SHREEKANT DISTRIBUTION WITH PROPERTIES AND APPLICATIONS TO MODEL STRESS AND STRENGTH DATA FROM ENGINEERING

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Abstract

Modeling and analyzing real lifetime data that are stochastic in nature with the existing lifetime distributions is really a challenging task for researchers. During the last decade several one parameter lifetime distributions have been proposed in statistics but almost each of them has been found having some problems regarding their goodness of fit on certain datasets. This might have been due to their distributional properties or due to stochastic nature of datasets. In this paper, a new lifetime distribution named Shreekant distribution has been proposed which has been found to give improved fits to lifetime datasets than those by other existing ones such as exponential, Lindley, Shanker, Akash, Sujatha and Uma distributions. This has been shown by presenting its fittings to two datasets relating to stress and strength data from engineering. Its various structural and other properties and estimation of parameters by different methods of estimation have also been studied.

Keywords: Lifetime distributions, statistical properties, estimation of parameter, applications.

I. Introduction

Due to stochastic nature of lifetime data, search for lifetime distribution in the field of lifetime data analysis seems to expand fast and getting popularity among researchers and policy makers to model data of various natures. In past decades several one parameter lifetime distributions have been suggested in statistics literature. For example, Lindley distribution by Lindley [1], Shanker distribution by Shanker [2], Akash distribution by Shanker [3] and Sujatha distribution by Shanker [4], is some among others. Shanker et al [5] discussed the modeling of lifetime data from various fields of knowledge using exponential and Lindley distributions and observed that there are some datasets to which these two distributions do not provide satisfactory fit. Further, Shanker et al [6] made an effort to have comparative study on modeling of lifetime data using exponential, Lindley and Akash distributions and found that Akash distribution gives much better fit than both exponential and Lindley distributions but still there are some datasets in which these three distributions do not provide satisfactory fit. Then, Shanker and Hagos [7] tried to model the real lifetime datasets arising from different fields of knowledge using exponential, Lindley, Shanker and Akash distributions and observed that still there are some data to which these distributions do not provide satisfactory fits. Shanker [8] proposed another lifetime distribution named Uma distribution which provides better fit than exponential, Lindley, Shanker, Akash and Sujatha distributions but there were some datasets where Uma distribution failed to provide better fit. Flexibility and tractability are the two important distributional characteristics of lifetime distributions and if the existing distributions are not flexible or tractable for given datasets, then search for a new distribution starts. It has been observed that sometimes data are transformed to satisfy some assumptions of the distribution so that distribution fits well. But this is not useful and should not be a preferred practice because the original nature of the dataset is lost. Also, many researchers try to modify the distributions by introducing some additional shape parameters to the existing distribution so that the distribution provides satisfactory fit. But again, this is not very much preferable because of the simple reason that with the introduction of additional parameters may create problems in finding estimates of parameters and also other computational complexities. Therefore, the most practical and preferable way is to search a lifetime distribution which fits the given data well than to modify the existing distributions or to transform the data.

In this paper, in course of search for a new distribution, an attempt has been made to propose a new one parameter lifetime distribution named Shreekant distribution which fits the data arising from different fields of knowledge well over the existing one parameter lifetime distributions. Its statistical properties, characterizations, estimation of parameter and applications to model stress and strength data from engineering have been presented systematically. It is hoped that Shreekant distribution would draw attention of researchers to model lifetime data from engineering and biomedical sciences and have preference over the existing one parameter lifetime distributions.

II. Shreekant Distribution

The Shreekant distribution is defined by its probability density function (pdf) and cumulative distribution function (cdf)

$$f\left(x;\theta\right)=\frac{\theta^5}{\theta^4+\theta^3+24}\left(1+x+x^4\right)e^{-\theta x};x>0,\theta>0$$

$$F\left(x;\theta\right)=1-\left[1+\frac{\theta x\left\{\theta^3x^3+4\theta^2x^2+12\theta x+\left(\theta^3+24\right)\right\}}{\theta^4+\theta^3+24}\right]e^{-\theta x};x>0,\theta>0\ .$$

It can be easily seen that the Shreekant distribution is a convex combination of exponential $\left(\theta\right)$, gamma $\left(2,\theta\right)$ and gamma $\left(5,\theta\right)$ distributions with respective mixing proportions $\frac{\theta^4}{\theta^4+\theta^3+24},\frac{\theta^3}{\theta^4+\theta^3+24}$ and $\frac{24}{\theta^4+\theta^3+24}$. Since it is a convex combination of exponential and gamma distributions, it is hoped to provide better fit over other one parameter lifetime distributions proposed in statistics literature using convex combinations of exponential distribution and gamma distribution with different shape parameter.

The pdf and the cdf of Shreekant distribution for varying values of parameter θ have been presented in figures 1 and 2 respectively. It is quite obvious that the distribution is positively skewed and its skewness depends upon the values of the parameter.

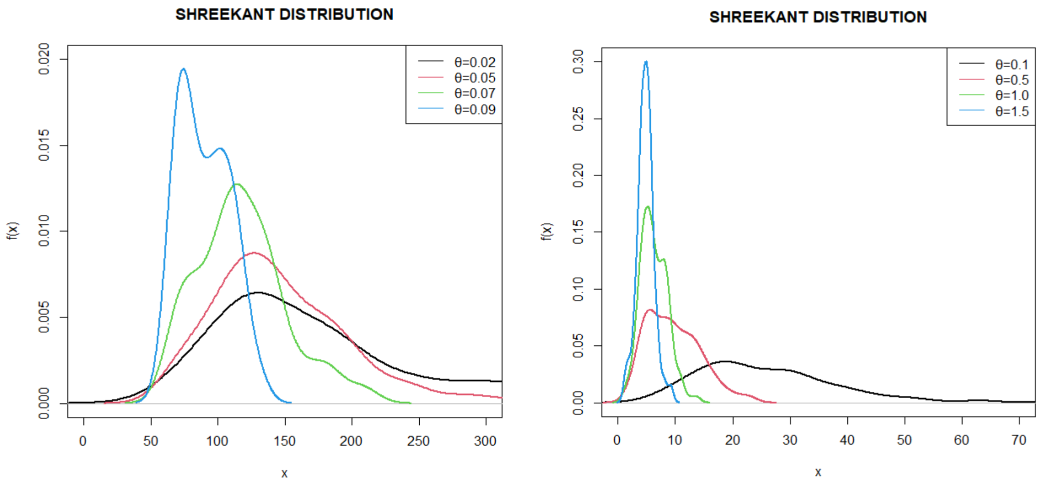

Figure1: *The pdf of Shreekant distribution*

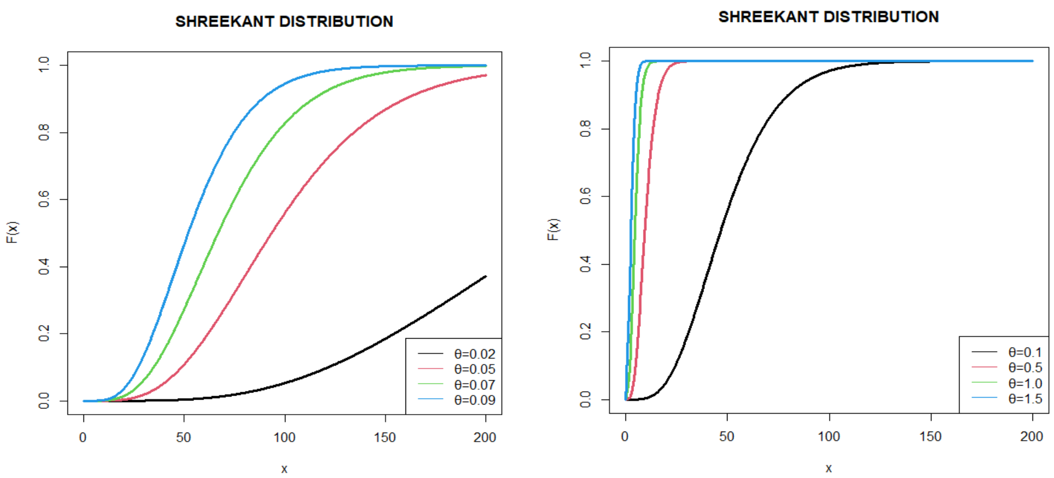

Figure2: *The cdf of Shreekant distribution*

III. Reliability Properties

I. Hazard function

The hazard function of a random variable X having pdf $f(x;\theta)$ and cdf $F(x;\theta)$ is defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X < x + \Delta x \mid X > x)}{\Delta x} = \frac{f(x;\theta)}{1 - F(x;\theta)}.$$

Thus, the hazard function of the Shreekant distribution can be obtained as

$$h(x,\theta) = \frac{\theta^5 (1 + x + x^4)}{\theta^4 x^4 + 4\theta^3 x^3 + 12\theta^2 x^2 + (\theta^3 + 24)\theta x + (\theta^4 + \theta^3 + 24)}.$$

This gives $h(0,\theta) = \dfrac{\theta^5}{\theta^4 + \theta^3 + 24} = f(0,\theta)$. The behaviour of the hazard function of the distribution for various values of parameter θ is shown in the following figure 3. It is quite obvious that for increasing values of x the hazard function is increasing and for increasing values of the parameter the hazard function scaled up.

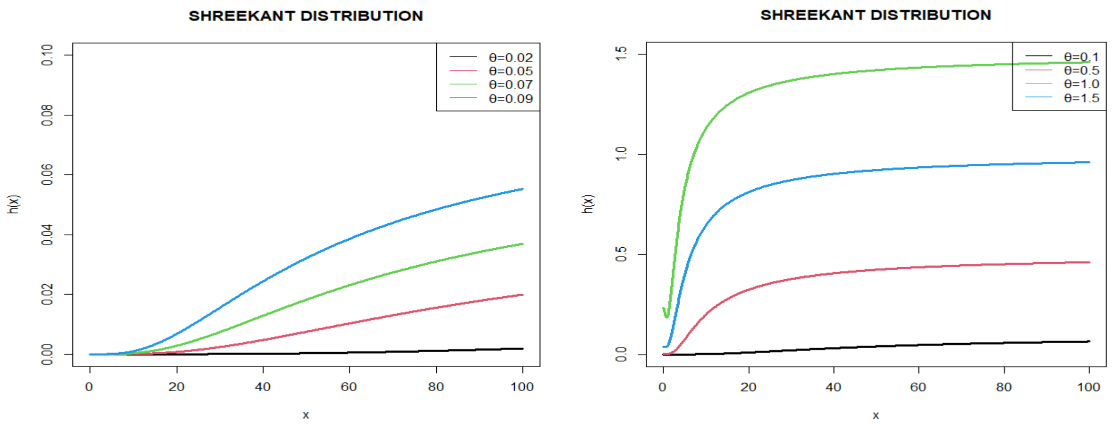

Figure 3: *The hazarad function of the Shreekant distribution.*

II. Mean Residual Life Function

Let X be a random variable over the support $(0,\infty)$ representing the lifetime of a system. Mean residual life (MRL) function measures the expected value of the remaining lifetime of the system, provided it has survived up to time x. Let us consider the conditional random variable $X_x = (X - x \mid X > x); x > 0$. Then, the MRL function, denoted by $m(x)$, is defined as

$$m(x) = E(X_x) = \frac{1}{S(x)} \int_x^\infty \left[1 - F(t)\right] dt \quad .$$

The MRL function of the Shreekant distribution can thus be obtained as

$$\begin{aligned} m(x,\theta) &= \frac{1}{\left\{\theta^4 x^4 + 4\theta^3 x^3 + 12\theta^2 x^2 + \left(\theta^3 + 24\right)\theta x + \left(\theta^4 + \theta^3 + 24\right)\right\} e^{-\theta x}} \int_x^\infty t\left(1 + t + t^4\right) e^{-\theta t}\, dt - x \\ &= \frac{\theta^4 x^4 + 8\theta^3 x^3 + 36\theta^2 x^2 + \left(\theta^4 + 96\theta\right) x + \left(\theta^4 + 2\theta^3 + 120\right)}{\theta\left\{\theta^4 x^4 + 4\theta^3 x^3 + 12\theta^2 x^2 + \left(\theta^3 + 24\right)\theta x + \left(\theta^4 + \theta^3 + 24\right)\right\}} . \end{aligned}$$

This gives $m(0,\theta) = \dfrac{\theta^4 + 2\theta^3 + 120}{\theta\left(\theta^4 + \theta^3 + 24\right)} = \mu_1'$. The behaviour of the mean residual life function of Shreekant distribution for various values of parameter θ is shown in the following figure 4. It is quite obvious that MRL function is decreasing for increasing values of x and it scaled down for increasing values of parameter.

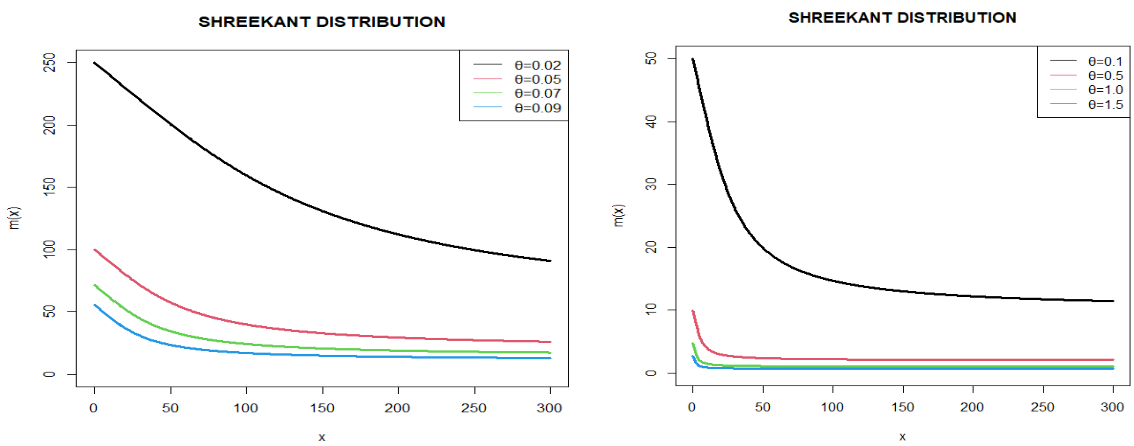

Figure 4: *The mean residual life function of the Shreekant distribution*

III. Reverse hazard function and Mill's ratio

The reverse hazard function of a random variable X having pdf $f(x;\theta)$ and cdf $F(x;\theta)$ is defined as

$$h_r(x,\theta)=\frac{f(x;\theta)}{F(x;\theta)}.$$

Thus, the reverse hazard rate function of the Shreekant distribution can be obtained as

$$h_r(x,\theta)=\frac{\theta^5\left(1+x+x^4\right)e^{-\theta x}}{\left(\theta^4+\theta^3+24\right)-\left[\theta^4x^4+4\theta^3x^3+12\theta^2x^2+\left(\theta^3+24\right)\theta x+\left(\theta^4+\theta^3+24\right)\right]e^{-\theta x}}.$$

Mill's ratio of a random variable X having pdf $f(x;\theta)$ and cdf $F(x;\theta)$ is defined as

$$\text{Mill's ratio}=\frac{1}{h(x)}=\frac{1-F(x;\theta)}{f(x;\theta)}.$$

Thus, the Mill's ratio of the Shreekant distribution can be obtained as

$$\frac{1}{h(x,\theta)}=\frac{\theta^4x^4+4\theta^3x^3+12\theta^2x^2+\left(\theta^3+24\right)\theta x+\left(\theta^4+\theta^3+24\right)}{\theta^5\left(1+x+x^4\right)}.$$

IV. Stochastic Ordering

The stochastic order, in probability theory and statistics, quantifies the concept of one random variable being bigger than another. A random variable X is said to be smaller than a random variable Y in the

(i) Stochastic order $\left(X\le_{st}Y\right)$ if $F_X(x)\ge F_Y(y)$ for all x

(ii) Hazard rate order $\left(X\le_{hr}Y\right)$ if $h_X(x)\ge h_Y(y)$ for all x

(iii) Mean residual life order $\left(X\le_{mrl}Y\right)$ if $m_X(x)\ge m_Y(y)$ for all x

(iv) Likelihood ratio order $\left(X\le_{lr}Y\right)$ if $\frac{f_X(x)}{f_Y(y)}$ decrease in x

The following results due to Shaked and Shantikumar [9] are well known for establishing stochastic ordering of distributions

$$\begin{array}{c}X<_{lr}Y\Rightarrow X<_{hr}Y\Rightarrow X<_{mrl}Y\\ \Downarrow\\ X<_{st}Y\end{array}$$

Theorem: Let X and Y have Shreekant distributions with their respective parameters θ_1 and θ_2. If $\theta_1>\theta_2$, then $X<_{lr}Y$ hence $X<_{hr}Y$, $X<_{mrl}Y$ and $X<_{st}Y$.

Proof: We have

$$\frac{f_X(x;\theta_1)}{f_Y(x;\theta_2)}==\frac{\theta_1^5\left(\theta_2^4+\theta_2^3+24\right)}{\theta_2^5\left(\theta_1^4+\theta_1^3+24\right)}e^{-(\theta_1-\theta_2)x}.$$

This gives

$$\log\left[\frac{f_X(x;\theta_1)}{f_Y(x;\theta_2)}\right]=\log\left[\frac{\theta_1^5\left(\theta_2^4+\theta_2^3+24\right)}{\theta_2^5\left(\theta_1^4+\theta_1^3+24\right)}\right]-(\theta_1-\theta_2)x.$$

Therefore, $\frac{d}{dx}\log\left[\frac{f_X(x;\theta_1)}{f_Y(x;\theta_2)}\right]=-(\theta_1-\theta_2)$

Thus, for $\theta_1>\theta_2$, $\frac{d}{dx}\log\left[\frac{f_X(x)}{f_Y(x)}\right]<0$. This means that $X<_{lr}Y$ hence $X<_{hr}Y$, $X<_{mrl}Y$ and $X<_{st}Y$. This proves the theorem.

V. Stress-Strength Reliability

Suppose X and Y be independent strength and stress random variables having Shreekant distribution with parameter θ_1 and θ_2 respectively. Then $R=P(Y<X)$ is known as stress-strength parameter and is a measure of the component reliability. Thus

$$R=P(Y<X)=\int_0^\infty P(Y<X\mid X=x)f_X(x)dx=\int_0^\infty f(x;\theta_1)\,F(x;\theta_2)\,dx$$

$$=1-\frac{\theta_1^5\left[\begin{array}{l}40320\theta_2^4+20160(\theta_1+\theta_2)\theta_2^3+8640(\theta_1+\theta_2)^2\theta_2^2+240(\theta_2^4+12\theta_2)(\theta_1+\theta_2)^3\\+24(2\theta_2^4+5\theta_2^3+24)(\theta_1+\theta_2)^4+24(\theta_2^3+3\theta_2^2)(\theta_1+\theta_2)^5+2(\theta_2^4+12\theta_2^2+24\theta_2)(\theta_1+\theta_2)^6\\+(2\theta_2^4+\theta_2^3+24\theta_2+24)(\theta_1+\theta_2)^7+(\theta_2^4+\theta_2^3+24)(\theta_1+\theta_2)^8\end{array}\right]}{(\theta_1^4+\theta_1^3+24)(\theta_2^4+\theta_2^3+24)(\theta_1+\theta_2)^9}$$

IV. Moments based descriptive measures

The r th moment about origin μ_r' of the Shreekant distribution can be obtained as

$$\mu_r'=E(X^r)=\frac{\theta^5}{\theta^4+\theta^3+24}\int_0^\infty x^r\left(1+x+x^4\right)e^{-\theta x}dx$$

$$=\frac{r!\left\{\theta^4+(r+1)\theta^3+(r+1)(r+2)(r+3)(r+4)\right\}}{\theta^r\left(\theta^4+\theta^3+24\right)};r=1,2,3,\cdots.$$

Substituting $r=1,2,3,4$ in the above equation, the first four moments about origin of the Shreekant distribution can be obtained as

$$\mu_1'=\frac{\theta^4+2\theta^3+120}{\theta\left(\theta^4+\theta^3+24\right)},\qquad \mu_2'=\frac{2\left(\theta^4+3\theta^3+360\right)}{\theta^2\left(\theta^4+\theta^3+24\right)}$$

$$\mu_3'=\frac{6\left(\theta^4+4\theta^3+840\right)}{\theta^3\left(\theta^4+\theta^3+24\right)},\qquad \mu_4'=\frac{24\left(\theta^4+5\theta^3+1680\right)}{\theta^4\left(\theta^4+\theta^3+24\right)}.$$

The moments about the mean of the Shreekant distribution, using relationship between moments about the mean and the moments about the origin, can thus be obtained as

$$\mu_2=\frac{\theta^8+4\theta^7+2\theta^6+528\theta^4+384\theta^3+2880}{\theta^2\left(\theta^4+\theta^3+24\right)^2}$$

$$\mu_3=\frac{2\left(\theta^{12}+6\theta^{11}+6\theta^{10}+2\theta^9+1512\theta^8+2160\theta^7+864\theta^6+1728\theta^4+6912\theta^3+69120\right)}{\theta^3\left(\theta^4+\theta^3+24\right)^3}$$

$$\mu_4 = \frac{3\begin{pmatrix} 3\theta^{16}+24\theta^{15}+44\theta^{14}+32\theta^{13}+7976\theta^{12}+19776\theta^{11}+16896\theta^{10}+4800\theta^{9} \\ +1762560\theta^{8}+3548160\theta^{7}+1589760\theta^{6}+37048320\theta^{4}+27648000\theta^{3}+11612160 \end{pmatrix}}{\theta^4\left(\theta^4+\theta^3+24\right)^4}.$$

The descriptive constants including coefficient of variation (CV), coefficient of skewness (CS), coefficient of kurtosis (CK) and the index of dispersion (ID) of the Shreekant distribution are thus obtained as

$$CV = \frac{\sqrt{\mu_2}}{\mu_1'} = \frac{\sqrt{\theta^8+4\theta^7+2\theta^6+528\theta^4+384\theta^3+2880}}{\theta^4+2\theta^3+120}$$

$$CS = \frac{\mu_3^{\,2}}{\mu_2^{\,3}} = \frac{4\left(\theta^{12}+6\theta^{11}+6\theta^{10}+2\theta^9+1512\theta^8+2160\theta^7+864\theta^6+1728\theta^4+6912\theta^3+69120\right)^2}{\left(\theta^8+4\theta^7+2\theta^6+528\theta^4+384\theta^3+2880\right)^3}$$

$$CK = \frac{\mu_4}{\mu_2^{\,2}} = \frac{3\begin{pmatrix} 3\theta^{16}+24\theta^{15}+44\theta^{14}+32\theta^{13}+7976\theta^{12}+19776\theta^{11}+16896\theta^{10}+4800\theta^{9} \\ +1762560\theta^{8}+3548160\theta^{7}+1589760\theta^{6}+37048320\theta^{4}+27648000\theta^{3}+11612160 \end{pmatrix}}{\left(\theta^8+4\theta^7+2\theta^6+528\theta^4+384\theta^3+2880\right)^2}$$

$$ID = \frac{\mu_2}{\mu_1'} = \frac{\theta^8+4\theta^7+2\theta^6+528\theta^4+384\theta^3+2880}{\theta\left(\theta^4+\theta^3+24\right)\left(\theta^4+2\theta^3+120\right)}.$$

Behaviour of coefficient of variation (CV), coefficient of skewness (CS), coefficient of kurtosis (CK) and index of dispersion (ID) of the Shreekant distribution for various values of parameter are shown in the figure 5. CV is increasing for increasing values of parameter. CS and CK are decreasing for increasing values of parameter whereas ID is decreasing for increasing values of parameter. The over-dispersion (mean less than the variance), under-dispersion (mean more than the variance) and equi-dispersion (mean equal to the variance) of the Shreekant, Uma, Sujatha, Akash, Shanker, Lindley and exponential distributions are presented in table 1

Table 1: *Over-dispersion, equi-dispersion and under-dispersion of distributions*

Distributions	**Over-dispersion** $\left(\mu<\sigma^2\right)$	**Equi-dispersion** $\left(\mu=\sigma^2\right)$	**Under-dispersion** $\left(\mu>\sigma^2\right)$
Shreekant	$\theta<2.21145$	$\theta=2.21145$	$\theta>2.21145$
Uma	$\theta<1.73845$	$\theta=1.73845$	$\theta>1.73845$
Sujatha	$\theta<1.36427$	$\theta=1.36427$	$\theta>1.36427$
Akash	$\theta<1.51540$	$\theta=1.51540$	$\theta>1.51540$
Shanker	$\theta<1.17153$	$\theta=1.17153$	$\theta>1.17153$
Lindley	$\theta<1.17008$	$\theta=1.17008$	$\theta>1.17008$
	$\theta<1$	$\theta=1$	$\theta>1$

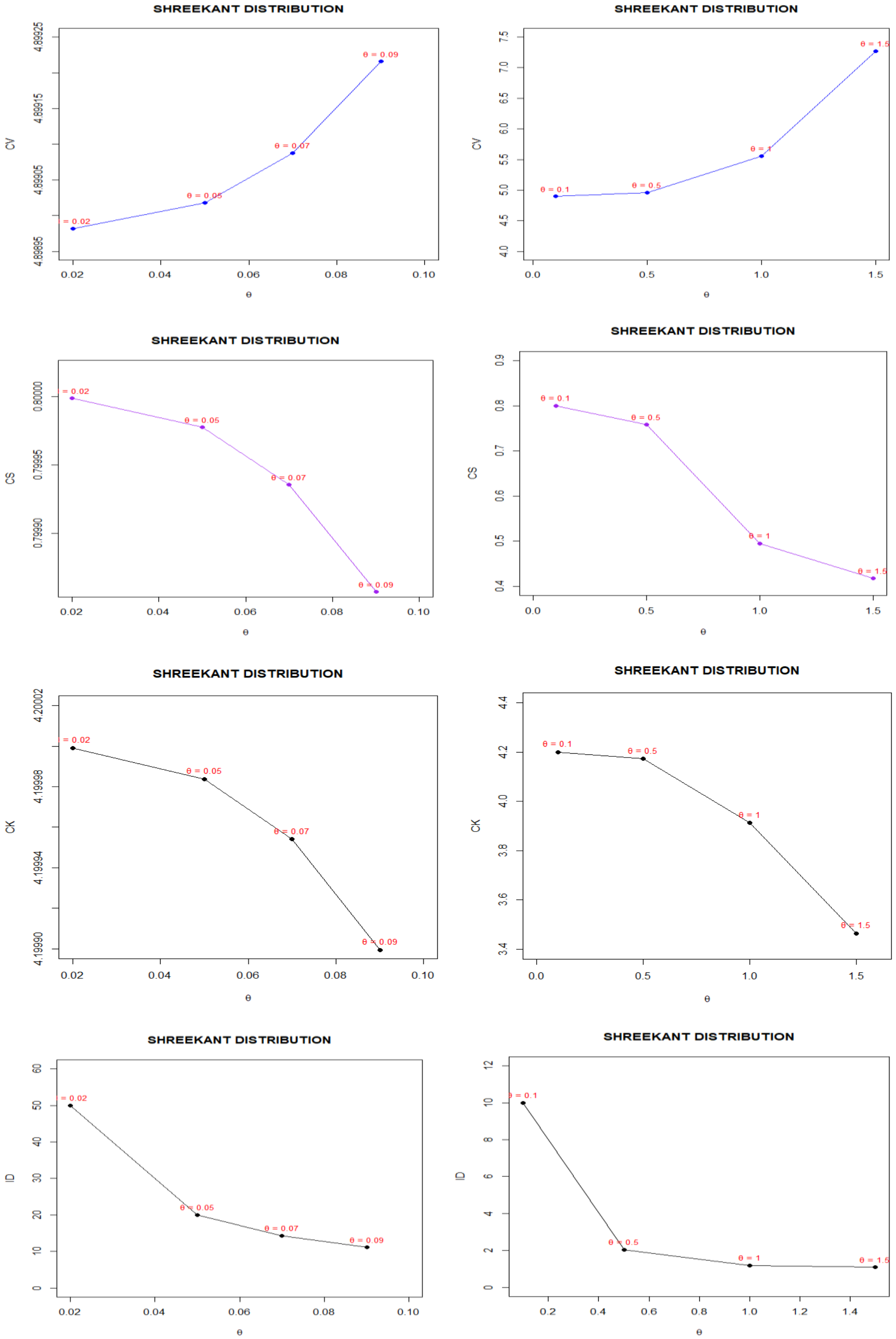

Figure 5. *The CV, CS, CK and ID of the Shreekant distribution*

V. Deviation from Mean and Median

Mean deviation about the mean and the mean deviation about median of a random variable X having pdf $f(x)$ and cdf $F(x)$ are defined by

$$\delta_1(x) = \int_0^\infty |x-\mu| \, f(x)dx = 2\mu F(\mu) - 2\int_0^\mu x\, f(x)dx$$

and $\delta_2(x) = \int_0^\infty |x-M| \, f(x)dx = -\mu + 2\int_M^\infty x\, f(x)dx$ respectively, where $\mu = E(X)$ and $M = Median(X)$.

Using the pdf and the expression for the mean of the Shreekant distribution, we get

$$\int_0^\mu x\, f(x;\theta)\,dx = \mu - \frac{\begin{bmatrix}\left(\mu^5+\mu^2+\mu\right)\theta^5+\left(5\mu^4+2\mu+1\right)\theta^4+\left(20\mu^3+2\right)\theta^3 \\ +60\mu^2\theta^2+120\mu\theta+120\end{bmatrix} e^{-\theta\mu}}{\theta\left(\theta^4+\theta^3+24\right)}$$

$$\int_0^M x\, f(x;\theta)\,dx = \mu - \frac{\begin{bmatrix}\left(M^5+M^2+M\right)\theta^5+\left(5M^4+2M+1\right)\theta^4+\left(20M^3+2\right)\theta^3 \\ +60M^2\theta^2+120M\theta+120\end{bmatrix} e^{-\theta M}}{\theta\left(\theta^4+\theta^3+24\right)}.$$

Using above expressions and after some algebraic simplifications, the mean deviation about the mean $\delta_1(x)$, and the mean deviation about the median $\delta_2(x)$ of the Shreekant distribution are obtained as

$$\delta_1(X) = \frac{2\left\{\left(\mu^4+\mu+1\right)\theta^4+\left(8\mu^3+2\right)\theta^3+36\mu^2\theta^2+96\mu\theta+120\right\}e^{-\theta\mu}}{\theta\left(\theta^4+\theta^3+24\right)}.$$

$$\delta_2(X) = \frac{2\begin{Bmatrix}\left(M^5+M^2+M\right)\theta^5+\left(5M^4+2M+1\right)\theta^4+\left(20M^3+2\right)\theta^3 \\ +60M^2\theta^2+120M\theta+120\end{Bmatrix} e^{-\theta M}}{\theta\left(\theta^4+\theta^3+24\right)} - \mu.$$

VI. Distribution of Order Statistics

Let $X_i \sim$ Shreekant distribution(θ), $i = 1,2,3,...,n$ be a *random* sample of size n. Let the corresponding order statistics of these n samples observations be $Y_1, Y_2, ..., Y_n$. The pdf of the r th order statistic Y_r of the Shreekant distribution can be obtained as

$$f_{Y_r}(y) = \frac{1}{B(r,n-r+1)}\left[F(y)\right]^{r-1}\left[1-F(y)\right]^{n-r} f(y)$$

$$= \frac{\theta^5}{\left(\theta^4+\theta^3+24\right)B(r,n-r+1)}\left[1-\left[1+\frac{\theta y\left\{\theta^3y^3+4\theta^2y^2+12\theta y+\left(\theta^3+24\right)\right\}}{\theta^4+\theta^3+24}\right]e^{-\theta y}\right]^{r-1}$$

$$\left[1-\left[1-\left[1+\frac{\theta y\left\{\theta^3y^3+4\theta^2y^2+12\theta y+\left(\theta^3+24\right)\right\}}{\theta^4+\theta^3+24}\right]e^{-\theta y}\right]\right]^{n-r}\left(1+y+y^4\right)e^{-\theta y}$$

The pdf of the nth order statistic Y_n and that of the 1st order statistics Y_1 can be obtained by putting $r=n$ and $r=1$ respectively in the above equation.

The pdf and cdf of the nth order statistic Y_n can be obtained as

$$f_{Y_n}(y)=\frac{n\theta^5}{\theta^4+\theta^3+24}\left(1+y+y^4\right)e^{-\theta y}\left[1-\left[1+\frac{\theta y\left\{\theta^3y^3+4\theta^2y^2+12\theta y+\left(\theta^3+24\right)\right\}}{\theta^4+\theta^3+24}\right]e^{-\theta y}\right]^{n-1}$$

$$F_{Y_n}(y)=\left[1-\left[1+\frac{\theta y\left\{\theta^3y^3+4\theta^2y^2+12\theta y+\left(\theta^3+24\right)\right\}}{\theta^4+\theta^3+24}\right]e^{-\theta y}\right]^{n}.$$

VII. Estimation of Parameter of the Shreekant Distribution

In this section, following five estimation techniques for estimating the parameter of the Shreekant distribution have been discussed.

I. Maximum Likelihood Estimation

Let $X_i \sim$ Shreekant distribution(θ) , $i=1,2,3,...,n$ be n random samples. The log-likelihood function, L of the Shreekant distribution is given by

$$\log L=\sum_{i=1}^{n}\log f\left(x_i;\theta\right)=n\left\{5\log\theta-\log\left(\theta^4+\theta^3+24\right)\right\}+\sum_{i=1}^{n}\log\left(1+x_i+x_i^{\,4}\right)-n\theta\,\overline{x}$$

The maximum likelihood estimate (MLE) $\left(\hat{\theta}\right)$ of the parameters $\left(\theta\right)$ of the Shreekant distribution is the solution of the following log likelihood equation

$$\frac{d\log L}{d\theta}=\frac{5n}{\theta}-\frac{n\left(4\theta^3+3\theta^2\right)}{\theta^4+\theta^3+24}-n\,\overline{x}=0$$

This gives

$$\overline{x}\theta^5+\left(\overline{x}-1\right)\theta^4-2\theta^3+24\overline{x}\theta-120=0\,.$$

This is a fifth-degree polynomial equation in θ. It should be noted that the method of moment estimate is also the same as that of the MLE. The above equation can easily be solved using Newton-Raphson method, taking the initial value of the parameter $\theta=0.5$.

II. Maximum product spacing estimation

The maximum product spacing estimates (MPSE) $\left(\hat{\theta}\right)$ of the parameter $\left(\theta\right)$ can be obtained numerically by maximizing the following function in relation to θ.

$$MPS=\frac{1}{n+1}\sum_{i=1}^{n+1}\log\left[F\left(x_i,\theta\right)-F\left(x_{i-1},\theta\right)\right]$$

III. Least Squares Estimation

Let $(x_1, x_2, ..., x_n)$ be a random sample from Shreekant (θ) distribution. The least square function of the parameter based on the pdf of Shreekant distribution can be given as

$$LSE = \sum_{i}^{n} \left(F\left(x_{(i)}\right) - \frac{i}{n+1} \right)^2$$

The least square estimates (LSE) $\hat{\theta}$ of θ can be obtained by minimizing the above equation with respect to θ

IV. Weighted Least Squares Estimation

Let $(x_1, x_2, ..., x_n)$ be a random sample from Shreekant (θ) distribution. The weighted least square function of the parameter based on the pdf of Shreekant distribution can be given as

$$WLSE = \sum_{i}^{n} w_i \left(F\left(x_{(i)}\right) - \frac{i}{n+1} \right)^2, \text{ where } w_i = \frac{(n+1)^2 (n+2)}{i(n-i+1)}$$

The weighted least square estimates (WLSE) $\hat{\theta}$ of θ can be obtained by minimizing the above equation with respect to θ .

V. Cramer-Von Mises Estimation

Let $(x_1, x_2, ..., x_n)$ be a random sample from Shreekant (θ) distribution. The Cramer-Von Mises function of the parameter based on the pdf of the Shreekant distribution can be given as

$$CVM = \frac{1}{12n} + \sum_{i=1}^{n} \left(F\left(x_{(i)}\right) - \frac{2i-1}{2n} \right)^2$$

The Cramer-Von Mises estimates (CVME) $\hat{\theta}$ of θ can be obtained by minimizing the above equation with respect to θ .

VIII. Numerical Simulation Study

A simulation study has been conducted to assess the consistency of the estimation of the parameter by different methods of estimation for Shreekant distribution. The study involved examining biases (B) and mean square errors (MSEs) of the different estimation methods viz. MLE, MPSE, LSE. WLSE and CVME discussed in section 7 for the Shreekant distribution using the formulas

$$B = \frac{1}{n} \sum_{i=1}^{n} \left(\hat{\theta} - \theta \right), \text{ and } \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} \left(\hat{\theta} - \theta \right)^2$$

The acceptance-rejection method of simulation study has been used to generate random samples from the Shreekant distribution.
The B and MSE of the parameter for the five estimation methods are presented in table 2. The values of B and MSE are decreasing for increasing values of sample size.

Table 2: *The Biases and MSE of Shreekant distribution for* $\theta = 0.07$ *and* $\theta = 0.16$

Parameter $(\hat{\theta})$	Sample size	MLE		MPSE		LSE		WLSE		CVME	
		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
0.07	**20**	-0.00762	0.00008	0.00168	0.00005	0.00216	0.00007	0.00147	0.00007	-0.00289	0.00001
	40	-0.00409	0.00005	0.00141	0.00004	0.00199	0.00007	0.00119	0.00007	-0.00259	0.00001
	60	-0.00238	0.00002	0.00115	0.00003	0.00156	0.00005	0.00104	0.00007	-0.00207	0.00000
	80	-0.00177	0.00001	0.00099	0.00002	0.00133	0.00004	0.00096	0.00007	-0.00183	0.00000
	100	-0.00139	0.00001	0.00089	0.00002	0.00119	0.00003	0.00075	0.00006	-0.00168	0.00000
0.16	**20**	0.01630	0.00036	-0.00031	0.00003	0.00036	0.00002	-0.01259	0.00041	-0.01110	0.00013
	40	0.01580	0.00033	-0.00024	0.00002	0.00028	0.00002	-0.01156	0.00031	-0.01083	0.00012
	60	0.01115	0.00019	-0.00017	0.00002	0.00018	0.00002	-0.01057	0.00027	-0.00985	0.00011
	80	0.00427	0.00008	-0.00008	0.00002	0.00008	0.00002	-0.00928	0.00023	-0.00898	0.00009
	100	0.00113	0.00003	-0.00006	0.00002	0.00005	0.00001	-0.00775	0.00020	-0.00767	0.00007

IX. Real Data Analysis

The applications and the goodness of fit of Shreekant distribution have been discussed with two datasets relating to strength and stress data from engineering. Keeping in mind the flexibility and tractability of the distribution with the dataset following two datasets have been considered. The total time on test (TTT) plot of dataset 1 and 2 and the simulated datasets of the Shreekant distribution are shown in figure 6 and 7 respectively. The descriptive summary of the datasets 1 and 2 are presented in the table 3.

Dataset 1: This dataset is the strength data of glass of the aircraft window reported by Fuller et al [10]:
18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.2, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381.

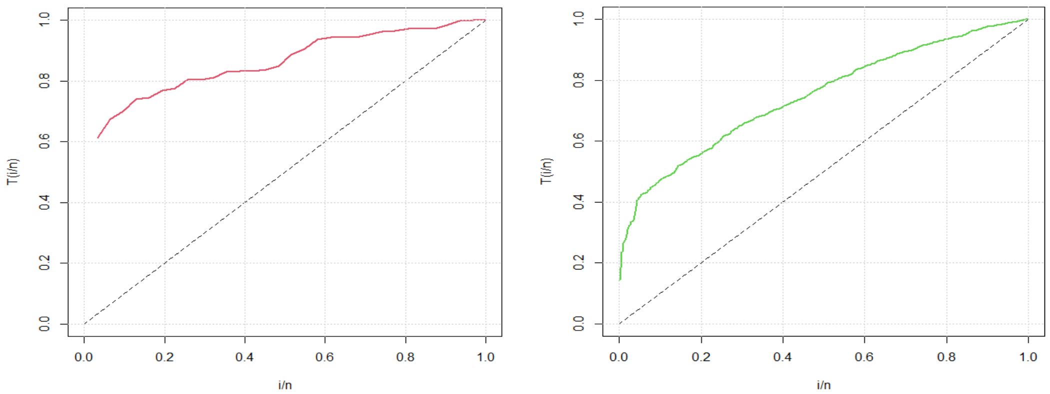

Figure 6: TTT plot of dataset 1 and simulated dataset of the Shreekant distribution.

Dataset 2: The data is given by Birnbaum & Saunders [11] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data ($\times 10^{-3}$) are presented below (after subtracting 65).

5, 25, 31, 32, 34, 35, 38, 39, 39, 40, 42, 43, 43, 43, 44, 44, 47, 47, 48, 49, 49, 49, 51, 54, 55, 55, 55, 56, 56, 56, 58, 59, 59, 59, 59, 59, 63, 63, 64, 64, 65, 65, 65, 66, 66, 66, 66, 66, 67, 67, 67, 68, 69, 69, 69, 69, 71, 71, 72, 73, 73, 73, 74, 74, 76, 76, 77, 77, 77, 77, 77, 77, 79, 79, 80, 81, 83, 83, 84, 86, 86, 87, 90, 91, 92, 92, 92, 92, 93, 94, 97, 98, 98, 99, 101, 103, 105 ,109 ,136, 147

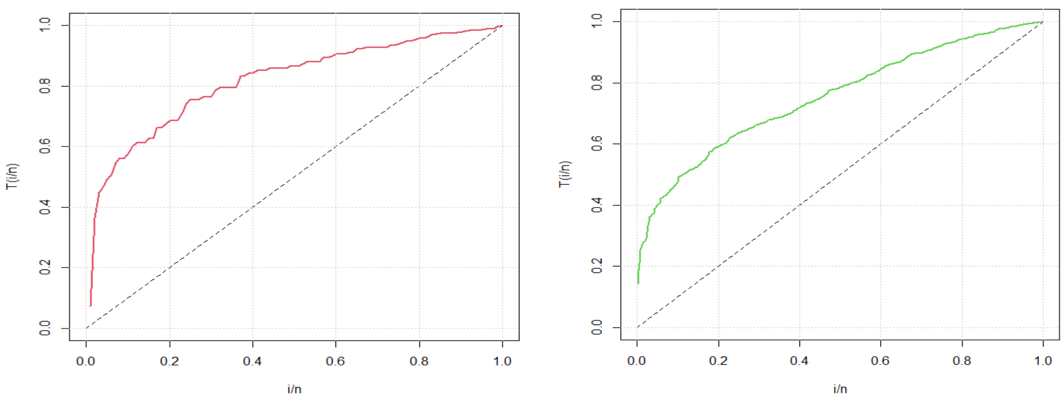

Figure 7: *TTT-plot of dataset 2 and simulated dataset of the Shreekant distribution.*

In order to compare the lifetime distributions, values of $-2\log L$, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Consistent Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Kolmogorov-Smirnov (K-S) Statistics and the corresponding probability value (p-value) for the above two datasets have been computed using the formulae

$$AIC = -2\log L + 2p\,,\;\; BIC = -2\log L + p\log(n)\,,\;\; CAIC = -2\log L + \frac{2pn}{n-p-1}$$

$$HQIC = -2\log L + 2p\log\lfloor \log(n) \rfloor,\quad K\text{-}S = \underset{x}{Sup}\,|\,F_m(x) - F_o(x)\,|$$

where, p = number of parameters, n = sample size, $F_m(x)$ = empirical cdf of considered distribution and $F_o(x)$ = cdf of considered distribution

The estimated values of parameter with standard error (SE) of the considered distributions for the given datasets are presented in the table 4 and 6 respectively. The values of $-2\log L$, AIC, BIC, CAIC, HQIC, K-S and p-values of the considered distributions for the given datasets are presented in the table 5 and 7 respectively. The confidence intervals of the parameter of the proposed distribution are presented in table 8.

Table 3: *Summary of the datasets 1 and 2*

Datasets	Minimum	1st Quartile	Median	Mean	Standard deviation	3rd Quartile	Maximum
1	18.83	25.51	29.90	30.81	7.25	30.81	45.38
2	5.00	55.00	67.00	68.33	22.41	80.25	147.00

Table 4: *Estimated values by different methods of estimation and standard error (S.E) of the parameter for the considered distributions of the dataset-1*

Distributions	MLE $\hat{\theta}$(S.E)	MPSE $\hat{\theta}$(S.E)	LSE $\hat{\theta}$(S.E)	WLSE $\hat{\theta}$(S.E)	CVME $\hat{\theta}$(S.E)
Shreekant	0.1622 (0.0130)	0.1611 (0.0128)	0.1545 (0.0046)	0.1555 (0.0015)	0.1557 (0.0239)
Uma	0.1296 (0.0116)	0.1285 (0.0114)	0.1212 (0.0039)	0.1225 (0.0012)	0.1224 (0.0206)
Sujatha	0.0956 (0.0099)	0.0946 (0.0097)	0.0868 (0.0032)	0.0881 (0.0010)	0.0879 (0.0168)
Akash	0.0970 (0.0100)	0.0961 (0.0099)	0.0881 (0.0032)	0.0896 (0.0010)	0.0893 (0.0171)
Shanker	0.0647 (0.0082)	0.0638 (0.0080)	0.0553 (0.0025)	0.0568 (0.0008)	0.0565 (0.0133)
Lindley	0.0629 (0.0080)	0.0621 (0.0078)	0.0537 (0.0024)	0.0551 (0.0007)	0.0548 (0.0129)
Exponential	0.0324 (0.0058)	0.0317 (0.0057)	0.0229 (0.0015)	0.0242 (0.0005)	0.0238 (0.0083)

Table 5: *Goodness of fit for the dataset-1*

Sl. No	Distributions	$-2\log L$	AIC	BIC	CAIC	HQIC	K-S	P-value
1	Shreekant	227.25	229.25	231.86	229.30	230.31	0.2120	0.1460
2	Uma	232.88	234.88	237.49	234.93	235.94	0.4502	0.0000
3	Sujatha	241.50	243.50	246.10	243.54	244.55	0.4501	0.0000
4	Akash	240.68	242.68	245.28	242.72	243.73	0.3026	0.0000
5	Shanker	252.35	254.35	256.95	254.39	255.40	0.3341	0.0000
6	Lindley	253.98	255.98	258.59	256.02	257.04	0.3444	0.0000
7	Exponential	274.52	276.52	279.13	276.56	277.58	0.4777	0.0000

Table 6: *Estimate values with standard error of the considered distributions for the dataset-2*

Distributions	MLE $\hat{\theta}(\text{S.E})$	MPSE $\hat{\theta}(\text{S.E})$	LSE $\hat{\theta}(\text{S.E})$	WLSE $\hat{\theta}(\text{S.E})$	CVME $\hat{\theta}(\text{S.E})$
Shreekant	0.0731	0.0728	0.0703	0.0717	0.0705
	(0.0032)	(0.0032)	(0.0007)	(0.0003)	(0.0064)
Uma	0.0585	0.0582	0.0553	0.0568	0.0556
	(0.0029)	(0.0029)	(0.0006)	(0.0002)	(0.0055)
Sujatha	0.0435	0.0433	0.0400	0.0416	0.0405
	(0.0025)	(0.0025)	(0.0004)	(0.0001)	(0.0045)
Akash	0.0438	0.0436	0.0403	0.0419	0.0408
	(0.0025)	(0.0025)	(0.0004)	(0.0001)	(0.0045)
Shanker	0.0292	0.0290	0.0254	0.0270	0.0259
	(0.0020)	(0.0020)	(0.0003)	(0.0001)	(0.0035
Lindley	0.0288	0.0286	0.0250	0.0266	0.0255
	(0.0020)	(0.0286)	(0.0003)	(0.0001)	(0.0034)
Exponential	0.0146	0.0144	0.0105	0.0119	0.0109
	(0.0014)	(0.0014)	(0.0002)	(0.0007)	(0.0021)

Table 7: *Goodness of fit for the dataset-2*

Sl. No	Distributions	$-2\log L$	AIC	BIC	CAIC	HQIC	K-S	P-value
1	Shreekant	924.19	926.19	928.80	926.23	927.25	0.1271	0.1390
2	Uma	934.03	936.03	938.63	936.07	937.08	0.1891	0.0050
3	Sujatha	951.78	953.78	956.38	953.82	954.83	0.2149	0.0000
4	Akash	950.97	952.97	955.57	953.01	954.02	0.1944	0.0010
5	Shanker	980.97	982.97	985.57	983.01	984.02	0.2333	0.0000
6	Lindley	983.10	985.10	987.70	985.14	986.15	0.2326	0.0000
7	Exponential	1044.87	1046.87	1049.47	1046.91	1047.92	0.3700	0.0000

Table 8: *Confidence interval of the parameter θ of the Shreekant distribution*

Dataset	90%Confidence interval		95%Confidence interval		99%Confidence interval	
	Lower	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
1	0.0731	0.0732	0.0730	0.0732	0.0730	0.0732
2	0.0679	0.0786	0.0669	0.0797	0.0650	0.0819

The fitted plots of the considered distributions, the Q-Q plot and the P-P plot of Shreekant distribution for the datasets 1 and 2 are presented in the figure 8 and 9 respectively. The profile plots of the parameters of Shreekant distribution are presented in the figure 10.

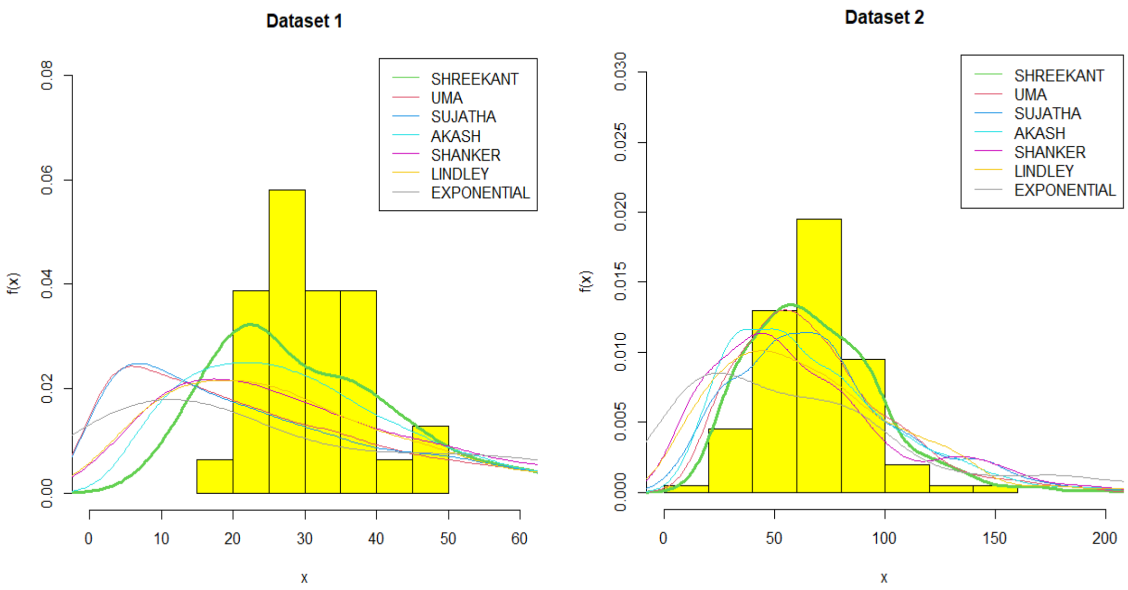

Figure 8. *Fitted plots of the considered distributions for the datasets 1 and 2.*

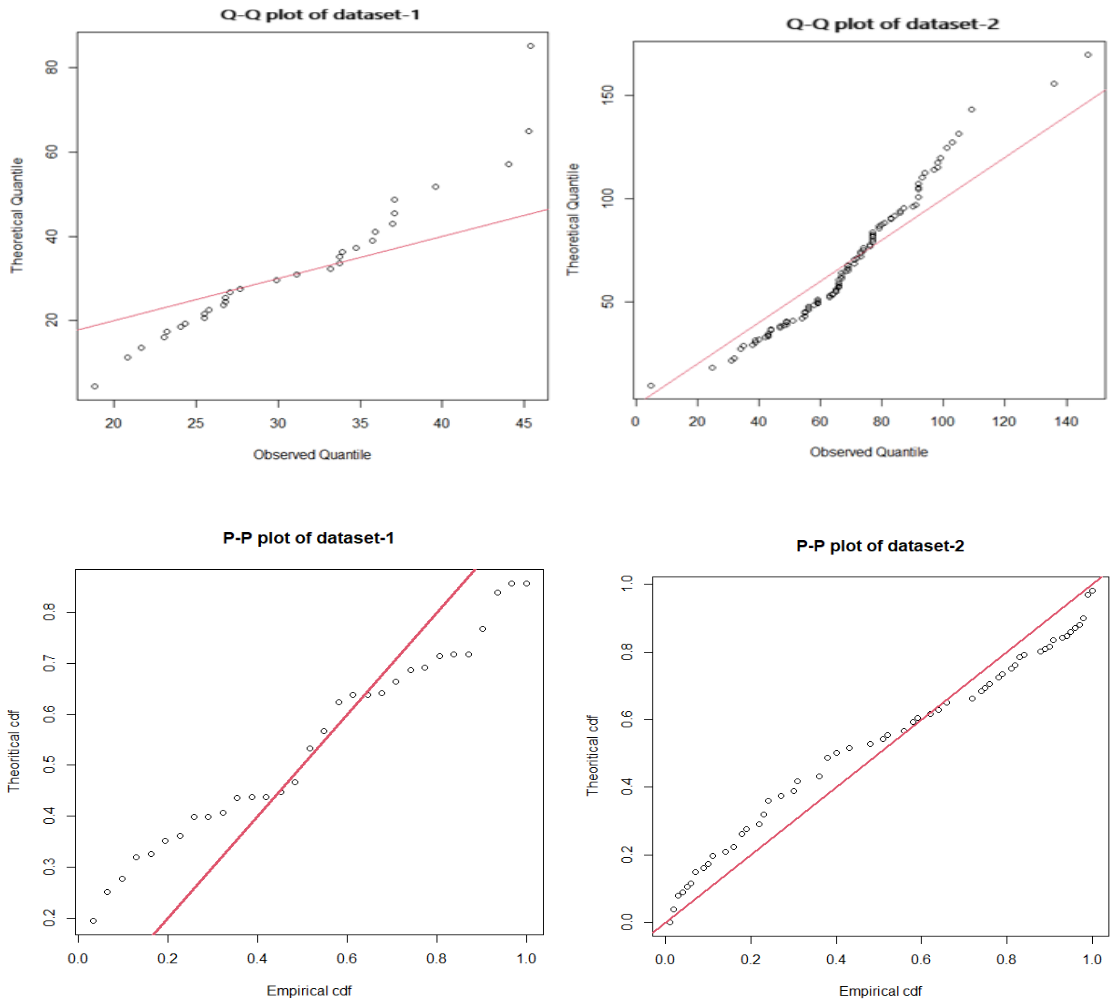

Figure 9: *Q-Q plot and P-P plot of the Shreekant distribution for the datasets 1 and 2.*

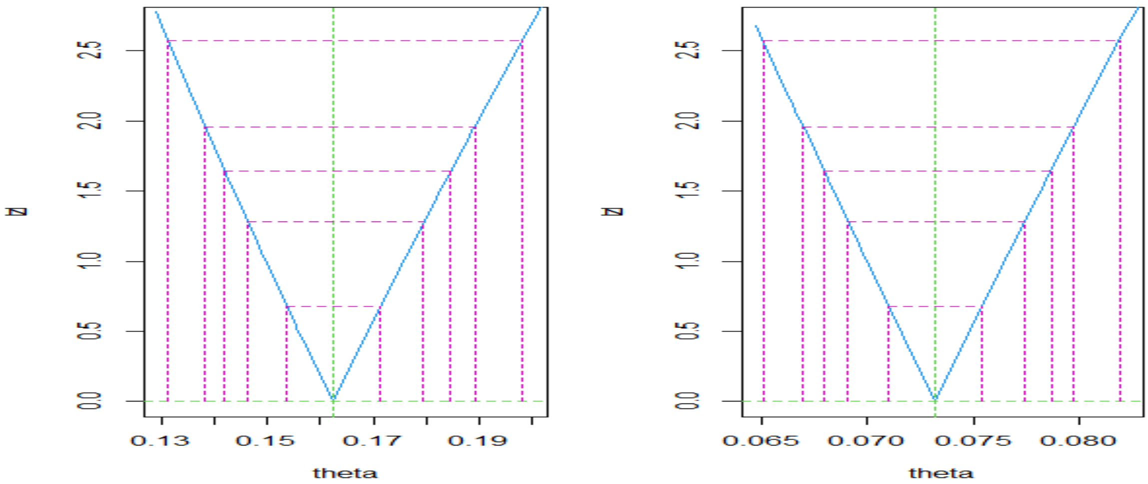

Figure 10: *Profile plot of the parameter of Shreekant distribution for the dataset 1 and 2*

It is clear from the goodness of fit in tables 5 and 7 and from the fitted plots of distributions in figure 8 that the Shreekant distribution gives much better fit over exponential, Lindley, Shanker, Akash, Sujatha and Uma distributions.

X. Conclusion and Future Work

A new lifetime distribution named Shreekant distribution has been suggested to model strength and stress data from engineering. Statistical properties, estimation of parameter and applications of the distribution have been presented. As the distribution is new one, it is expected and hoped that it will be of great use to statisticians and policy makers working in the field of modeling lifetime data from different fields of knowledge.

Being a new lifetime distribution having flexibility, tractability and practicability, the distribution is expected to generate much scope for future works on it.

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