Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY STABILITY OF K-OUT-OF-N ACYCLIC TRANSMISSION NETWORK

Hemalatha.G[1], Vijayalakshmi.G[2*]

•

[1, 2*] Department of Mathematics, College of Engineering and Technology,
SRM Institute of Science and Technology, Kattankulathur,
Tamil Nadu- 603 203, India.
[1] hg9080@srmist.edu.in, [2*]vijayalg02@gmail.com

Abstract

Networks are used in many areas of engineering, including computers and communication, transportation and electric transmission. Component importance measures are applicable to improve the system design. The reliability characteristics of k-out-of-n acyclic transmission network have been evaluated using universal generating function (UGF). This paper explores and analyses Mean Time To Failure(MTTF), Birnbaum importance measurement, critical component importance, component risk growth factor, average risk growth factor and MTTF sensitivity for acyclic transmission network. Finally, The stability illustrates the impact of component failure on the reliability of the network system. The network reliability stability for acyclic transmission network is computed. In this work, we have evaluated reliability characteristics and of the acyclic transmission network using UGF method. The network reliability stability for k-out-of-n acyclic transmission network is computed. Furthermore, numerical illustrations are provided.

Keywords: Birnbaum importance measurement, risk growth factor, average risk growth factor, MTTF sensitivity, universal generating function, critical component importance.

I. Introduction

Reliability is a critical aspect of network systems, ensuring their ability to function under various conditions. Over the years, significant efforts have been made to improve network reliability using advanced methodologies. The universal generating function (UGF), introduced by Ushakov [15], was a pivotal development for system reliability evaluation. Levitin and Lisnianski [5] utilized UGF for significance and sensitivity analysis of multistate systems, while Levitin[6] extended its application to acyclic networks. Lisnianski and Levitin [7] further applied UGF to analyze real-world systems with distinct failure modes. Consistency in signal transmission from source to destination, known as terminal reliability *or* network reliability, has been extensively discussed in Colbourn [2]. Yeh [16] evaluated multi-state node network reliability using traditional binary-state methods, while Yeh [17] enhanced the UGF method for general multi-state networks, offering practical solutions for reliability evaluation.

Birnbaum [1] first introduced component importance measures, highlighting the role of individual components in system reliability. Building on this, Gao and Yao [3] explored the importance indices of components in uncertain systems, while Zhu et al. [21] tackled multi-type component assignment problems using Birnbaum importance heuristics. He et al. [4] employed cellular automata to measure component importance in network systems, advancing the understanding of individual component contributions.

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

In network reliability studies, Trivedi et al. [14] emphasized the mathematical modeling of reliability in modern communication systems, while Negi and Singh [9] analyzed non-repairable complex systems with weighted subsystems. Yeh [18] used a path-based UGF approach to examine network reliability and multistate connectivity, particularly in binary-state systems.

Complex system reliability has been studied using diverse modeling techniques. Ram and Singh [10] applied copula-based approaches to assess engineering systems, and Ram [11] reviewed various reliability methods. Ram and Manglik [12], [13] used stochastic and extra variable techniques to evaluate industrial systems, emphasizing critical components.

Graph-theoretic methods were applied by Zarghami et al. [20], who evaluated infrastructure network reliability by analyzing series and parallel structures. Yeh [19] further refined the UGF algorithm for acyclic binary-state network reliability, enabling better performance assessment.

These studies underscore the value of tools like UGF, importance measures, and advanced modeling techniques in improving network reliability. Their contributions have significantly enhanced system robustness, minimizing failures and ensuring stable operations across multiple domains.

Notations and Abbreviations:

$u(z)$ Universal Generating Function of zth vertex of the network

$U(z)$ Universal Generating Function of zth subnet vertex of the network

$\otimes$ Composition operator

$I_i^{\text{BM}}(t)$ Birnbaum component importance of ith component

$I_i^{\text{CIM}}(t)$ Critical importance measure of ith component

$RGF_i(t)$ Risk growth factor of ith component

ξ Function operator

R Reliability of the suggested network

N Number of sink vertices in the suggested network

L Number of vertices in the suggested network

$\eta_{i:j}$ Failure rate of flow from vertex i to j

$h(t)$ Hazard rate

t Mission time

p_n Probability that x which equ als x_n

$p_{n,\chi^{m:n}}$ Probability of a set of vertices $\chi^{m:n}$ getting a signal directly from the vertex at f_m

$q_{n,\chi^{m:n}}$ Probability that the set of vertex $\chi^{m:n}$ cannot receive a signal directly from the vertex at f_m

f_m m th vertex of the suggested network

$\chi^{m:n}$ A signal is received by a set of vertices from the vertex at f_m

UGF   Universal generating function

MTTF   Mean Time To Failure

II. Network Reliability using the UGF

Ushakov[15] first presented the UGF method for determining a system's or network's reliability. A detailed discussion of UGF, composition operator, and network reliability is provided by Levitin[8].

The UGF of a discrete random variable is defined by a polynomial as follows:

$$u(z) = \sum_{n=1}^{N} p_n z^{x_n} \tag{1}$$

where p_n is the probability of x, which is equal to x_n and x has n probable values.

Let us suppose that a vertex is placed at f_m. If in-state n is the vertex available for signal transmission from f_m to a set of vertices $\chi^{m:n}$, then $v_{m:n} = \begin{cases} 1 & if\ f_m \in \chi^{m:n} \\ 0 & if\ f_m \notin \chi^{m:n} \end{cases}$ (2)

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

The vertex-UGF of $v_{m:n}$ elements are expressed as:

$$u_i(z) = \sum_{n=1}^{N} p_{n,\chi^{m:n}} q_{n,\chi^{m:n}} z^{v_{m:n}} \tag{3}$$

The subnet UGF of $U_{i+1}(z)$ elements are expressed as:

$$U_{i+1}(z) = \{\xi[U_i(z)], u_{i+1}(z)\} \tag{4}$$

- If the path fails to pass through the regarded vertex (unit) and if the path fails to complete the connection from the source vertex to the considered vertex, remove the term from UGF from the operator η.
- Gather every term that is similar for every vertex in the generated UGF.

I. Algorithm for the Reliability of Networks

The following algorithm is created to assess network reliability for a binary state network:

Step 1: Find $v_{m:n}$ corresponding to $\chi^{m:n}$ for the vertices located at the positions $f_1, \ldots\ldots , f_{L-N}$ in the network.
Step 2: Determine $u_i(z)$ of the vertices located at $f_1, \ldots\ldots. , f_{L-N}$ places using equation(3).
Step 3: Set $U_1(z) = u_1(z)$.
Step 4: Determine the value of $U_{i+1}(z) = \{\xi[U_i(z)], u_{i+1}(z)\}$ where $i = 1$ to $L - N - 1$.
Step 5: Obtain the network reliability at the sink vertices by simplifying polynomial $U_{L-N}(z)$ and applying operator ξ.

II. Model Description

Moving signal flows can occur under the following conditions:

- If the signal flow from vertex i to vertex j is successful and vertex i to vertex k fails then the probability becomes $p_{i:j}(1 - p_{i:k}) = p_{i:j} q_{i:k}$ (5)

- If signal flow at a vertex k is interrupted, then the probability becomes $p_{k:\varphi}$, where $k = 1, 2, \ldots\ldots, L - 1$.

- If the signal flows from vertex 1 to vertices 2 and 3 then probability becomes

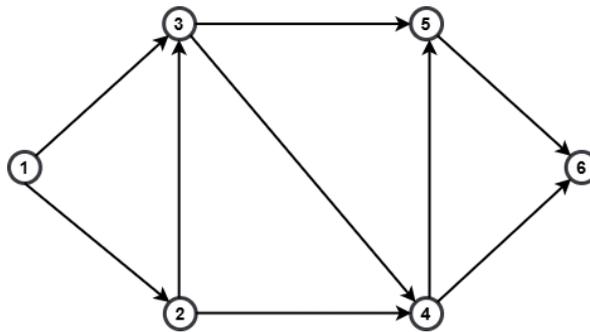$$p_{1:[2,3]} = p_{1:2} p_{1:3}. \tag{6}$$

III. Numerical Illustration

Figure 1: *k-out-of-n network*

Step 1: Consider the k-out-of-n network depicted in Figure 1, where signal flow starts at vertex 1 and ends at vertex 6. Here $L = 6$ and $N = 1$
Step 2: The Universal generating function of all the vertices are expressed as:

$$u_1(z) = p_{1:\varphi} q_{1:2} q_{1:3} z^{000000} + p_{1:2} q_{1:3} z^{010000} + p_{1:3} q_{1:2} z^{001000} + p_{1:[2,3]} z^{011000} \tag{7}$$

$$u_2(z) = p_{2:\varphi} q_{2:3} q_{2:4} z^{000000} + p_{2:3} q_{2:4} z^{001000} + p_{2:4} q_{2:3} z^{000100} + p_{2:[3,4]} z^{001100} \tag{8}$$

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

$$u_3(z) = p_{3:\varphi}\, q_{3:4}\, q_{3:5}\, z^{000000} + p_{3:4}\, q_{3:5}\, z^{000100} + p_{3:5}\, q_{3:4}\, z^{000010} + p_{3:[4,5]}\, z^{000110} \tag{9}$$

$$u_4(z) = p_{4:\varphi}\, q_{4:5}\, q_{4:6}\, z^{000000} + p_{4:5}\, q_{4:6}\, z^{000010} + p_{4:6}\, q_{4:5}\, z^{000001} + p_{4:[5,6]}\, z^{000011} \tag{10}$$

$$u_5(z) = p_{5:\varphi}\, q_{5:6}\, z^{000000} + p_{5:6}\, z^{000001} \tag{11}$$

Step 3: The UGF of vertex 1 is $U_1(z) = u_1(z)$

$$U_1(z) = p_{1:\varphi}\, q_{1:2}\, q_{1:3}\, z^{000000} + p_{1:2}\, q_{1:3}\, z^{010000} + p_{1:3}\, q_{1:2}\, z^{001000} + p_{1:[2,3]}\, z^{011000} \tag{12}$$

$$\xi[U_1(z)] = p_{1:2}\, q_{1:3}\, z^{010000} + p_{1:3}\, q_{1:2}\, z^{001000} + p_{1:[2,3]}\, z^{011000} \tag{13}$$

Step 4: The UGF of vertex 2 is $U_2(z) = \{\xi[U_1(z)], u_2(z)\}$

$$U_2(z) = [\, p_{1:2}\, q_{1:3}\, z^{010000} + p_{1:3}\, q_{1:2}\, z^{001000} + p_{1:[2,3]}\, z^{011000}] \otimes [p_{2:\varphi}\, q_{2:3}\, q_{2:4}\, z^{000000} + p_{2:3}\, q_{2:4}\, z^{001000}$$
$$+ p_{2:4}\, q_{2:3}\, z^{000100} + p_{2:[3,4]}\, z^{001100}\,]$$

$$= p_{1:2}\, p_{2:3}\, q_{2:4}\, q_{1:3}\, z^{001000} + p_{1:[2,3]}\, p_{2:3}\, q_{2:4}\, z^{001000} + p_{1:2}\, p_{2:4}\, q_{2:3}\, q_{1:3}\, z^{000100} + p_{1:[2,3]}\, p_{2:4}$$
$$q_{2:3}\, z^{001100} + p_{1:2}\, p_{2:[3,4]}\, q_{1:3}\, z^{001100} + p_{1:[2,3]}\, p_{2:[3,4]}\, z^{001100} \tag{14}$$

$$\xi[U_2(z)] = \{\, p_{1:2}\, p_{2:3}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, q_{2:4}\}z^{001000} + p_{1:2}\, p_{2:4}\, q_{2:3}\, q_{1:3}\, z^{000100} + \{p_{1:[2,3]}\, p_{2:4}\, q_{2:3}$$
$$+ p_{1:2}\, p_{2:[3,4]}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, \}z^{001100} \tag{15}$$

The UGF of vertex 3 is $U_3(z) = \{\xi[U_2(z)], u_3(z)\}$

$$U_3(z) = [\, \{p_{1:2}\, p_{2:3}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, q_{2:4}\}z^{001000} + p_{1:2}\, p_{2:4}\, q_{2:3}\, q_{1:3}\, z^{000100} + \{p_{1:[2,3]}\, p_{2:4}\, q_{2:3} + p_{1:2}$$
$$p_{2:[3,4]}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\}z^{001100}] \otimes [p_{3:\varphi}\, q_{3:4}\, q_{3:5}\, z^{000000} + p_{3:4}\, q_{3:5}\, z^{000100} + p_{3:5}\, q_{3:4}\, z^{000010}$$
$$+ p_{3:[4,5]}\, z^{000110}\,]$$

$$= \{\, p_{1:2}\, p_{2:3}\, p_{3:4}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, q_{3:5}\, q_{2:4}\}z^{000100} + \{p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, q_{3:5}\, q_{2:3} + p_{1:2}$$
$$p_{2:[3,4]}\, p_{3:4}\, q_{3:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:4}\, q_{3:5}\}z^{000100} + \{p_{1:2}\, p_{2:3}\, p_{3:5}\, q_{3:4}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}$$
$$p_{3:5}\, q_{3:4}\, q_{2:4}\}z^{000010} + \{p_{1:[2,3]}\, p_{2:4}\, p_{3:5}\, q_{3:4}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:5}\, q_{3:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:5}$$
$$q_{3:4}\}z^{000110} + \{p_{1:2}\, p_{2:3}\, p_{3:[4,5]}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:[4,5]}\, q_{2:4}\}z^{000110} + \{p_{1:[2,3]}\, p_{2:4}\, p_{3:[4,5]}\, q_{2:3}$$
$$+ p_{1:2}\, p_{2:[3,4]}\, p_{3:[4,5]}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:[4,5]}\, \}z^{000110} \tag{16}$$

$$\xi[U_3(z)] = \{\, p_{1:2}\, p_{2:3}\, p_{3:4}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, q_{3:5}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, q_{3:5}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}$$
$$p_{3:4}\, q_{3:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:4}\, q_{3:5}\}z^{000100} + \{p_{1:2}\, p_{2:3}\, p_{3:5}\, q_{3:4}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:5}$$
$$q_{3:4}\, q_{2:4}\}z^{000010} + \{p_{1:[2,3]}\, p_{2:4}\, p_{3:5}\, q_{3:4}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:5}\, q_{3:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:5}\, q_{3:4}$$
$$+ p_{1:2}\, p_{2:3}\, p_{3:[4,5]}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:[4,5]}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:[4,5]}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:[4,5]}$$
$$q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:[4,5]}\, \}z^{000110} \tag{17}$$

The UGF of vertex 4 is $U_4(z) = \{\xi[U_3(z)], u_4(z)\}$

$$\xi[U_4(z)] = \{p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, p_{4:5}$$
$$q_{4:6}\, q_{3:5}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5} + p_{1:[2,3]}$$
$$p_{2:4}\, p_{3:5}\, p_{4:5}\, q_{4:6}\, q_{3:4}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:5}\, p_{4:5}\, q_{4:6}\, q_{3:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:5}\, p_{4:5}\, q_{4:6}$$
$$q_{3:4} + p_{1:2}\, p_{2:3}\, p_{3:[4,5]}\, p_{4:5}\, q_{4:6}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:[4,5]}\, p_{4:5}\, q_{4:6}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:[4,5]}$$
$$p_{4:5}\, q_{4:6}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:5}\, q_{4:6}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:5}\, q_{4:6}\}z^{000010} + \{p_{1:2}$$
$$p_{2:3}\, p_{3:4}\, p_{4:6}\, q_{4:5}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, p_{4:6}\, q_{4:5}\, q_{3:5}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, p_{4:6}\, q_{4:5}$$
$$q_{3:5}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:6}\, q_{4:5}\, q_{3:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:6}\, q_{4:5}\, q_{3:5}\}z^{000001}$$
$$+ \{p_{1:[2,3]}\, p_{2:4}\, p_{3:5}\, p_{4:6}\, q_{4:5}\, q_{3:4}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:5}\, p_{4:6}\, q_{4:5}\, q_{3:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}p_{3:5}$$
$$p_{4:6}\, q_{4:5}\, q_{3:4} + p_{1:2}\, p_{2:3}\, p_{3:[4,5]}\, p_{4:6}\, q_{4:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:[4,5]}\, p_{4:6}\, q_{4:5}\, q_{2:4} + p_{1:[2,3]}$$
$$p_{2:4}\, p_{3:[4,5]}p_{4:6}\, q_{4:5}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:6}\, q_{4:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:6}\, q_{4:5}$$
$$+ p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{4:[5,6]}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, p_{4:[5,6]}\, q_{3:5}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, p_{4:[5,6]}$$
$$q_{3:5}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:[5,6]}\, q_{3:5}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:4}\, p_{4:[5,6]}\, q_{3:5} + p_{1:[2,3]}\, p_{2:4}\, p_{3:5}$$
$$p_{4:[5,6]}\, q_{3:4}\, q_{2:3} + p_{1:2}\, p_{2:[3,4]}\, p_{3:5}\, p_{4:[5,6]}\, q_{3:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}p_{3:5}\, p_{4:[5,6]}\, q_{3:4} + p_{1:2}\, p_{2:3}$$
$$p_{3:[4,5]}\, p_{4:[5,6]}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:[4,5]}\, p_{4:[5,6]}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:[4,5]}\, p_{4:[5,6]}\, q_{2:3} + p_{1:2}$$
$$p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:[5,6]}\, q_{1:3} + p_{1:[2,3]}\, p_{2:[3,4]}\, p_{3:[4,5]}\, p_{4:[5,6]}\}\, z^{000011} \tag{18}$$

The UGF of vertex 5 is $U_5(z) = \{\xi[U_4(z)], u_5(z)\}$

$$\xi[U_5(z)] = \{\, p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5}\, q_{2:4}\, q_{1:3} + p_{1:[2,3]}\, p_{2:3}\, p_{3:4}\, p_{4:5}\, q_{4:6}\, q_{3:5}\, q_{2:4} + p_{1:[2,3]}\, p_{2:4}\, p_{3:4}\, p_{4:5}$$

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

$q_{4:6}$ $q_{3:5}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:4}$ $p_{4:5}$ $q_{4:6}$ $q_{3:5}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}$ $p_{3:4}$ $p_{4:5}$ $q_{4:6}$ $q_{3:5}$ + $p_{1:[2,3]}$ $p_{2:4}$ $p_{3:5}$ $p_{4:5}$ $q_{4:6}$ $q_{3:4}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:5}$ $p_{4:5}$ $q_{4:6}$ $q_{3:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}p_{3:5}$ $p_{4:5}$ $q_{4:6}$ $q_{3:4}$ $+p_{1:2}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:5}$ $q_{4:6}$ $q_{2:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:5}$ $q_{4:6}$ $q_{2:4}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:[4,5]}$ $p_{4:5}$ $q_{4:6}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:5}$ $q_{4:6}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:5}$ $q_{4:6}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:5}$ $p_{4:6}$ $q_{4:5}$ $q_{3:4}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:5}$ $p_{4:6}$ $q_{4:5}$ $q_{3:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}p_{3:5}$ $p_{4:6}$ $q_{4:5}$ $q_{3:4}$ $+p_{1:2}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:6}$ $q_{4:5}$ $q_{2:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:6}$ $q_{4:5}$ $q_{2:4}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:[4,5]}p_{4:6}$ $q_{4:5}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:6}$ $q_{4:5}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:6}$ $q_{4:5}$ $+p_{1:2}$ $p_{2:3}$ $p_{3:4}$ $p_{4:[5,6]}$ $q_{3:5}$ $q_{2:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:3}$ $p_{3:4}$ $p_{4:[5,6]}$ $q_{3:5}$ $q_{2:4}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:4}$ $p_{4:[5,6]}$ $q_{3:5}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:4}$ $p_{4:[5,6]}$ $q_{3:5}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}$ $p_{3:4}$ $p_{4:[5,6]}$ $q_{3:5}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:5}$ $p_{4:[5,6]}$ $q_{3:4}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:5}$ $p_{4:[5,6]}$ $q_{3:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}p_{3:5}$ $p_{4:[5,6]}$ $q_{3:4}$ $+p_{1:2}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:[5,6]}$ $q_{2:4}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:3}$ $p_{3:[4,5]}$ $p_{4:[5,6]}$ $q_{2:4}$ $+p_{1:[2,3]}$ $p_{2:4}$ $p_{3:[4,5]}$ $p_{4:[5,6]}$ $q_{2:3}$ $+p_{1:2}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:[5,6]}$ $q_{1:3}$ $+p_{1:[2,3]}$ $p_{2:[3,4]}$ $p_{3:[4,5]}$ $p_{4:[5,6]}$ $\}p_{5:6}$ z^{000001} (19)

Step 5:

$$R = p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{4:5}\, p_{5:6} - p_{1:2}\, p_{1:3}\, p_{2:3}\, p_{2:4}\, p_{4:5}\, p_{5:6} + p_{1:2}\, p_{1:3}\, p_{2:4}\, p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\, p_{1:3}\, p_{2:4}\, p_{3:5}$$

$$p_{4:6}\, p_{5:6} + p_{1:2}\, p_{2:3}\, p_{2:4}\, p_{3:5}\, p_{4:6}\, p_{5:6} - p_{1:2}\, p_{1:3}\, p_{2:3}\, p_{2:4}\, p_{3:5}\, p_{4:6}\, p_{5:6} - p_{1:2}\, p_{2:3}\, p_{2:4}\, p_{3:5}\, p_{4:5}\, p_{4:6}$$

$$p_{5:6} + p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{3:5}\, p_{4:6}\, p_{5:6} - p_{1:2}\, p_{2:3}\, p_{2:4}\, p_{3:4}\, p_{3:5}\, p_{4:6}\, p_{5:6} - p_{1:2}\, p_{2:3}\, p_{3:4}\, p_{3:5}\, p_{4:5}\, p_{4:6}$$

$$p_{5:6} + p_{1:2}\, p_{2:3}\, p_{2:4}\, p_{3:4}\, p_{3:5}\, p_{4:5}\, p_{4:6}\, p_{5:6} - p_{1:2}\, p_{1:3}\, p_{2:4}\, p_{3:4}\, p_{3:5}\, p_{4:5}\, p_{4:6}\, p_{5:6} + p_{1:2}\, p_{1:3}\, p_{2:3}\, p_{2:4}$$

$$p_{3:4}\, p_{3:5}\, p_{4:5}\, p_{4:6}\, p_{5:6}$$ (20)

III. Weibull Distribution

$P_{i:j}(t) = e^{-\eta_{i:j}t^{\beta}}$ where η is the scale parameter and β is the shape parameter.

I. Mean Time To Failure (MTTF)

Mean Time To Failure in network reliability refers to the average time elapsed between the start of network operation and the occurrence of a failure in one of its components or nodes. It's a crucial metric used to assess the reliability and availability of a network infrastructure over time. MTTF helps in predicting the frequency of failures within a network and aids in planning maintenance activities to minimize downtime.

When considering the failure rate for different network components, the MTTF is calculated as:

$$\text{MTTF} = M(t) = \int_0^{\infty} R(t)\, dt = \frac{1}{\eta}\Gamma\left(1+\frac{1}{\beta}\right)$$ (21)

II. Sensitivity of MTTF

The partial derivative of the function with respect to that factor is the definition of sensitivity analysis. The sensitivity analysis of MTTF can be obtained by differentiating the MTTF equation with respect to $\eta_{1:2}, \eta_{1:3}, \eta_{2:3}, \eta_{2:4}, \eta_{3:4}, \eta_{3:5}, \eta_{4:5}, \eta_{4:6}$ and $\eta_{5:6}$ the failure rates, respectively [10].

$$\text{Sensitivity of MTTF} = \frac{\partial MTTF}{\partial \eta_{i:j}}$$ (22)

We have the following three cases.

Case 1: $\beta < 1 \Rightarrow$ h(t) is a decreasing function of time.

If $\beta = 0.5$ then $P_{i:j}(t) = e^{-\eta_{i:j}t^{0.5}}$ substitute in equation (20),

$$R(t) = e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^{0.5}} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^{0.5}} +$$

$$e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^{0.5}} + e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} +$$

$$e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} -$$

$$e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} + e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} -$$

$$e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} - e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} +$$

$$e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}} +$$

$$e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^{0.5}}$$ (23)

We consider that the failure rates are as follows: $\eta_{1:2} = 0.1,\ \eta_{1:3} = 0.1, \eta_{2:3} = 0.2, \eta_{2:4} = 0.2, \eta_{3:4} = 0.3, \eta_{3:5} = 0.3, \eta_{4:5} = 0.4, \eta_{4:6} = 0.4, \eta_{5:6} = 0.5$. The reliability variation over time for the proposed network, derived from equation (23), is presented in Table 1.

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

Table 1: *Reliability of k-out-of-n network*

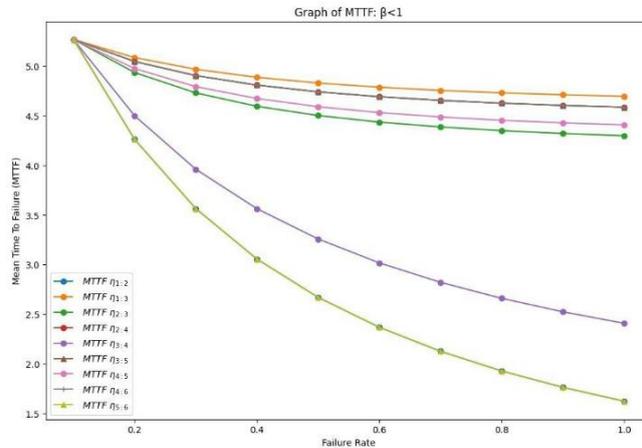| t	R
1	0.348257
2	0.208999
3	0.138394
4	0.096688
5	0.070000
6	0.052018
7	0.039447
8	0.030410
9	0.023765
10	0.018790

$$\text{MTTF}= \frac{1}{\eta}\Gamma\left(1+\frac{1}{0.5}\right)=\frac{1}{\eta}\Gamma(3)=\frac{2}{\eta} \tag{24}$$

$$\text{MTTF} = \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} - \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} + \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}}$$
$$+ \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} + \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}}$$
$$- \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}}$$
$$- \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{2}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}}$$
$$- \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{2}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} \tag{25}$$

The variation in MTTF for parameters $\eta_{1:2}$, $\eta_{1:3}$, $\eta_{2:3}$, $\eta_{2:4}$, $\eta_{3:4}$, $\eta_{3:5}$, $\eta_{4:5}$, $\eta_{4:6}$ and $\eta_{5:6}$ is derived from equation (25). The values are presented in Table 2, with a graphical representation in Figure 2.

Table 2: *MTTF of k-out-of-n network*

Failure rate	MTTF $\eta_{1:2}$	MTTF $\eta_{1:3}$	MTTF $\eta_{2:3}$	MTTF $\eta_{2:4}$	MTTF $\eta_{3:4}$	MTTF $\eta_{3:5}$	MTTF $\eta_{4:5}$	MTTF $\eta_{4:6}$	MTTF $\eta_{5:6}$
0.1	5.2698	5.2698	5.2698	5.2698	5.2698	5.2698	5.2698	5.2698	5.2698
0.2	4.2619	5.0873	4.9365	5.0476	4.5000	5.0476	4.9762	5.0476	4.2619
0.3	3.5642	4.9690	4.7309	4.9055	3.9610	4.9055	4.7944	4.9055	3.5642
0.4	3.0556	4.8882	4.5960	4.8095	3.5635	4.8095	4.6746	4.8095	3.0556
0.5	2.6698	4.8307	4.5031	4.7419	3.2585	4.7419	4.5919	4.7419	2.6698
0.6	2.3680	4.7884	4.4367	4.6926	3.0173	4.6926	4.5325	4.6926	2.3680
0.7	2.1259	4.7564	4.3878	4.6557	2.8218	4.6557	4.4885	4.6557	2.1259
0.8	1.9277	4.7317	4.3507	4.6273	2.6603	4.6273	4.4551	4.6273	1.9277
0.9	1.7626	4.7122	4.3221	4.6050	2.5245	4.6050	4.4292	4.6050	1.7626
1.0	1.6230	4.6965	4.2995	4.5873	2.4087	4.5873	4.4087	4.5873	1.6230

Figure 2: *MTTF of k-out-of-n network*

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

The sensitivity of MTTF for each edge will then be determined. By varying the failure rates to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 in the partial derivatives of MTTF, we can determine the sensitivity of MTTF and Table 3 have been generated. Table on the sensitivity of MTTF shows that the system is very sensitive with respect to the failure.

Table 3: *Sensitivity of MTTF*

Failure rate	$\partial MTTF/\partial\eta_{1:2}$	$\partial MTTF/\partial\eta_{1:3}$	$\partial MTTF/\partial\eta_{2:3}$	$\partial MTTF/\partial\eta_{2:4}$	$\partial MTTF/\partial\eta_{3:4}$	$\partial MTTF/\partial\eta_{3:5}$	$\partial MTTF/\partial\eta_{4:5}$	$\partial MTTF/\partial\eta_{4:6}$	$\partial MTTF/\partial\eta_{5:6}$
0.1	-12.2832	-2.2920	-4.2971	-2.8093	-9.3353	-2.8093	-9.3353	2.7463	-12.2832
0.2	-8.2571	-1.4441	-2.5630	-1.7449	-6.3438	-1.7449	-6.3438	2.3367	-8.2571
0.3	-5.8888	-0.9646	-1.6388	-1.1513	-4.5771	-1.1513	-4.5771	1.9737	-5.8888
0.4	-4.3904	-0.6743	-1.1051	-0.7963	-3.4522	-0.7963	-3.4522	1.6728	-4.3904
0.5	-3.3881	-0.4888	-0.7770	-0.5719	-2.6939	-0.5719	-2.6939	1.4281	-3.3881
0.6	-2.6875	-0.3650	-0.5652	-0.4235	-2.1595	-0.4235	-2.1595	1.2294	-2.6875
0.7	-2.1802	-0.2794	-0.4228	-0.3218	-1.7693	-0.3218	-1.7693	1.0671	-2.1802
0.8	-1.8018	-0.2184	-0.3239	-0.2499	-1.4758	-0.2499	-1.4758	0.9335	-1.8018
0.9	-1.5127	-0.1738	-0.2531	-0.1977	-1.2496	-0.1977	-1.2496	0.8227	-1.5127
1.0	-1.2870	-0.1405	-0.2013	-0.1590	-1.0717	-0.1590	-1.0717	0.7299	-1.2870

Case 2: $\beta = 1 \Rightarrow$ h(t) is constant.

If $\beta = 1$ then $P_{i:j}(t) = e^{-\eta_{i:j}(t)}$, substitute in equation (20)

$$
\begin{aligned}
R(t) = \ & e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t} + \\
& e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t} + e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t} + \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t} - \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t} + e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t} - \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t} - e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t} + \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t} + \\
& e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t}
\end{aligned}
\tag{26}
$$

We consider that the failure rates are as follows: $\eta_{1:2} = 0.1$, $\eta_{1:3} = 0.1, \eta_{2:3} = 0.2, \eta_{2:4} = 0.2, \eta_{3:4} = 0.3, \eta_{3:5} = 0.3, \eta_{4:5} = 0.4, \eta_{4:6} = 0.4, \eta_{5:6} = 0.5$. The reliability variation over time for the proposed network, derived from equation (26), is presented in Table 4.

Table 4: *Reliability of k-out-of-n network*

t	R
1	0.348257
2	0.096688
3	0.023765
4	0.005470
5	0.001214
6	0.000264
7	0.000057
8	0.000012
9	0.000003
10	0.018790

$$
\begin{aligned}
MTTF = \ & \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} - \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} + \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} \\
& + \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} + \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{1}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{1}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}}
\end{aligned}
\tag{27}
$$

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

From equation (27), we can obtain variation in MTTF for the parameters $\eta_{1:2}$, $\eta_{1:3}$, $\eta_{2:3}$, $\eta_{2:4}$, $\eta_{3:4}$, $\eta_{3:5}$, $\eta_{4:5}$, $\eta_{4:6}$ and $\eta_{5:6}$. All values are listed in Table 5, and their corresponding graph is shown in Figure 3.

Table 5: *MTTF of the k-out-of-n network*

Failure rate	MTTF $\eta_{1:2}$	MTTF $\eta_{1:3}$	MTTF $\eta_{2:3}$	MTTF $\eta_{2:4}$	MTTF $\eta_{3:4}$	MTTF $\eta_{3:5}$	MTTF $\eta_{4:5}$	MTTF $\eta_{4:6}$	MTTF $\eta_{5:6}$
0.1	2.6349	2.6349	2.6349	2.6349	2.6349	2.6349	2.6349	2.6349	2.6349
0.2	2.1310	2.5437	2.4683	2.5238	2.25	2.5238	2.4881	2.5238	2.1310
0.3	1.7821	2.4845	2.3654	2.4527	1.9805	2.4527	2.3972	2.4527	1.7821
0.4	1.5278	2.4441	2.2980	2.4048	1.7817	2.4048	2.3373	2.4048	1.5278
0.5	1.3349	2.4153	2.2516	2.3710	1.6293	2.3710	2.2959	2.3710	1.3349
0.6	1.1840	2.3942	2.2184	2.3463	1.5087	2.3463	2.2662	2.3463	1.1840
0.7	1.0629	2.3782	2.1939	2.3278	1.4109	2.3278	2.2442	2.3278	1.0629
0.8	0.9638	2.3658	2.1754	2.3136	1.3301	2.3136	2.2275	2.3136	0.9638
0.9	0.8813	2.3561	2.1610	2.3025	1.2622	2.3025	2.2146	2.3025	0.8813
1.0	0.8115	2.3483	2.1497	2.2937	1.2044	2.2937	2.2043	2.2937	0.8115
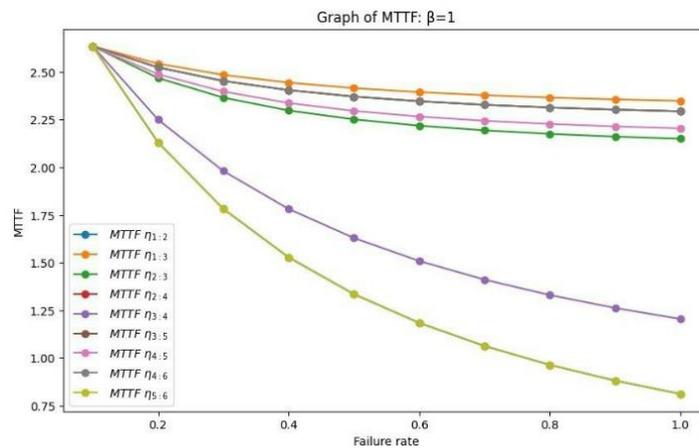
Figure 3: *MTTF of k-out-of-n network*

By varying the failure rates to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 in the partial derivatives of MTTF, we can determine the sensitivity of MTTF and Table 6 have been generated. Table on the sensitivity of MTTF shows that the system is very sensitive with respect to the failure.

Table 6: *Sensitivity of MTTF*

Failure rate	∂MTTF/ $\partial\eta_{1:2}$	∂MTTF/ $\partial\eta_{1:3}$	∂MTTF/ $\partial\eta_{2:3}$	∂MTTF/ $\partial\eta_{2:4}$	∂MTTF/ $\partial\eta_{3:4}$	∂MTTF/ $\partial\eta_{3:5}$	∂MTTF/ $\partial\eta_{4:5}$	∂MTTF/ $\partial\eta_{4:6}$	∂MTTF/ $\partial\eta_{5:6}$
0.1	-6.1416	-1.1460	-2.1485	-1.4046	-4.6677	-1.4046	-4.6677	1.3731	-6.1416
0.2	-4.1285	-0.7221	-1.2815	-0.8724	-3.1719	-0.8724	-3.1719	1.1684	-4.1285
0.3	-2.9444	-0.4823	-0.8194	-0.5757	-2.2886	-0.5757	-2.2886	0.9868	-2.9444
0.4	-2.1952	-0.3371	-0.5525	-0.3981	-1.7261	-0.3981	-1.7261	0.8364	-2.1952
0.5	-1.6941	-0.2444	-0.3885	-0.2859	-1.3469	-0.2859	-1.3469	0.7141	-1.6941
0.6	-1.3438	-0.1825	-0.2826	-0.2118	-1.0798	-0.2118	-1.0798	0.6147	-1.3438
0.7	-1.0901	-0.1397	-0.2114	-0.1609	-0.8846	-0.1609	-0.8846	0.5335	-1.0901
0.8	-0.9009	-0.1092	-0.1619	-0.1250	-0.7379	-0.1250	-0.7379	0.4668	-0.9009
0.9	-0.7563	-0.0869	-0.1265	-0.0989	-0.6248	-0.0989	-0.6248	0.4113	-0.7563
1.0	-0.6435	-0.0703	-0.1006	-0.0795	-0.5359	-0.0795	-0.5359	0.3650	-0.6435

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

Case 3: $\beta > 1 \Rightarrow h(t)$ is an increasing function of time.

If $\beta = 2$ then $P_{i:j}(t) = e^{-\eta_{i:j}t^2}$, substitute in equation (20)

$$
\begin{aligned}
R(t) = \ & e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^2} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^2} + \\
& e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6})t^2} + e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^2} + \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^2} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^2} - \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^2} + e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^2} - \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6})t^2} - e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^2} + \\
& e^{-(\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^2} - e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^2} + \\
& e^{-(\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6})t^2}
\end{aligned}
$$
(28)

We consider that the failure rates are as follows: $\eta_{1:2} = 0.1$, $\eta_{1:3} = 0.1, \eta_{2:3} = 0.2, \eta_{2:4} = 0.2, \eta_{3:4} = 0.3, \eta_{3:5} = 0.3, \eta_{4:5} = 0.4, \eta_{4:6} = 0.4, \eta_{5:6} = 0.5$. The reliability variation over time for the proposed network, derived from equation (28), is presented in Table 7.

Table 7: *Reliability of k-out-of-n network*

t	R
1	0.348257
2	0.083151
3	0.044262
4	0.027324
5	0.017865
6	0.012166
7	0.008545
8	0.006151
9	0.004517
10	0.003373

$$\text{MTTF} = \frac{1}{\eta}\Gamma\left(1+\frac{1}{2}\right) = \frac{1}{\eta}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{\eta} \tag{29}$$

$$
\begin{aligned}
\text{MTTF} = \ & \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{4:5}+\eta_{5:6}} \\
& + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} \\
& - \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}} + \frac{\sqrt{\pi}}{\eta_{1:2}+\eta_{1:3}+\eta_{2:3}+\eta_{2:4}+\eta_{3:4}+\eta_{3:5}+\eta_{4:5}+\eta_{4:6}+\eta_{5:6}}
\end{aligned}
$$
(30)

From equation (30), we can obtain variation in MTTF for the parameters $\eta_{1:2}, \eta_{1:3}, \eta_{2:3}, \eta_{2:4}, \eta_{3:4}, \eta_{3:5}, \eta_{4:5}, \eta_{4:6}$ and $\eta_{5:6}$. All values are listed in Table 8, and their corresponding graph is shown in Figure 4.

By varying the failure rates to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1 in the partial derivatives of MTTF, we can determine the sensitivity of MTTF and Table 9 have been generated. Table on the sensitivity of MTTF shows that the system is very sensitive with respect to the failure.

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

Table 8: *MTTF of k-out-of-n network*

Failure rate	MTTF$\eta_{1:2}$	MTTF$\eta_{1:3}$	MTTF$\eta_{2:3}$	MTTF$\eta_{2:4}$	MTTF$\eta_{3:4}$	MTTF$\eta_{3:5}$	MTTF$\eta_{4:5}$	MTTF$\eta_{4:6}$	MTTF$\eta_{5:6}$
0.1	4.6701	4.6701	4.6701	4.6701	4.6701	4.6701	4.6701	4.6701	4.6701
0.2	3.7769	4.5084	4.3747	4.4732	3.9879	4.4732	4.4099	4.4732	3.7769
0.3	3.1586	4.4035	4.1925	4.3472	3.5103	4.3472	4.2488	4.3472	3.1586
0.4	2.7078	4.3319	4.0729	4.2622	3.1580	4.2622	4.1426	4.2622	2.7078
0.5	2.3660	4.2809	3.9907	4.2023	2.8877	4.2023	4.0693	4.2023	2.3660
0.6	2.0985	4.2435	3.9318	4.1586	2.6739	4.1586	4.0167	4.1586	2.0985
0.7	1.8839	4.2151	3.8884	4.1259	2.5007	4.1259	3.9777	4.1259	1.8839
0.8	1.7083	4.1932	3.8556	4.1007	2.3575	4.1007	3.9481	4.1007	1.7083
0.9	1.5620	4.1759	3.8302	4.0810	2.2372	4.0810	3.9252	4.0810	1.5620
1.0	1.4383	4.1621	3.8102	4.0653	2.1346	4.0653	3.9070	4.0653	1.4383
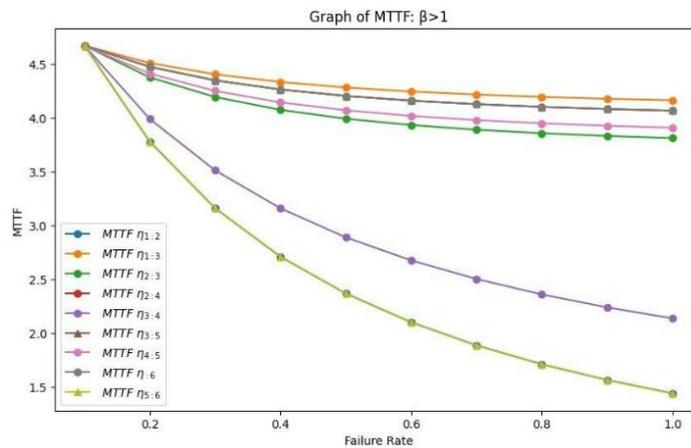
Figure 4: *MTTF of k-out-of-n network*

Table 9: *Sensitivity of MTTF*

Failure rate	∂MTTF/$\partial\eta_{1:2}$	∂MTTF/$\partial\eta_{1:3}$	∂MTTF/$\partial\eta_{2:3}$	∂MTTF/$\partial\eta_{2:4}$	∂MTTF/$\partial\eta_{3:4}$	∂MTTF/$\partial\eta_{3:5}$	∂MTTF/$\partial\eta_{4:5}$	∂MTTF/$\partial\eta_{4:6}$	∂MTTF/$\partial\eta_{5:6}$
0.1	-10.8854	-2.0312	-3.8081	-2.4896	-8.2730	-2.4896	-8.2730	2.4338	-10.8854
0.2	-7.3174	-1.2798	-2.2713	-1.5463	-5.6219	-1.5463	-5.6219	2.0708	-7.3174
0.3	-5.2187	-0.8548	-1.4523	-1.0203	-4.0562	-1.0203	-4.0562	1.7490	-5.2187
0.4	-3.8908	-0.5975	-0.9793	-0.7057	-3.0593	-0.7057	-3.0593	1.4825	-3.8908
0.5	-3.0025	-0.4331	-0.6886	-0.5068	-2.3873	-0.5068	-2.3873	1.2656	-3.0025
0.6	-2.3817	-0.3234	-0.5009	-0.3753	-1.9138	-0.3753	-1.9138	1.0895	-2.3817
0.7	-1.9321	-0.2476	-0.3747	-0.2852	-1.5679	-0.2852	-1.5679	0.9456	-1.9321
0.8	-1.5968	-0.1936	-0.2870	-0.2215	-1.3078	-0.2215	-1.3078	0.8273	-1.5968
0.9	-1.3405	-0.1540	-0.2243	-0.1752	-1.1074	-0.1752	-1.1074	0.7291	-1.3405
1.0	-1.1405	-0.1245	-0.1784	-0.1409	-0.9498	-0.1409	-0.9498	0.6469	-1.1405

Reliability comparison results from tables 1, 4 and 7 indicate that the system's reliability generally decreases as the failure rate increases is presented in Figure 5. For all time points (t), the reliability is highest when β < 1, indicating a decreasing failure rate over time. When β = 1, representing a constant failure rate, the reliability is significantly lower than when β < 1. The lowest reliability is observed when β > 1, indicating an increasing failure rate over time. Specifically, the reliability drops sharply as β transitions from 1 to values greater than 1. This trend suggests that

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

systems with a higher failure rate experience more rapid degradation and reduced reliability over time. The reliability values decrease by orders of magnitude as β increases, indicating a substantial impact on the system's ability to maintain functionality over an extended period. These results highlight the importance of managing and reducing failure rates to ensure in critical systems.
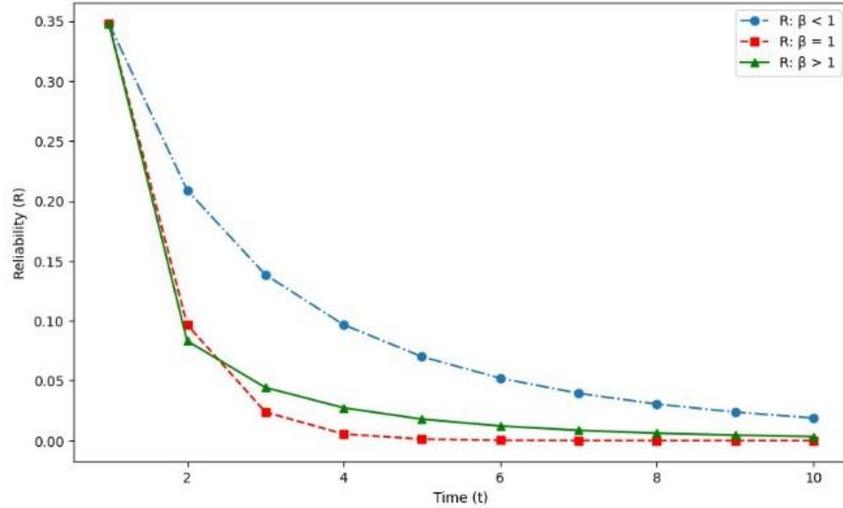

Figure 5: *Reliability comparison analysis*

IV. Birnbaum Measure

The significance of network system components importance is determined by the reliability of the network system and components can be expressed by the Birnbaum measure[1] which indicates the influence degree of network system components on network system connectivity. Due to the mathematical structure of this definition, the reliability of the network system is partially derived from the reliability of each of its components; in other words, changes in the reliability of its components will also affect the reliability of the network system. The Birnbaum measure of a network system with N components is defined as:

$$I_i^{BM}(t) = \frac{\partial R_N(t)}{\partial R_i(t)} \tag{31}$$

Here, $R_N(t)$ is the reliability function of the network and $R_i(t)$ is the working function of the network component i. The Birnbaum component importance of the components in the network is discussed below:

$$
\begin{aligned}
I_{1:2}^{BM}(t) = \ & p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{5:6} - p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + \\
& p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{2:3}\,p_{3:4} \\
& p_{3:5}\,p_{4:6}\,p_{5:6} - p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5} \\
& p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{32}
\end{aligned}
$$

$$
\begin{aligned}
I_{1:3}^{BM}(t) = \ & -p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5} \\
& p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{33}
\end{aligned}
$$

$$
\begin{aligned}
I_{2:3}^{BM}(t) = \ & p_{1:2}\,p_{3:4}\,p_{4:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6} \\
& p_{5:6} - p_{1:2}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2} \\
& p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{34}
\end{aligned}
$$

$$
\begin{aligned}
I_{2:4}^{BM}(t) = \ & -p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{1:3}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:5}\,p_{4:6} \\
& p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + \\
& p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5} \\
& p_{4:6}\,p_{5:6} \tag{35}
\end{aligned}
$$

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

$$I_{3:4}^{BM}(t) = p_{1:2}\,p_{2:3}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}$$
$$p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}$$
$$p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{36}$$

$$I_{3:5}^{BM}(t) = p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}$$
$$p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{4:6}$$
$$p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}$$
$$p_{3:4}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{37}$$

$$I_{4:5}^{BM}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}$$
$$p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}$$
$$p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} \tag{38}$$

$$I_{4:6}^{BM}(t) = p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}$$
$$p_{3:5}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}$$
$$p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}$$
$$p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{5:6} \tag{39}$$

$$I_{5:6}^{BM}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}$$
$$+ p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6} + p_{1:2}\,p_{2:3}$$
$$p_{3:4}\,p_{3:5}\,p_{4:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}$$
$$p_{3:5}\,p_{4:5}\,p_{4:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6} \tag{40}$$

We consider that the various probabilities for the various components are as follows: $p_{1:2} = 0.4$, $p_{1:3} = 0.45$, $p_{2:3} = 0.5$, $p_{2:4} = 0.55$, $p_{3:4} = 0.6$, $p_{3:5} = 0.65$, $p_{4:5} = 0.7$, $p_{4:6} = 0.75$, $p_{5:6} = 0.8$. Different values of Birnbaum component importance for different edges are obtained by entering different probabilities for various components, as shown in Table 10.

Table 10: *Birnbaum component importance measure*

	BCIM
$I_{1:2}^{BM}(t)$	0.1184
$I_{1:3}^{BM}(t)$	0.0372
$I_{2:3}^{BM}(t)$	0.0787
$I_{2:4}^{BM}(t)$	0.1406
$I_{3:4}^{BM}(t)$	0.1645
$I_{3:5}^{BM}(t)$	0.0468
$I_{4:5}^{BM}(t)$	0.0284
$I_{4:6}^{BM}(t)$	0.0405
$I_{5:6}^{BM}(t)$	0.1289

V. Critical Important Measure

The concept of critical importance in network reliability describes the significance of specific network components that significantly affect the overall performance and stability of the network. Determining the critical components is essential to maintaining network stability.

The critical importance of components can be computed based on the Birnbaum measurement.

$$\text{Since } I_i^{CIM}(t) = \frac{I_i^{BM}(t)(1-p_i(t))}{(1-R(t))} \tag{41}$$

Entering different probabilities for different components yields varied values of the critical importance of components for different edges, as Table 11 illustrates.

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

Table 11: *Critical importance measure*

CIM	
$I_{1:2}^{CIM}(t)$	0.0799
$I_{1:3}^{CIM}(t)$	0.0230
$I_{2:3}^{CIM}(t)$	0.0443
$I_{2:4}^{CIM}(t)$	0.0712
$I_{3:4}^{CIM}(t)$	0.0740
$I_{3:5}^{CIM}(t)$	0.0184
$I_{4:5}^{CIM}(t)$	0.0096
$I_{4:6}^{CIM}(t)$	0.0114
$I_{5:6}^{CIM}(t)$	0.0290

VI. Risk Growth Factor

Network system average risk growth factor has a significant impact on network system stability and reliability, according to the definition of network system stability. When $R(t) \to 1$, the network system component failure has little impact on the network system reliability, and vice versa.
The reliability stability depends on the risk growth fact of the component, so the reliability stability can be calculated:

$$RGF_{i:j}(t) = R(t) - R_{e_{i:j=0}}(t) \tag{42}$$

$$RGF_{1:2}(t) = R(t) - R_{e_{1:2=0}}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{43}$$

$$RGF_{1:3}(t) = R(t) - R_{e_{1:3=0}}(t) = -p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{44}$$

$$RGF_{2:3}(t) = R(t) - R_{e_{2:3=0}}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{45}$$

$$RGF_{2:4}(t) = R(t) - R_{e_{2:4=0}}(t) = -p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{46}$$

$$RGF_{3:4}(t) = R(t) - R_{e_{3:4=0}}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{47}$$

$$RGF_{3:5}(t) = R(t) - R_{e_{3:5=0}}(t) = p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{48}$$

$$RGF_{4:5}(t) = R(t) - R_{e_{4:5=0}}(t) = p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{4:5}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{4:5}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}p_{4:5}p_{5:6} - p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{2:3}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{1:3}\,p_{2:3}\,p_{2:4}\,p_{3:4}\,p_{3:5}\,p_{4:5}\,p_{4:6}\,p_{5:6} \tag{49}$$

$$RGF_{4:6}(t) = R(t) - R_{e_{4:6=0}}(t) = p_{1:2}\,p_{1:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} + p_{1:2}\,p_{2:3}\,p_{2:4}\,p_{3:5}\,p_{4:6}\,p_{5:6} - p_{1:2}\,p_{1:3}\,p_{2:3}$$

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

$$p_{2:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3} \ p_{2:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{2:3} \ p_{3:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3}$$
$$p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{2:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6}$$
$$p_{5:6} - p_{1:2} \ p_{1:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{1:3} \ p_{2:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} \quad (50)$$

$$RGF_{5:6}(t) = R(t) - R_{e_{5:6}=0}(t) = \ p_{1:2} \ p_{2:3} \ p_{3:4} \ p_{4:5} \ p_{5:6} - p_{1:2} \ p_{1:3} \ p_{2:3} \ p_{2:4} \ p_{4:5} \ p_{5:6} + p_{1:2} \ p_{1:3} \ p_{2:4}$$
$$p_{3:4} p_{4:5} p_{5:6} + p_{1:2} \ p_{1:3} \ p_{2:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{2:3} \ p_{2:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{1:3} \ p_{2:3} \ p_{2:4}$$
$$p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3} \ p_{2:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{2:3} \ p_{3:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3} \ p_{2:4}$$
$$p_{3:4} \ p_{3:5} \ p_{4:6} \ p_{5:6} - p_{1:2} \ p_{2:3} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{2:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} -$$
$$p_{1:2} \ p_{1:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} + p_{1:2} \ p_{1:3} \ p_{2:3} \ p_{2:4} \ p_{3:4} \ p_{3:5} \ p_{4:5} \ p_{4:6} \ p_{5:6} \quad (51)$$

Table 12: *Risk growth factor*

RGF	
$RGF_{1:2}(t)$	0.1031
$RGF_{1:3}(t)$	0.0635
$RGF_{2:3}(t)$	0.0475
$RGF_{2:4}(t)$	0.2189
$RGF_{3:4}(t)$	0.0987
$RGF_{3:5}(t)$	0.0304
$RGF_{4:5}(t)$	0.0199
$RGF_{4:6}(t)$	0.0304
$RGF_{5:6}(t)$	0.1031

Average risk growth factor: $ARGF(t) = \frac{\sum_{i=1}^{d} RGF_i(t)}{n} = 0.0795$ (52)

Network reliability stability: $NR_S(t) = \frac{R(t) - ARGF(t)}{R(t)} = \frac{0.1112 - 0.0795}{0.1112} = 0.2851$ (53)

VII. Conclusion

The network, structured as a directed acyclic graph, was evaluated based on performance metrics including MTTF, sensitivity analysis, and reliability stability indices. Our findings indicate a noticeable decline in system reliability over time, as illustrated in Tables 1, 4, and 7. The relationship between increasing failure rates and the decreasing MTTF, depicted in Figures 2, 3, and 4, emphasizes the significant influence that variations in failure rates exert on system lifespan. Node 1 and its connections are identified as the most critical, while node 3 also plays a vital role in sustaining reliability. A reliability stability index of 0.2851 highlights moderate operational deterioration, necessitating proactive measures to address vulnerabilities. By applying MTTF sensitivity, organizations can prioritize maintenance on critical components, particularly nodes 1, 2 and their edges, to enhance system stability and resilience.

Declarations

Hemalatha.G, Vijayalakshmi.G
ENHANCING SENSITIVITY OF MTTF AND NETWORK RELIABILITY
STABILITY OF K-OUT-OF-N ACYCLIC RANSMISSION NETWORK

RT&A, No 2 (84)
Volume 20, June 2025

References

[1] Birnbaum, Z. W. (1968). On the importance of different components in a multicomponent system, 591–592.

[2] Colbourn, C. J. (1987). *The combinatorics of network reliability*. Oxford University Press, Inc.

[3] Gao, R., & Yao, K. (2016). Importance index of components in uncertain random systems. *Knowledge-Based Systems, 109*, 208–217.

[4] He, L., Cao, Q., & Shang, F. (2019). Measuring component importance for network system using cellular automata. *Complexity*, 397–597.

[5] Levitin, G., & Lisnianski, A. (1999). Importance and sensitivity analysis of multi-state systems using the universal generating function method. *Reliability Engineering & System Safety, 65*(3), 271–282.

[6] Levitin, G. (2001). Reliability evaluation for acyclic consecutively connected networks with multistate elements. *Reliability Engineering & System Safety, 73*(2), 137–143.

[7] Lisnianski, A., & Levitin, G. (2003). *Multi-state system reliability: Assessment, optimization and applications*. World Scientific.

[8] Levitin, G. (2005). *The universal generating function in reliability analysis and optimization* (Vol. 6). London: Springer.

[9] Negi, S., & Singh, S. B. (2015). Reliability analysis of non-repairable complex system with weighted subsystems connected in series. *Applied Mathematics and Computation, 262*, 79–89.

[10] Ram, M., & Singh, S. B. (2009). Analysis of reliability characteristics of a complex engineering system under copula. *Journal of Reliability and Statistical Studies*, 91–102.

[11] Ram, M. (2013). On system reliability approaches: A brief survey. *International Journal of System Assurance Engineering and Management, 4*, 101–117.

[12] Ram, M., & Manglik, M. (2016). Reliability measures analysis of an industrial system under standby modes and catastrophic failure. *International Journal of Operations Research and Information Systems (IJORIS, 7*(3), 36–56.

[13] Ram, M., & Manglik, M. (2016). Stochastic biometric system modelling with rework strategy. *International Journal of Mathematical, Engineering and Management Sciences, 1*(1), 1–7.

[14] Trivedi, S., & Andrea Bobbio. (2017). *Reliability and availability engineering: Modeling, analysis, and applications*. Cambridge University Press.

[15] Ushakov, I. A. (1986). A universal generating function. *Soviet Journal of Computer and Systems Sciences, 24*(5), 118–129.

[16] Yeh, W. C. (2003). An evaluation of the multi-state node networks reliability using the traditional binary-state networks reliability algorithm. *Reliability Engineering & System Safety, 81*(1), 1-7.

[17] Yeh, W. C. (2008). A simple universal generating function method for estimating the reliability of general multi-state node networks. *Iie Transactions, 41*(1), 3-11.

[18] Yeh, W. C. (2009). A simple universal generating function method to search for all minimal paths in networks. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 39*(6), 1247-1254.

[19] Yeh, W. C. (2012). A modified universal generating function algorithm for the acyclic binary-state network reliability. *IEEE Transactions on Reliability, 61*(3), 702-709.

[20] Zarghami, S. A., Gunawan, I., & Schultmann, F. (2020). Exact reliability evaluation of infrastructure networks using graph theory. *Quality and Reliability Engineering International, 36*(2), 498–510.

[21] Zhu, X., Fu, Y., Yuan, T., & Wu, X. (2017). Birnbaum importance-based heuristics for multi-type component assignment problems. *Reliability Engineering & System Safety, 165*, 209–221.