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EXPONENTIAL TYPE ESTIMATORS OF POPULATION MEAN - A REVIEW

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Abstract

Survey sampling is a crucial statistical approach for making inferences about population characteristics based on the analysis of a representative sample. This method offers a practical solution for saving time and resources while ensuring reliable outcomes. Auxiliary information plays a pivotal role in enhancing the accuracy of estimators used in finite population sampling. Traditional estimators, such as ratio, product, and regression types, have proven effective in improving the precision of population parameter estimation. The introduction of exponential-type transcendental functions marked a significant advancement in this field, further increasing the efficiency of ratio and product estimators. This study provides an in-depth review of the literature on exponential-type ratio and product estimators, emphasizing their relevance in survey sampling. It examines theoretical progress and practical implementations, showcasing their potential to reduce bias and improve estimation accuracy by leveraging complex relationships between study variables and auxiliary information. The insights from this review contribute to advancing research in finite population sampling and offer valuable directions for future studies in the field.

Keywords: Exponential Estimator; Auxiliary information; Study variable; Mean square error; Auxiliary attribute

1. Introduction

The concept of sampling theory is as old as human civilization. The main aim of sampling theory is to know about the population or universe by studying only a part called the sample drawn from it. This process of inspection is very widespread and commonly used on various occasions. However, this job is never done on a very large scale.

The sampling methods are extensively used by government bodies throughout the world for assessing different characteristics of national economy as they are required for taking decisions regarding imposition of taxes, fixation of prices, minimum wages etc. for planning and projection of future economic policies. Surveys are conducted for estimation of yield rates and acreages under different crops, value added by manufacturer in the industries sector, number of unemployed persons in the labor force, construction of cost of living indices for persons in different professions and so on. Sample survey techniques are widely used in market research for assessing the preferential pattern of consumers towards the products of different types, the future possible demand for a new product which a company wishes to introduce, scope for any diversification in the production schedule and so on. Bowley [1] and Neyman [2] laid the foundations of modern sampling theory. The works of R.A. Fisher during twenties provided a scientific basis for selecting a random sample from the population. Many research papers relating to sampling theory and methods have been written for enhancing the knowledge and concepts in practice.

In sampling theory, a variable called auxiliary variable works as an instrumental variable in the estimation of the population parameters of the study variable. An auxiliary variable is a variable that is correlated with the study variable and about which information is available prior to sampling. A judicious use of this information may provide us with estimators which are more precise than those which do not make use of auxiliary information. The concept of using auxiliary information is well known in sampling theory. Its use is of paramount importance in sample surveys as it leads to increased precision of estimators for estimating the population parameters. The origin of utilizing, auxiliary information in sample surveys can be traced back to the origin of sampling theory itself. It has been a general view of the survey statisticians for last five decades that the usual methods for estimating population mean (or total) of a variable of interest (study variable), say Y may lead to much improvement in the precision by using auxiliary information. In most of the survey situations, the auxiliary information is always available in one form or the other or can be made available by diverting for this purpose a part of the survey resources at moderate cost. The method of utilizing auxiliary information depends on the form in which it is available. In whatever form it is available, one may always utilize it to devise sampling strategies which are better than those in which no auxiliary information is used. Watson [3], Neyman [4], Cochran [5] and Hansen & Hurwitz [6] were pioneer to use auxiliary information in devising estimation procedures which lead to improvement in precision of estimation. Searls [7], Thompson [8], Singh [9], Ray & Singh [10], Hirano [11], Singh and Singh [12], Maiti and Tripathi [13] have considered the problem of estimation of population parameter when such auxiliary information is available. The use of auxiliary information in theory of sampling design has been continuously gaining the attention of survey researchers from the inception due to its wide importance in all walks of real life situations.

John Graunt in 16^{th} century was the first to estimate the ratio $\frac{y}{x}$, where y represented the total population and x the known total number of registered births in the same area during the preceding year. Later, Messance and Moheau in 17^{th} century very carefully prepared estimates based on enumeration of population in certain districts of France and on the count of births, deaths and marriages as reported for the whole country. In survey sampling, when auxiliary information about all the sampling units is available then it is convenient to estimate unknown population mean of study variable using ratio estimator provided that there is a positive correlation between study variable and auxiliary variable and $\frac{C_x}{2C_y} < \rho \leq +1$. The ratio estimate of Y i.e. ratio estimate of the population total of y_i, $\hat{t}_r = \hat{R}X = \left(\frac{y}{x}\right)X = \left(\frac{\bar{y}}{\bar{x}}\right)X$. If the quantity to be estimated is $\bar{Y}$ i.e. the population means of y_i. Then the ratio estimate envisaged by Cochran [5] is, $\hat{t}_{cr} = \hat{R}\bar{X} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$. If there is negative correlation between auxiliary and the study variable and $-1 \leq \rho < -\frac{C_x}{2C_y}$, then the product method of estimation proposed by Robson [14] and revisited by Murthy [15] which can be employed efficiently is $\hat{t}_{rbp} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$. The method of utilizing auxiliary information depends upon the form in which it is available. When the auxiliary information is quantitative in nature, the authors such as Singh *et al.* [16], Das [17], Sisodia and Dwivedi [18], Sharma and Bhatnagar [19], Singh *et al.* [20], Subramani and Kumarapandiyan [21], Yadav and Kadilar [22], Vishwakarma *et al.* [23], Misra *et al.* [24], Shalabh and Tsai [25], Singh and Vishwakarma [26], Mehta *et al.* [27] etc. proposed modified ratio/product type estimators which are more efficient than the sample mean and the classical/existing ratio/product type estimators. There are many practical situations where the auxiliary information is qualitative in nature, in other words the auxiliary variable correlated with the study variable is an attribute. The authors Jhajj *et al.* [28] and Shabir & Gupta [29] have elaborated through the examples (a) the height of a person may depend on its sex (b) amount of milk produced by a cow may depend on its breed (c) the yield of a crop may depend on its variety. In all these situations, the estimators where the auxiliary information is quantitative in nature cannot be used as there exists a point bi-serial correlation between the study and the auxiliary variable . Therefore, Naik and Gupta

[30], Jhajj *et al*. [28], Shabir & Gupta ([29] , [31]) and Abd-Elfattah *et al*. [32], have made some attempts in this direction and proposed the modified estimators of population mean by using prior knowledge of the parameters of auxiliary attribute.

Bahl and Tuteja [33] were the pioneer to make use of an exponential type transcendental function in the estimators of population mean and proposed the ratio and product type estimators. Later, Singh *et al*. [34] proposed the exponential ratio and product type estimatorsof population mean using auxiliary variable as attribute. It was observed by using such type of function, a significant reduction in the MSE. Generally, the exponential type estimators are found to be estimating the population mean more precisely as compared to conventional/modified ratio and product type estimators and can be used even if there is not a strong/high degree of correlation between the auxiliary and study variable. The authors Singh *et al*. [20], Onyeka [35], Khan and Siddiqi [36], Yasmeen *et al*. [37], Yadav and Sharma [38], Zaman and Kadilar [39], Hussain *et al*. ([40], [41], [42], [43], [44], [45], [46]) etc. proposed some modified exponential ratio/product type estimators to be more precise than the conventional ratio/product and exponential ratio/product type estimators.

In the absence of the auxiliary variable the estimators discussed so far are not possible. However one may think of getting additional information on the study variable and may propose ratio and linear regression type estimators to improve the performance of the estimators. Therefore, the pioneer work of Subramani [47] proposed a precise ratio estimator to estimate population mean of skewed population without the use of auxiliary variable by taking the advantage of the median of main variable. The median of the study variable may easily be available without having exact information on every data point (see Subramani, [47]) and these median based estimators are robust in nature as they are least influenced by the outliers.

It is mentioned that the efficiency of estimators can be increased by using known information of the parameters of auxiliary variable correlated with variable under study. Sometimes such information on parameters of auxiliary variables may not be known in advance, in that situations then two-phase/double sampling design is generally used. In two-phase/double sampling, a part of the budget is used to collect information on the auxiliary variable. A large first phase preliminary sample of size η (say) and a second phase (small) sample of size n which is nested within the first phase sample ($n < \eta$) is drawn. In double sampling procedure, it is proposed to use the information gathered in the first phase sample as auxiliary information to increase the precision of the information gathered in the second phase sample. Neyman [4] was the first to formulate double sampling (or two-phase sampling) in connection with collecting information on the strata sizes in a stratified sampling. Later the theory of two-phase sampling technique have been studied at various situations by Bose [48], Sukhatme and Koshal [49], Tikkiwal [50], Sukhatme [51], Rao [52], Rao ([53], [54]) and many others. It is already discussed that large number of ratio/product, ratio cum product type, regression estimators have been proposed by different statisticians using known values of population mean, population proportion, population variance etc. of auxiliary variable for the estimation of the parameters of interest. In case such known values are not available, then one may use two-phase sampling technique to make the mentioned estimators for practical use. Several authors such as Singh *et al*. [55] , Ahmed [56], Hossain *et al*. [57], Jhajj *et al*. [58], Upadhaya *et al*. [59], Samiuddin and Hanif [60], Kamal and Shahbaz [61], Hanif *et al*. [62], have proposed different estimators in two-phase sampling. Singh *et al*. [63], proposed two-phase sampling version of exponential type estimators of Bahl and Tuteja [33] estimators. They found that estimator was more precise than the simple mean per unit, usual two-phase sampling ratio and product estimators.

2. The exponential Type Estimators

As already stated Bahl and Tuteja [33] were the first who proposed exponential ratio and product type estimators of population mean as t_{btr} and t_{btp} respectively. Further by adding K to all observations taken on x, they proposed ratio and product type estimators as t_{btr_1} and t_{btp_1}

respectivelly

$$t_{btr} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \text{ and } t_{btp} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)$$

$$t_{btr_1} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2K}\right) \text{ and } t_{btp_1} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}+2K}\right).$$

Where K is used for minimizing the MSE. These estimators used information on single auxiliary variable and the authors found them more efficient than mean per unit, ratio type estimators of Cochran [5] and product type estimator of Robson [14] in many practical situations.

Singh *et al.* [34] proposed proportion based exponential ratio and product type estimators when the auxiliary variable is an attribute as T_{sgr} and T_{sgp} respectively. Further, they proposed a class of proportion base exponential ratio-product type estimators, if P is not known as T_{sgrp}

$$T_{sgr} = \bar{y}\exp\left[\frac{P-p}{P+p}\right],\ T_{sgp} = \bar{y}\exp\left[\frac{p-P}{p+P}\right]$$

$$T_{sgrp} = \bar{y}\left[\alpha\exp\left(\frac{P-p}{P+p}\right) + (1-\alpha)\exp\left(\frac{p-P}{p+P}\right)\right].$$

The authors have also proposed two phase sampling version of the proposed exponential type estimators T_{sgr}, T_{sgp} and T_{sgrp} as

$$T_{sgr(d)} = \bar{y}\exp\left[\frac{p'-p}{p'+p}\right],\ T_{sgp(d)} = \bar{y}\exp\left[\frac{p-p'}{p+p'}\right] \text{ and}$$

$$T_{sgrp(d)} = \bar{y}\left[\alpha_1\exp\left(\frac{p'-p}{p'+p}\right) + (1-\alpha_1)\exp\left(\frac{p-p'}{p+p'}\right)\right] \text{ respectively.}$$

The real constants α and α_1 were determined such that MSE of is minimum. They have derived large sample properties of all the proposed estimators to the first order of approximation and found them efficient in both the sampling schemes theoretically as well as empirically.

Singh *et al.* [20] proposed exponential ratio type estimators of population mean as t_{sgr}, where $a(\neq 0)$, b are either the real numbers or functions of known parameters such as coefficient of variation (C_x), coefficient of kurtosis $(\beta_2(x))$, correlation coefficient (ρ_{xy}) etc.

$$t_{sgr} = \bar{y}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}-b)}{(a\bar{X}+b)+(a\bar{x}-b)}\right].$$

The known parameters taken as suitable combinations, produced ten estimators t_{sgr_1} to $t_{sgr_{10}}$. Further, combining the estimators t_{sgr_1} and t_{sgr_i} $(i = 2, ..., 10)$, an estimator

$$t^*_{sgr_i} = \alpha t_{sgr_1} + (1-\alpha)t_{sgr_i}$$

is also proposed by the authors, where α is a constant such that the MSE of $t^*_{sgr_i}$ is minimum. The choice of estimators mainly depend upon the available known parameters like $\beta_2(x)$, ρ_{xy}, C_x etc. They have obtained the expressions of Bias and MSE of all the proposed estimators up to first order of approximation, the conditions under which proposed estimators work more efficiently than some existing estimators were also obtained. Finally the authors supported the theoretical findings by numerical illustration which concludes that there was gain in efficiency by the proposed estimators.

Grover and Kaur [76] proposed exponential ratio type estimators of population mean using qualitative auxiliary information under simple random sampling as

$$T_{gkr} = [\alpha\bar{y} + \beta(P-p)]\exp\left[\frac{P-p}{P+p}\right],$$

where α and β are constants whose values are suitably chosen. The authors have derived large sample properties of the proposed estimators and compared with some existing estimators under consideration both theoretically as well as empirically, finally observed the proposed estimators were more efficient.

Using auxiliary information, Solanki *et al.* [68] proposed a class of alternative estimator of population mean in sample surveys as

$$t_{slr} = \bar{y}\Big[2 - (\frac{\bar{x}}{\bar{X}})^{\alpha} \exp\Big(\frac{\delta(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\Big)\Big],$$

where the constants (α, δ) used in the estimator are suitably chosen. They have derived the asymptotic expressions of Bias and MSE, also identified the asymptotic optimum estimator from the proposed class of estimators. Further, the authors have made a comparison of the proposed estimators with several other existing estimators and found them efficient.

Sanaullah *et al.* [81] proposed some improved exponential ratio type estimators using two auxiliary variables in case of double sampling as

$$t_{su_1(d)} = \bar{y}_2 \exp\left[\alpha\left(\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right) - (1-\alpha)\left(\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right)\right],$$

$$t_{su_2(d)} = \bar{y}_2 \exp\left[\sqrt{\rho_{xy}}\left(\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right) - (1-\sqrt{\rho_{xy}})\left(\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right)\right],$$

$$t_{su_3(d)} = \bar{y}_2 \exp\left[\sqrt{\rho_{yz}}\left(\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right) - (1-\sqrt{\rho_{yz}})\left(\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right)\right],$$

$$t_{su_4(d)} = \bar{y}_2 \exp\left[\sqrt{\rho_{zx}}\left(\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right) - (1-\sqrt{\rho_{zx}})\left(\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right)\right],$$

$$t_{su_5(d)} = \bar{y}_2 \exp\left[\left(\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right) + \exp\left(\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2} + \frac{\bar{x}_1-\bar{x}_2}{\bar{x}_1+\bar{x}_2}\right)\right],$$

where α is a non-zero constant whose value is to be determined such that MSE of $t_{su_1(d)}$ is minimum and $\rho_{ij} > 0$, where $i \neq j$. They have derived the properties of the proposed estimators to the first order of approximation for dependent and independent sample cases and compared them with some existing estimators. They found the proposed estimators efficient both theoretically as well as empirically.

Yadav *et al.* [70] considered the problem of the estimation of finite population mean of the study variable and proposed an estimator which is generalization of the ratio and product type estimators proposed by the authors Bahl and Tuteja [33] as

$$t_{ydg} = \bar{y} \exp\left[c\left(\frac{\bar{X}-\bar{x}}{\bar{X}-\bar{x}}\right)\right],$$

where c is non zero constant. Further, they proposed an almost unbiased exponential estimator of population mean of study variable as

$$t^{(u)}_{ydg} = \bar{y}\Big[\frac{3K+1}{4K}\exp\{2K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\} + \frac{K-1}{2K}\exp\{4K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\} + \frac{1-K}{4K}\exp\{6K\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\}\Big].$$

The authors have derived theoretical expressions of Bias and MSE for the proposed estimators to the first order of approximation and found them efficient than the existing estimators under consideration.

Onyeka [35] proposed a class of product type exponential estimators of population mean as t_{onp}. Further a modified ratio-cum product type exponential estimator as t_{onrp} is also proposed by the author in the paper

$$t_{onp} = \bar{y} \exp\Big[\frac{(a\bar{x}+b)-(a\bar{X}+b)}{(a\bar{x}+b)+(a\bar{X}+b)}\Big].$$

$$t_{onrp} = \alpha_1\left(\bar{y}\exp\left[\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right]\right) + \alpha_2\left(\bar{y}\exp\Big[\frac{(a\bar{x}+b)-(a\bar{X}+b)}{(a\bar{x}+b)+(a\bar{X}+b)}\Big]\right)$$

where α_1 and α_2 are weighted fractions such that $\alpha_1 + \alpha_2 = 1$. The scalars $a(\neq 0)$ and b are either real numbers or functions of known values of the auxiliary variable such as coefficient of skewness $(\beta_1(x))$, coefficient of kurtosis $(\beta_2(x))$, standard deviation (σ_x) and the correlation coefficient (ρ_{xy}). In this study, the author has derived theoretical expressions of Bias and MSE of the estimators to the first order of approximation and compared them with some estimators existing in literature both theoretically and empirically. He found both the estimators t_{onp} and t_{onrp} more efficient than the estimators considered for comparision in the study.

Solanki and Singh [73] proposed a class of exponential ratio estimators of population mean using known population proportion of an auxiliary character highly correlated with the study variable as

$$T_{ssr} = \bar{y} \exp\left(\frac{\alpha(p - \hat{p})}{p + \hat{p}}\right).$$

This proposed class of estimators is observed to be more generall and it includes the usual unbiased estimator and the the one proposed by Singh *et al.* [34]. They proposed two phase sampling version of the proposed class of estimators as

$$T_{ssr(d)} = \bar{y} \exp\left(\frac{\alpha(\hat{p}' - \hat{p})}{\hat{p}' + \hat{p}}\right)$$

in the paper. Regions of preferences for the constant used (α) under which the proposed estimators work efficiently than the existing estimators have been investigated by the authors. The authors have identified asymptotic optimum estimators (AOE) along with their mean square error formula in both simple random sampling and in two phase sampling schemes.

Yadav and Kadilar [22] proposed a family of exponential ratio estimators of population mean as

$$t_{ykr} = k\bar{y} \exp\left[\frac{(a\bar{X} + b) - (a\bar{x} - b)}{(a\bar{X} + b) + (a\bar{x} - b)}\right],$$

where k is a constant chosen suitably and a ($\neq 0$) & b are either real numbers or may be the functions of known parameters of auxiliary variable, such as coefficient of variation, coefficient of skewness, correlation coefficient etc. They have derived the large sample properties of the proposed estimators to the first order of approximation and made their efficiency comparison with some existing estimators, which shows that every member of the family of estimators t_{ykr} has an improvement over the corresponding family of estimators as proposed by Singh *et al.* [20].

Saini and Kumar [79] suggested modified exponential ratio and product type estimators using information on auxiliary attribute in simple random sampling as

$$T_{skr} = \bar{y} - (e^{A_1} - 1) \text{ and } T_{skp} = \bar{y} - (e^{A_2} - 1) \text{ respectively,}$$

$$\text{where } A_1 = P - \frac{NP - np}{N - n} \text{ and } A_2 = \frac{NP - np}{N - n} - P.$$

The authors have obtained unbiasedness and variance of θ_5 and θ_6 to the first order of approximation. They have derived the conditions under which the proposed estimators work more efficiently than some existing estimators considered. They have also carried a comparative analysis of efficiency between the proposed and some existing estimators both theoretically as well as empirically, which concludes that there is gain in efficiency by the proposed estimators.

Using the information on single auxiliary variable, Shabir *et al.* [82] proposed difference-cum-exponential type estimator for estimating population mean of the variable under study as

$$t_{shd} = \left[\frac{\bar{y}}{2}\{\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right)\}w_1(\bar{X} - \bar{x}) + w_2\bar{y}\right] \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right).$$

They have obtained large sample properties of the proposed estimators to the first order of approximation. The authors found that the proposed estimators always perform better than some existing estimators considered regardless of positive or negative correlation between the study and auxiliary variable both theoretically as well as empirically.

Khan *et al.* [36] proposed exponential ratio and product type estimators as

$$t_{khr} = \bar{y} \exp\left(\frac{\sqrt{\bar{X}} - \sqrt{\bar{x}}}{\sqrt{\bar{X}} + \sqrt{\bar{x}}}\right) \text{ and } t_{khp} = \bar{y} \exp\left(\frac{\sqrt{\bar{x}} - \sqrt{\bar{X}}}{\sqrt{\bar{x}} + \sqrt{\bar{X}}}\right) \text{ respectively.}$$

They have also proposed a generalized exponential type estimator for estimating population mean by introducing some real constants (α, h, a) as

$$t^{g}_{kh} = \bar{y} \exp\left[\alpha\left(1 - \frac{a\bar{x}^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}} + (a - 1)\bar{x}^{\frac{1}{h}}}\right)\right],$$

where the constants $\alpha(-\infty < \alpha < +\infty)$ and $h(h > 0)$ are assumed to be known and the constant a ($a \neq 0$) is estimated such that the proposed estimator has minimum MSE. The authors have derived theoretical expressions of mean MSE and Bias to the first order of approximation and analyzed the properties for independent units under simple random sampling. Finally, the efficiency comparison of the proposed estimators with some existing estimators was done which clearly depicted that the proposed estimator are most efficient among all other estimators considered for comparison.

Saini and Kumar [79] proposed a new modified exponential type ratio estimator using auxiliary attribute as $T_{skr} = \left[\bar{y} - a(E_1 - 1)\right]$, where a is any constant and

$$E_1 = \exp\left[P - \frac{NP - np}{N - n}\right].$$

They have obtained the unbiasedness and variance properties to the first order of approximation in the study. The authors worked out the conditions under which T_{skr} will be more efficient than existing estimators. Using different population data sets they authors concluded that the there is gain in efficiency by the proposed estimator.

Haq and Shabir [66] proposed improved estimators of population mean in simple random sampling and in two phase sampling scheme using information on two auxiliary attributes as

$$T_{hs(d)} = w_1\frac{\bar{y}}{4}\left(\frac{P_1}{p_1} + \frac{p_1}{P_1}\right)\left(\frac{P_2}{p_2} + \frac{p_2}{P_2}\right) + w_2(P_1 - p_1) + w_3(P_2 - p_2) \text{ and}$$

$$T^*_{hs(d)} = d_1\frac{\bar{y}}{4}\left(\frac{p^*_1}{p_1} + \frac{p_1}{p^*_1}\right)\left(\frac{P_2}{p^*_2} + \frac{p^*_2}{P_2}\right) + d_2(p^*_1 - p_1) + d_3(P_2 - p^*_2) \text{ respectively,}$$

where w_1, w_2, w_3, d_1, d_2 and d_3 are suitably chosen constants. The authors found proposed estimators more efficient than some of the estimators existing in literature both theoretically as well as empirically under under both the sampling schemes.

Khan and Siddiqi [36] proposed an exponential ratio type estimator, a general class of exponential ratio and product type estimators of population mean using variance of an auxiliary variable in single phase sampling as

$$t_{ksr_1} = \bar{y}\exp\left(\frac{S^2_x - s^2_x}{S^2_x + s^2_x}\right), t_{ksr_2} = \bar{y}\exp\left(\frac{S^2_x - s^2_x}{S^2_x + (a-1)s^2_x}\right) \text{ and } t_{ksr_3} = \bar{y}\exp\left(\frac{s^2_x - S^2_x}{S^2_x + (b-1)s^2_x}\right),$$

where a and b $(a, b \neq 0)$ are unknown constants to be determined such tha MSE of the proposed estimators is minimum. The large sample properties of the proposed estimators have been obtained and analyzed by the authors for independent units under SRSWOR ignoring finite population corrections. They carried an efficiency comparison of the proposed estimators with some existing estimators which revealed that the proposed estimators are efficient than the existing estimators under consideration.

Grover and Kaur [77] proposed a generalized class of ratio type exponential estimators of population mean in SRS under linear transformation of auxiliary variable as

$$t_{gkr} = \left[\omega_1\bar{y} + \omega_2(\bar{X} - \bar{x})\right]\exp\left[\frac{\eta(\bar{X} - \bar{x})}{\eta(\bar{X} + \bar{x}) + 2\lambda}\right],$$

where $\eta(\neq 0)$ and λ are either any known constants or the functions of known population parameters like coefficient of variation, coefficient of skewness, coefficient of kurtosis etc. and the value of constants ω_1 and ω_2 is suitably chosen. The theoretical expressions of Bias and MSE were obtained up to first order. The performance of the proposed estimators is compared with some existing estimators by the authors by conducting a simulation study and observed the proposed estimators efficient.

John and Inyang [64] proposed two exponential ratio estimators of population mean as

$$t_{jir_1} = \theta_1\bar{y} + \theta_2(\bar{X} - \bar{x})\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \text{ and } t_{jir_2} = \psi_1\bar{y} + \psi_2(\bar{X} - \bar{x})\exp\left[\frac{2(\bar{X} - \bar{x})}{\bar{X} + \bar{x}}\right],$$

where θ_1, ψ_1 and θ_1, θ_2 are suitably chosen scalars θ_1, $\psi_1 > 0$, $-\infty < \theta_2$, θ_2,$\psi_2 < +\infty$. They found the estimators t_1 and t_2 at their optimal conditions always more efficient than some existing estimators including the linear regression estimator which is assumed to be the best linear unbiased estimator in presence of auxiliary variable.

Singh and Pal [83] proposed chain Ratio-Ratio exponential type estimator of population mean as

$$t_{sgcr} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$$

and a generalized chain Ratio-Ratio-Type exponential estimator as
$$y_{sggcr} = \bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)\exp\left(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right),$$
where a and b are the real constants or the functions of the parameters such as coefficient of skewness $(\beta_1(x))$, coefficient of variation (C_x), standard deviation (S_x), coefficient of kurtosis $(\beta_2(x))$, quartiles Q_i $(i = 1,2,3)$, deciles (D_i) $(i = 1,2,...,10)$ and $\Delta = \beta_2(x)\ \beta_1(x) - 1$ of the auxiliary variable x, coefficient of kurtosis $(\beta_2(y))$, coefficient of variation (C_y) of the study variable and the correlation coefficient between the auxiliary and study variable (ρ_{xy}). The Bias and MSE expressions of the proposed estimators were derived to the first order of approximation by the authors. They observed proposed estimators were efficient as concluded through the efficiency comparison with some existing estimators.

Saini and Kumar [80] proposed a new modified exponential product type estimator of population mean using bi-serial negative correlation between the study and auxiliary variable as
$$T_{skp} = \left[\bar{y} - k(m-1)\right], \text{ where k is a constant and } m = \exp\left[\frac{NP-np}{N-n} - P\right].$$
They have obtained expression of Bias and MSE to the first order of approximation and also derived the conditions under which the proposed modified estimator works efficiently than existing estimators. The authors supported theoretical findings of the study by numerical illustrations.

Yasmeen *et al.* [37] proposed the dual to ratio-cum-exponential ratio and dual to product-cum-exponential product type estimators of population mean as
$$t_{ysrr} = \bar{y}\left[\frac{\bar{x}_t}{\bar{X}}\right] + \exp\left[\frac{\bar{z}_t - \bar{Z}}{\bar{z}_t + \bar{Z}}\right] \text{ and } t_{yspp} = \bar{y}\left[\frac{\bar{X}}{\bar{x}_t}\right] + \exp\left[\frac{\bar{Z} - \bar{z}_t}{\bar{Z} + \bar{z}_t}\right] \text{ respectively.}$$
These estimators are based on the information from two transformed auxiliary variables using simple random sampling without replacement. The large sample properties of the proposed estimators were obtained to the first order of approximation and the conditions under which the proposed estimators work efficiently than some existing estimators were also obtained in the paper. Further they carried an empirical study for efficiency comparison using different population data sets which supported the claim of gain in efficiency by the proposed class of estimators.

Etuk *et al.* [75] proposed a modified class of ratio estimators for estimating the population mean using a combination of scalars and known population mean of auxiliary variable as
$$t_{etr} = \bar{y}\left[\lambda - r\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left(\frac{\alpha(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right)\right],$$
where λ, r, α and δ are suitably chosen scalars such that λ and r satisfy the condition $\lambda = 1 + r$; $-\infty < r < +\infty$. The large sample properties of the proposed class of estimators were derived up to first order of approximation and some existing members were also identified by them. They found the proposed estimator at their optimal condition superior than some existing estimators both theoretically as well as empirically.

In the absence of auxiliary variable, Subramani [47] proposed median based ratio type estimators of finite population mean as
$$t_{sbr} = \bar{y}\left(\frac{M}{m}\right).$$
The large sample properties and the conditions for which the proposed estimators are more efficient than the considered estimators were derived by the author. He found proposed estimator more efficient than some existing estimators making use of auxiliary variable and the linear regression estimator which is considered as the best linear unbiased estimator in the presence of auxiliary variable.

Vishwakarma *et al.* [23] proposed exponential ratio and product type estimators as
$$t_{vkr} = \alpha\bar{y} + (1-\alpha)\bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \text{ and } t_{vkp} = \beta\bar{y} + (1-\beta)\bar{y}\exp\left[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right] \text{ respectively,}$$
where α and β are the constants to be obtained so that the MSE of t_3 and t_4 are minimized. The expressions for MSE and Bias of the proposed estimators were obtained to the first order of approximation. On assigning suitable values to the constants α and β, they found the proposed

estimators reducing to the estimators $\bar{y}$, ratio and product type estimators of Bahl and Tuteja [33].

Kadilar [65] proposed a new exponential type estimator in SRS for population mean as

$$t_{kdr} = \bar{y}\Big[\frac{\bar{x}}{\bar{X}}\Big]^{\alpha} \exp\Big(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\Big).$$

This estimator at its optimal condition is equal efficient as that of usual linear regression estimator. The expressions of Bias, MSE and the optimal value of α were obtained. They found the proposed estimator better than the mentioned existing estimators both theoretically as well as empirically and therefore the author recommended the proposed estimator for its practical use when the auxiliary information is available.

Enang *et al.* [74] proposed a class of exponential ratio-product type estimator of population mean as

$$t_{egpr} = \bar{y}\Big[\alpha_1 \frac{\bar{x}}{\bar{X}} \exp\Big(\frac{\delta_1(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\Big) + \alpha_2 \frac{\bar{X}}{\bar{x}} \exp\Big(\frac{\delta_2(\bar{x}-\bar{X})}{\bar{X}+\bar{x}}\Big)\Big],$$

where $\alpha_1+\alpha_2=1$ while δ_1 and δ_2 are suitably chosen scalars. They found the proposed class of estimators at its optimal condition equal efficient as that of linear regression estimator.

Yadav *et al.* [38] proposed median based exponential estimator of population mean using population and sample meadian of study variable as

$$t_{ydr} = \bar{y} \exp\left(\frac{M-m}{M+(a-1)m}\right),$$

where the constant a is is determined such that MSE of proposed estimator is minimum. The expressions of Bias and MSE were obtained. It was found that the proposed estimator have minimum MSE among all other competing estimators considered and therefore is most efficient.

Enang *et al.* [74] proposed a modified class of exponential ratio-product type estimators for estimating the population mean in simple random sampling (SRS) as

$$t_{enrp} = \bar{y}\big[\alpha_1 \frac{\bar{x}}{\bar{X}} \exp\left(\frac{\delta_1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right) + \alpha_2 \frac{\bar{X}}{\bar{x}} \exp\left(\frac{\delta_2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right)\big],$$

where $\alpha_1+\alpha_2=1$ and δ_1 & δ_2 are suitably chosen constants. The authors have derived the bias and MSE expressions for the proposed estimators to the first order of approximation. The proposed class of estimators were found efficient than the estimators considered for comparison.

Yadav *et al.* [38] proposed two new exponential ratio type estimators for estimating the finite population mean using sample and population median of study variable as

$$t_{ydr_1} = \bar{y}\left(\frac{m}{M}\right)\exp\big(\frac{a(M-m)}{M+m}\big) \text{ and } t_{ydr_2} = \bar{y}\left(\frac{m}{M}\right)\exp\left(\frac{M-m}{a(M+m)}\right),$$

where the arbitrary constant a is determined such that the MSE of the proposed estimators is minimum. The large sample properties of the proposed estimators were derived to the first order of approximation and compared with some existing estimators. The said comparison concluded that the proposed estimators are most efficient among all other estimators considered.

Panigrahi and Mishra [78] proposed some modified exponential ratio estimators of population mean as

$$t_{pmr_1} = \frac{\bar{y}}{1+\theta_1 C_y^2} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),$$

if C_y^2 is known. Further, if the coefficient of variation of the auxiliary variable (C_y^2) is not known, they proposed the estimators

$$t_{pmr_2} = \frac{\bar{y}}{1+\theta_1 \hat{C}_y^2} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \text{ and } t_{pmr_3} = (1+\theta_1 \hat{C}_y^2)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),$$

where $\hat{C}_y^2 = \frac{S_y^2}{\bar{y}^2}$. The large sample properties of all the proposed estimators were derived to the first order of approximation and compared with the estimators considered in the study. They observed from both theoretical and empirical study that the proposed estimators are efficient.

Panigrahi and Mishra [78] proposed modified exponential product type estimators of population mean when the population coefficient of variation of y is known in advance as t_{pmp_1}.

Further in the absence of coefficient of variation, they proposed a product type estimator as t_{pmp_2}

$$t_{pmp_1} = \frac{\bar{y}}{1+\theta_1 C_y^2} \exp\left[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right].$$

$$t_{pmp_2} = \frac{\bar{y}}{1+\theta_1 \hat{C}_y^2} \exp\left[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right].$$

The theoretical expressions of Bias and MSE were obtained by the authors to the first order of approximation. The numerical efficiency comparison conducted by the authors concluded that the proposed estimators are most efficient among all other estimators considered.

Singh *et al.* [67] noted that nearly sixty five existing estimators are the members of the proposed estimator

$$t_{sg} = \left[\bar{y} + \hat{\beta}(\bar{X}-\bar{x})\right]\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)$$

and identified some one hundred sixty three unknown members of t. They have also proposed a general class of exponential ratio type estimators of population mean

$$t_{sggr} = \left[\bar{y} + \hat{\beta}(\bar{X}-\bar{x})\right]\left(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right)$$

and identified some members of t_{sggr}, where $\hat{\beta}$ is the regression coefficient of y on x, a and b are real constants or the functions of population parameters such as population standard deviation (S_x), total (X), variance (S_x^2), coefficient of skewness $(\beta_1(x))$, coefficient of kurtosis $(\beta_2(x))$, qartiles, median, midrange, mode, deciles, tri-mean, Δ, correlation coefficient (ρ), coefficient of variation (C_x) etc. Further, the authors have derived the large sample properties of the proposed estimators and observed the proposed class of exponential ratio type estimators t_{sggr} provides efficient estimators which are less biased than the some estimators considered in the paper.

Kumar *et al.* [69] proposed a class of generalized exponential ratio type estimators for estimating the population mean using known median (M) of study variable as

$$t_{kmr} = \bar{y}\left[\alpha + (1-\alpha)\exp\left(\frac{M-m}{M+m}\right)\right],$$

where α is a constant obtained such that the MSE of the proposed estimator is minimum. The Bias and MSE expressions of the proposed estimators are obtained by the authors to the first order of approximation. The proposed estimators were found efficient than the estimators making use of auxiliary information which is collected on additional cost of the survey.

Singh *et al.* [26] proposed generalized exponential ratio estimator under simple random sampling using auxiliary variable as

$$t_{sggr} = \bar{y}\ A^{\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]} \text{ for A } > \ 0.$$

They found the estimator of Bahl and Tuteja [33] a particular case of t_{sggr} for A = e = 2.71 ... . The authors have derived theoretical expressions of Bias and MSE to the first order of approximation and proved that the proposed estimators have highest efficiency among the other estimators considered in the study both theoretically as well as empirically.

Using the information on auxiliary variable as attribute, Zaman & Kadilar (2019) [39] proposed a family of exponential ratio type estimators as

$$T_{zkr} = \bar{y}\exp\left[\frac{(kP+l)-(kp+l)}{(kP+l)-(kp+l)}\right],$$

where the constants $(k \neq 0)$ are either real numbers or some functions of the known auxiliary attribute such as C_p, $\beta_2(\phi)$ and ρ_{pb}. They used the suitable combinations of the constants l and k which have generated various types of estimators. The authors have obtained large sample properties of the proposed estimators to the first order of approximation and compared these properties with the existing estimators considered concluding the proposed estimators are most efficient.

Zaman [72] proposed a class of exponential ratio type estimators using information on population proportion which possesses a certain attribute as

$$T_{zmr} = \bar{y}(\frac{p}{P})^{\alpha}\exp\left[\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right]; i = 1,2,...,10,$$

where k and l are the constants like $0, 1, C_p, \beta_2(\phi)$ & ρ_{pb} taken in suitable combinations. The author has derived theoretical expressions of Bias and MSE up to first order for the proposed estimators and found the proposed estimators efficient than the estimators under consideration through numerical illustration.

Hussain *et al.* [71] proposed dual type exponential estimators of population mean using auxiliary information as

$$t_{de1} = \bar{y}\left[\alpha \exp\left(\frac{\bar{X}-\bar{x}}{p\bar{X}}\right) + (1-\alpha)\exp\left(\frac{\bar{x}-\bar{X}}{p\bar{X}}\right)\right] \quad \text{and}$$

$$t_{de2} = \bar{y}\left[\beta \exp\left(\frac{\bar{X}-\bar{x}}{q\bar{x}}\right) + (1-\beta)\exp\left(\frac{\bar{x}-\bar{X}}{q\bar{x}}\right)\right].$$

Where p and q are non zero constants whose value is chosen such that the estimators t_{de1} and t_{de2} should be unbiased. The value of constants α and β are chosen such that the MSE of t_{de1} and t_{de2} is minimum.

Hussain [46] proposed proportion based unbiased exponential ratio-cum-product estimators in double sampling plan as

$$T_{rp1} = \bar{y}\left[\omega_1 \exp\left(\frac{p_1-p}{lp_1}\right) + (1-\omega_1)\exp\left(\frac{p-p_1}{lp_1}\right)\right] \quad \text{and}$$

$$T_{rp2} = \bar{y}\left[\omega_2 \exp\left(\frac{p_1-p}{gp}\right) + (1-\omega_2)\exp\left(\frac{p-p_1}{gp}\right)\right].$$

Where $l(\neq 0)$, $g(\neq 0)$, ω_1 and ω_2 are constants. The values of l and g are chosen such that the proposed estimators are unbiased and the values of ω_1 & ω_2 are chosen such that MSE (T_{rpi}) $(i = 1, 2)$ is minimum.

Conclusion

In conclusion, exponential type estimators play a pivotal role in the accurate estimation of the population mean. The utilization of exponential type transcendental functions in these estimators proves to be an effective strategy for achieving precise and reliable estimates. By harnessing the mathematical properties of exponential functions, these estimators significantly reduce bias and variance, enhancing the robustness of the population mean estimates. This approach not only outperforms traditional estimation methods but also adapts well to various data distributions, making it a valuable tool in statistical analysis. The findings of this research underscore the importance and efficacy of exponential type estimators in achieving precise population mean estimates, highlighting their potential for broad application in statistical practices.

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