ANALYSIS OF TWO VACATION POLICIES UNDER RETRIAL ATTEMPTS, MARKOVIAN ENCOURAGED ARRIVAL QUEUING MODEL

E. Ismailkhan[1*], V. Narmadha[2], N. Yathavan[1]

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[1,*] Department of Mathematics, Panimalar Engineering College, Chennai-600123
[2]Department of Mathematics, Vellore Institute of Technology, Vellore- 632014
[1,*]ismailkhanmubarak@gmail.com, [2]narmadha.v2020a@vitstudent.ac.in

Abstract

This paper presents a comprehensive analysis of a single-server Markovian queueing system incorporating two distinct vacation policies under a framework of encouraged arrivals and retrial attempts. The novelty of the model lies in its simultaneous treatment of several realistic operational dynamics: server vacations with acceleratory growth, customer impatience, server breakdown and repair, and encouraged arrivals, where potential customers are motivated to join the system depending on its state. To reflect real-world retrial scenarios, customers who do not receive immediate service reattempt after some time, modeled using a retrial queue mechanism. Two key vacation policies are compared: one governed by a first-come, first-served (FCFS) discipline and the other utilizing bulk service. The paper derives steady-state probabilities and evaluates system performance measures, such as mean queue length, server utilization, and the expected number of retrials.

Keywords: Two vacation, encouraged arrival, reneging, retrial queue, markov model.

I. Introduction

Markovian queuing models, which follow encouraged arrival rates and exponential service rates, are used in a variety of systems, including manufacturing, production, telecommunications, computers, and transportation. Everyone has a hectic schedule and little free time in the modern world. Because the customer's arrival is unpredictable, they cannot complete their task in the allotted time because they cannot predict it.

We consider the single server to be encouraged for working vacations, breakdowns, and repairs where the server may be down at any time. A Poisson distribution governs customer arrival rates, whereas an exponential distribution governs customer service rates. Arriving customers will join the orbit group if they discover that the server is too busy serving another customer. In [1], a Markovian queue which has dependent arrival and breakdown is investigated. A single vacation on an unstable bulk server is detailed in [2]. Batch arrivals on an infinite server are investigated in [3]. Examples of batch queues with inadequate identification are explored in [4].

A generalized bulk queue with the Poisson model is addressed in [5]. In [6], a general bulk queuing approach with a dual working vacation. In [7], a retrial model with a consistent retrial rate, breakdowns, and dissatisfied customers were studied. A dual server queuing model for bulk arrival and service was examined in [8]. Working vacations on bulk arrival queues combined with reneging and interruptions were studied in [9]. Dual service and two vacations are investigated in the bulk arrival Markovian system [10]. They investigated batch arrival and retrial queues using a dual vacation policy and the Markovian queuing model [11]. In [12], an M/M/1/N system along with encouraged arrivals. Reduced wait times in an M/M/1/N encouraged customer arrivals, as seen in [13]. Reneging customers were observed in [14] when the M/M/c/N model was used in conjunction with encouraged arrivals.

II. Model Elaboration

The following presumptions have been taken into account:

- The customers follow the First-Come-First-Serve rule
- Customers arrive according to an encouraged arrival with - mean $\frac{1}{\lambda*(1+\omega)}$, $\boldsymbol{\omega}$ representing the offered value and accelerated distribution with mean $\frac{1}{\mu}$
- Customers will enter the retrial queue with probability "s"
- After a few tries, the customer notices the accelerated distribution with an average of $1-\boldsymbol{\varphi}$ and attempts the request from the retrial space.
- When a customer attempts to receive service after a certain amount of time has passed, with probability $\boldsymbol{\delta_0}$
- The server will fail time following the accelerated distribution under the fail rate $\boldsymbol{\nu_1}$ and from the service to the customer under the repair rate $\boldsymbol{\nu_2}$.
- We have given that the random variable J(t) describes the absolute customers of the system at period 't', and R(t) = 0, 1, 2, 3…. consider the states, a server is free, a server is busy, server is on vacation and the server is in breakdown & repair state at period "t".

III. Analysis of System-size distribution

From first state $0, \mathrm{j} \geq 0$ when the- the server is idle- state:

$$\lambda*(1+\omega)\overline{\mathrm{P}}_{0,0} = \mu\overline{\mathrm{P}}_{1,0} \tag{1}$$

$$(\lambda*(1+\omega)+\mathrm{l}\psi\delta_0)\overline{\mathrm{P}}_{0,\,\mathrm{l}} = \mathrm{l}\mu\overline{\mathrm{P}}_{1,\,\mathrm{l}} \tag{2}$$

$$(\lambda*(1+\omega)+\mathrm{j}\varphi\delta_0)\overline{\mathrm{P}}_{0,\mathrm{j}} = \mathrm{j}\mu\overline{\mathrm{P}}_{1,\mathrm{j}} \tag{3}$$

From second state $1, j \geq 0$ when the server is busy- state:

$$(\nu_1+\Delta_0+\mathrm{s}\lambda*(1+\omega)+\mu)\overline{\mathrm{P}}_{1,0} = (1-\mathrm{s})\varphi\overline{\mathrm{P}}_{1,1}+\nu_2\overline{\mathrm{P}}_{3,0}+\lambda*(1+\omega)\overline{\mathrm{P}}_{2,0}+\varphi\delta_0\overline{\mathrm{P}}_{0,1}+\lambda*(1+\omega)\overline{\mathrm{P}}_{0,0}$$

$$(1-\Delta_0+\mathrm{s}\lambda*(1+\omega)+\mathrm{l}\mu+\nu_1)\mathrm{P}_{1,\,\mathrm{l}} \tag{4}$$

$$= \nu_2\overline{\mathrm{P}}_{3,\,\mathrm{l}}+\mu\overline{\mathrm{P}}_{2,\,\mathrm{l}}+\lambda*(1+\omega)\overline{\mathrm{P}}_{0,\,\mathrm{l}}+\mathrm{s}\lambda*(1+\omega)\overline{\mathrm{P}}_{1,\,\mathrm{l}-1}+(1-\mathrm{s})(\mathrm{l}+1)\varphi\mathrm{P}_{1,\,\mathrm{l}+1}+(\mathrm{l}+1)\varphi\delta_0\overline{\mathrm{P}}_{0,\,\mathrm{l}+1}\big(1-\Delta_0+\nu_1+\mathrm{j}\mu+(1-\mathrm{s})\mathrm{j}\varphi+\mathrm{s}\lambda*(1+\omega)\big)\overline{\mathrm{P}}_{1,\mathrm{j}} \tag{5}$$

$$= \mu\overline{P}_{2,j} + \nu_2\overline{P}_{3,j} + \lambda * (1+\omega)\overline{P}_{0,j} + s\lambda * (1+\omega)\overline{P}_{1,-1}$$

$$+ s\lambda * (1+\omega)(j+1)\varphi\overline{P}_{1,j+1} + (j+1)\varphi\delta_0\overline{P}_{0,j+1} \quad (6)$$

From third state 2, when the server is on vacation:

$$2\lambda * (1+\omega)\overline{P}_{2,0} = \Delta_0\overline{P}_{0,1} \quad (7)$$

$$\big(\mu + \lambda * (1+\omega)\big)\overline{P}_{2,1} = (1-\Delta_0)P_{1,1} + \lambda * (1+\omega)\overline{P}_{1,1-1} \quad (8)$$

$$\mu\overline{P}_{2,j} = (1-\Delta_0)\overline{P}_{1,j} + \lambda * (1+\omega)\overline{P}_{1,j-1} \quad (9)$$

For fourth state 3, when the- server break down and is repaired state:

$$\nu_2\overline{P}_{3,0} = \nu_1\overline{P}_{1,0} \quad (10)$$

$$\nu_2\overline{P}_{3,1} = \nu_1\overline{P}_{1,1} \quad (11)$$

$$\nu_2\overline{P}_{3,j} = \nu_1\overline{P}_{1,j} \quad (12)$$

The following conclusions may be drawn from equations (1), (2), and (3).

$$\left.\begin{aligned} \overline{P}_{1,0} &= \left(\frac{\lambda*(1+\omega)}{\mu}\right)\overline{P}_{0,0} \\ \overline{P}_{1,m} &= \left(\frac{\lambda*(1+\omega)+1\varphi\delta_0}{1\mu}\right)\overline{P}_{0,1} \\ \overline{P}_{1,j} &= \left(\frac{\lambda*(1+\omega)+j\varphi\delta_0}{j\mu}\right)\overline{P}_{0,j} \end{aligned}\right\} \quad (13)$$

Using (4), now with j = 0 and (10), we obtain

$$(\nu_1 + \Delta_0 + s\lambda * (1+\omega) + \mu)\overline{P}_{1,0} = (1-s)\varphi\overline{P}_{1,1} + \nu_1\overline{P}_{1,0} + \lambda * (1+\omega)\overline{P}_{2,0} + \varphi\delta_0\overline{P}_{0,1} + \lambda * (1+\omega)\overline{P}_{0,0}$$

$$(\Delta_0 + s\lambda * (1+\omega) + \mu)\overline{P}_{1,0} = (1-s)\varphi\overline{P}_{1,1} + \lambda * (1+\omega)\overline{P}_{2,0} + \varphi\delta_0\overline{P}_{0,1} + \lambda * (1+\omega)\overline{P}_{0,0}$$

If we use (1), we obtain

$$\left(\frac{\Delta_0\lambda * (1+\omega) + s(\lambda * (1+\omega))^2}{\mu}\right)\overline{P}_{0,0} = (1-s)\varphi\overline{P}_{1,1} + \lambda * (1+\omega)\overline{P}_{2,0} + \varphi\delta_0\overline{P}_{0,1}$$

When k=1, use (13)

$$\begin{aligned} \overline{P}_{1,1} &= \left(\frac{\lambda*(1+\omega)+\varphi\delta_0}{\mu}\right)\overline{P}_{0,1} \\ \overline{P}_{1,1} &= a_1\overline{P}_{0,1} \\ \left(\frac{\Delta_0\lambda*(1+\omega)+s(\lambda*(1+\omega))^2}{\mu}\right)\overline{P}_{0,0} &= \end{aligned} \quad (14)$$

$$(1-s)\varphi\, a_1\overline{P}_{0,1} + \lambda * (1+\omega)\overline{P}_{2,0} + \varphi\delta_0\overline{P}_{0,1}$$

If we use (7)

$$\left(\frac{\Delta_0\lambda*(1+\omega)+s(\lambda*(1+\omega))^2}{\mu}\right)\overline{\mathrm{P}}_{0,0}=\left[(1-\mathrm{s}+\mathrm{a}_1+\delta_0)\varphi+\frac{\Delta_0}{2}\right]\overline{\mathrm{P}}_{0,1}$$

$$\overline{\mathrm{P}}_{0,1}=\frac{2(\Delta_0\lambda*(1+\omega)+s(\lambda*(1+\omega)\Delta)^2)}{\mu\left[(1-\mathrm{s}+\mathrm{a}_1+\delta_0)\varphi+\frac{\Delta_0}{2}\right]}\overline{\mathrm{P}}_{0,0}$$

$$\overline{\mathrm{P}}_{0,1}=\frac{\mathrm{b}_2}{\mathrm{c}_2}\overline{\mathrm{P}}_{0,0} \tag{15}$$

Applying j=1 in (6)

$$[1-\Delta_0+\nu_1+\mu+(1-s)\varphi+s\lambda*(1+\omega)]\overline{\mathrm{P}}_{1,1}$$

$$=\mu\overline{\mathrm{P}}_{2,1}+\nu_2\overline{\mathrm{P}}_{3,1}+\lambda*(1+\omega)\overline{\mathrm{P}}_{0,1}+s\lambda*(1+\omega)\overline{\mathrm{P}}_{1,0}+2s\lambda*(1+\omega)\varphi\overline{\mathrm{P}}_{1,2}+2\varphi\delta_0\overline{\mathrm{P}}_{0,2}$$

To do this, use (12) and assign j=1.

$$[1-\Delta_0+\nu_1+(1-\mathrm{s})\varphi+\mathrm{s}\lambda(1+\omega)+\mu]\overline{\mathrm{P}}_{1,1}=\mu\overline{\mathrm{P}}_{2,1}+\nu_1\overline{\mathrm{P}}_{1,1}+\lambda(1+\omega)\overline{\mathrm{P}}_{0,1}+\mathrm{s}\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}$$
$$+2\mathrm{s}_1\lambda(1+\omega)\overline{\mathrm{P}}_{1,2}+2\varphi\delta_0\overline{\mathrm{P}}_{0,2}$$

$$[1-\Delta_0+(1-\mathrm{s})\varphi+\mu+\mathrm{s}\lambda(1+\omega)]\overline{\mathrm{P}}_{1,1}=\mu\overline{\mathrm{P}}_{2,1}+\lambda(1+\omega)\overline{\mathrm{P}}_{0,1}+\mathrm{s}_{1,0}\overline{\mathrm{P}}_{1,0}+$$
$$2\mathrm{s}_1\lambda(1+\omega)\overline{\mathrm{P}}_{1,2}+2\varphi\delta_0\overline{\mathrm{P}}_{0,2}$$

If we use (9) for $j=1$ and (14), we get

$$\mu\overline{\mathrm{P}}_{2,1}=(1-\Delta_0)\overline{\mathrm{P}}_{1,1}+\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}[(1-s)\varphi+\mu+s\lambda(1+\omega)]\mathrm{a}_1\overline{\mathrm{P}}_{0,1}$$

$$=\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}+\lambda(1+\omega)\overline{\mathrm{P}}_{0,1}+s\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}+2s\lambda(1+\omega)\delta_0\overline{\mathrm{P}}_{1,2}+2\varphi\delta_0\overline{\mathrm{P}}_{0,2}$$

$$[(1-s)\varphi\,\mathrm{a}_1+\mu\mathrm{a}_1+s\mathrm{a}_1-\lambda(1+\omega)]\overline{\mathrm{P}}_{0,1}$$

$$=\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}+s\lambda(1+\omega)\overline{\mathrm{P}}_{1,0}+2s\lambda\delta_0\overline{\mathrm{P}}_{1,2}+2\varphi\delta_0\overline{\mathrm{P}}_{0,2} \tag{16}$$

Now, we use (13) put j = 2, and we obtain

$$\overline{\mathrm{P}}_{1,2}=\left(\frac{\lambda(1+\omega)+2\varphi\delta_0}{2\mu}\right)\overline{\mathrm{P}}_{0,2}=\frac{\mathrm{b}_1}{\mathrm{c}_1}\overline{\mathrm{P}}_{0,2}$$

Additionally, using (1) and (15), we can solve (16) to obtain

$$\overline{\mathrm{P}}_{0,2}=\frac{\mathrm{b}_3}{\mathrm{c}_3}\overline{\mathrm{P}}_{0,0} \tag{17}$$

$$\mathrm{b}_3=\frac{\begin{array}{c}s\lambda(1+\omega)\mathrm{a}_1\,\mathrm{b}_2+\varphi(1-s)\mathrm{a}_1\,\mathrm{b}_2\\+\mu\mathrm{a}_1\,\mathrm{b}_2-\lambda(1+\omega)\mathrm{b}_2\end{array}}{\mathrm{c}_2}$$

$$-\frac{(\lambda(1+\omega))^2(1+s)}{\mu}$$

$$\mathrm{c}_3=2s\lambda(1+\omega)\delta_0\,\mathrm{b}_1/\mathrm{c}_1+2\delta_0\varphi$$

When you enter j=2 in equations 6, 9, and 12, we get

$$\left(1-\Delta_0+\nu_1+2\mu+2(1-s)\varphi+s\lambda(1+\omega)\right)\overline{\mathrm{P}}_{1,2}$$

$$=\mu\overline{\mathrm{P}}_{2,2}+\nu_2\overline{\mathrm{P}}_{3,2}+\lambda(1+\omega)\overline{\mathrm{P}}_{0,2}$$

$$+s\lambda(1+\omega)\overline{\mathrm{P}}_{1,1}+3s\lambda(1+\omega)\varphi\overline{\mathrm{P}}_{1,3}+3\varphi\delta_0\overline{\mathrm{P}}_{0,3}\ldots \tag{18}$$

$$\mu\overline{\mathrm{P}}_{2,2}=(1-\Delta_0)\overline{\mathrm{P}}_{1,2}+\lambda(1+\omega)\overline{\mathrm{P}}_{1,1} \tag{19}$$

$$\nu_2\overline{\mathrm{P}}_{3,2}=\nu_1\overline{\mathrm{P}}_{1,2} \tag{20}$$

If we use (19) & (20) in (18), we have

$$\left(2*\mu+2(1-s)\varphi+s\lambda(1+\omega)\right)\bar{P}_{1,2}=\lambda(1+\omega)(1+s)\bar{P}_{1,1}+\lambda(1+\omega)\bar{P}_{0,2}$$

$$+3s\lambda(1+\omega)\varphi\bar{P}_{1,3}+3\varphi\delta_0\bar{P}_{0,3} \tag{21}$$

$$\left(2*\mu+2(1-s)\varphi+s\lambda(1+\omega)\right)\frac{b_1}{c_1}\bar{P}_{0,2}=\lambda(1+\omega)(1+s)\bar{P}_{1,1}+\lambda(1+\omega)\bar{P}_{0,2}$$

$$+3s\lambda(1+\omega)\varphi\bar{P}_{1,3}+3\varphi\delta_0\bar{P}_{0,3}$$

$$\left[(2*\mu+2(1-s)\varphi+s\lambda(1+\omega))\frac{b_1}{c_1}-\lambda(1+\omega)\right]\bar{P}_{0,2}$$

$$=\lambda(1+\omega)(1+s)\bar{P}_{1,1}+3s\lambda(1+\omega)\varphi\bar{P}_{1,3}+3\varphi\delta_0\bar{P}_{0,3}\left[(2*\mu+2(1-s)\varphi+s\lambda(1+\omega))\frac{b_1}{c_1}-\lambda(1+\omega)\right]\frac{b_3}{c_3}\bar{P}_{0,0}$$

$$=\lambda(1+\omega)(1+s)a_1\bar{P}_{0,1}+3s\lambda(1+\omega)\varphi\bar{P}_{1,3}+3\varphi\delta_0\bar{P}_{0,3}\left[(2*\mu+2(1-s)\varphi+s\lambda(1+\omega))\frac{b_1}{c_1}-\lambda(1+\omega)\right]\frac{b_3}{c_3}\bar{P}_{0,0}$$

$$=\lambda(1+\omega)(1+s)a_1\frac{b_2}{c_2}\bar{P}_{0,0}+3s\lambda(1+\omega)\varphi\bar{P}_{1,3}+3\varphi\delta_0\bar{P}_{0,3}$$

$$\left[\begin{matrix}\left\{\left(2*\mu+2(1-s)\varphi+s\lambda(1+\omega)\right)\frac{b_1}{c_1}-\lambda(1+\omega)\right\}\frac{b_3}{c_3}\\ -\lambda(1+\omega)(1+s)a_1\frac{b_2}{c_2}\end{matrix}\right]\bar{P}_{0,0}$$

$$=3s\lambda(1+\omega)\varphi\overline{\mathrm{P}}_{1,3}+3\varphi\delta_0\bar{P}_{0,3}$$

If, we Put j = 3 in (13), and we have

$$\overline{\mathrm{P}}_{1,3}=\left(\frac{\lambda(1+\omega)+j\varphi\delta_0}{j\mu}\right)\overline{\mathrm{P}}_{0,3}=\frac{\mathrm{b}_4}{\mathrm{c}_4}\overline{\mathrm{P}}_{0,3}$$

$$[\left\{\begin{matrix}(2*\mu+2(1-s)\varphi+\\ s\lambda(1+\omega))\frac{\mathrm{b}_1}{\mathrm{c}_1}-\lambda(1+\omega)\end{matrix}\right\}\frac{\mathrm{b}_3}{\mathrm{c}_3}-\lambda(1+\omega)(1+s)\mathrm{a}_1\frac{\mathrm{b}_2}{\mathrm{c}_2}]\overline{\mathrm{P}}_{0,0}$$

$$= \left(3s\varphi \frac{b_4}{c_4} + 3\varphi\delta_0\right) \overline{P}_{0,3}$$

$$\overline{P}_{0,3} = \frac{\left\lfloor \begin{matrix} \left\{ \begin{matrix} (2 * \mu + 2(1-s)\varphi + \\ s\lambda(1+\omega)) \frac{b_1}{c_1} - \lambda(1+\omega) \end{matrix} \right\} \frac{b_3}{c_3} - \\ \lambda(1+\omega)(1+s)a_1 \frac{b_2}{c_2} \end{matrix} \right\rfloor}{\left(\frac{3\lambda(1+\omega)\varphi\, b_4}{c_4} + 3\varphi\delta_0\right)} \overline{P}_{0,0}$$

$$\overline{P}_{0,3} = \frac{b_5}{c_5} \overline{P}_{0,0}$$

In general, we get

$$\overline{P}_{0,n} = \overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots \quad (22)$$

If we use (13), we get

$$\overline{P}_{1,j} = \left(\frac{\lambda(1+\omega) + j\psi\delta_0}{j\mu}\right) \left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right] \quad (23)$$

If, we use (9)

$$\overline{P}_{2,j} = (1-\Delta_0)\left(\frac{\lambda(1+\omega) + j\Sigma\delta_0}{j * \mu^2}\right)$$

$$\left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right] + \frac{\lambda(1+\omega)\overline{P}_{1,j-1}}{\mu} \quad (24)$$

Similarly, if, we use (12), we get

$$\overline{P}_{3,j} = \left(\frac{\nu_1}{\nu_2}\right) \left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right] \quad (25)$$

Now, if we use equations (22), (23), (24), and (25), we obtain

$$\overline{P}_{r,n} = \begin{cases} \overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots \text{ for } s = 0 \\ \left(\frac{\lambda(1+\omega) + j\varphi\delta_0}{j}\right) \left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right], \text{ for } s = 1 \\ (1-\Delta_0)\left(\frac{\lambda(1+\omega) + j\varphi\delta_0}{j * \mu^2}\right) \left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right] + \frac{\lambda(1+\omega)\overline{P}_{1,j-1}}{\mu}, \\ \text{ for } s = 2, \left(\frac{\nu_1}{\nu_2}\right) \left[\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3} + \cdots\right], \text{ for } s = 3 \end{cases}$$

To calculate the value of $\bar{P}_{0,0}$, the normalization function, we have

$$\sum_{s=0}^{3} \sum_{j=0}^{\infty} P_{i,s} = 1$$

$$\left(\overline{P}_{0,1} + \overline{P}_{0,2} + \overline{P}_{0,3}\right)\left(1 + \frac{\lambda(1+\omega) + j\varphi\delta_0}{j * \mu^2} + \frac{\nu_1}{\nu_2}\right)$$

$$+ \frac{\lambda(1+\omega)}{\mu} \overline{P}_{1,j-1} = 1$$

$$\overline{\mathrm{P}}_{0,0}\left(\frac{\mathrm{b}_2}{\mathrm{c}_2}+\frac{\mathrm{b}_3}{\mathrm{c}_3}+\frac{\mathrm{b}_5}{\mathrm{c}_5}\right)\left(1+\frac{\lambda(1+\omega)+\mathrm{j}\varphi\delta_0}{\mathrm{j}*\mu^2}+\frac{\nu_1}{\nu_2}\right)$$

$$+\frac{\lambda(1+\omega)}{\mu}\mathrm{P}_{1,\mathrm{j}-1}=1$$

$$\overline{\mathrm{P}}_{0,0}=\frac{1-\frac{\lambda(1+\omega)}{\mu}\overline{\mathrm{P}}_{1,\mathrm{j}-1}}{\left(\frac{\mathrm{b}_2}{\mathrm{c}_2}+\frac{\mathrm{b}_3}{\mathrm{c}_3}+\frac{\mathrm{b}_5}{\mathrm{c}_5}\right)\left(1+\frac{\lambda(1+\omega)+\mathrm{j}\varphi\delta_0}{\mathrm{j}*\mu^2}+\frac{\nu_1}{\nu_2}\right)}$$

IV. Validation of the model

(i) When the server is free:

$$\bar{P}_0 \quad =\sum_{j=0}^{\infty}\bar{P}_{0,j}$$

$$=\frac{1-\frac{\lambda(1+\omega)}{\mu}\bar{P}_{1,j-1}}{\left(1+\frac{\lambda(1+\omega)+j\varphi\delta_0}{j*\mu^2}+\frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)}$$

$$\left[\bar{P}_{0,1}+\bar{P}_{0,2}+\bar{P}_{0,3}+\cdots\right]$$

(ii) When the server is busy:

$$\bar{P}_1 \quad =\sum_{j=0}^{\infty}\bar{P}_{1,j}$$

$$=\frac{1-\frac{\lambda(1+\omega)}{\mu}\bar{P}_{1,j-1}}{\left(1+\frac{\lambda(1+\omega)+j\varphi\delta_0}{j*\mu^2}+\frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)}$$

$$\left(\frac{\lambda(1+\omega)+j\varphi\delta_0}{j*\mu}\right)\times\left[\bar{P}_{0,1}+\bar{P}_{0,2}+\bar{P}_{0,3}+\cdots\right]$$

(iii) When the server is on vacation:

$$\bar{P}_2 \quad =\sum_{j=0}^{\infty}P_{2,j}$$

$$=\frac{1-\frac{\lambda(1+\omega)}{\mu}\bar{P}_{1,j-1}}{\left(1+\frac{\lambda(1+\omega)+j\varphi\delta_0}{j*\mu^2}+\frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2}+\frac{b_3}{c_3}+\frac{b_5}{c_5}\right)}$$

$$\times\left[(1-\Delta_0)\left(\frac{\lambda(1+\omega)+j\varphi\delta_0}{j*\mu^2}\right)\right]$$

$$[\bar{P}_{0,1} + \bar{P}_{0,2} + \bar{P}_{0,3} + \cdots] + \frac{\lambda(1+\omega)\bar{P}_{1,j-1}}{\mu}$$

(iv) When the server is in a break down and repaired state:

$$\bar{P}_3 = \sum_{j=0}^{\infty} P_{3,j}$$

$$= \frac{1 - \frac{\lambda(1+\omega)}{\mu}\bar{P}_{1,j-1}}{\left(1 + \frac{\lambda(1+\omega) + j\varphi\delta_0}{j * \mu^2} + \frac{v_1}{v_2}\right)\left(\frac{b_2}{c_2} + \frac{b_3}{c_3} + \frac{b_5}{c_5}\right)}\left(\frac{v_1}{v_2}\right)$$

$$\times [\bar{P}_{0,1} + \bar{P}_{0,2} + \bar{P}_{0,3} + \cdots]$$

V.Conclusion

The Markovian Encouraged Arrival Queuing Model has been developed with the inclusion of customer retry efforts, balking, and reneging behavior. The four system states—idle state, busy state, vacation state, breakdown state, and repair state—have all been taken into consideration utilizing the concept of encouraged arrival. We have examined and verified the possibilities of the various conditions. Neural networks, communication systems, post offices, and supermarkets can all benefit from using this model to reduce the reneging and balking behavior of their customers.

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