Ajay Singh, et al. "Optimization of an inventory model for deteriorating items with both selling price and time-sensitive demand and carbon emission under green technology investment." International Journal on Interactive Design and Manufacturing (IJIDeM) (2023): 1-17.
[41] Yadav, Krishan Kumar, Ajay Singh Yadav, and Shikha Bansal. "Interval number approach for two-warehouse inventory management of deteriorating items with preservation technology investment using analytical optimization methods." International Journal on Interactive Design and Manufacturing (IJIDeM) (2024a): 1-17.
[42] Yadav, Krishan Kumar, Ajay Singh Yadav, and Shikha Bansal. "OPTIMIZATION OF AN INVENTORY MODEL FOR DETERIORATING ITEMS ASSUMING DETERIORATION DURING CARRYING WITH TWO-WAREHOUSE FACILITY." Reliability: Theory & Applications 19.3 (79) (2024b): 442-459.
[43] Yadav, Krishan Kumar, Ajay Singh Yadav, and Shikha Bansal. "OPTIMIZATION OF A TWO-WAREHOUSE INVENTORY MANAGEMENT FOR DETERIORATING ITEMS WITH TIME AND RELIABILITY-DEPENDENT DEMAND UNDER CARBON EMISSION CONSTRAINTS." Reliability: Theory & Applications 19.4 (80) (2024c): 404-418.
[44] Jiang, XiaoWei, et al. "Effect of exponential decay in hydraulic conductivity with depth on regional groundwater flow." Geophysical Research Letters 36.24 (2009).
[45] Rana, Upasana, and Tanuj Kumar. "OPTIMIZING AN INVENTORY MODEL FOR PERISHABLE PRODUCTS WITH PRODUCT RELIABILITY AND TIME DEPENDENT DEMAND USING PENTAGONAL-FUZZY NUMBER." Reliability: Theory & Applications 19.4 (80) (2024): 374-384.
[46] Maiti, A. K., M. K. Maiti, and M. Maiti. "An EOQ model of an item with imprecise seasonal time via genetic algorithm." International Journal of Operational Research 19.3 (2014): 358-384.
[47] Negi, Ashish, and Ompal Singh. "INVENTORY MODEL FOR PROBABILISTIC DETERIORATION WITH RELIABILITY-DEPENDENT DEMAND AND TIME USING CLOUDY-FUZZY ENVIRONMENT." Reliability: Theory & Applications 19.4 (80) (2024): 385-403.
[48] Dutta, Anurag, et al. "Evaluation modelling of asteroids™ hazardousness using chaosNet." 2023 IEEE 8th International Conference for Convergence in Technology (I2CT). IEEE, 2023.
[49] Bhunia, Asoke Kumar, and Ali Akbar Shaikh. "An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies." Applied Mathematics and Computation 256 (2015): 831-850.