MATHEMATICAL ANALYSIS OF THE MECHANICAL PART OF THE DESIGN SCHEME OF THE ELECTRIC DRIVE OF A HYBRID ELECTRIC MACHINE

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Abstract

The paper analyzes the mechanical part of the design scheme of the electric drive of a hybrid electric machine, which is a key stage in the design and research of automatic control systems. The main elements of a mechanical system, a model of a real mechanical system connected to an electric drive, including all moving elements, transmission mechanisms, and actuators that convert electrical energy into mechanical work, are considered. The presented calculation scheme allows you to analyze dynamic processes, i.e. to study the system's behavior over time, to determine stability, fluctuations, and other characteristics. Calculations of various mass systems are performed using the capabilities of the MATLAB/Simulink software package for a three-mass and two-mass system. These models can be used for different systems with different parameters. To draw up a structural diagram, the elements of the mechanical part and the connections between the elements, the types of these connections (rigid, elastic) and the directions of motion transmission are determined. Structural diagrams are used to analyze the dynamic characteristics of the system, determine transients, stability, and vibrations.

Key words: hybrid electric machine, mechanical system, electric drive, inertial element, moment of inertia, elastic element, transfer function, mathematical model, disturbing effect.

I. Introduction

The mathematical analysis of the mechanical part of the design scheme of the electric drive of a hybrid electric machine (HEM) includes various methods of theoretical mechanics, dynamics, control, and electronics to evaluate and optimize the operation of the electric drive. In the context of a hybrid electric machine, where both a traditional internal combustion engine (ICE) and an electric motor are used, it is important to carry out complex calculations for all system elements to interact correctly [2, 4, 10]. The main steps for the mathematical analysis of the mechanical part of the electric drive are:

1. Modeling of a mechanical system - the mechanical part of the electric drive of a hybrid electric machine consists of various elements such as engines (internal combustion engine and electric), transmission, wheels, etc. For each of these elements, a mathematical model is created describing their movement, interaction, and reactions to external forces.

The main elements of the mechanical system include:

- Models of drive motors (for both electric and internal combustion engines).
- Transmission models (if the hybrid system uses a manual transmission).

– Wheel and suspension models to account for their inertial characteristics and impact on the road surface.

Mathematically, this can be represented using differential equations describing the motion and interaction of these elements.

2. System dynamics - mathematical analysis of the dynamics of the mechanical part of an electric drive includes the study of the system's response to various external influences, such as: acceleration; load change; transitions between different operating modes (for example, from working on an electric motor to working on an internal combustion engine).

To describe these processes, a system of second-order differential equations is usually used, which simulate the movement of a body under the influence of forces. An important element here is to take into account inertia, friction and other factors that affect the operation of the system.

3. Energy analysis - energy analysis allows you to evaluate the efficiency of the system. For a hybrid car, it is important to switch between the operating modes of the electric motor and the internal combustion engine, which is associated with energy consumption management.

For the purpose of mathematical analysis, the following are required: power calculation: an estimate of the power transferred from the engine to the wheels; transmission efficiency: calculation of energy losses in transmission from the motor to the wheels; energy consumption optimization: a system operation strategy that uses the optimal balance between the internal combustion engine and the electric motor to minimize fuel consumption and maximize efficiency [1, 3, 7].

4. Control models - for the efficient operation of the electric drive of a hybrid machine, a control system is required that will optimize the use of an electric and mechanical drive, for example, torque control on wheels depending on the operating modes of the machine, a system that optimizes the transition between an electric motor and an internal combustion engine depending on the load and battery condition.

5. Transient modeling - for a HEM, it is also important to take into account transients such as: switching between the electric motor and the internal combustion engine; changes in the speed of rotation of the motor when the load changes; the effect of accelerations and decelerations.

For this purpose, methods of numerical solution of differential equations and modeling using specialized software packages can be used.

8. Vibration and noise calculation - The hybrid electric machine must operate with minimal vibrations and noises, which is also part of the analysis. This includes modeling vibrations arising from the operation of the engine and other transmission elements, as well as noise characteristics, which are important to take into account when designing. As a result of mathematical analysis of the mechanical part of the electric drive of a hybrid electric machine, it is possible to obtain optimal system operation parameters, evaluate its dynamic characteristics, energy efficiency, and make predictions about the durability and wear of key system components.

To perform calculations of various mass systems, it is advisable to use the capabilities of the MATLAB/Simulink software package [2, 4, 6, 12]. For three- and two-mass systems. These models can be used for different systems with different parameters. To draw up a structural diagram, it is necessary to determine all the elements of the mechanical part, establish connections between the elements, types of connections and directions of motion transmission. With the help of structural diagrams, it is possible to analyze the dynamic characteristics of the system, determine transients, stability, and vibrations. At the same time, it is necessary to take into account the factors influencing the choice of a block diagram: the type of engine, the nature of the load. Drawing up and analyzing structural diagrams is an important stage in the design and operation of electric drives. A properly selected and constructed scheme allows you to ensure the effective operation of the system and avoid undesirable phenomena.

II. Formulation of the problem

The mechanical part of the design scheme of the electric drive is a model of a real mechanical system connected to an electric drive. It includes all moving elements, transmission mechanisms and actuators that convert electrical energy into mechanical work. The main elements of the mechanical part consist of the following [9, 13]:

- Inertial elements with mass and moment of inertia, such as engine rotors, working bodies of machines and mechanisms.
- Elastic elements that can deform under the influence of external forces and restore their shape after removing the load (for example, shafts, springs).
- Damping elements that convert mechanical energy into thermal energy, reducing system vibrations (e.g. friction).

The calculation scheme allows you to analyze dynamic processes, that is, to study the behavior of the system over time, to determine stability, fluctuations and other characteristics. In addition, it is possible to determine the parameters of the electric drive, select an engine, calculate its power, select a control system, optimize the operation of the system, minimize energy losses, improve positioning accuracy, reduce vibrations [8, 11]. To build a calculation scheme, it is necessary to pay attention to the following factors:

1. Identification of elements, i.e. determination of all elements of a mechanical system and their characteristics (masses, moments of inertia, stiffness, coefficients of friction).
2. Connecting the elements according to their physical connection, forming a chain.
3. Simplifying calculations, for example, neglecting some masses or friction.

There are various types of calculation schemes: kinematic schemes reflecting only the geometric connections between the elements of the system and dynamic circuits that take into account not only geometric connections, but also inertial, elastic and damping properties of the elements.

Various mathematical methods are used to analyze computational circuits, such as: the method of differential equations, in which the differential equations of motion of the system are compiled and solved analytically or numerically, the method of transfer functions, the method of state matrices, which is described by a system of first-order differential equations in matrix form.

III. Problem solution

Simplification methods can be used when constructing the design scheme of an electric drive. The system of equations for a three-mass system is described as follows [7, 9, 12]:

$$\left.\begin{aligned} M_m - \Delta M_1 - M_{12} &= J_1 \frac{d\omega_1}{dt} \\ M_{12} - \Delta M_2 - M_{23} &= J_2 \frac{d\omega_2}{dt} \\ M_{23} - \Delta M_3 - M_d &= J_3 \frac{d\omega_3}{dt} \end{aligned}\right\} \tag{1}$$

In these equations, M_m - moment of the engine; J_1, J_2, J_3 - moments of inertia; ω_1, ω_2, ω_3 -rotational velocities; M_{12} and M_{23} - elastic moments acting between masses; ΔM_1, ΔM_2, ΔM_d, - disturbing effects. For the first mass, the moment M_{12} is used as the moment of resistance. This moment of motion for the second mass is used as a torque:

$$M_{12} = C_{12}\left(\varphi_1 - \varphi_2\right) \tag{2}$$

$$M_{23} = C_{23}\left(\varphi_2 - \varphi_3\right) \tag{3}$$

Here, φ_1, φ_2, φ_3 – rotation angles during rotation of the mass; C_{12}, C_{23} – stiffness of elastic bonds. The stiffness of elastic bond is determined as follows:

$$C_h = \frac{M_{sh}}{\Delta\varphi_i}, \tag{4}$$

where M_{sh} - torque of the motor shaft, $\Delta\varphi_i$ - difference in the angles of rotation at the ends of the elastic elements, and the deformation of the part. Similarly, we can write a system of equations for a two-mass system:

$$\left.\begin{aligned} &M_m - \Delta M_1 - M_{12} = J_1\frac{d\omega_1}{dt} \\ &M_{12} - \Delta M_2 - M_{23} = J_2\frac{d\omega_2}{dt} \\ &M_{12} = C_{12}\left(\varphi_1 - \varphi_2\right) \end{aligned}\right\} \tag{5}$$

For a single-mass system, the equation is represented as follows:

$$M_m - M_d = J_\Sigma \frac{d\omega}{dt} \tag{6}$$

To perform calculations of various mass systems, it is advisable to use the capabilities of the MATLAB/Simulink software package. Figure 1 shows a Simulink model of the equation system for a three-mass system, and Figure 2 for a two-mass system. These models can be used for different systems with different parameters.

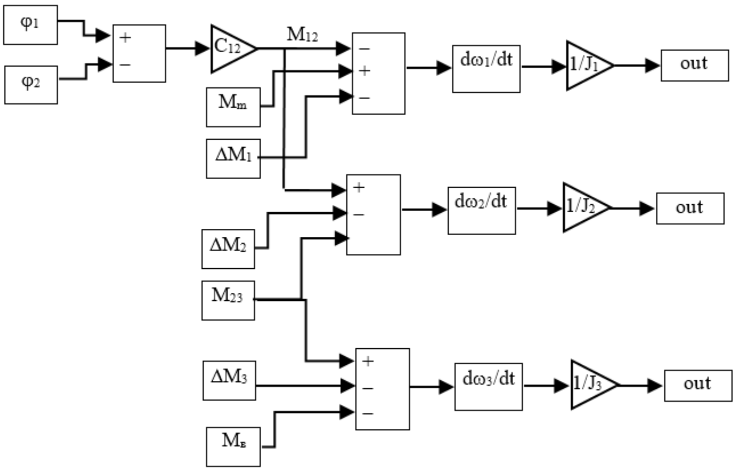


Figure 1. Simulink model of the equation system for a three-mass system

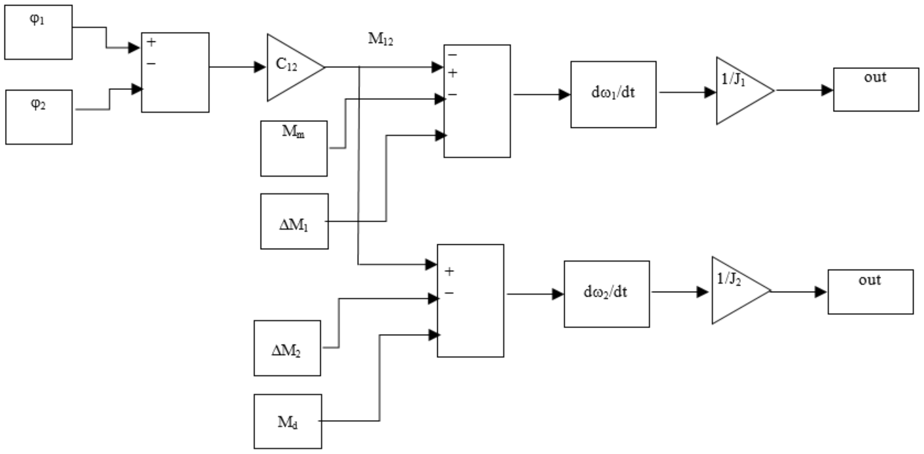


Figure 2. Simulink model of the equation system for a two-mass system

The block diagram of the mechanical part of an electric drive of the HEM is a graphical representation of the relationships between the elements of a mechanical system that converts the electrical energy of the engine into mechanical work. It allows you to visualize and analyze the processes taking place in the system, evaluate its dynamic characteristics and select the optimal parameters [6, 9, 10]. To draw up a structural diagram, it is necessary to determine all the elements of the mechanical part, the establishment of connections between the elements, types of connections (rigid, elastic) and directions of motion transmission. With the help of structural diagrams, it is possible to analyze the dynamic characteristics of the system, determine transients, stability, and vibrations. At the same time, it is necessary to take into account the factors influencing the choice of a block diagram: the type of engine, and the nature of the load (constant, variable, shock).

Drawing up and analyzing structural diagrams is an important stage in the design and operation of electric drives. A properly selected and constructed scheme allows you to ensure the effective operation of the system and avoid undesirable phenomena.

Figure 3 shows a block diagram of the mechanical part of a two-mass electric drive of the HEM.

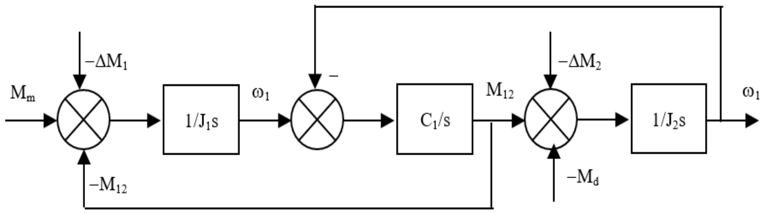


Figure 3. Block diagram of the mechanical part of a two-mass electric drive

Equivalent transfer functions for the mechanical part of the drive, depending on the adjustable parameters, can be expressed in various forms [3, 12, 14]. For example, the output value of the transfer function $\omega_1(s)$ for control is expressed in this way:

$$W_{\omega_1}(s)=\frac{\omega_1(s)}{M_m(s)}=\frac{s^2+\dfrac{J_1}{J_1+J_2}\Omega_{12}^2}{J_1 s(s^2+\Omega_{12}^2)}, \tag{7}$$

where, Ω_{12} - specific oscillation frequency of a two - mass mechanical system:

$$\Omega_{12}=\sqrt{\frac{C_{12}(J_1+J_2)}{J_1 J_2}} \tag{8}$$

Similarly, for $\omega_2(s)$ we get:

$$W_{\omega_2}(s)=\frac{\omega_2(s)}{M_m(s)}=\frac{\Omega_{12}^2}{s(J_1+J_2)(s^2+\Omega_{12}^2)} \tag{9}$$

The transfer function for a two-mass electric drive of the HEM is determined by solving the ratio of the polynomials of the system of equations of motion written in operator form. Similarly, the transfer functions of the perturbation can be expressed as follows:

$$W_{\omega_1}(s)=\frac{\omega_2(s)}{(\Delta M_2+M_d)(s)}=\frac{\Omega_{12}^2}{s(J_1+J_2)(s^2+\Omega_{12}^2)}=W_{\omega_2}(s) \tag{10}$$

$$W_{\omega_2}(s)=\frac{\omega_3(s)}{(\Delta M_3+M_d)(s)} \tag{11}$$

It should be noted that the transfer function of the system does not depend on either the shape or the amplitude of the input signal. This function depends only on the internal structure of the system and the circuit parameters.

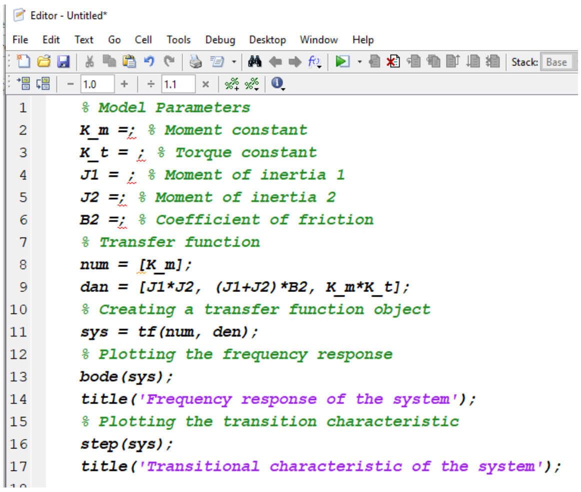
These steps will help you create and configure a three-mass electric drive model in Simulink and visualize its behavior. If the system has more complex interactions or additional elements, you may need to adapt the model depending on specific requirements [4, 10].

Consider the standard model of a two-mass electric drive. It is usually described by the following equation:

$$\frac{\Theta_1(s)}{U(s)}=\frac{K_m}{s((J_1J_2s^2+J_1+J_2)B_2s+K_mK_t)} \tag{12}$$

Where, $\Theta_1(s)$ - angle of rotation at the output (in radians); $U(s)$ - control voltage; K_m - torque constant of the electric motor; K_t - constant of the torque of the load; J_1 and J_2 - moments of inertia of the electric motor and the load, respectively; B_2 - coefficient of friction of the load.

Next, the following algorithm (code) of equation (12) is written in the MATLAB program indicated below. This code will create a transfer function, plot its frequency and transient response (Figure 4). Substituting the real values of the parameters into the code, depending on the required task, you can create a transfer function, plot its frequency and transient response. To construct the amplitude-phase characteristic of the system, it is necessary to replace the symbol of the operator s with the symbol $j\Omega$ in the expression (9) of the transfer function of the system.



```
% Model Parameters
K_m =; % Moment constant
K_t = ; % Torque constant
J1 = ; % Moment of inertia 1
J2 =; % Moment of inertia 2
B2 =; % Coefficient of friction
% Transfer function
num = [K_m];
dan = [J1*J2, (J1+J2)*B2, K_m*K_t];
% Creating a transfer function object
sys = tf(num, den);
% Plotting the frequency response
bode(sys);
title('Frequency response of the system');
% Plotting the transition characteristic
step(sys);
title('Transitional characteristic of the system');
```

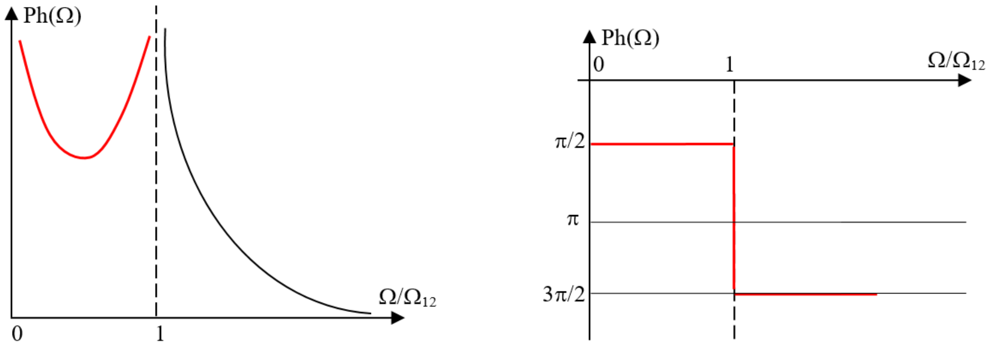
Figure 4. Algorithm (code) of equation (12)

$$W_{\omega_1}(j\Omega)=\frac{1}{\Omega_{12}(J_1+J_2)}+\frac{j}{(\Omega/\Omega_{12})(1-(\Omega\Omega_{12})^2)} \tag{13}$$

$$W_{\omega_1}(j\Omega)=Re(\Omega)+jIm(\Omega)$$

here, $Re(\Omega)$ – real number; $Im(\Omega)$ – imaginary number.

Using expression (13), we obtain the amplitude-phase (Figure 5,a) and phase-frequency (Figure 5,b) characteristics of a two-mass system.



a)

b)

Figure 5. Amplitude-phase and phase-frequency characteristics of a two-mass system

When the frequency value changes in the range $0 \div \Omega_{12}$, the amplitude of the output signal has a U-shaped appearance [2, 9]. But in the range of $0 \div \infty$, the amplitude of the variable value will tend to infinity. In subsequent increases, the amplitude of the variable always decreases. When changing the phase variable in the range $0 \div \Omega_{12}$ this value abruptly changes from $-\pi/2$ to $3\pi/2$, and will be a constant in phase $3\pi/2$: $\Omega > \Omega_{12}$.

Suppose that the excitation of the electric drive under study affects the formation of the amplitude of the electromagnetic moment, which varies according to the harmonic law of time, and the amplitude value of this electromagnetic moment is slightly more than the value of the static torque. When the frequency of the electromagnetic torque is close to zero, the difference between static and dynamic torques, that is, the dynamic torque will be positive. As a result, after a certain time you will return the value of the angular velocity, which is the output value, will be equal to infinity.

With an increase in the frequency of change in the angle of rotation of the electromagnetic torque, its speed decreases due to the action of moments of inertia on the mechanical parts of the electric drive of the HEM. Here, the inertia of the mechanical parts plays the role of a damper. When the amplitude of the forced frequency fluctuations approaches a certain frequency of the system, this value begins to increase again. In this case, a mechanical resonance is detected, and at a value of $\Omega=\Omega_{12}$, the amplitude of the oscillations again takes on an infinite value [7, 9, 13].

With a subsequent increase in the amplitude of the forced frequency, the inertia of the mechanical parts leads to a decrease in the amplitude of the output value, which plays the role of a damper that prevents movement. Under the condition $\Omega>>\Omega_{12}$, the inertia of the parts of the mechanism completely disables the oscillatory motion. Vibrations during movement, along with dynamic influences, dissipating forces arise (for example, friction against the lining), the speed also increases as the resistance force increases, as a result of which the dynamic torque decreases to zero, and the speed naturally reaches a large value.

IV. Conclusions

1. The article presents a mathematical analysis of the mechanical part of the design circuit of an electric drive, which is a key stage in the design and research of automatic control systems, which allows to obtain an accurate description of the dynamic processes occurring in the system and evaluate its characteristics.

2. The presented calculation scheme allows analyzing dynamic processes, i.e. studying the behavior of the system over time, determining stability, fluctuations and other characteristics. At the same time, it is possible to determine the parameters of the electric drive, select an engine, calculate its power, select a control system, optimize the operation of the system, minimize energy losses, improve positioning accuracy, reduce vibrations.

3. To draw up a structural diagram, the elements of the mechanical part and the connections between the elements, the types of these connections (rigid, elastic) and the directions of motion transmission are determined. The analysis of the dynamic characteristics of the system, the determination of transients, stability, and vibrations was performed using structural diagrams.

4. With an increase in the frequency of change in the angle of rotation of the electromagnetic torque, its speed decreases due to the action of moments of inertia on the mechanical parts of the electric drive. When the amplitude of the forced frequency fluctuations approaches a certain frequency of the system, this value begins to increase again. With a subsequent increase in the amplitude of the forced frequency, the inertia of the mechanical parts leads to a decrease in the amplitude of the output value, which plays the role of a damper that prevents movement.

5. Vibrations during movement, along with dynamic influences, dissipating forces occur (for example, friction against the lining), the speed also increases as the resistance force increases, as a result of which the dynamic torque decreases to zero, and the speed naturally reaches a large value.

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