REPETITIVE SAMPLING INSPECTION PLAN UNDER TRUNCATED LIFETEST BASED ON ONE PARAMETER POLYNOMIAL EXPONENTIAL DISTRIBUTION

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Abstract

This article constructs a Repetitive Sampling Inspection Plan under Truncated life test (RSIPTL) when the lifetime follows the One Parameter Polynomial Exponential (OPPE) family of distributions. In RSIPTL, a lot can be accepted or rejected in the first, second, and so on, based on the number of defective items in each sample. The OPPE has infinite support. It has transformed into its unit form to utilize finite support, i.e., having the support (0, 1). The Lindley distribution, a particular choice of the OPPE, has been studied in detail. We obtained the minimum number of items required in a lot to satisfy the consumer risk. Extensive tables are prepared for easy understanding and use of the plan for industrial workers. The RSIPTL is compared with a single sampling plan (SSP) and a two-stage reliability acceptance sampling plan (TSRASP) for Lindley and Exponential distributions. Two data sets are discussed and comparative statements are made with respect to the proposed plan.

Keywords: Consumer's risk, Operating characteristic function, Scale-invariant family of distributions, Truncated life test, unit-Lindley distribution.

1. Introduction

When a product is prepared to be sold to a consumer, it is crucial to verify its quality. Because reputation on the global market largely depends on product quality, it is imperative to confirm the quality of each product. This means that the manufacturers should focus more on each product's quality. It is possible to use an acceptance sampling plan to guarantee the items' quality. Several types of acceptance sampling are there, like attribute sampling plans, variable sampling plans, sequential sampling plans, etc. In these acceptance sampling plans, we aim to reduce sample size and make decisions about the lot to protect both the producer and consumer interests. We also know that any experiment is not free from error; here, we face two types of errors: 1st type of error known as producer error(α), where a good lot can reject; 2nd type of error known as consumer error (β), where a lousy lot can be accepted. So, in the sampling plan, we have to minimize these risks.

In real-life experiments like in medical or life testing experiments, we can see two types of censoring. In the first case, we fixed the time duration of the test, and in the second case, the number of failures was fixed. To reduce the cost of the experiment and testing time, we performed the 1st type of censoring, i.e., fixed the test duration. In the life testing process, researchers truncated the testing procedure at a specific time to minimize the cost and time required for the experiment. The product's lifetime is the quality of interest in reliability acceptance sampling strategies. All consumers anticipate that their products will last for an extended period. In other words, a product should be expected to last a long time. Furthermore, the buyer will accept the goods with trust if it is demonstrated that their real life exceeds its stated lifetime. Because of the expense and time involved, waiting until every sampled item fails in a life test is not ideal. A time-truncated life test, which ends at a particular time, can be helpful in this situation to guarantee a lifetime with the least amount of money and effort.

Many authors, including [10], [29], [12], [14], [13], [17], [24], [21], [1], [8], [2], [28], [4] proposed acceptance sampling for truncated life tests. Few studies have been done on acceptance sampling inspection plans assuming the Lindley distribution [see [22], [25]. [3] and [30] devised an acceptance sampling technique based on the assumptions of a two-parameter Lindley distribution and a truncated life test. [27] first designed the attribute repetitive sampling plan for normal distribution. According to him, this sampling scheme gives producers and consumers the minimum sample size and the required protection. He also said that a repetitive sampling plan is more effective than a single one and less effective than a sequential one. [6] and[7] designed the repetitive group sampling (RGS) plan for variable inspection. Some other authors have also studied the design of RGS plans under various situations, including reliability concepts like [11], [15], [16]. In this paper, we construct a repetitive sampling plan for One Parameter Polynomial Exponential (OPPE) distribution.

Most works on repetitive sampling inspection plans for truncated life (RSIPTL) tests are done for scale-invariant distributions. Minimum sample size (n) are determined for different times per unit mean $(\frac{t}{\mu_0})$. Our study aims to determine the RSIPTL for the OPPE distribution and compare it to that for the exponentially distributed quality characteristics. The OPPE distribution does not belong to a scale-invariant family of distributions. Therefore, the utilization of time per mean is beyond our scope. We may directly chalk the plan with plan parameters $(n,\ c_1,\ c_2,\ t)$. Since the OPPE distribution has support (0,∞), we may utilize it with the finite support by transforming into its unit form, i.e., having the support (0,1) with the transformation $V = e^{-T}$. Using the unit-Lindley form, we make tables by choosing V and the mean μ_v in the interval (0,1). Utilizing this benefit, we choose optimal $(n,\ c_1,\ c_2)$ and then revert to plan parameter t from the relation of the transformation. So, in a nutshell, our objective is to develop an RSIPTL for the OPPE distributed quality characteristic. Based on the time-truncated life test, the plan saves the organization's time and cost while being very helpful in determining whether to accept or reject a lot. The OC is derived for choosing the optimal plan based on the consumer's confidence level. Tables of minimum sample sizes are examples for easy understanding and execution of the proposed plan. It is implemented for real-life experimental data, and the OC surface is depicted to provide a clear picture of the plan.

The following is the arrangement of the rest of the paper. The OPPE and unit-OPPE distributions are described in section 2. Section 3 describes the sampling design, operating characteristics function, and operating procedure. The sampling results for the Lindley and exponential distributions, in particular, are presented in tabular form. Section 4 compares the sample size for the repetitive, two-stage, and single-sampling plans. In section 5, we use the said sampling plan to work on real-world data. Section 6 concludes.

2. The One Parameter Polynomial Exponential Distribution And Its Unit Version

The probability density function (PDF) of a random variable T of the OPPE distribution can be written as

$$f_T(t;\theta) = h(\theta)p(t)e^{-\theta t}t > 0, \theta > 0, \tag{1}$$

where, $h(\theta) = \frac{1}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}$, $p(t) = \sum_{k=0}^{r} a_k t^k$, $a_k's$ are known non-negative constants and r is known non-negative integer. The distribution is also known as

$$\begin{aligned} f_T(t,\theta) &= h(\theta)\sum_{k=0}^{r} a_k t^k e^{-\theta t} \\ &= \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}} f_{GA}(t;k+1,\theta)}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}, \end{aligned} \quad (2)$$

where $f_{GA}(t;k+1,\theta)$ is the PDF of a gamma distribution with shape parameter (k+1) and scale parameter θ. The distribution is a finite mixture of (r+1) gamma distributions. The cumulative density function (CDF) is given by

$$F_T(t,\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1,\theta t)}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right),\ t,\ \theta > 0, \quad (3)$$

where $\Gamma(m,t) = \frac{1}{\Gamma(m)} \int_t^{\infty} e^{-u} u^{m-1} du$.

The s-th order raw moment of OPPE is given by

$$\mu_s' = E(T^s) = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+s+1)}{\theta^{k+s+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}. \quad (4)$$

Now, if we take a transformation V= e^{-T}, then the OPPE turns into unit-OPPE in the range of (0,1). The PDF and CDF of unit-OPPE are given by,

$$\begin{aligned} f_V(v,\theta) &= h(\theta)\sum_{k=0}^{r} a_k (-\ln v)^k v^{\theta-1} \\ &= \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}} f_{UGA}(v;k+1,\theta)}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}},\ 0 < v < 1, \end{aligned} \quad (5)$$

where $f_{UGA}(v;k+1,\theta) = \frac{\theta^{k+1}}{\Gamma(k+1)}(-\ln v)^{k+1} v^{\theta-1}$ is the PDF of the unit-gamma distribution with shape parameter (k+1) and scale parameter θ, and

$$F_v(v,\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1,-\theta ln v)}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right), 0 < v < 1, \theta > 0, \quad (6)$$

respectively.

The s-th order raw moment of unit-OPPE is given by

$$\mu_s' = E(V^s) = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{(s+\theta)^{k+1}}}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}. \quad (7)$$

The Lindley distribution (for r=1,$a_0 = a_1 = 1$), introduced by [9] to analyze failure time data, has the PDF, CDF, and hazard rate function (HRF) as

$$f_T(t;\theta) = \frac{\theta^2}{\theta+1}(1+t)e^{-\theta t} \qquad t > 0, \theta > 0, \quad (8)$$

$$F_T(t;\theta) = 1 - \frac{1+\theta+\theta t}{\theta+1} e^{-\theta t} \quad (9)$$

and

$$h_T(t;\theta) = \frac{\theta^2(1+t)}{1+\theta+\theta t} \quad t > 0, \theta > 0, \tag{10}$$

respectively.

The mean of the random variable T is

$$\mu = \frac{\theta+2}{\theta(1+\theta)}. \tag{11}$$

The unit-Lindley distribution with parameter θ has the PDF , CDF and HRF respectively , as follows:

$$f(v;\theta) = \frac{\theta^2}{1+\theta}(1-logv)\left(v^{\theta-1}\right) \quad 0 < v < 1, \theta > 0, \tag{12}$$

$$F(v;\theta) = \frac{v^\theta(1+\theta(1-logv))}{1+\theta} \quad 0 < v \leq 1, \theta > 0, \tag{13}$$

$$h(v;\theta) = \frac{\theta^2(1-logv)}{v(\theta logv - (1+\theta)(1-v^{-\theta}))} \quad 0 < v < 1, \theta > 0. \tag{14}$$

We will choose the exponential distribution and its corresponding unit version for comparison purposes. The unit version of the exponential distribution with parameter θ has the PDF, CDF, and HRF as

$$f_v(v,\theta) = \theta v^{\theta-1}, \quad 0 < v < 1, \theta > 0, \tag{15}$$

$$F_v(v,\theta) = v^\theta, \quad 0 < v \leq 1, \theta > 0, \tag{16}$$

$$h_v(v,\theta) = \frac{\theta v^{\theta-1}}{1-v^\theta}, \quad 0 < v < 1, \theta > 0, \tag{17}$$

respectively. In this case , $\mu_v = \frac{\theta}{1+\theta}$ implying $\theta = \frac{\mu_v}{1-\mu_v}$.

3. Sampling Process of Repetitive Sampling Inspection Plan under Truncated life test (RSIPTL) for OPPE distribution

According to the product's mean life, this sampling plan labels a lot (of products) as good or bad. The RSIPTL has the plan parameters n, c_1, c_2, and t. Engineers and practitioners use the tabulated value or algorithm to implement the plan properly. Tables are presented for fixed t and c_1, c_2, the optimal value of n. Since the value of $t \in (0, \infty)$, fixing t is tedious, whereas choosing $v \in (0, 1)$ is easy and comprehensive.

3.1. Design of the sampling plan

A product is defective if it fails before truncation time v. The fraction defective, i.e., the probability that a product is defective, is

$$p(v) = F(v;\theta) = 1 - \left(\frac{\sum_{k=0}^{r} \frac{a_k \Gamma(k+1)\Gamma(k+1,v^\theta)}{\theta^{k+1}}}{\sum_{k=0}^{r} a_k \frac{k!}{\theta^{k+1}}} \right), \; 0 < v < 1, \; \theta > 0. \tag{18}$$

In particular, for the unit-Lindley distribution,

$$p(v) = F(v;\theta) = \frac{v^\theta(1+\theta(1-logv))}{1+\theta} \quad 0 < v < 1, \theta > 0. \tag{19}$$

In this equation, we replace the shape parameter θ as the function of the product's mean life (μ). We can say that θ = g(μ), and we get the value of θ by solving the equation by numerical method, and hence we have $p(v) = F(v, \mu_v)$.

The Operating Characteristic (OC) function, i.e., the probability of accepting the lot according to the RSIPTL plan (also see [27]), is

$$L(p) = \frac{P_a}{P_r + P_a} \tag{20}$$

and the average sample number (ASN) of RSP is obtained as

$$ASN = \frac{n}{P_r + P_a} \tag{21}$$

where the lot acceptance probability is

$$P_a = \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i}, \tag{22}$$

The lot rejection probability is

$$P_r = 1 - \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i}, \tag{23}$$

where p=$F(v, \mu_v)$.

3.2. Sampling Procedure

We propose the sampling procedure as follows.

1. Put n items in the test for t_0 times.
2. Count the number of defectives (d) and, if the number of defectives is less than or equal to the acceptance number, c_1, then accept the lot. If the number of defectives exceeds the acceptance number, c_2, then reject the lot.
3. If $c_1 < d \leq c_2$, then repeat the test.

The triplet (n, c_1, c_2) is the plan parameters of the proposed plan, which are to be estimated. The minimum sample size n is estimated subject to constraints

$$L(p) \leq 1 - P^*, \tag{24}$$

$$0 \leq c_1 < c_2, \tag{25}$$

$$n > 1, c_1, c_2 \text{ are integers.} \tag{26}$$

3.3. Sampling plan result

The minimum values of n satisfying the inequality are obtained and shown in Tables 1-4 for the unit-Lindley (Lindley) and that for the unit-exponential (exponential) in Tables 5-6 for P^*= 0.95 and 0.99, and the truncation time $t = 2, 3, 4, 5$ and $\mu_0 = 2, 3, 4, 5$.

Table 1: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.95, c_1 = 1, c_2 = 2$			
$\mu_v^0(\mu_0)$ | v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3002(2)	4	6	7	9
0.2065(3)	3	4	5	6
0.1515(4)	3	3	4	4
0.1162(5)	-	3	3	4
	$P^* = 0.95, c_1 = 1, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	5	6	7	10
0.2065(3)	-	4	5	6
0.1515(4)	-	-	4	5
0.1162(5)	-	-	-	4
	$P^* = 0.95, c_1 = 1, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	5	6	8	10
0.2065(3)	-	5	6	7
0.1515(4)	-	-	-	5
0.1162(5)	-	-	-	-

Table 2: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.95, c_1 = 2, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3002(2)	6	8	9	11
0.2065(3)	5	6	7	8
0.1515(4)	-	5	5	7
0.1162(5)	-	-	5	7
	$P^* = 0.95, c_1 = 2, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	6	8	9	12
0.2065(3)	-	6	7	8
0.1515(4)	-	-	6	6
0.1162(5)	-	-	-	-
	$P^* = 0.95, c_1 = 3, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	7	9	11	14
0.2065(3)	7	7	9	10
0.1515(4)	-	-	7	8
0.1162(5)	-	-	-	7

Table 3: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.99, c_1 = 1, c_2 = 2$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	5	7	9	11
0.2065(3)	4	5	6	7
0.1515(4)	4	4	5	6
0.1162(5)	3	4	4	5
	$P^* = 0.99, c_1 = 1, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	5	7	9	11
0.2065(3)	4	5	6	7
0.1515(4)	4	4	5	6
0.1162(5)	-	4	4	5
	$P^* = 0.99, c_1 = 1, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	6	7	9	11
0.2065(3)	5	6	7	8
0.1515(4)	-	5	5	6
0.1162(5)	-	-	5	5

Table 4: *Determination of optimal sample size for Lindley set up*

	$P^* = 0.99, c_1 = 2, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	7	9	10	16
0.2065(3)	6	7	8	9
0.1515(4)	5	6	6	7
0.1162(5)	5	5	6	8
	$P^* = 0.99, c_1 = 2, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	7	9	12	17
0.2065(3)	6	7	8	10
0.1515(4)	-	6	7	7
0.1162(5)	-	-	6	6
	$P^* = 0.99, c_1 = 3, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3001(2)	9	11	14	18
0.2065(3)	7	9	10	12
0.1515(4)	7	7	8	9
0.1162(5)	-	-	7	8

Table 5: *Determination of optimal sample size for Exponential set up*

	$P^* = 0.95, c_1 = 1, c_2 = 2$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	12	20	34	58
0.25(3)	8	12	17	24
0.2(4)	6	9	12	15
0.1667(5)	6	7	9	12
	$P^* = 0.95, c_1 = 1, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	12	21	36	60
0.25(3)	8	12	18	25
0.2(4)	7	9	12	16
0.1667(5)	6	8	10	12
	$P^* = 0.95, c_1 = 1, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	13	22	38	74
0.25(3)	8	13	19	27
0.2(4)	7	10	13	17
0.1667(5)	6	8	10	13

Table 6: *Determination of optimal sample size for Exponential set up*

	$P^* = 0.95, c_1 = 2, c_2 = 3$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	16	27	46	76
0.25(3)	11	16	23	32
0.2(4)	9	12	16	21
0.1667(5)	7	10	12	16
	$P^* = 0.95, c_1 = 2, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	16	28	47	78
0.25(3)	11	16	23	33
0.2(4)	9	12	16	21
0.1667(5)	8	10	13	16
	$P^* = 0.95, c_1 = 3, c_2 = 4$			
$\mu_v^0(\mu_0)$v(t)	0.1353(2)	0.0498(3)	0.0183(4)	0.0067(5)
	n	n	n	n
0.3334(2)	19	33	56	94
0.25(3)	13	19	28	40
0.2(4)	11	14	19	25
0.1667(5)	9	12	15	19

Few observations:

1. If c_1 increases, the sample size increases. For example, Tables 1 and 2.
2. If c_2 increases, the sample size decreases. For example, Tables 1 and 2.

3. If the confidence level increases, the sample size increases. For example, Tables 1 and 3.
4. The sample required is less in this plan for the Lindley distributed quality characteristic than the exponential one. For example, Tables 2 and 6.

4. Comparison of sample sizes among different sampling plans

We compared this RSIPTL with a single sampling plan (SSP) and two-stage reliability acceptance sampling plan (TSRASP) at a fixed time point 5, 10, and 15 and mean lives at 5, 10, and 15, respectively, for Lindley and Exponential distributions. The optimal sample size comparison for confidence level $P^* = 0.95$ is shown in Table 7. It is observed that RSIPTL has a smaller sample size than others if it exists for a specified confidence level. Moreover, the sample sizes for the Lindley distribution are less than that of the exponential distribution.

Table 7: *Optimal Sample Size Comparison for Different Sampling Plans*

		$P^* = 0.95$							
		RSIPTL			TSRASP			SSP	
Distribution	μ	$6.74 \times 10^{-3}(5)$			$6.74 \times 10^{-3}(5)$			$6.74 \times 10^{-3}(5)$	
		n	c_1	c_2	n	c_1	c_2	n	c
Lindley	0.1162(5)	3	0	2	4	0	2	5	2
	0.0447(10)	-	0	2	-	0	2	4	2
	0.0236(15)	-	0	2	-	0	2	3	2
Exponential	0.1667(5)	8	0	2	9	0	2	15	2
	0.0909(10)	4	0	2	8	0	2	8	2
	0.0625(15)	4	0	2	6	0	2	7	2
Distribution	μ	$4.54 \times 10^{-5}(10)$			$4.54 \times 10^{-5}(10)$			$4.54 \times 10^{-5}(10)$	
		n	c_1	c_2	n	c_1	c_2	n	c
Lindley	0.1162(5)	4	0	2	6	0	2	8	2
	0.0447(10)	-	0	2	-	0	2	4	2
	0.0236(15)	-	0	2	-	0	2	4	2
Exponential	0.1667(5)	24	0	2	52	0	2	45	2
	0.0909(10)	8	0	2	14	0	2	15	2
	0.0625(15)	5	0	2	10	0	2	10	2
Distribution	μ	$3.059 \times 10^{-7}(15)$			$3.059 \times 10^{-7}(15)$			$3.059 \times 10^{-7}(15)$	
		n	c_1	c_2	n	c_1	c_2	n	c
Lindley	0.1162(5)	6	0	2	8	0	2	12	2
	0.0447(10)	-	0	2	-	0	2	5	2
	0.0625(10)	-	0	2	-	0	2	4	2
Exponential	0.1667(5)	67	0	2	150	0	2	197	2
	0.0909(10)	14	0	2	26	0	2	34	2
	0.0625(15)	8	0	2	14	0	2	8	2

5. Real-life data analysis

This section considers two cited data sets for applying the proposed RSIPTL under the exponential and Lindley setup. Since these are cited data sets, the sample sizes are known. Therefore, we have found out optimal $(c_1,\ c_2)$ for given n and P^* and decision tables are shown different choices of $v(t)$ and $\mu_v^0(\mu_0)$.

Using the maximum likelihood technique, we estimate the parameters of the distributions. The trial-and-error method determines the non-negative constants a_i of the OPPE. The selection criteria for each data set is Akaike's Information Criterion ($AIC = -2\log -likelihood + 2k$, where k is the number of parameters in the model). The lower the value of AIC, the better the model fit.

The goodness-of-fit of the distribution to a data set is made through Kolmogorov-Smirnov (K-S) test.

Data set 1: Let us consider data on the mileages at which 19 military personnel carriers failed in service. There is no censoring, and the mileages are ([19], page 194) 162, 200, 271, 320, 393, 508, 539, 629, 706, 777, 884, 1008, 1101, 1182, 1463, 1603, 1984, 2355, 2880. Table 8 shows that the Lindley distribution better fits the data based on the AIC value and the p-value. It is reflected in the Histogram, fitted distributions, and the P-P plot in Figure 1.

Table 8: *Comparison of Exponential and Lindley Distribution for Data Set 1.*

Distribution	Estimate of θ	Negative Log-likelihood	AIC	K-S Statistic	p-value
Exponential	0.001000977	150.2123	302.4247	0.14983	0.7328
Lindley	0.0020019	148.4087	298.8174	0.075969	0.9995

The failure times of the military carriers are 400,800,1600 and 2400. To make the decision table, we select the specified mean life as 400, 800, 1600, and 2400. The average failure time of the military carriers is 998.1579. To find the optimal c_1 and c_2 (the acceptance and rejection number of items for a lot), Table 9 and 10 are constructed for the Lindley and exponential distributions.

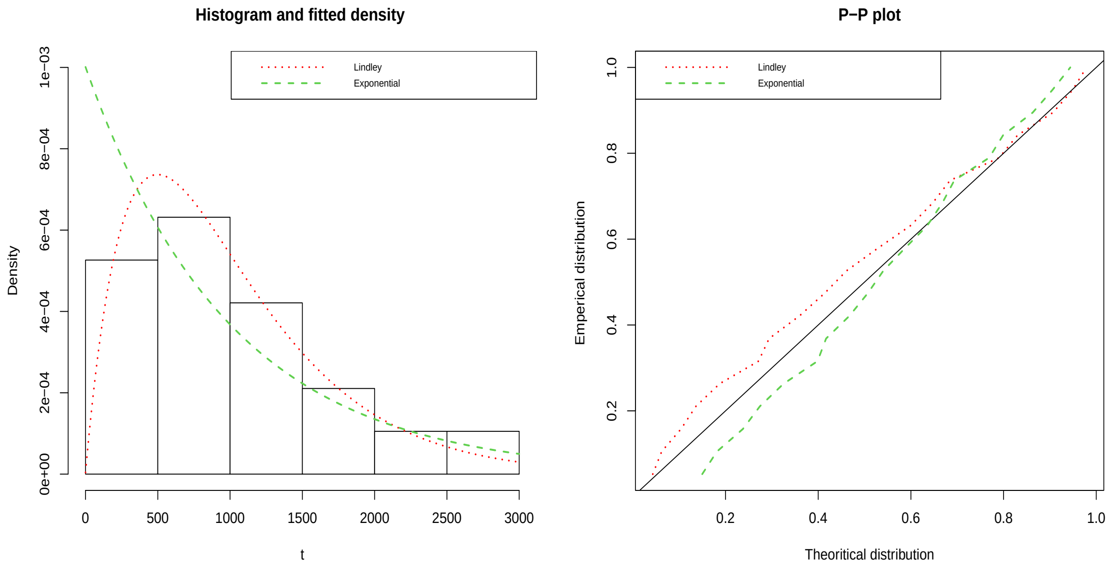

Figure 1: *Histogram, fitted distributions, and P-P plot for Data Set 1.*

If $d \leq c_1$, we accept the lot; if $d > c_2$, we reject the lot, and if d lies between c_1 and c_2, we repeat the plan until we get any result. In table 9, we choose a failure time point 1600 and a specified mean life of 800. Since the number of defective, $d = 10$ and c_1=10, we accept the lot. The OC surface of RSIPTL under Lindley set up has been shown in Figure 2 for n=19, $c_1 = 12$, $c_2 = 14$.

Table 9: *Determination of optimal $(c_1,\ c_2)$ for Lindley set up when n=19*

	$P^* = 0.95$												
$\mu_v^0(\mu_0)$v(t)		$1.8312 \times 10^{-2}(400)$			$3.3546 \times 10^{-4}(800)$			$1.1253 \times 10^{-6}(1600)$			$3.7751 \times 10^{-11}(2400)$		
	d	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D
0.15155(400)	5	12	14	A	7	10	R	2	4	R	2	3	R
0.06207(800)	10	16	17	A	13	14	A	10	12	A	6	8	R
0.02122(1600)	14	17	18	A	16	17	A	16	17	A	14	15	Rep
0.01062(2400)	17	-	-	No	17	18	Rep	17	18	Rep	16	17	Rep

[a]

Table 10: *Determination of optimal $(c_1,\ c_2)$ for Exponential distribution set up when n=19*

	$P^* = 0.95$												
$\mu_v^0(\mu_0)$v(t)		$1.8312 \times 10^{-2}(400)$			$3.3546 \times 10^{-4}(800)$			$1.1253 \times 10^{-6}(1600)$			$3.7751 \times 10^{-11}(2400)$		
	d	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D
0.2(400)	5	-	-	No	-	-	No	-	-	No	-	-	No
0.1111(800)	10	7	8	R	-	-	No	-	-	No	-	-	No
0.05882(1600)	14	11	12	R	7	9	R	4	5	R	-	-	No
0.040(2400)	17	12	14	R	9	12	R	6	8	R	2	6	R

[b]

[a] R=Reject, A=Accept, Rep= Repetition, No= No Decision
[b] R=Reject, A=Accept, Rep= Repetition, No= No decision

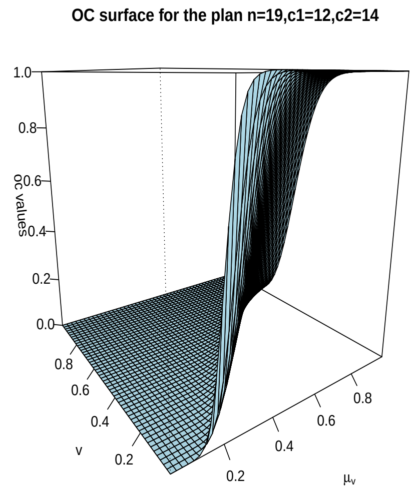

Figure 2: *OC surface of RSIPTL under Lindley set up for n=19,* $c_1 = 12$, $c_2 = 14$

Data set 2: Sixteen units are drawn randomly from a population and put on a time-truncated life test. After testing for 92 hours, which is prespecified, the test is terminated, by which time 15 fail. The failure times, in hours, are 13.4, 14.2, 28.8, 29, 29.8, 33, 37.8, 39.6, 43.4, 49.8, 54.8, 58.2, 70.2, 91.2, from [18]. Table 11 indicates the OPPE ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) distribution better fits the data. Histogram, fitted distributions, and P-P plot in Figure 3 are in the same tune.

Table 11: *Comparison of Exponential and OPPE* ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) *Distribution for Data Set 2.*

Distribution	Estimate of θ	Negative Log-likelihood	AIC	K-S Statistic	p-value
Exponential	0.02359375	66.45064	134.9013	0.35038	0.04846
OPPE(9,4,0.1)	0.05615	62.82411	127.64822	0.24319	0.3246

To make the decision table, we select the specified mean life as 20,40,60,80. The average failure time is 42.37143. To find the optimal c_1 and c_2 , table 12 and 13 are constructed for the OPPE ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) and exponential distributions. In table 13, if we choose a failure time point 20 and specified mean life 40, we can say that the lot is accepted because d (=8) is less than $c_1 (= 10)$. The OC surface of RSIPTL under OPPE ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) set up has been shown in Figure 4 for n=15, $c_1 = 7$, $c_2 = 9$.

For all these data sets, the RSIPTL under the exponential set-up, the optimal sample sizes are enormous and are not recommended from an economic point of view.

Table 12: *Determination of optimal (c_1, c_2) for Exponential set up when n=15*

	$P^* = 0.95$												
$\mu_v^0(\mu_0)$v(t)		0.1353(20)			0.0183(40)			0.0025(60)			0.0003(80)		
	d	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D
0.3333(20)	2	-	-	No	-	-	No	-	-	No	-	-	No
0.2(40)	8	5	6	R	-	-	No	-	-	No	-	-	No
0.1428(60)	12	7	8	R	3	5	R	1	5	R	-	-	No
0.1111(80)	14	8	10	R	5	6	R	3	5	R	-	-	No

[a]

Table 13: *Determination of optimal (c_1, c_2) for OPPE ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) set up when n=15*

	$P^* = 0.95$												
$\mu_v^0(\mu_0)$v(t)		0.1353(20)			0.0183(40)			0.0025(60)			0.0003(80)		
	d	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D	c_1	c_2	D
0.3164(20)	2	7	9	A	4	5	A	1	5	A	-	-	No
0.1764(40)	8	10	11	A	8	9	A	6	7	R	4	7	R
0.1089(60)	12	12	13	A	10	11	R	9	10	R	8	9	R
0.0760(80)	14	13	14	Rep	12	13	R	11	12	R	10	11	R

[b]

[a]R=Reject, A= Accept , Rep= Repetition, No= No decision
[b]R=Reject, A= Accept , Rep= Repetition, No= No decision

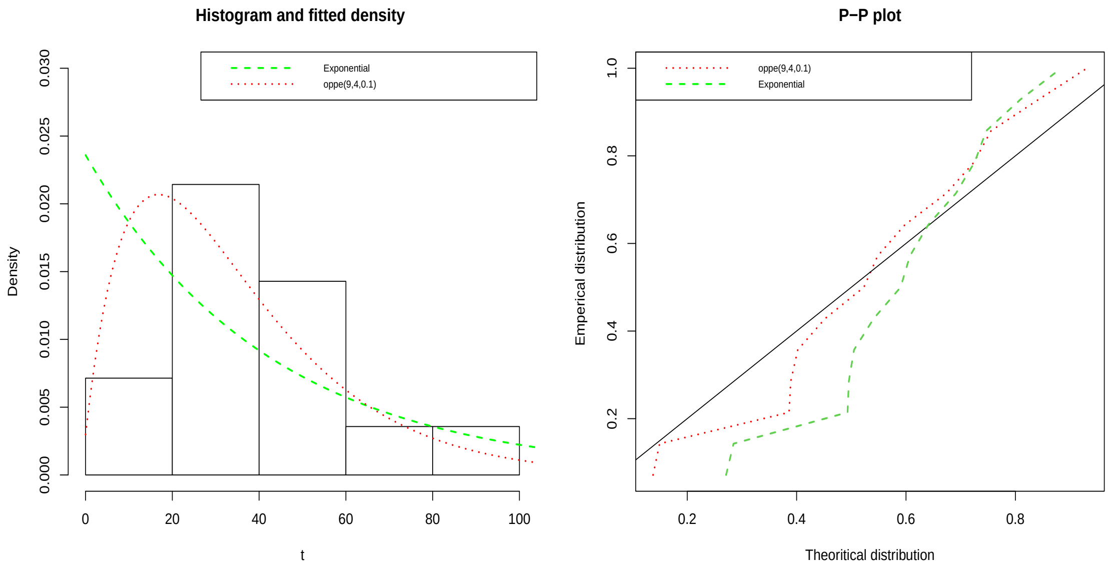

Figure 3: *Histogram, fitted distribution, and P-P plot for data set 2*

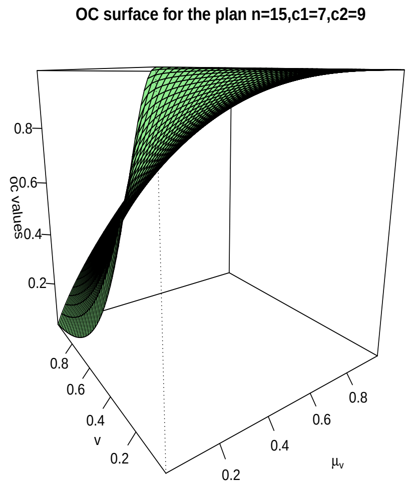

Figure 4: *OC surface of RSIPTL under OPPE ($a_0 = 9,\ a_1 = 4,\ a_2 = 0.1$) set up for n=15, $c_1 = 7$, $c_2 = 9$*

6. Concluding remarks

This article proposes a Repetitive Sampling Inspection Plan under Truncated life test (RSIPTL) when the lifetime follows the One Parameter Polynomial Exponential (OPPE) family of distributions. It is a more general setup than a two-stage reliability acceptance sampling plan, where a lot can be accepted or rejected based only on the first or second sample. Since the OPPE has infinite support, it has transformed into its unit form to utilize finite support, i.e., having the support (0, 1) for preparing tables for industrial workers to determine optimal sample sizes. The Lindley distribution, a particular choice of the OPPE ($r = 1, a_0 = a_1 = 1$), has been studied in detail. The Lindley distribution does not belong to the scale-invariant family, whereas the works so far on RSIPTL are chalked out for the scale-invariant family in the literature. The optimal plan parameters are estimated by transforming the Lindley distribution into its unit form. A few representative examples are presented for an easy and comprehensive understanding of the scientists and quality practitioners for implementation to find the optimal sample size of the lot to satisfy the consumer risk. Two data sets are analyzed to implement the proposed plan and compared with the widely used exponential model. The approach may be adopted to construct RSIPTL for other lifetime quality characteristic distributions that do not belong to the scale-invariant family.

Declarations

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Ethical Approval: This article does not contain any studies with human participants performed by any of the authors.
Code availability: Codes are available on request.

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