SURVIVAL ANALYSIS OF A STOCHASTIC MODEL ON CARDIOVASCULAR SYSTEM CONSIDERING POSSIBILITES OF DAMAGE, FAILURE AND RECOVERY OF HEART

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Abstract

The present paper deals with survival analysis of a stochastic model on cardiovascular system considering possibilities of damage, failure and recovery of heart. The analysis is based upon a stochastic model for the system considering different kinds of damage and failure of heart at different situations. The treatments and recovery of heart are taken in to account. On complete failure of heart, transplantation of the heart is also considered. The model has been analyzed by determining important measures of effectiveness using Markov process and regenerative point technique. Sensitivity analysis has also been done to select important parameters for enhancing the survivability of the system.

Keywords: Cardiovascular system, mean survival time, survivability, sensitivity analysis, Markov process and regenerative point technique.

1. Introduction

The cardiovascular system in the human body is a complex network of organs, vessels and tissues that transport blood, oxygen, nutrients and other vital substances throughout the body. The main components of cardiovascular system include the heart, blood vessels and blood, the major driving force of the system is the heart. The heart is a vital organ of the system responsible for pumping blood throughout the body, supplying oxygen and nutrients to the tissues and organs. It is a muscular organ located slightly left of the center of the chest and is protected by the ribcage.

The survivability of the system plays an important role for the functioning of the body. For investigations of heart related issues including morbidity and mortality attempts have been made by many researchers applying prospective and different methods. Kannel and McGee [3] discussed the relation between diabetes and cardiovascular disease. Bertoni et al. [1] studied diabetes impacts on the elderly in terms of the prevalence, incidence and mortality related to heart failure. Lerma et al. [6] discussed the stochastic aspects of cardiac arrhythmias. Smith et al. [11] predicted the outcomes among heart failure patients. Meenaxi et al. [7] studied a model for the progression of chronic heart failure. Tanna et al. [12] considered risk prediction models for adults with heart failure. Pierce et al. [8] explained the patterns of cardiovascular mortality associated with heart failure, comparing rural and urban counties across the United States. Rajeswari and Kausika [9] developed a machine learning model to predict future possibility of heart disease. Khan et al. [4] presented machine learning algorithm-based cardiovascular disease prediction. Roger [10] studied the epidemiology of heart failure. Deng [2] provided a modern viewpoint on cardiac transplantation.

Over the time, researches in the epidemiology of heart failure make simple the understanding of the syndrome's complexity. The prevalence of disease like diabetes mellitus, obesity, hypertension, and problems in supplementary organs like chronic kidney disease, and cancer comorbidities at heart failure are increasing, and these factors may be associated with the increasing prevalence of heart failure with preserved ejection fraction [5]. In motor vehicle accidents and vehicle air bags heart injury due to rib fractures and sucking chest wound are very common [14]. Further, according to American Heart Association, heart transplantation is a kind of surgery in which failing heart of patient is replaced by donor's heart. It gives better life to heart patient. Heart transplants have seen a remarkable rise in recent years. For instance, the number of transplant procedures in India grew from 53 in 2014 to 241 in 2018, and there were 187 transplants in 2019 [13].

2. Model Description and State Transition Diagram

It has been noticed that considering the above aspects of heart damages, failure and recovery/transplantation, survivability and sensitivity analysis of cardiovascular system has not been reported in the literature. To fill this gap, the present paper deals with the survivability and sensitivity analysis of cardiovascular system considering various causes of damage, failure and recovery of heart. It is considered that the heart may damage/fail due to prevalence of some disease, other organ issues and some severe accidents. The person having some heart problem reaches the hospital in negligible time for the treatments. After successful recovery, heart in the cardiovascular system works as good as healthy whereas transplantation of a heart does not make the patient healthy. A stochastic model has been developed for the cardiovascular system and analyzed by determining some measures of effectiveness using Markov process and regenerative point technique. The other assumptions of the model are

- Minor heart damages are recovered by some medicine/exercise/therapy.
- The severe accidents result into major heart damage.
- The single diagnose/treatment facility is available in the hospital.
- The time to damages and failure is considered to be exponentially distributed while other time distributions are general.
- All random variables are mutually independent.

3. States and Notations of the System

States & Notations	Description
S_0	healthy state
S_1, S_2	minor damage state
S_3	major damage state
S_4	failed state
λ_{01}	damage rate due to prevalence of some disease
λ_{02}	damage rate due to other organ issues
λ_{04}	failure rate due to severe accidents
λ_{13}	major damage rate from minor heart damage

λ_{24}	failure rate due to minor heart damage
λ_{34}	failure rate due to major heart damage
$g_1(t)/G_1(t)$	p.d.f/c.d.f of time of recovery through medicine/exercise/therapy
$g_2(t)/G_2(t)$	p.d.f/c.d.f of time of recovery from other organ issues
$g_3(t)/G_3(t)$	p.d.f/c.d.f of time of recovery due to surgery/operation
$h(t)$	p.d.f/c.d.f of time of heart transplantation

State Transition Diagram is depicted in fig. 1

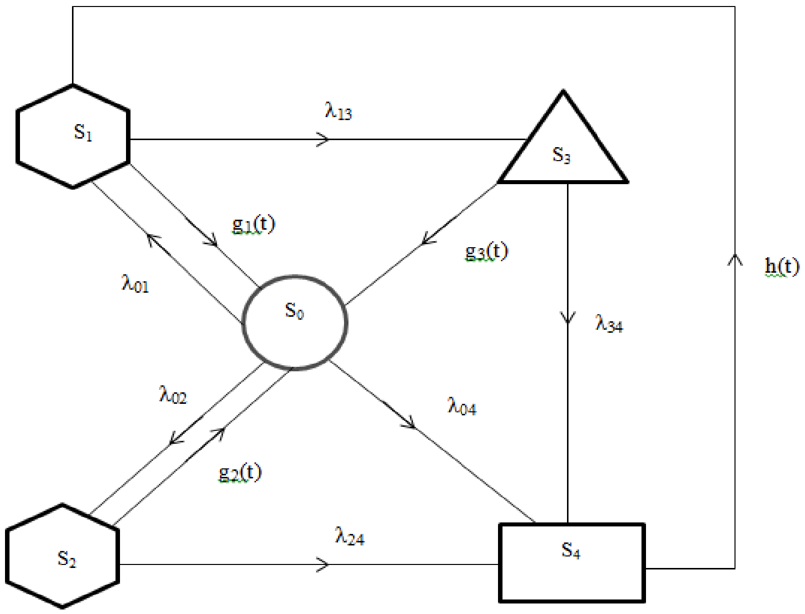
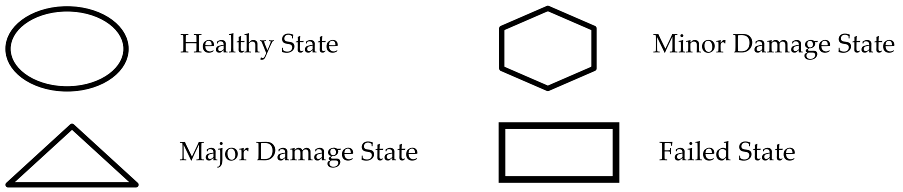


Figure 1. *State Transition Diagram*



4. Transition Probabilities

The transition probabilities are given by

$$dQ_{01}(t) = \lambda_{01}e^{-(\lambda_{01}+\lambda_{02}+\lambda_{04})t}dt; \qquad dQ_{02}(t) = \lambda_{02}e^{-(\lambda_{01}+\lambda_{02}+\lambda_{04})t}dt;$$

$$dQ_{04}(t) = \lambda_{04}e^{-(\lambda_{01}+\lambda_{02}+\lambda_{04})t}dt; \qquad dQ_{10}(t) = g_1(t)e^{-\lambda_{13}t}dt;$$

$$dQ_{13}(t) = \lambda_{13}e^{-\lambda_{13}t}\overline{G_1(t)}dt; \qquad dQ_{20}(t) = g_2(t)e^{-\lambda_{24}t}dt;$$

$$dQ_{24}(t) = \lambda_{24}e^{-\lambda_{24}t}\overline{G_2(t)}dt; \qquad dQ_{30}(t) = g_3(t)e^{-\lambda_{34}t}dt;$$

$$dQ_{34}(t) = \lambda_{34}e^{-\lambda_{34}t}\overline{G_3(t)}dt; \qquad dQ_{41}(t) = h(t)dt.$$

The non-zero element p_{ij} are given by

$$p_{ij} = \lim_{s \to 0} q^*_{ij}(s) = \lim_{s \to 0} Q^{**}_{ij}(s).$$

Therefore,

$p_{01} = \dfrac{\lambda_{01}}{\lambda_{01} + \lambda_{02} + \lambda_{04}}$; $p_{02} = \dfrac{\lambda_{02}}{\lambda_{01} + \lambda_{02} + \lambda_{04}}$; $p_{04} = \dfrac{\lambda_{04}}{\lambda_{01} + \lambda_{02} + \lambda_{04}}$;

$p_{10} = g_1^*(\lambda_{13})$; $p_{13} = 1 - g_1^*(\lambda_{13})$; $p_{24} = 1 - g_2^*(\lambda_{24})$;

$p_{30} = g_3^*(\lambda_{34})$; $p_{34} = 1 - g_3^*(\lambda_{34})$; $p_{41} = h^*(0)$.

It can be observed that

$p_{01} + p_{02} + p_{04} = 1$; $p_{10} + p_{13} = 1$; $p_{20} + p_{24} = 1$;

$p_{30} + p_{34} = 1$; $p_{41} = 1$.

5. Mean Sojourn Time

Expected time taken by the patient in state i before transiting to any other state is termed as mean sojourn time in that state and it is denoted by μ_i . Mean sojourn time μ_i in the i th state is given by

$$\mu_i = \int_0^\infty \Pr(\mathrm{T}_i > t)dt,$$

where T_i is the sojourn time in state i .

$\mu_0 = \dfrac{1}{\lambda_{01} + \lambda_{02} + \lambda_{04}}$; $\mu_1 = \dfrac{1 - g_1^*(\lambda_{13})}{\lambda_{13}}$; $\mu_2 = \dfrac{1 - g_2^*(\lambda_{24})}{\lambda_{24}}$;

$\mu_3 = \dfrac{1 - g_3^*(\lambda_{34})}{\lambda_{34}}$; $\mu_4 = \int_0^\infty \overline{H(t)dt}$.

The unconditional mean time is, mathematically, defined as

$$m_{ij} = \int_0^\infty \mathrm{t q}_{ij}(\mathrm{t})\,\mathrm{dt} = -\mathrm{q}^{*'}_{ij}(0).$$

Therefore,

$m_{01} = \dfrac{\lambda_{01}}{(\lambda_{01} + \lambda_{02} + \lambda_{04})^2}$; $m_{02} = \dfrac{\lambda_{02}}{(\lambda_{01} + \lambda_{02} + \lambda_{04})^2}$; $m_{04} = \dfrac{\lambda_{04}}{(\lambda_{01} + \lambda_{02} + \lambda_{04})^2}$;

$m_{10} = -g_1^{*'}(\lambda_{13})$; $m_{13} = \dfrac{1 - g_1^*(\lambda_{13})}{\lambda_{13}} + g_1^{*'}(\lambda_{13})$; $m_{20} = -g_2^{*'}(\lambda_{24})$;

$m_{24} = \dfrac{1 - g_2^*(\lambda_{24})}{\lambda_{24}} + g_2^{*'}(\lambda_{24})$; $m_{30} = -g_3^{*'}(\lambda_{34})$; $m_{34} = \dfrac{1 - g_3^*(\lambda_{34})}{\lambda_{34}} + g_3^{*'}(\lambda_{34})$;

$m_{41} = -h^{*'}(0)$.

From above we observed that

$m_{01} + m_{02} + m_{04} = \mu_0$; $m_{10} + m_{13} = \mu_1$; $m_{20} + m_{24} = \mu_2$;

$m_{30} + m_{34} = \mu_3$; $m_{41} = \mu_4$.

6. Mean Survival Time

Let $\Phi_i(t)$ denotes the cumulative distribution function of first passage time from S_i to the failed state. The following recursive relations are obtained for $\Phi_i(t)$:

$\Phi_0(t) = Q_{01}(t) \,\&\, \Phi_1(t) + Q_{02}(t) \,\&\, \Phi_2(t) + \mathrm{Q}_{04}(t)$;

$\Phi_1(t) = Q_{10}(t) \,\&\, \Phi_0(t) + Q_{13}(t) \,\&\, \Phi_3(t)$;

$\Phi_2(t) = Q_{20}(t) \,\&\, \Phi_0(t) + Q_{24}(t)$;

$\Phi_3(t) = Q_{30}(t) \,\&\, \Phi_0(t) + Q_{34}(t)$.

Taking L.S.T and solving for $\Phi_0^{**}(s)$, we get

$$\Phi_0^{**}(s) = \frac{N(s)}{D(s)},$$

where

$$N(s) = Q_{04}^{**}(s) + Q_{02}^{**}(s)Q_{24}^{**}(s) + Q_{01}^{**}(s)Q_{13}^{**}(s)Q_{34}^{**}(s),$$

and

$$D(s) = 1 - Q_{01}^{**}(s)Q_{10}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) - Q_{01}^{**}(s)Q_{13}^{**}(s)Q_{30}^{**}(s).$$

Now the expression for the mean survival time is

$$T_0 = \lim_{s\to 0}\frac{1-\Phi_0^{**}(s)}{s}.$$

Using L'Hospital's rule and solving for $\Phi_0^{**}(s)$, we get

$$T_0 = \frac{N}{D},$$

where

$N = \mu_0 + p_{01}(\mu_1 + p_{13}\mu_3) + p_{02}\mu_2,$ and $D = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{01}p_{13}p_{30}.$

7. Survivability

Let $S_i(t)$ denotes the probability that patient survive at instant t given that the patient entered the state i at time $t = 0$. The recursive relations for $S_i(t)$ as given below:

$$S_0(t) = M_0(t) + q_{01}(t)©S_1(t) + q_{02}(t)©S_2(t) + q_{04}(t)©S_4(t)$$
$$S_1(t) = M_1(t) + q_{10}(t)©S_0(t) + q_{13}(t)©S_3(t)$$
$$S_2(t) = M_2(t) + q_{20}(t)©S_0(t) + q_{24}(t)©S_4(t)$$
$$S_3(t) = M_3(t) + q_{30}(t)©S_0(t) + q_{34}(t)©S_4(t)$$
$$S_4(t) = q_{41}(t)©S_1(t),$$

where

$M_0(t) = e^{-(\lambda_1+\lambda_2+\lambda_3)t}$; $M_1(t) = e^{-\lambda_1 t}\overline{G_1(t)}$; $M_2(t) = e^{-\lambda_2 t}\overline{G_2(t)}$; $M_3(t) = e^{-\lambda_1 t}\overline{G_3(t)}$.

By using LT of these equations and then computing for $S_0^*(s)$,

$$S_0^*(s) = \frac{N_1(s)}{D_1(s)},$$

where

$$N_1(s) = M_0^*(s) + M_1^*(s)q_{01}^*(s) + M_2^*(s)q_{02}^*(s) + M_3^*(s)q_{01}^*(s)q_{13}^*(s) + M_1^*(s)q_{04}^*(s)q_{41}^*(s)$$
$$+ M_3^*(s)q_{04}^*(s)q_{13}^*(s)q_{41}^*(s) + M_1^*(s)q_{02}^*(s)q_{24}^*(s)q_{41}^*(s) + M_3^*(s)q_{02}^*(s)q_{13}^*(s)q_{24}^*(s)q_{41}^*(s)$$
$$- M_0^*(s)q_{13}^*(s)q_{34}^*(s)q_{41}^*(s) - M_2^*(s)q_{02}^*(s)q_{13}^*(s)q_{34}^*(s)q_{41}^*(s)$$

and

$$D_1(s) = 1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{01}^*(s)q_{13}^*(s)q_{30}^*(s) - q_{04}^*(s)q_{10}^*(s)q_{41}^*(s)$$
$$- q_{02}^*(s)q_{10}^*(s)q_{24}^*(s)q_{41}^*(s) - q_{04}^*(s)q_{13}^*(s)q_{30}^*(s)q_{41}^*(s) - q_{02}^*(s)q_{13}^*(s)q_{24}^*(s)q_{30}^*(s)q_{41}^*(s)$$
$$- q_{13}^*(s)q_{34}^*(s)q_{41}^*(s) + q_{02}^*(s)q_{13}^*(s)q_{20}^*(s)q_{34}^*(s)q_{41}^*(s).$$

Expected survivability is derived as

$$S_0 = \lim_{s\to 0} sS_0^*(s) = \frac{N_1}{D_1},$$

where

$$N_1 = \mu_0(1 - p_{13}p_{34}) + \mu_1(1 - p_{02}p_{20}) + \mu_2 p_{02}(1 - p_{13}p_{34}) + \mu_3 p_{13}(1 - p_{02}p_{20})$$

and

$$D_1 = \mu_0(1 - p_{13}p_{34}) + \mu_1(1 - p_{02}p_{20}) + \mu_2 p_{02}(1 - p_{13}p_{34}) + \mu_3 p_{13}(1 - p_{02}p_{20}) + \mu_4(p_{04} + p_{02}p_{24} + p_{01}p_{13}p_{34}).$$

8. Numerical Computations and Graphical Interpretations

The expressions derived for mean sojourn times, mean survival time and survivability are analytic and computationally tedious involving several parameters. Therefore, following particular case is considered for computations and analysis purpose:

$$g_1(t)=\beta_1 e^{-\beta_1 t};\quad g_2(t)=\beta_2 e^{-\beta_2 t};\quad g_3(t)=\beta_3 e^{-\beta_3 t};\quad h(t)=\frac{1}{\gamma}.$$

Numerical computations have been done for the above particular case and various graphs have been plotted for mean survival time and survivability giving different estimated values to the parameters $\lambda_{01}, \lambda_{02}, \lambda_{04}, \lambda_{01}, \lambda_{13}, \lambda_{24}, \lambda_{34}, \beta_1, \beta_2, \beta_3, \gamma$. The following interpretations and conclusions have been drawn from the plotted graphs.

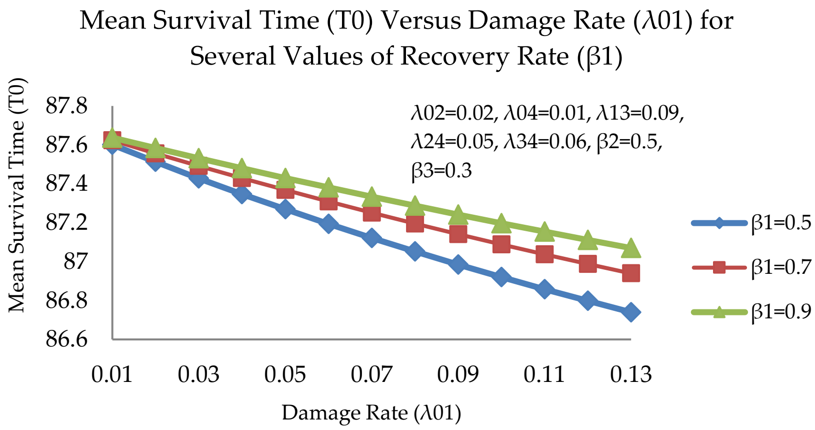


Figure 2. *Mean survival time* (T_0) *with respect to damage rate* (λ_{01})

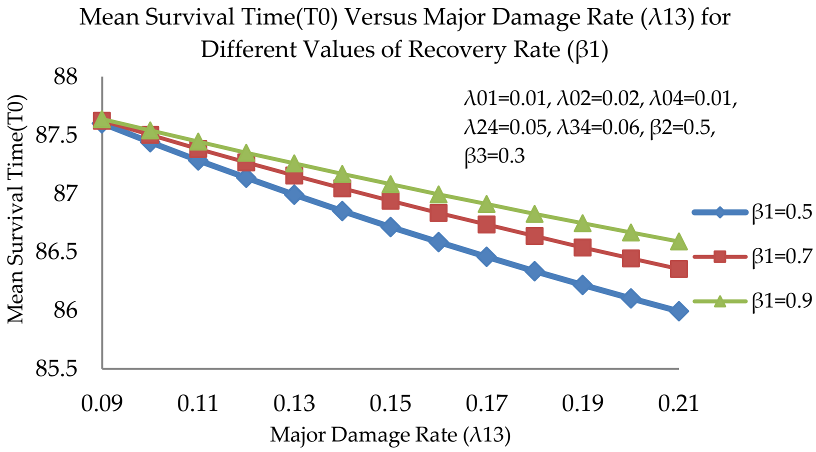


Figure 3. *Mean survival time* (T_0) *with respect to major damage rate* (λ_{13})

From the fig. 2 and fig. 3, it can be observed that mean survival time (T_0) of the system decreases as the damage rates λ_{01} and λ_{13} increases and has greater value for greater value of recovery rate β_1.

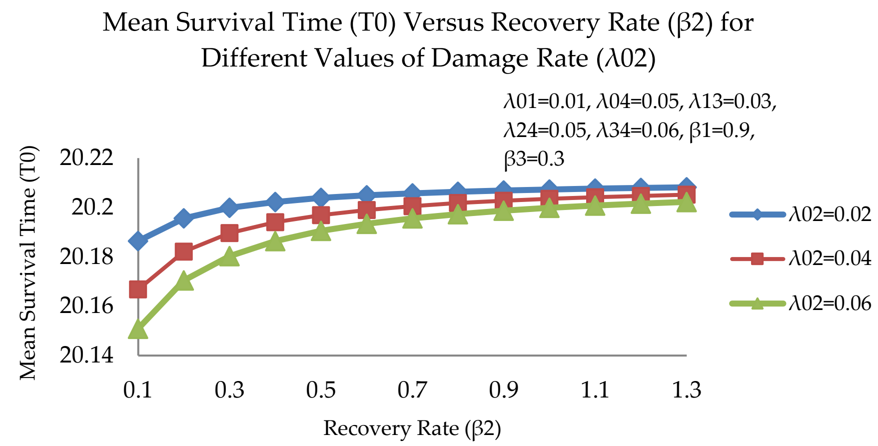


Figure 4. *Mean survival time* (T_0) *with respect to recovery rate* (β_2)

From the fig. 4, we can observe that mean survival time (T_0) of the system is more for higher recovery rate β_2 and it has lesser value for higher value of damage rate λ_{02}.

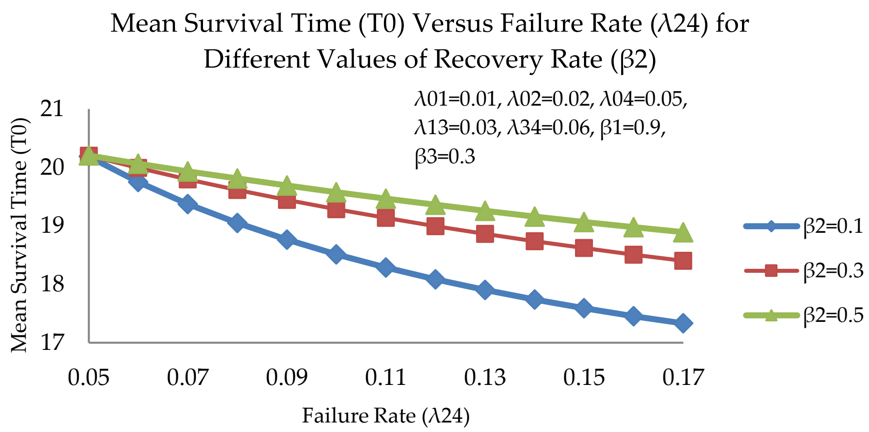


Figure 5. *Mean survival time* (T_0) *with respect to failure rate* (λ_{24})

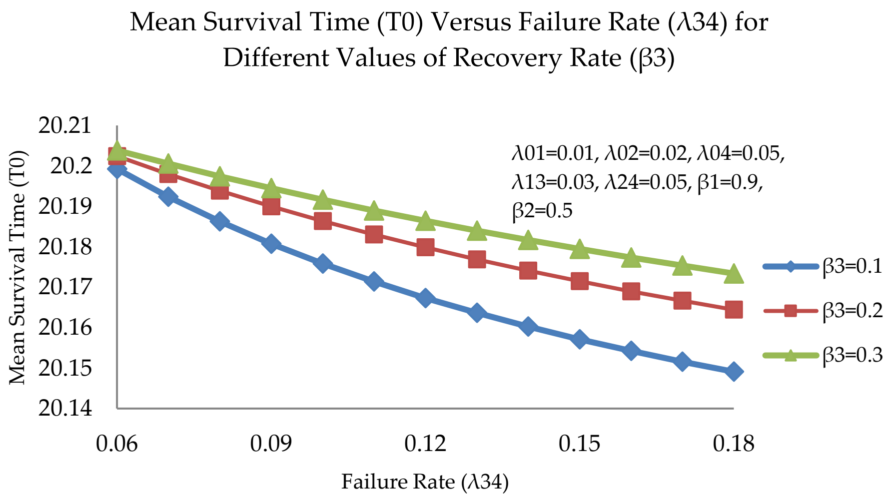


Figure 6. *Mean survival time* (T_0) *with respect to failure rate* (λ_{34})

From the fig. 5, we observed that mean survival time (T_0) of the system declines with the failure rate λ_{24} increases and it improves with the recovery rate β_2.

From the fig. 6, we observed that mean survival time (T_0) of the system decreases as the failure rate λ_{34} increases and has greater value for greater value of recovery rate β_3.

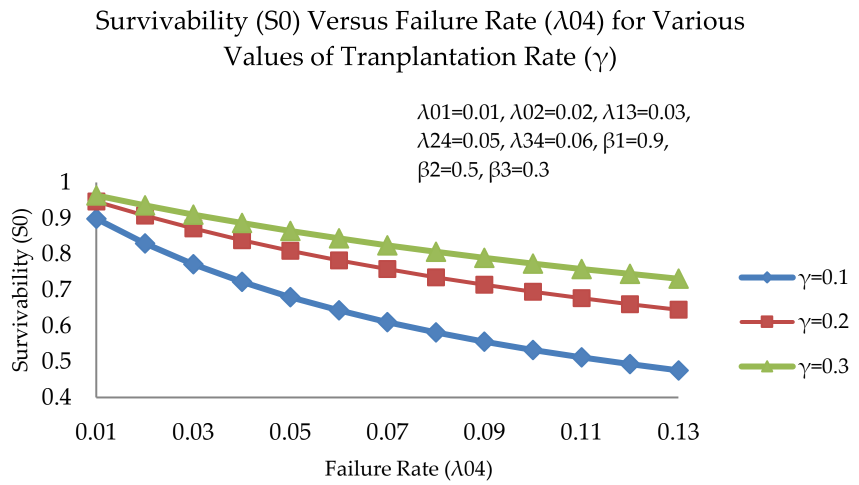


Figure 7. *Survivability* (S_0) *with respect to failure rate* (λ_{04})

From fig. 7 and fig. 8, it can be observed that survivability (S_0) of the system decreases as the failure rates λ_{04} and λ_{24} increases and has greater value for greater value of transplantation rate γ.

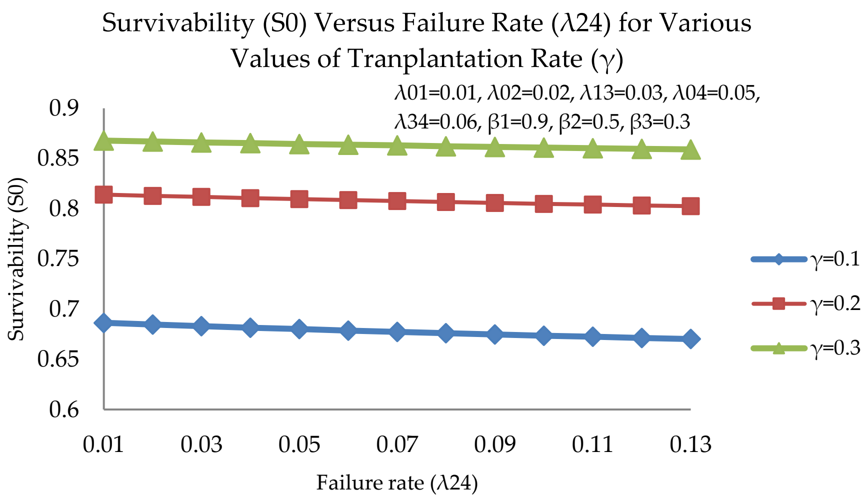


Figure 8. *Survivability* (S_0) *with respect to failure rate* (λ_{24})

9. Sensitivity and Relative Sensitivity Analysis

Sensitivity analysis determines how different values of an independent variable affect a particular dependent variable under a given set of assumptions. As there is significant difference among the values of parameters, we can use the concept of sensitivity analysis for comparing their effects on mean survival time (T_0) and survivability (S_0). The sensitivity and relative sensitivity function for mean survival time (T_0) and survivability (S_0) of the system are given below:

$$\pi_k = \frac{\partial(T_0)}{\partial k}, \quad \delta_k = \pi_k(\frac{k}{T_0}) \quad \text{and} \quad \pi_k = \frac{\partial(S_0)}{\partial k}, \quad \delta_k = \pi_k(\frac{k}{S_0}),$$

where

$k = \lambda_{01}, \lambda_{02}, \lambda_{04}, \lambda_{13}, \lambda_{24}, \lambda_{34}, \beta_1, \beta_2, \beta_3, \gamma.$

The fixed values of parameters are as follows:

$\lambda_{01} = 0.01, \lambda_{02} = 0.02, \lambda_{04} = 0.01, \lambda_{13} = 0.09, \lambda_{24} = 0.05, \lambda_{34} = 0.06, \beta_1 = 0.9, \beta_2 = 0.5, \beta_3 = 0.3, \gamma = 0.1$

Table 1: *Sensitivity and relative sensitivity analysis of mean survival time (T_0) of the system with respect to damage rate λ_{02}*

λ_{02}	$\pi_{\lambda_{02}} = \frac{\partial T_0}{\partial \lambda_{02}}$	$\delta_{\lambda_{02}} = \pi_{\lambda_{02}}(\frac{\lambda_{02}}{T_0})$
0.01	-601.6138	-0.0646
0.02	-513.6997	-0.1172
0.03	-443.7370	-0.1607
0.04	-387.1513	-0.1967
0.05	-340.7376	-0.2269
0.06	-302.1962	-0.2523
0.07	-269.8426	-0.2737

Table 2: *Sensitivity and relative sensitivity analysis of mean survival time (T_0) of the system with respect to failure rate λ_{04}*

λ_{04}	$\pi_{\lambda_{04}} = \frac{\partial T_0}{\partial \lambda_{04}}$	$\delta_{\lambda_{04}} = \pi_{\lambda_{04}}(\frac{\lambda_{04}}{T_0})$
0.01	-0.007	-0.8354
0.02	-0.002	-0.9103
0.03	-0.001	-0.9384
0.04	-595.5245	-0.9531
0.05	-388.3926	-0.9621
0.06	-273.1571	-0.9682
0.07	-202.5219	-0.9726
0.08	-156.1222	-0.9760
0.09	-124.0172	-0.9786

Table 3: *Sensitivity and relative sensitivity analysis of mean survival time (T_0) of the system with respect to recovery rate β_1*

β_1	$\pi_{\beta_1} = \frac{\partial T_0}{\partial \beta_1}$	$\delta_{\beta_1} = \pi_{\beta_1}(\frac{\beta_1}{T_0})$
0.1	1.3466	0.0015
0.2	0.6038	0.0014
0.3	0.3412	0.0012
0.4	0.2189	0.0009
0.5	0.1523	0.0008
0.6	0.1120	0.0007
0.7	0.0859	0.0006

Table 4: *Sensitivity and relative sensitivity analysis of mean survival time* (T_0) *of the system with respect to recovery rate* β_3

β_3	$\pi_{\beta_3} = \frac{\partial T_0}{\partial \beta_3}$	$\delta_{\beta_3} = \pi_{\beta_3}(\frac{\beta_3}{T_0})$
0.1	12.2427	0.0141
0.2	4.7379	0.0109
0.3	2.4954	0.0085
0.4	1.5368	0.0070
0.5	1.0407	0.0059
0.6	0.7571	0.0051
0.7	0.5675	0.0045

Table 5: *Sensitivity and relative sensitivity analysis of survivability* (S_0) *of the system with respect to damage rate* λ_{01}

λ_{01}	$\pi_{\lambda_{01}} = \frac{\partial(S_0)}{\partial \lambda_{01}}$	$\delta_{\lambda_{01}} = \pi_{\lambda_{01}}(\frac{\lambda_{01}}{S_0})$
0.01	-0.0056	-0.00006
0.02	-0.0055	-0.00012
0.03	-0.0054	-0.00017
0.04	-0.0052	-0.00023
0.05	-0.0051	-0.00028
0.06	-0.0050	-0.00033
0.07	-0.0049	-0.00038

Table 6: *Sensitivity and relative sensitivity analysis of survivability* (S_0) *of the system with respect to damage rate* λ_{02}

λ_{02}	$\pi_{\lambda_{02}} = \frac{\partial(S_0)}{\partial \lambda_{02}}$	$\delta_{\lambda_{02}} = \pi_{\lambda_{02}}(\frac{\lambda_{02}}{S_0})$
0.01	-0.5580	-0.0062
0.02	-0.5314	-0.0118
0.03	-0.5067	-0.0170
0.04	-0.4837	-0.0218
0.05	-0.4622	-0.0262
0.06	-0.4421	-0.0302
0.07	-0.4233	-0.0339

Table 7: *Sensitivity and relative sensitivity analysis of survivability* (S_0) *of the system with respect to major damage rate* λ_{13}

λ_{13}	$\pi_{\lambda_{13}} = \frac{\partial S_0}{\partial \lambda_{13}}$	$\delta_{\lambda_{13}} = \pi_{\lambda_{13}}(\frac{\lambda_{13}}{S_0})$
0.01	-0.0265	-0.0003
0.02	-0.0260	-0.0006
0.03	-0.0255	-0.0009
0.04	-0.0250	-0.0011
0.05	-0.0245	-0.0014
0.06	-0.0241	-0.0016
0.07	-0.0236	-0.0018

Table 8: *Sensitivity and relative sensitivity analysis of survivability (S_0) of the system with respect to failure rate λ_{34}*

λ_{34}	$\pi_{\lambda_{34}} = \frac{\partial S_0}{\partial \lambda_{34}}$	$\delta_{\lambda_{34}} = \pi_{\lambda_{34}} (\frac{\lambda_{34}}{S_0})$
0.01	-0.0484	-0.0005
0.02	-0.0456	-0.0010
0.03	-0.0431	-0.0014
0.04	-0.0408	-0.0018
0.05	-0.0386	-0.0022
0.06	-0.0367	-0.0025
0.07	-0.0348	-0.0027

Table 9: *Sensitivity and relative sensitivity analysis of survivability (S_0) of the system with respect to recovery rate β_1*

β_1	$\pi_{\beta_1} = \frac{\partial S_0}{\partial \beta_1}$	$\delta_{\beta_1} = \pi_{\beta_1} (\frac{\beta_1}{S_0})$
0.1	0.0031	0.00034
0.2	0.0014	0.00031
0.3	0.0008	0.00026
0.4	0.0005	0.00022
0.5	0.0004	0.00020
0.6	0.0003	0.00017
0.7	0.0002	0.00015

Table 10: *Sensitivity and relative sensitivity analysis of survivability (S_0) of the system with respect to recovery rate β_2*

β_2	$\pi_{\beta_2} = \frac{\partial S_0}{\partial \beta_2}$	$\delta_{\beta_2} = \pi_{\beta_2} (\frac{\beta_2}{S_0})$
0.1	0.2040	0.0234
0.2	0.0836	0.0189
0.3	0.0452	0.0152
0.4	0.0283	0.0126
0.5	0.0193	0.0108
0.6	0.0140	0.0094
0.7	0.0107	0.0083

Table 11: *Sensitivity and relative sensitivity analysis of survivability (S_0) of the system with respect to recovery rate β_3*

β_3	$\pi_{\beta_3} = \frac{\partial S_0}{\partial \beta_3}$	$\delta_{\beta_3} = \pi_{\beta_3} (\frac{\beta_3}{S_0})$
0.1	0.0295	0.0033
0.2	0.0110	0.0025
0.3	0.0057	0.0019
0.4	0.0035	0.0016
0.5	0.0024	0.0013
0.6	0.0017	0.0011
0.7	0.0013	0.0009

Table 12: *Sensitivity and relative sensitivity analysis of survivability* (S_0) *of the system with respect to transplantation rate* γ

γ	$\pi_\gamma = \frac{\partial S_0}{\partial \gamma}$	$\delta_\gamma = \pi_\gamma (\frac{\gamma}{S_0})$
0.1	0.9198	0.1025
0.2	0.2555	0.0540
0.3	0.1177	0.0367
0.4	0.0675	0.0278
0.5	0.0437	0.0223
0.6	0.0305	0.0187
0.7	0.0266	0.0161

The sensitivity analysis of model for mean survival time and survivability of the system with respect to heart damage rates, failure rates and recovery rates are explained in table 1to table 12. The sign of sensitivity of mean survival time and survivability of the system with respect to heart damage rates $(\lambda_{01}, \lambda_{02})$, major damage rate (λ_{13}) and failure rates $(\lambda_{04}, \lambda_{24}, \lambda_{34})$ are negative. This shows that increase in these parameters decrease the value of mean survival time and survivability of the system. The sign of sensitivity of mean survival time and survivability of the system with respect to recovery rates $(\beta_1, \beta_2, \beta_3)$ and transplantation rate (γ) of the heart are positive which means increase in these parameters improve the value of mean survival time and survivability of the system. For example, in case of sensitivity of mean survival time (T_0) and survivability (S_0) of the system with respect to heart damage rate λ_{02} shows that increase in the heart damage rate due to some organ issues decrease the value of mean survival time and survivability of the system. Further, it can be observed from the analyses that mean survival time is more sensitive towards the values of heart damage rate due to some organ issues/failure rate of heart due to severe accidents/recovery from other organ issues and survivability of the system is more sensitive towards heart transplantation rate.

10. Conclusion

In the human body, the cardiovascular system is the most vital system. The stochastic model and analysis presented in the paper is a simple and concise approach for understanding and investigating the human cardiovascular system considering its various causes heart damage and failure issues. This study is quite helpful to make prediction about patients' mean survival time and survivability and accordingly to take appropriate measure to treat/cure the patients. Further graphical and sensitivity analyses of the proposed model highlight the impacts of different rates of damage, failure, recovery and transplantation of heart on mean survival time and survivability of the system. The important factors/rates that can help to enhance survivability of the system can be easily selected. The investigation through the stochastic analysis of the system considering various causes of heart damage and failure concludes that the mean survival time of the system deceases with the increase in the rates of prevalence of heart diseases, other organ issues and severe accidents. However, the mean survival time and survivability of the system increases with the increase in the recovery/transplantation rates through medicine/exercise/therapy/surgery of the heart. The evaluated expressions for mean sojourn time in the different states of the system gives estimates of the times for cardiac patient remains in a particular state. Investigations also conclude that heart failure rate of due to severe accidents and heart transplantation rate play crucial roles as far as mean survival time and survivability of the system is concerned. Thus, survivability of the patient may be enhanced controlling these rates taking appropriate measures.

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