RELIABILITY ANALYSIS OF THE SHAFT SUBJECTED TO FLUCTUATING LOADS UNDER TORSION AND BENDING MOMENTS WHEN SHEAR STRESS FOLLOWS EXPONENTIAL DISTRIBUTION

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Abstract

The reliability of shafts subjected to fluctuating loads is essential for ensuring the safe and efficient operation of rotating machinery in various engineering applications, including automotive, aerospace, and industrial systems. Accurate reliability assessments are crucial to preventing failures and extending the lifespan of mechanical components. This study examines the reliability analysis of shafts subjected to combined torsional and bending moments, which create complex stress states that significantly affect shaft performance and failure behaviour. A statistical model is considered since the shear stress generated is random in nature. Analytical expressions for reliability are derived by changing the parameters twisting moment, bending moment, and shaft diameter.

Keywords: Reliability, circular solid shaft, combined twisting and bending moment, fluctuating loads, exponential distribution.

1. Introduction

Shafts are critical components in mechanical systems, designed to transmit power between rotating elements. These components are commonly subjected to complex loading conditions, including torsion and bending moments, during operation. In conventional design approaches, it is often assumed that the torque and bending moments acting on a shaft are constant. However, in practical applications, such as line shafts and counter shafts, shafts frequently experience fluctuating loads due to dynamic operating conditions. These fluctuations can induce combined effects of shock and fatigue, significantly influencing the reliability and lifespan of the shaft. To ensure the safe and reliable operation of rotating machinery, it is essential to perform a reliability analysis that accounts for the stochastic nature of the loading conditions. Shear stress, a critical factor influencing shaft failure, can vary unpredictably under fluctuating loads. Modeling shear stress as a random variable using an exponential distribution provides a practical framework for evaluating the reliability of shafts under such conditions.

Adekunle A. A. et al. [1] studied the development of CAD software for shafts under various loading conditions. Dr. Edward E. Osakue et al. [2] studied fatigue shaft design verification for bending and torsion. Dr. Edward E. Osakue et al. [3] studied the probabilistic fatigue design of shafts for bending and torsion. Fatima K. et al. [4] discussed the statistical properties of the exponential Rayleigh distribution and its application to medical sciences and engineering. Frydrysek K. et al. [5] studied performance-based design applied to a shaft subjected to combined stress.

Gowtham et al. [6] studied drive shaft design and analysis. Kamboh, M. S., et al. [8] discussed the design and analysis of the drive shaft with a critical review of advanced composite materials and the root causes of shaft failure. Misra, A., et al. [9] studied the reliability analysis of drilled shaft behaviour using the finite difference method and Monte Carlo simulation. Nadarajah S. [10] studied reliability for lifetime distributions through mathematical and computer modelling. Nayek et al. [11] studied the reliability approximation for a solid shaft under a gamma setup. Patel, B., et al. [12] studied a critical review of the design of a shaft with multiple discontinuities and combined loadings. T. S. Uma Maheswari et al. [14] obtained reliability analysis of unsymmetrical columns subjected to eccentric loads for stress following exponential distribution. Villa-Covarrubias, B., et al. [15] discussed the probabilistic method to determine the shaft diameter and design reliability.

If the shaft is subjected to fluctuating loads that include both torsion (twisting moment) and bending moment, the diagram illustrates the relationship between time and torsion, as well as between time and bending moment.

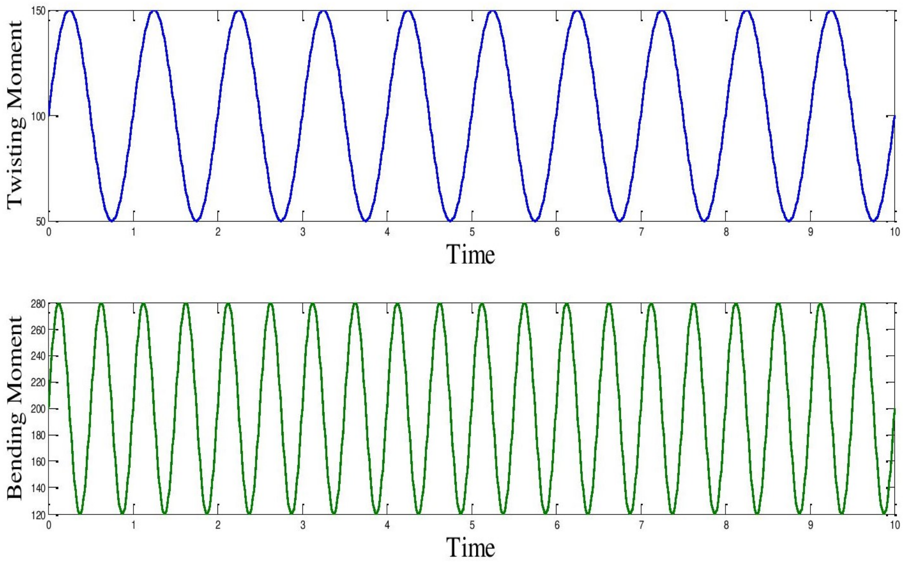

Figure 1: *Shaft subjected to fluctuating loads*

2. Statistical model

The probability density function for exponentially distributed strength ξ and for exponentially stress χ is given by [7]

$$f_\xi(\xi) = \lambda_\xi \, e^{-\lambda_\xi \times \xi} \text{ for } 0 \leq \xi < \infty,\ \lambda_\xi > 0$$

$$f_\chi(\xi) = \lambda_\chi \, e^{-\lambda_\chi \times \chi} \text{ for } 0 \leq \chi < \infty,\ \lambda_\chi > 0$$

Reliability of the stress-strength system is the probability that the strength ξ is greater than the stress χ applied on it is given by [13]

$$R = \int_0^\infty f_\chi(\chi) \left[\int_\chi^\infty f_\xi(\xi)\, d\xi \right] d\chi = \int_0^\infty \lambda_\chi \exp\left[-(\lambda_\xi + \lambda_\chi)\chi \right] \, d\chi \tag{1}$$

Then the reliability is

$$R = \frac{\bar{\xi}}{\bar{\xi} + \bar{\chi}} \tag{2}$$

Let T be the torsion, M be the maximum bending moment, and d be the diameter of the shaft. For

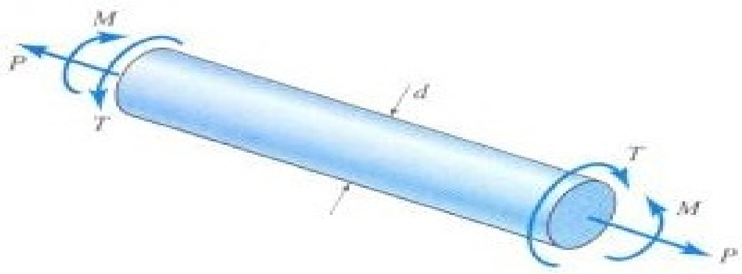

Figure 2: *Shaft subjected to combined torsion and bending moment*

a shaft subjected to combined torsion and maximum bending, the equivalent twisting moment is given by [13]

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} \tag{3}$$

and equivalent bending moment is given by [13]

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] \tag{4}$$

where K_m is combined shock and fatigue factor for bending and K_t is combined shock and fatigue factor for torsion.

Recommended values for K_m and K_t

Nature of load for stationary shafts	K_m	K_t
(a) Gradually applied load	1.0	1.0
(b) Suddenly applied load	1.5 to 2.0	1.5 to 2.0

Nature of load for rotating shafts	K_m	K_t
(a) Gradually applied load or steady load	1.5	1.0
(b) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(c) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

The equivalent twisting moment is given by [13]

$$T_e = \frac{\pi}{16} \times \sigma_t \times d^3$$

Then the shear stress due to twisting moment is

$$\sigma_t = \frac{16 \times T_e}{\pi \times d^3} \tag{5}$$

The equivalent bending moment is given by [13]

$$M_e = \frac{\pi}{32} \times \sigma_b \times d^3$$

Then the shear stress due to bending moment is

$$\sigma_b = \frac{32 \times M_e}{\pi \times d^3} \tag{6}$$

From equations (2) and (5), the reliability of the shaft subjected to torsion with the exponentially distributed strength and stress is

$$R_t = \frac{\bar{\xi}}{\bar{\xi} + \left(\dfrac{16 \times T_e}{\pi \times d^3}\right)} \tag{7}$$

From equations (2) and (6), the reliability of the shaft subjected to bending moment with exponentially distributed strength and stress is

$$R_b = \frac{\bar{\xi}}{\bar{\xi} + \left(\dfrac{32 \times M_e}{\pi \times d^3}\right)} \tag{8}$$

3. Numerical results and discussion

Numerical results and discussions on the shaft reliability under combined torsion and bending moments, where stress and strength follows exponential distribution have been presented.

Table 1: $M = 100000$ *N-mm*, $d = 52$ *mm*, $\bar{\xi} = 119.6584$ *N/mm*^{2}.

T	$R_{t(K_m=K_t=0.1)}$	$R_{b(K_m=K_t=0.1)}$	$R_{t(K_m=K_t=1.5)}$	$R_{b(K_m=K_t=1.5)}$
100000	0.995739087	0.992748016	0.939684303	0.901246468
2500000	0.929622187	0.927014622	0.468255256	0.458510624
5000000	0.868567560	0.866290841	0.305827816	0.301640839
7500000	0.815025042	0.813020043	0.227048647	0.224732753
10000000	0.767697469	0.765918310	0.180539737	0.179072384
13000000	0.717685823	0.716130686	0.144916817	0.143969874
18000000	0.647393843	0.646128147	0.109053373	0.108516258
20000000	0.622986891	0.621814744	0.099230436	0.098785527
25000000	0.569327148	0.568348068	0.080992053	0.080695416
30000000	0.524177967	0.523347903	0.068417071	0.068205276

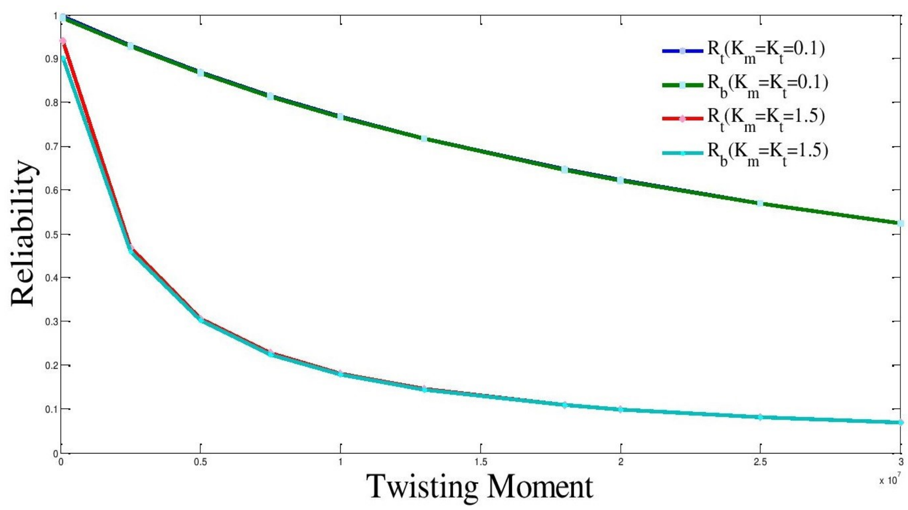

Figure 3: *As the twisting moment increases, reliability of the shaft decreases.*

Table 2: $T = 100000$ *N-mm*, $d = 52$ *mm*, $\bar{\xi} = 119.6584$ *N/mm*^{2}.

M	$R_{t(K_m=K_t=0.1)}$	$R_{b(K_m=K_t=0.1)}$	$R_{t(K_m=K_t=1.5)}$	$R_{b(K_m=K_t=1.5)}$
100000	0.995739087	0.992748016	0.939684303	0.901246468
1200000	0.964845268	0.932187659	0.646607085	0.478198768
2000000	0.942869764	0.891974623	0.523867963	0.355035226
2400000	0.932242274	0.873132117	0.478414437	0.314511825
3800000	0.896844329	0.813007064	0.366930648	0.224717879
4800000	0.873156128	0.774887696	0.314558563	0.186649213
6500000	0.835630527	0.717679829	0.253131424	0.144913151
7800000	0.809040000	0.679326476	0.220240429	0.123751748
8600000	0.793501206	0.657696792	0.203933346	0.113547822
12000000	0.733616403	0.579304536	0.155119255	0.084082248

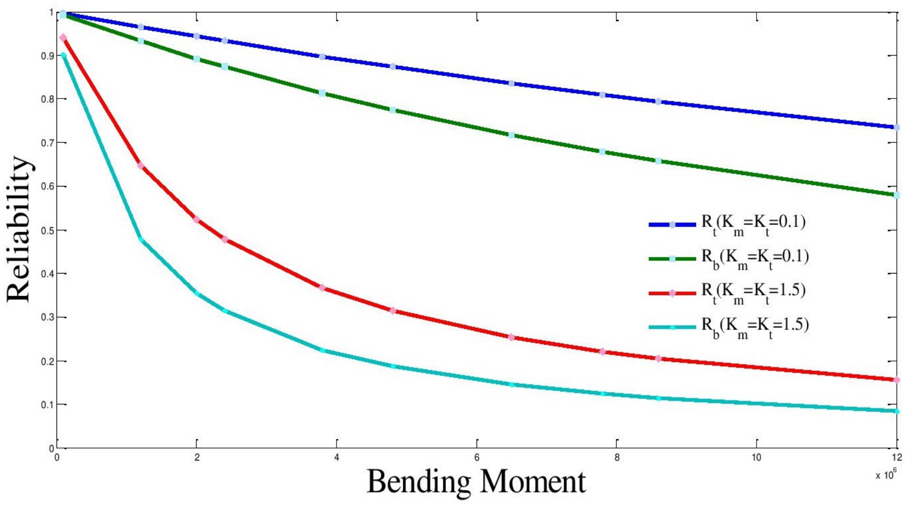

Figure 4: *As the bending moment increases, reliability of the shaft decreases.*

Table 3: $M = 100000$ *N-mm*, $T = 150000$ *N-mm*, $\bar{\xi} = 119.6584$ *N/mm*^{2}.

d	$R_{t(K_m=K_t=0.1)}$	$R_{b(K_m=K_t=0.1)}$	$R_{t(K_m=K_t=1.5)}$	$R_{b(K_m=K_t=1.5)}$
12	0.692585922	0.551516771	0.130583035	0.075770650
16	0.842278898	0.744567852	0.262547956	0.162709726
20	0.912513114	0.850595313	0.410152097	0.275125262
24	0.947433525	0.907731222	0.545778442	0.396084264
28	0.966239826	0.939839580	0.656126570	0.510159686
32	0.977128452	0.958880686	0.740135750	0.608554105
36	0.983826470	0.970762730	0.802187593	0.688815533
40	0.988157591	0.978515805	0.847626441	0.752253845
44	0.991076342	0.983771903	0.881010619	0.801643759
48	0.993112390	0.987453455	0.905771942	0.839920119

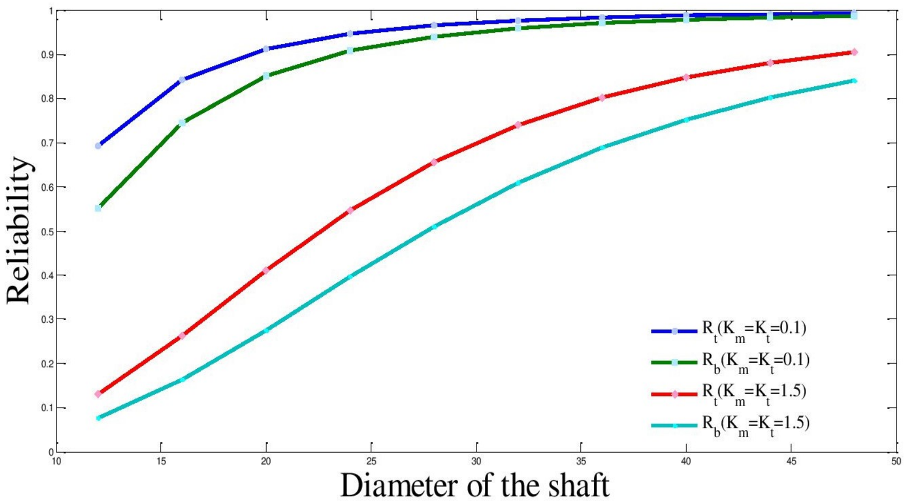

Figure 5: *As the diameter of the shaft increases, reliability of the shaft increases.*

Table 4: $M = 200000$ *N-mm*, $T = 400000$ *N-mm*, $d = 52$ *mm*.

ξ	$R_{t(K_m=K_t=0.1)}$	$R_{b(K_m=K_t=0.1)}$	$R_{t(K_m=K_t=1.5)}$	$R_{b(K_m=K_t=1.5)}$
20.5	0.926796658	0.897417429	0.457711994	0.368374369
40.5	0.961556741	0.945304732	0.625115863	0.535360857
65.5	0.975875765	0.965459724	0.729496231	0.650770585
102.5	0.984448600	0.977649248	0.808436196	0.744642702
136.5	0.988276798	0.983122507	0.848944006	0.795223260
162.5	0.990134004	0.985784519	0.869970282	0.822160764
234.5	0.993142450	0.990106003	0.906147153	0.869646132
396.5	0.995932878	0.994124692	0.942279746	0.918568479
684.5	0.997640061	0.996588265	0.965733068	0.951156933
984.5	0.998358009	0.997625431	0.975923594	0.965527473

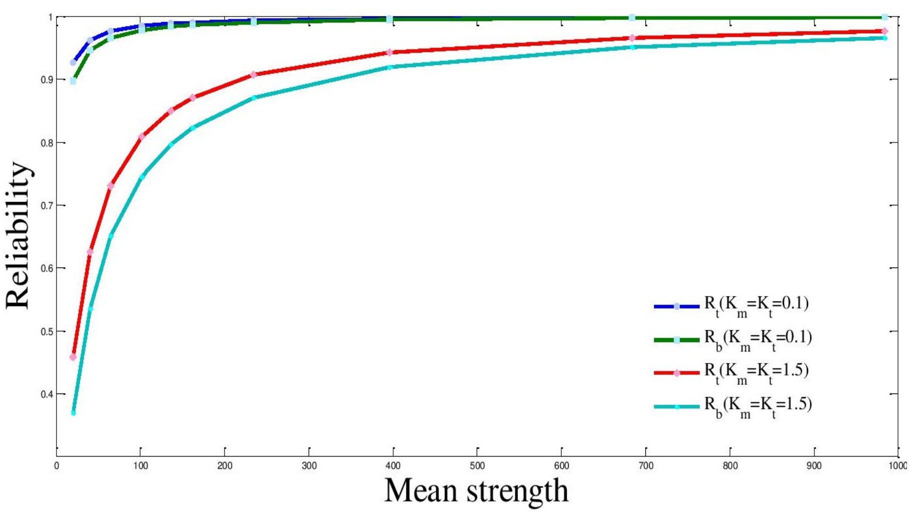

Figure 6: *As the mean strength of the shaft increases, reliability of the shaft also increases.*

4. Conclusion

The reliability of a shaft is affected when subjected to combined torsion and bending moments. According to the computations, reliability of the shaft decreases as the twisting and bending moments increase. Conversely, the reliability increases when the diameter and strength of the shaft increase.

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