STOCHASTIC ANALYSIS THROUGH MATLAB OF TWO-UNIT PARALLEL SYSTEM

Sonia[1*], Shakuntla Singla[2]

•

[1*]Department of mathematics, Dr. B. R. Ambedkar Govt. College Kaithal (Jagdishpura), Haryana, India
[1*,2]Departent of mathematics and humanities, MMEC, Maharishi Markandeshwar(Deemed to be University), Mullana, Ambala, Haryana, India
[1*]Sonia_garg99@yahoo.com, [2]shakus25@gmail.com
Corresponding Author Email: sonia_garg99@yahoo.com

Abstract

This paper presents a reliability modelling of a two parallel unit of manufacturing Industry. Original conservation figures of the manufacturing industry have been applied for this determination. Four kinds of failure were observed: developmental, electronical, motorized and infrastructural failures. Both units of manufacturing plant work in parallel and independently. Various reliability parameters of the plant such as availability, busy period for repair, expected number of repairs and profitability for each type of failure are examined. Graphical Technique of Regenerative Point, process of Markov and Genetic Algorithm are applied for analysis. Profit analysis for the plant is examined with respect to returns of the plant and repair rate along with a graphical representation.

Keywords: Manufacturing, Markov process, regenerative point graphical techniques, repairs, failures

I. Introduction

Many investigators have studied reliability modelling and analysis of huge complex industrial systems under various operating conditions and presumptions for contributing to the discipline. Al Rahbi et al. [1,2] analyzed the reliability of single unit/ multiple units of a rodding anode plant in the aluminum industry with single/numerous repairs and optimized maintenance strategies, improved system reliability, and reduced downtime, leading to increased productivity and cost savings. Rizwan et al. [3] presented a Sensitivity analysis of reliability of Fuel cell. Sonia et al [4] discussed in her paper analysis of reliability through multiple way like normal distribution, exponential distribution, weibull distribution and observe that the normal distribution is the best fit for reliability analyze. Shakuntla Singla, Sonia [5] analysis the reliability of mushroom plant by the use of regenerative point graphical technique. Shakuntla Singla et al [6] represent a accurate model for explainng the availability through the use of chapman kolomogorov approach under the reduced capacity. Sunita Kumari et al [7] examining the performance and profit of thresher plant under steady state condition. Sunita Kumari et al [8] discussed the dependability of rubber plant with the help of scrutiny of results with and COGA with the limitation of cost and RAP illustration. In this paper non-poison problem of system were analyzed. Shakuntla Singla et al [9] discussed single unit subdivision with complete failure and in working condition with two types of malfunction one is direct and the second is in partial mode.

Sara Salim Al Oraimi et al [10] discussed in their paper sensitivity of Ammonia/Urea plant by taking parallel system. Thus, the literature has widely discussed for analysis of reliability and modelling of complex industrial systems in various failure/maintenance circumstances. But the concept of reliability analysis for manufacturing plants has yet to be discussed.

The demand for manufacturing items is growing day by day around the world to meet the daily requirements. The most used or consumed items of manufacturing plant is called modernization or fashion in daily life which is manufactured from manufacturing plant, from different industries. To meet the growing demand of manufacturing items in the current market, manufacturer must keep production continuously with maximum capacity to meet the market's growing demand. For continuous operation, these facilities must function the plant and equipment efficiently throughout the year without significant technical or maintenance issues. Any unexpected operational failure, breakdown, or downtime may affect plant productivity and efficiency. For that, the operation and maintenance strategies are critical as they help maintain the life and smooth operation of the equipment. These strategies also help reduce plant downtime. Also, further analysis and research techniques for plant performance, productivity, reliability, availability, maintainability, sensitivity, etc., may be carried out to ensure continuous and smooth plant operations. This paper provides profitability analysis, along with reliability analysis of parallel units of plant worldwide that have operated for more than 15 years. The research is based on the actual plant data of the probability of various failures and repair rates with some assumptions.

II. Assumptions and Notation

These are some assumptions which are as follows:

- Initially, we have an operative manufacturing plant composed of two parallel units: Unit 1 and Unit 2.
- The four types of failures are observed in both units, i.e., developmental, electronical, motorized, and infrastructural.
- Both units work independently.
- By repairing of units after failure as new one.
- Failure rate and repair rates all are taken as general.

The following are the notations used in the analysis:

λ_u : failure rate of manufacturing plant;

$p_1/\ p_2/\ p_3/\ p_4$: probability of developmental/ electronical/motorized/infrastructural in unit 1;

$p_5/\ p_6/\ p_7/\ p_8$: probability of developmental/ electronical/motorized/ infrastructural failure in unit 2;

$\alpha_1/\ \alpha_2/\ \alpha_3/\ \alpha_4$: repair rate of developmental/ electronical/motorized/ infrastructural failure in unit 1;

$\alpha_5/\ \alpha_6/\ \alpha_7/\ \alpha_8$: repair rate of developmental/ electronical/motorized/ infrastructural failure in unit 2;

$f^{1,2}(t)$: p.d.f. of failure time;

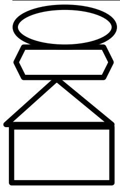
$g_1(t)/\ g_2(t)/\ g_3(t)/\ g_4(t)$: p.d.f. of repair time due to developmental/electronical/ motorized/ infrastructural failure in unit 1;

$g_5(t)/g_6(t)/\ g_7(t)/\ g_8(t)$: p.d.f. of repair time due to developmental/electronical/ motorized/ infrastructural failure in unit 2;

A_0^u : Availability of the System;

$DB^{u1}/\ EB^{u1}/\ MB^{u1}/IB^{u1}$: Busy period of the server due to developmental/ electronical/ motorized/ infrastructural failure in unit 1;

$DB^{u2}/\ EB^{u2}/\ MB^{u2}/IB^{u2}$: Busy period of the server due to developmental/ electronical/ motorized/ infrastructural failure in unit 2;



: Developmental Failure of unit '1' and unit '2';
: Electronical Failure of unit '1' and unit '2';
: Motorized Failure of unit '1' and unit '2';
: Infrastructural Failure of unit '1' and unit '2';

III. Data Summary

The real data from a manufacturing company is summarized as follows:

- Probability of developmental failure in unit 1, p_1 = 0.3044;
- Probability of electronical failure in unit 1, p_2 = 0.02;
- Probability of motorized failure in unit 1, p_3=0.21;
- Probability of infrastructural failure in unit 1, p_4 =0.0124;
- Probability of developmental failure in unit 2, p_5 =0.1648;
- Probability of electronical failure in unit 2, p_6 = 0.152;
- Probability of motorized failure in unit 2, p_7 = 0.0112;
- Probability of infrastructural failure in unit 2, p_8 = 0.1252;
- Failure rate of manufacturing plant, λ_u= 0.00034 per hour;
- Repair rate of developmental failure in unit 1, α_1 = 0.0149 per hour;
- Repair rate of electronical failure in unit 1, α_2 = 0.12 per hour;
- Repair rate of motorized failure in unit 1, α_3 = 0.0072 per hour;
- Repair rate of infrastructural failure in unit 1, α_4= 0.0253 per hour;
- Repair rate of developmental failure in unit 2, α_5 = 0.006 per hour;
- Repair rate of electronical failure in unit 2, α_6 = 0.0275 per hour;
- Repair rate of motorized failure in unit 2, α_7 = 0.0047 per hour;
- Repair rate of infrastructural failure in unit 2, α_8 = 0.0192 per hour;

IV. Stochastic Model and Model Description

There are different transitions from different stages to different stages which shows in Figure 1. Figure 1 shows the transitions from stage i (S_i) to stage j (S_j). The set of stages {0,1,2,3...,8} all are functioning and regenerative.

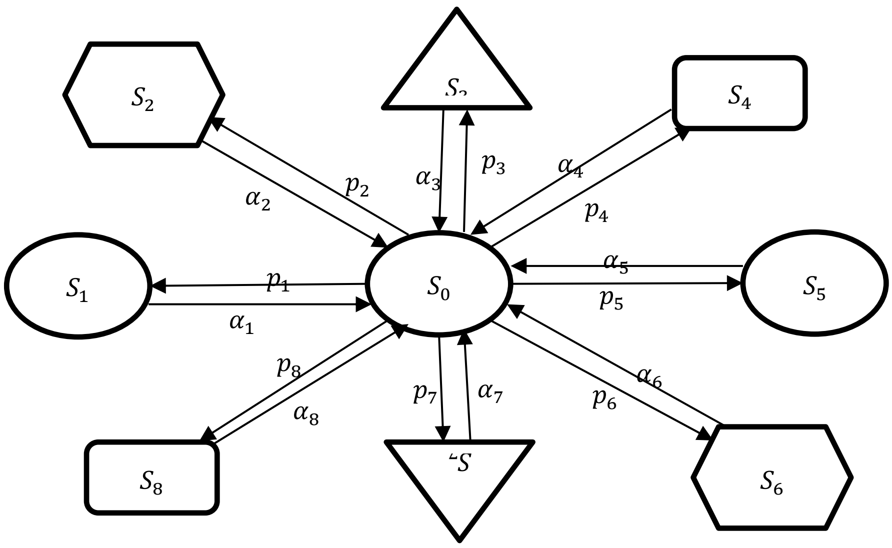


Figure 1: *State Transition Diagram*

where,

Stage 0 (S_0) – Both machines of manufacturing Industry unit 1 and unit 2 are functioning.
Stage 1 (S_1) - Unit 1 failed due to developmental failure, and Unit 2 is still functioning.
Stage 2 (S_2) - Unit 1 failed due to electronical failure, and Unit 2 is still functioning.
Stage 3 (S_3) - Unit 1 failed due to motorized failure, and Unit 2 is still functioning.
Stage 4 (S_4) - Unit 1 failed due to infrastructural failure, and Unit 2 is still functioning.
Stage 5 (S_5)- Unit 2 failed due to developmental failure and Unit 1 is still functioning.
Stage 6 (S_6) - Unit 2 failed due to electronical failure and Unit 1 is still functioning.
Stage 7 (S_7) - Unit 2 failed due to motorized failure and Unit 1 is still functioning.
Stage 8 (S_8) - Unit 2 failed due to infrastructural failure and Unit 1 is still functioning.

I. Transition Probabilities

The transition probabilities Table 1 from stage i (S_i) to stage j (S_j), $q_{ij}(t)$ where (i, j=1 to 8) are given by:

Table 1: *Transition Probabilities*

$q_{0i}(t)$	$q_{i0}(t)$
$q_{01}(t) = p_1\, f^{1,2}(t)$	$q_{10}(t) = g_1(t)$
$q_{02}(t) = p_2\, f^{1,2}(t)$	$q_{20}(t) = g_2(t)$
$q_{03}(t) = p_3\, f^{1,2}(t)$	$q_{30}(t) = g_3(t)$
$q_{04}(t) = p_4\, f^{1,2}(t)$	$q_{40}(t) = g_4(t)$
$q_{05}(t) = p_5\, f^{1,2}(t)$	$q_{50}(t) = g_5(t)$
$q_{06}(t) = p_6\, f^{1,2}(t)$	$q_{60}(t) = g_6(t)$
$q_{07}(t) = p_7\, f^{1,2}(t)$	$q_{70}(t) = g_7(t)$
$q_{08}(t) = p_8 f^{1,2}(t)$	$q_{80}(t) = g_8(t)$

The steady-stage probability is p_{ij} as $p_{0j} = p_j$ and $p_{i0} = 1$ where i and j vary from 1 to 8 and $\sum_{j=1}^{8} p_j$=1

II. Mean Sojourn time

(μ_i), i.e., mean stay time in particular stage i, is given in Table 2 as

Table 2: *Mean sojourn time*

$\mu_i = \int_0^\infty tp.d.f dt$	
$\mu_0 = \int_0^\infty tf^{1,2}(t)dt$	$\mu_5 = \int_0^\infty tg_5(t)dt$
$\mu_1 = \int_0^\infty tg_1(t)dt$	$\mu_6 = \int_0^\infty tg_6(t)dt$
$\mu_2 = \int_0^\infty tg_2(t)dt$	$\mu_7 = \int_0^\infty tg_7(t)dt$
$\mu_3 = \int_0^\infty tg_3(t)dt$	$\mu_8 = \int_0^\infty tg_8(t)dt$
$\mu_4 = \int_0^\infty tg_4(t)dt$	

V. System Performance Measures

I. Availability of the System

Define $A_0^u(t)$ is the possibility of the system at time t, when the structure is in stage i at time t = 0. By using the state transitions, we get the following equation:

$$A_0^{u*}(s) = \frac{N_1^u(s)}{D_1^u(s)} \tag{1}$$

Where

$$N_1^u(s) = M_0^*(s) + q_{01}^*(s)M_1^*(s) + q_{02}^*(s)M_2^*(s) + q_{03}^*(s)M_3^*(s) + q_{04}^*(s)M_4^*(s) + q_{05}^*(s)M_5^*(s) + q_{06}^*(s)M_6^*(s) + q_{07}^*(s)M_7^*(s) + q_{08}^*(s)M_8^*(s)$$

$$D_1^u(s) = 1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s) - q_{04}^*(s)q_{40}^*(s) - q_{05}^*(s)q_{50}^*(s) - q_{06}^*(s)q_{60}^*(s) - q_{07}^*(s)q_{70}^*(s) - q_{08}^*(s)q_{80}^*(s)$$

where

$M_i(t)$= probability that the system stays in stage i while operating rather than transferring to any other stage.

And (*) denotes the transformation of laplace of the respective function.

For solving $A_0^{u*}(s)$, we get the steady-stage accessibility of the structure is given by

$$A_0^u = \lim_{s\to 0} s.A_0^{u*}(s) = \lim_{s\to 0} s.\frac{N_1^u(s)}{D_1^u(s)} = \frac{N_1^{u'}(0)}{D_1^u(0)} = \frac{N_1^u}{D_1^u}(say) \tag{2}$$

Where

$$N_1^u = \mu_0 + p_1\mu_1 + p_2\mu_2 + p_3\mu_3 + p_4\mu_4 + p_5\mu_5 + p_6\mu_6 + p_7\mu_7 + p_8\mu_8$$
$$D_1^u = p_1\mu_1 + p_2\mu_2 + p_3\mu_3 + p_4\mu_4 + p_5\mu_5 + p_6\mu_6 + p_7\mu_7 + p_8\mu_8 + \mu_0$$

II. Busy Period for Repair

The expected time for which the repair man is in demanding for the reoperative of unit 1 and unit 2 due to developmental failure in steady state is given by:

$$DB^{u1} = DN^{u1}/D_1^u \qquad DB^{\mu 2} = DN^{u2}/D_1^u \tag{3}$$

where

$DN^{u1} = p_1\mu_1 \qquad DN^{u2} = p_5\mu_5$

The expected time for which the repair man is in demanding for the reoperative of unit 1 and unit 2 due to electronical failure in steady state is given by:

$$EB^{u1} = EN^{u1}/D_1^\mu \qquad EB^{u2} = EN^{u2}/D_1^\mu \tag{4}$$

Where

$EN^{u1} = p_2\mu_2 \qquad EN^{u2} = p_6\mu_6$

The expected time for which the repair man is in demanding for the reoperative of unit 1 and unit 2 due to motorized failure in steady state is given by:

$$MB^{u1} = MN^{u1}/D_1^\mu \qquad MB^{u2} = MN^{u2}/D_1^\mu \tag{5}$$

Where

$MN^{u1} = p_3\mu_3 \qquad MN^{u2} = p_7\mu_7$

The expected time for which the repair man is in demanding for the reoperative of unit 1 and unit 2 due to infrastructural failure in steady state is given by:

$$IB^{u1} = IN^{u1}/D_1^\mu \qquad IB^{u2} = IN^{u2}/D_1^\mu \tag{6}$$

Where

$IN^{u1} = p_4\mu_4$ $IN^{u2} = p_8\mu_8$

III. Expected Number of Repairs

The expected number of repairs in unit 1 and unit 2 due to developmental failure in steady stage is as follow:

$$DR^{u1} = DS^{u1}/D_1^u \qquad DR^{\mu 2} = DS^{u2}/D_1^u \tag{7}$$

where

$DS^{u1} = p_1$ $DS^{u2} = p_5$

The expected number of repairs in unit 1 and unit 2 due to electronical failure in steady state is given by:

$$ER^{u1} = ES^{u1}/D_1^u \qquad ER^{\mu 2} = ES^{u2}/D_1^u \tag{8}$$

where

$ES^{u1} = p_2$ $ES^{u2} = p_6$

The expected number of repairs in unit 1 and unit 2 due to motorized failure in steady state is given by:

$$MR^{u1} = MS^{u1}/D_1^u \qquad MR^{\mu 2} = MS^{u2}/D_1^u \tag{9}$$

Where

$MS^{u1} = p_3$ $MS^{u2} = p_7$

The expected number of repairs in unit 1 and unit 2 due to infrastructural failur e in steady state is given by:

$$IR^{u1} = IS^{u1}/D_1^u \qquad IR^{\mu 2} = IS^{u2}/D_1^u \tag{10}$$

Where

$IS^{u1} = p_4$ $IS^{u2} = p_8$

IV. Profit Function of the System

The profit function of the system is as follows:

$$P^u = C_0 A_0^u - C_1(DB^{u1} + DR^{u1}) - C_2(DB^{u2} + DR^{u2}) - C_3(EB^{u1} + ER^{u1}) - C_4(EB^{u2} + ER^{u2}) - C_5(MB^{u1} + MR^{u1}) - C_6(MB^{u2} + MR^{u2}) - C_7(IB^{u1} + IR^{u1}) - C_8(IB^{u2} + IR^{u2}) \tag{11}$$

Where

C_0= Revenue generated by the system and

$C_1(C_2)$/ $C_3(C_4)$/ $C_5(C_6)$/ $C_7(C_8)$: - Cost per unit time for engaging the repair man and cost for repair due to developmental/electronical/motorized/infrastructural failure in unit 1 (unit 2).

VI. Result and Discussion

In this section, we numerically solve all the system parameters in equation (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) and (11) by using all the equations of table 1, table 2 and given as below. Let us adopt all the failures and repair times follow the distribution of exponential along with their p.d.f. as:

$$f^{1,2}(t) = \lambda_u e^{-\lambda_u t}$$
$$g_i(t) = \alpha_i e^{-\alpha_i t}, \text{ i = 1 to 8}$$

Using the values as written in equation (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) and (11) that is calculated from real data of a manufacturing company, we get system effectiveness measures in Table 3 as:

Table 3: *System Parameters*

Numerical Analysis of system parameters		
System Parameters	Unit-1	Unit-II
Availability of the manufacturing plant	1	1
Busy period of Repair	Developmental Failure $DB^{u1} = 0.0067$	Developmental Failure $DB^{u2} = 0.009$
	Electronical Failure $EB^{u1} = 5.495E - 5$	Electronical Failure $EB^{u2} = 0.0018$
	Motorized Failure $MB^{u1} = 0.0096$	Motorized Failure $MB^{u2} = 0.0008$
	Infrastructural Failure $IB^{u1} = 0.0002$	Infrastructural Failure $IB^{u2} = 0.0021$
Expected number of Repair	Developmental Failure $DR^{u1} = 0.0001$	Developmental Failure $DR^{u2} = 5.433E - 5$
	Electronical Failure $ER^{u1} = 6.5934 - 6$	Electronical Failure $ER^{u2} = 5.011E - 5$
	Motorized Failure $MR^{u1} = 6.9230 - 5$	Motorized Failure $MR^{u2} = 3.6923 - 6$
	Infrastructural Failure $IR^{u1} = 4.0879E - 6$	Infrastructural Failure $IR^{u2} = 4.1274E - 5$

I. Profit Analysis:

The graph of profit function (P^u) w.r.t. returns (C_0) for different kind of values of regenerative rate (α_i) has been shown in Figure 1.

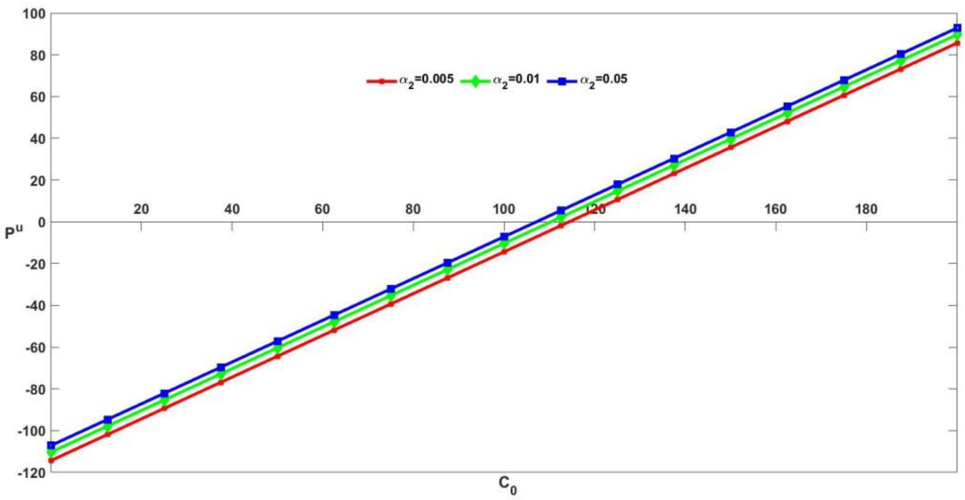


Figure 2: *Change in Profit w.r.t. Returns and Repair Rate*

Figure 2 shows that the increase in returns and repair rate increases profit Also, the cut-off points for the system to be profitable can be observed in Fig. 2.:

- For C_0> 108.2567 and α_2= 0.05, P^u> 0.
- For C_0 > 112.6834 and α_2= 0.01, P^u > 0.
- For C_0 > 115.8273 and α_2 = 0.005, P^u > 0

Similarly, we can draw graphs of the profit function with other parameters to see its effect and cut-off points when the system is profitable.

VII. Conclusion

In this paper, reliability of the parallel system of two units manufacturing plant has been examined and all these parameters such as accessibility of the plant, the busy period for repairs and the anticipated number of repairs for each type of failure are calculated with the help of regenerative technique of graphical point. Also, Profit analysis for the plant is also carried out along with the graphical representation with respect to various parameters. Profit increases when returns and repair rates both increases. The cut-off point is also drawn to determine when a system is profitable. It demonstrates that, in comparison to other factors, returns and system failure rate have the most significant impact on the profit function. The model forecasts the failure and repair conditions based on the optimized reliability and profitability results.

References

[1] Al Rahbi, Y., Rizwan, S., Alkali, B., Cowell, A., & Taneja, G. (2019). Reliability analysis of multiple units with multiple repairmen of rodding anode plant in aluminum industry. *Advances and Applications in Statistics*, *54*(1), 151-178.

[2] Taj, S. Z., Rizwan, S., Alkali, B., Harrison, D., & Taneja, G. (2018). Reliability analysis of a 3-unit subsystem of a cable plant. *Advances and Applications in Statistics*, *52*(6), 413-429.

[3] Rizwan, S. M., Sachdeva, K., Al Balushi, N., Al Rashdi, S., & Taj, S. Z. (2023). Reliability and sensitivity analysis of membrane biofilm fuel cell. *International Journal of Engineering Trends and Technology*, *71*(3), 73-80.

[4] Sonia, Singla, S. (2024). EXPLORE THE DYNAMICS OF MANUFACTURING INDUSTRIES: RELIABILITY ANALYSIS THROUGH STOCHASTIC PROCESS MODELING. *Reliability: Theory & Applications*, *19*(2 (78)), 467-471.

[5] Singla, S., Sonia & Panwar, P. (2024). STOCHASTIC OPTIMIZATION AND RELIABILITY ANALYSIS OF MUSHROOM PLANT. *Reliability: Theory & Applications*, *19*(1 (77)), 729-743.

[6] Singla, S., Modibbo, U. M., Mijinyawa, M., Malik, S., Verma, S., & Khurana, P. (2022). Mathematical model for analysing availability of threshing combine machine under reduced capacity. *Yugoslav Journal of Operations Research*, *32*(4), 425-437.

[7] Kumari, S., Khurana, P., & Singla, S. (2022). Behavior and profit analysis of a thresher plant under steady state. *International Journal of System Assurance Engineering and Management*, 1-6.

[8] Kumari, S., Khurana, P., & Singla, S. (2021). RAP via constraint optimization genetic algorithm. *Life Cycle Reliability and Safety Engineering*, *10*(4), 341-345.

[9] Singla, S., & Dhawan, P. (2022). Mathematical analysis of regenerative point graphical technique (RPGT). *Mathematical Analysis and its Contemporary Applications*, *4*(4), 49-56.

[10] Al Oraimi, S. S., Rizwan, S. M., & Sachdeva, K. (2024). SENSITIVITY AND PROFITABILITY ANALYSIS OF TWO-UNITS AMMONIA/UREA PLANT. *Reliability: Theory & Applications*, *19*(1 (77)), 376-386.