RELIABILITY ANALYSIS OF A POWER DISTRIBUTION SYSTEM WITH TWO TRANSFORMERS AND SIX FEEDERS

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Abstract

The article explores the reliability and sensitivity of a power distribution substation. It includes an analysis based on real maintenance data collected from a 33/11kV electrical power distribution substation, which features a set of two 6 MVA power transformers supplying power through a total of six outgoing feeders (three feeders per transformer). The study documents faults observed in both transformers and all six outgoing feeders. The reliability of the substation is evaluated using various indices such as availability, repair durations, and expected repair frequencies for different failure types. The analysis employs Markov processes and regenerative point techniques. In addition to reliability, the study includes a profit analysis of the substation. It presents graphical representations of key parameters. Furthermore, a sensitivity analysis is conducted to assess how variations in parameters impact the availability and profitability of the substation components. Substation economics is also established to assess the operational viability.

Keywords: failure, reliability, transformers, Markov process, regenerative processes.

I. Introduction

Reliability modeling and analysis of industrial systems have become pivotal in ensuring the optimal performance and longevity of complex machinery and processes. As industries become increasingly dependent on sophisticated technology, the need to understand and predict system reliability has become more critical. This field involves developing mathematical and statistical models that simulate various failure and repair scenarios, allowing engineers to predict system behavior under different conditions. These models are essential for designing maintenance strategies that minimize downtime and costs while maximizing operational efficiency and profitability. By analyzing real failure and maintenance data, these models can provide valuable insights into the reliability indices of systems, such as system availability, repair durations, and expected repair frequencies for different failure types. This comprehensive approach helps industries to not only improve their maintenance policies but also enhance the overall reliability and performance of their systems, thereby ensuring sustained productivity and economic operational benefits. Over the years, numerous studies have contributed to this domain, providing various models and methodologies to enhance system reliability and economic efficiency.

Parashar and Taneja [1] conducted a seminal study on the reliability and profit evaluation of a hot standby PLC system utilizing a master-slave configuration and dual repair facilities, establishing a foundational approach to reliability studies in complex systems. In the same year, Nilsson and Bertling [2] explored the maintenance management of wind power systems using condition monitoring systems and life cycle cost analysis, highlighting the importance of maintenance in energy sustainability. Mathew et al. [3] extended these concepts into the industrial manufacturing sector by modeling the reliability of a single-unit continuous casting (CC) plant with scheduled maintenance, providing insights into maintenance strategies that enhance operational continuity. Further expanding the scope, Rizwan et al. [4] analyzed a hot standby industrial system, emphasizing the critical balance between reliability and operational efficiency. The reliability modeling of a two-unit continuous casting plant was advanced by Mathew et al. [5], who offered detailed insights into the maintenance strategies necessary for operational stability in industrial settings. In another study, Shakuntla et al. [6] utilized supplementary variable techniques for reliability analysis in the polytube industry, demonstrating the utility of advanced mathematical techniques in predicting system behavior. In the realm of communication systems, Kumar and Kapoor [7] assessed the profitability of a base transceiver system considering hardware failures and system congestion, reflecting the intricate balance between operational efficiency and reliability. Similarly, Rizwan et al. [8] conducted a reliability analysis of a seven-unit desalination plant, incorporating both major and minor failures and highlighting the seasonal impact on system performance. Further research by Padmavathi et al. [9] on a desalination plant with major and minor failures underscored the importance of probabilistic analysis in understanding system reliability under varying conditions. This theme was continued by Rizwan et al. [10] in their analysis of an anaerobic batch reactor treating fruit and vegetable waste, which provided valuable insights into the reliability and availability of biogas production systems. The comparative analysis of reliability models for a desalination plant by Padmavathi et al. [11] and the performance analysis of a desalination plant with mandatory shutdowns by Rizwan et al. [12] further contributed to the understanding of maintenance strategies and their economic impacts. Additionally, Ahmad and Kumar [13] analyzed the profit implications of operational halts in a two-unit centrifuge system, illustrating the financial consequences of reliability. Adlakha et al. [14] explored the reliability and cost-benefit analysis of a two-unit cold standby system used for satellite communication, emphasizing the critical nature of reliability in high-stake environments. Naithani et al. [15] prioritized repair in their analysis of a three-unit induced draft fan system with a warm standby, enhancing operational reliability. Al Rahbi et al. [16] investigated the reliability challenges in the aluminum industry, specifically in a rodding anode plant with multiple units and a single repairman, highlighting the complexities of maintaining operational continuity amidst multiple failures. Taj et al. [17] provided a comparative analysis of three reliability models of a building cable manufacturing plant, illustrating the ongoing evolution and refinement of reliability assessment techniques. Kaur et al. [18] analyzed the reliability of a gravity die casting system, addressing diverse failure types that impact production processes. Sachdeva et al. [19] analyzed the reliability and sensitivity of an insured system where the warranty duration exceeds the insurance duration. Most recently, Oraimi et al. [20] conducted a sensitivity and profitability analysis of a two-units ammonia/urea plant, providing insights into optimizing system performance and economic viability under various conditions. Hussien and El-Sherbeny [21] examined the reliability and availability of a single-unit system under random shocks and varying demand, adding to the body of knowledge on stochastic behaviour in production systems. Finally, Rani et al. [22] explored the reliability of a two non-identical unit standby system with correlated failures, further enriching the literature on system reliability and maintenance optimization. These studies collectively underscore the critical importance of advanced reliability modeling across various industrial and environmental applications. They offer profound insights into system design and maintenance optimization, ultimately enhancing overall operational efficacy and sustainability.

In the context of electrical energy transmission and distribution, the reliability of power transformers is crucial for maintaining consistent and efficient power delivery. This paper presents a reliability and the profit analysis of a power distribution system comprising transformers feeding power through feeders, aiming to obtain reliability indices that reflect the system's behaviour and conduct a sensitivity analysis that underscores the economic viability of the substation operations. The study explores the causes of power unavailability from transformers, which may arise from environmental conditions such as severe weather, including cuts or heavy winds, and electrical faults like short circuits. Markovian processes are employed which are well-suited for analyzing systems with probabilistic state transitions that adhere to the memoryless property, where future states are influenced solely by the current state. The effectiveness of the model based on the Markovian process has been proven in various reliability applications [19], making them a preferred tool for systems with state-based transitions, the analysis provides a detailed examination of the system's stochastic behavior over time. Key reliability metrics, including system availability are obtained to evaluate the system's performance, utilizing real transformer data on failures and repairs. The analysis reveals the significant impact of environmental conditions and electrical faults on transformer unavailability. A sensitivity analysis further evaluates how variations in transformer failure and repair rates influence overall reliability and profitability of the system providing valuable insights into the determinants of reliability for the distribution system under consideration. The findings form the basis for enhancing system robustness by addressing key determinants of system reliability. Additionally, the paper opens the directions for future research to further explore and mitigate reliability challenges in power distribution systems, thereby contributing to the development of more reliable and resilient power infrastructure.

II. Model Description and Assumptions

I. Model Description

The electrical distribution substation, which divides and distributes electrical power to various areas of the power distribution region, resembles an enormous junction as shown in Fig.1. The two major cables, which we refer to as incoming feeders, provide this substation with power at 33 kV. The two 6 MVA transformers are used to make the power available at 11 kV for its subsequent distribution to loads in different areas. Three outgoing feeders receive power from each transformer. As a result, there are six outgoing feeders that are carrying power at 11 kV to different areas. Various household as well as industrial establishments receive the power from each of these feeders. Typically, the load on each feeder is nearly balanced.

II. Assumptions

- Initially the system is operative.
- Both the transformers are working well and can’t fail simultaneously.
- The three feeders connected from each transformer are working properly and only one can fail at one time from three feeders connected to one transformer.
- All the states are regenerative.
- All the failure and repair times follows exponential distribution.

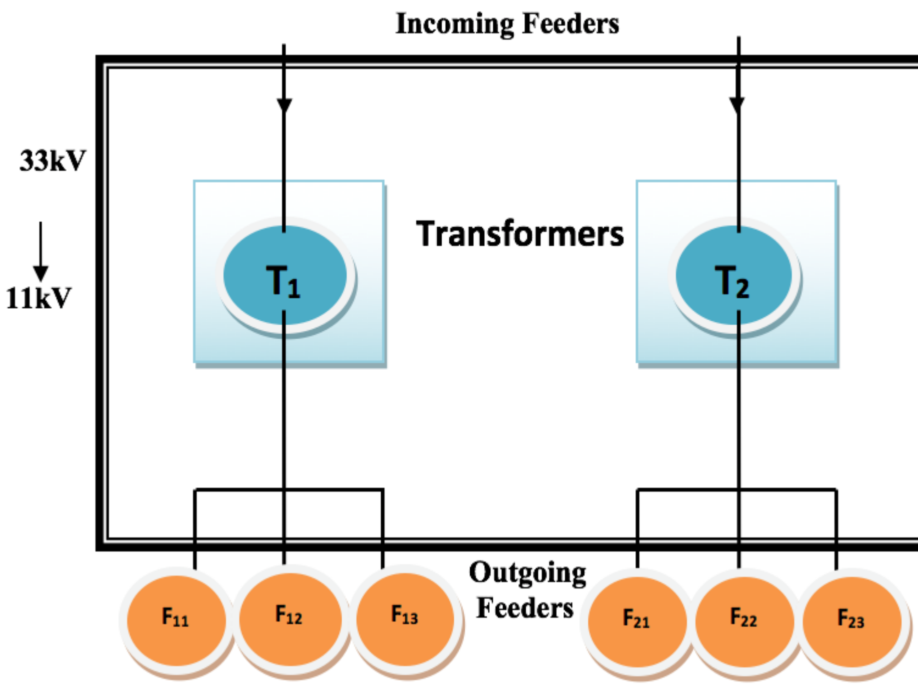

Figure 1: *Simplistic schematic for a Power Distribution Substation*

III. Notations

The following notations are used for rates of failure and repair of system

$\lambda_{t_1}, \lambda_{t_2}$ = Failure rate of transformers 1, transformers 2.

$\lambda_{f_{11}}, \lambda_{f_{12}}, \lambda_{f_{13}}$= Failure rate of feeder 1, feeder 2, feeder 3 that are connected from transformer 1.

$\lambda_{f_{21}}, \lambda_{f_{22}}, \lambda_{f_{23}}$= Failure rate of feeder 1, feeder 2, feeder 3 that are connected from transformer 2.

μ_{t_1}, μ_{t_2} = Repair rate of transformers 1, transformer 2.

$\mu_{f_{11}}, \mu_{f_{12}}, \mu_{f_{13}}$= Repair rate of feeder 1, feeder 2, feeder 3 that are connected from transformer 1.

$\mu_{f_{21}}, \mu_{f_{22}}, \mu_{f_{23}}$= Repair rate of feeder 1, feeder 2, feeder 3 that are connected from transformer 2.

IV. Data Summary

From the component wise failure and maintenance data obtained for the 33/11 kV electrical distribution substation for the previous 10 years, the following failure as well as repair rates are estimated:

Failure rate of transformer 1, λ_{t_1}= 1.14889 x 10^{-5}

Failure rate of transformer 2, λ_{t_2}= 1.90981 x 10^{-5}

Failure rate of feeder 1 that is connected from transformer 1, $\lambda_{f_{11}}$=3.05518 x 10^{-5}

Failure rate of feeder 2 that is connected from transformer 1, $\lambda_{f_{12}}$= 5.7282 x 10^{-5}

Failure rate of feeder 3 that is connected from transformer 1, $\lambda_{f_{13}}$=2.6732 x 10^{-5}

Failure rate of feeder 1 that is connected from transformer 2, $\lambda_{f_{21}}$ = 1.1456 x10^{-5}

Failure rate of feeder 2 that is connected from transformer 2, $\lambda_{f_{22}}$ = 1.5275 x 10^{-5}

Failure rate of feeder 3 that is connected from transformer 2, $\lambda_{f_{23}}$ = 2.2913 x 10^{-5}

Repair rate of transformer 1, μ_{t_1}= 0.3891

Repair rate of transformer 2, μ_{t_2}= 0.0881

Repair rate of feeder 1 that is connected from transformer 1, $\mu_{f_{11}}$= 0.6525
Repair rate of feeder 2 that is connected from transformer 1, $\mu_{f_{12}}$= 0.6310
Repair rate of feeder 3 that is connected from transformer 1, $\mu_{f_{13}}$= 0.5254
Repair rate of feeder 1 that is connected from transformer 2, $\mu_{f_{21}}$ =1.3793
Repair rate of feeder 2 that is connected from transformer 2, $\mu_{f_{22}}$ = 1.2766
Repair rate of feeder 3 that is connected from transformer 2, $\mu_{f_{23}}$ = 0.6173

V. Stochastic Model

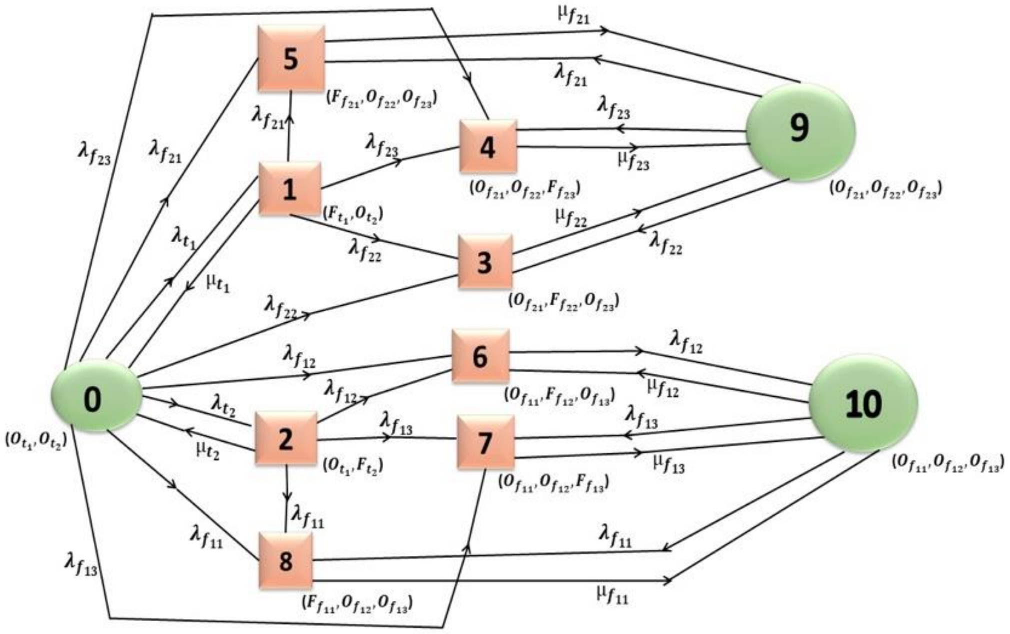

Figure 2: *State Transition Diagram*

Table 1: *State symbol and their meaning*

State No.	State Symbol	Description
State 0	(O_{t_1}, O_{t_2})	Both the transformers are operative.
State 1	(F_{t_1}, O_{t_2})	Transformer 1 failed and Transformer 2 operative.
State 2	(O_{t_1}, F_{t_2})	Transformer 1 operative and Transformer 2 failed.
State 3	$(O_{f_{21}}, F_{f_{22}}, O_{f_{23}})$	Feeders that are connected from transformer 2, first and third are operative, second feeder failed.
State 4	$(O_{f_{21}}, O_{f_{22}}, F_{f_{23}})$	Feeders that are connected from transformer 2, first and second are operative, third feeder failed.
State 5	$(F_{f_{21}}, O_{f_{22}}, O_{f_{23}})$	Feeders that are connected from transformer 2, first feeder failed while second and third are operative.
State 6	$(O_{f_{11}}, F_{f_{12}}, O_{f_{13}})$	Feeders that are connected from transformer 1, second feeder failed while first and third are operative.
State 7	$(O_{f_{11}}, O_{f_{12}}, F_{f_{13}})$	Feeders that are connected from transformer 1, third feeder failed while first and second are operative.
State 8	$(F_{f_{11}}, O_{f_{12}}, O_{f_{13}})$	Feeders that are connected from transformer 1, first feeder failed while second and third are operative.
State 9	$(O_{f_{21}}, O_{f_{22}}, O_{f_{23}})$	All the feeders that are connected from transformer 2 are operative.
State 10	$(O_{f_{11}}, O_{f_{12}}, O_{f_{13}})$	All the feeders that are connected from transformer 1 are operative.

Figure 2 shows the transition between states of system. The set of states {0, 9, 10} are operative states and set {1, 2, 3, 4, 5, 6, 7, 8} are partially operative states. All the states are regenerative states. The description of states along with their symbols are given in Table 1.

The transition densities from state r to state s (q_{rs}) are,

$q_{01}(t) = \lambda_{t_1} e^{-(\lambda_{t_1}+\lambda_{t_2})t} e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$; $q_{02}(t) = \lambda_{t_2} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$

$q_{03}(t) = \lambda_{f_{22}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$; $q_{04}(t) = \lambda_{f_{23}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$

$q_{05}(t) = \lambda_{f_{21}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$; $q_{06}(t) = \lambda_{f_{12}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$

$q_{07}(t) = \lambda_{f_{13}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$; $q_{08}(t) = \lambda_{f_{11}} e^{-(\lambda_{t_1}+\lambda_{t_2})t}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})t}$

$q_{13}(t) = \lambda_{f_{22}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $q_{14}(t) = \lambda_{f_{23}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $q_{15}(t) = \lambda_{f_{21}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$;

$q_{10}(t) = \mu_{t_1}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $q_{26}(t) = \lambda_{f_{12}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $q_{27}(t) = \lambda_{f_{13}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$

$q_{28}(t) = \lambda_{f_{11}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $q_{20}(t) = \mu_{t_2}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $q_{39}(t) = \mu_{f_{22}}. e^{-\mu_{f_{22}}(t)}$;

$q_{49}(t) = \mu_{f_{23}}. e^{-\mu_{f_{23}}(t)}$; $q_{59}(t) = \mu_{f_{21}}. e^{-\mu_{f_{21}}(t)}$; $q_{6,10}(t) = \mu_{f_{12}}. e^{-\mu_{f_{12}}(t)}$; $q_{7,10}(t) = \mu_{f_{13}}. e^{-\mu_{f_{13}}(t)}$;

$q_{8,10}(t) = \mu_{f_{11}}. e^{-\mu_{f_{11}}(t)}$; $q_{93}(t) = \lambda_{f_{22}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})t}$; $q_{94}(t) = \lambda_{f_{23}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})t}$;

$q_{95}(t) = \lambda_{f_{21}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{2i}})t}$; $q_{10,6}(t) = \lambda_{f_{12}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})t}$; $q_{10,7}(t) = \lambda_{f_{13}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})t}$;

$q_{10,8}(t) = \lambda_{f_{11}}. e^{-\sum_{i=1}^{3}(\lambda_{f_{1i}})t}$

Steady-state probability, p_{ij} as

$p_{01} = \frac{\lambda_{t_1}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{02} = \frac{\lambda_{t_2}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{03} = \frac{\lambda_{f_{22}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$;

$p_{04} = \frac{\lambda_{f_{23}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{05} = \frac{\lambda_{f_{21}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{06} = \frac{\lambda_{f_{12}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$;

$p_{07} = \frac{\lambda_{f_{13}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{08} = \frac{\lambda_{f_{11}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}}$; $p_{13} = \frac{\lambda_{f_{22}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $p_{14} = \frac{\lambda_{f_{23}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$;

$p_{15} = \frac{\lambda_{f_{21}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $p_{10} = \frac{\mu_{t_1}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}}$; $p_{26} = \frac{\lambda_{f_{12}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $p_{27} = \frac{\lambda_{f_{13}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$;

$p_{28} = \frac{\lambda_{f_{11}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $p_{20} = \frac{\mu_{t_2}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}}$; $p_{39} = p_{49} = p_{59} = p_{6,10} = p_{7,10} = p_{8,10} = 1$

$p_{93} = \frac{\lambda_{f_{22}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})}$; $p_{94} = \frac{\lambda_{f_{23}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})}$; $p_{95} = \frac{\lambda_{f_{21}}}{\sum_{i=1}^{3}(\lambda_{f_{2i}})}$; $p_{10,6} = \frac{\lambda_{f_{12}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})}$; $p_{10,7} = \frac{\lambda_{f_{13}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})}$;

$p_{10,8} = \frac{\lambda_{f_{11}}}{\sum_{i=1}^{3}(\lambda_{f_{1i}})}$

The contribution to mean sojourn time (m_{ij}) is given by $m_{ij} = \int_0^\infty t\, q_{ij}(t)dt$, we get

$m_{01} = \frac{\lambda_{t_1}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{02} = \frac{\lambda_{t_2}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{03} = \frac{\lambda_{f_{22}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$

$m_{04} = \frac{\lambda_{f_{23}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{05} = \frac{\lambda_{f_{21}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{06} = \frac{\lambda_{f_{12}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$

$m_{07} = \frac{\lambda_{f_{13}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{08} = \frac{\lambda_{f_{11}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}}+\lambda_{f_{2i}})+\lambda_{t_1}+\lambda_{t_2}\right)^2}$; $m_{13} = \frac{\lambda_{f_{22}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}\right)^2}$

$m_{14} = \frac{\lambda_{f_{23}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}\right)^2}$; $m_{15} = \frac{\lambda_{f_{21}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}\right)^2}$; $m_{10} = \frac{\mu_{t_1}}{\left(\sum_{i=1}^{3}(\lambda_{f_{2i}})+\mu_{t_1}\right)^2}$; $m_{26} = \frac{\lambda_{f_{12}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}\right)^2}$

$m_{27} = \frac{\lambda_{f_{13}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}\right)^2}$; $m_{28} = \frac{\lambda_{f_{11}}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}\right)^2}$; $m_{20} = \frac{\mu_{t_2}}{\left(\sum_{i=1}^{3}(\lambda_{f_{1i}})+\mu_{t_2}\right)^2}$; $m_{39} = \frac{1}{\mu_{f_{22}}}$; $m_{49} = \frac{1}{\mu_{f_{23}}}$; $m_{59} = \frac{1}{\mu_{f_{21}}}$;

$m_{6,10} = \frac{1}{\mu_{f_{12}}}$; $m_{7,10} = \frac{1}{\mu_{f_{13}}}$; $m_{8,10} = \frac{1}{\mu_{f_{11}}}$; $m_{93} = \frac{\lambda_{f_{22}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{2i}})\right]^2}$; $m_{94} = \frac{\lambda_{f_{23}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{2i}})\right]^2}$;

$m_{95} = \frac{\lambda_{f_{21}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{2i}})\right]^2}$; $m_{10,6} = \frac{\lambda_{f_{12}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{1i}})\right]^2}$; $m_{10,7} = \frac{\lambda_{f_{13}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{1i}})\right]^2}$; $m_{10,8} = \frac{\lambda_{f_{11}}}{\left[\sum_{i=1}^{3}(\lambda_{f_{1i}})\right]^2}$.

Now if μ_i is the mean stay time in particular state i, then

$m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_{06} + m_{07} + m_{08} = \mu_0$

$m_{13} + m_{14} + m_{15} + m_{10} = \mu_1$; $m_{26} + m_{27} + m_{28} + m_{20} = \mu_2$; $m_{39} = \mu_3$; $m_{49} = \mu_4$; $m_{59} = \mu_5$,

$m_{6,10} = \mu_6$; $m_{7,10} = \mu_7$; $m_{8,10} = \mu_8$; $m_{93} + m_{94} + m_{95} = \mu_9$; $m_{10,6} + m_{10,7} + m_{10,8} = \mu_{10}$

VI. System Measures

I. System Availability

Let $Ad_i(t)$ = Pr {system is operative at time t, given that it is in the state i at time t=0}. By the transition of states and definition of $Ad_i(t)$, we get the following equations as

$Ad_0(t) = M_0(t) + q_{01}(t)©Ad_1(t) + q_{02}(t)©Ad_2(t) + q_{03}(t)©Ad_3(t) + q_{04}(t)©Ad_4(t) + q_{05}(t)©Ad_5(t) + q_{06}(t)©Ad_6(t) + q_{07}(t)©Ad_7(t) + q_{08}(t)©Ad_8(t)$ (1)

$Ad_1(t) = q_{13}(t)©Ad_3(t) + q_{14}(t)©Ad_4(t) + q_{15}(t)©Ad_5(t) + q_{10}(t)©Ad_0(t)$ (2)

$Ad_2(t) = q_{26}(t)©Ad_6(t) + q_{27}(t)©Ad_7(t) + q_{28}(t)©Ad_8(t) + q_{20}(t)©Ad_0(t)$ (3)

$Ad_3(t) = q_{39}(t)©Ad_9(t)$ (4)

$Ad_4(t) = q_{49}(t)©Ad_9(t)$ (5)

$Ad_5(t) = q_{59}(t)©Ad_9(t)$ (6)

$Ad_6(t) = q_{6,10}(t)©Ad_{10}(t)$ (7)

$Ad_7(t) = q_{7,10}(t)©Ad_{10}(t)$ (8)

$Ad_8(t) = q_{8,10}(t)©Ad_{10}(t)$ (9)

$Ad_9(t) = M_9(t) + q_{93}(t)©Ad_3(t) + q_{94}(t)©Ad_4(t) + q_{95}(t)©Ad_5(t)Ad_{10}(t) = M_{10}(t) + q_{10,6}(t)©Ad_6(t) + q_{10,7}(t)©Ad_7(t) + q_{10,8}(t)©Ad_8(t)$ (10)

where,
$M_i(t)$ = probability that the system stays in state i while operating rather than transferring to any other state.

Taking Laplace Transform of the above equations and solving for $Ad_0^*(s)$, we get

$Ad_0^*(s) = \frac{N_1(s)}{D_1(s)}$ (11)

where, $N_1(s)$ & $D_1(s)$ as as obtained solving equations (1) to (10)

Now, the availability of the system is steady state is given as

$Ad_0 = \lim_{t\to\infty} Ad_0(t) = \lim_{s\to 0} s\, Ad_0^*(s) = \lim_{s\to 0} s\, \frac{N_1(s)}{D_1(s)} = \frac{N_1}{D_1}$ (12)

is evaluated using determinants in $N_1(s)$ and $D_1(s)$.

II. Busy Period for Repair

Similarly, the expected duration during which the repairman is occupied with the repair of transformer 1 and transformer 2, respectively, can be determined in steady state and is given by

$Bd_{t_1} = \lim_{s\to 0} s\, Bd_{t_1}^*(s) = \lim_{s\to 0} s\, \frac{N_{t_1}^b(s)}{D_1(s)} = \frac{N_{t_1}^b}{D_1}$ (13)

$Bd_{t_2} = \lim_{s\to 0} s\, Bd_{t_2}^*(s) = \lim_{s\to 0} s\, \frac{N_{t_2}^b(s)}{D_1(s)} = \frac{N_{t_2}^b}{D_1}$ (14)

where,

$N_{t_1}^b(s) = q_{01}^*(s)W_1^*(s)(q_{93}^*(s)q_{10,6}^*(s) - q_{94}^*(s) - q_{95}^*(s) - q_{10,6}^*(s) - q_{10,7}^*(s) - q_{10,8}^*(s) - q_{93}^*(s) + q_{93}^*(s)q_{10,7}^*(s) + q_{94}^*(s)q_{10,6}^*(s) + q_{93}^*(s)q_{10,8}^*(s) + q_{94}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,6}^*(s) + q_{94}^*(s)q_{10,8}^*(s) + q_{95}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,8}^*(s) + 1)$;

$N_{t_2}^b(s) = q_{02}^*(s)W_2^*(s)(q_{93}^*(s)q_{10,6}^*(s) - q_{94}^*(s) - q_{95}^*(s) - q_{10,6}^*(s) - q_{10,7}^*(s) - q_{10,8}^*(s) - q_{93}^*(s) + q_{93}^*(s)q_{10,7}^*(s) + q_{94}^*(s)q_{10,6}^*(s) + q_{93}^*(s)q_{10,8}^*(s) + q_{94}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,6}^*(s) + q_{94}^*(s)q_{10,8}^*(s) + q_{95}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,8}^*(s) + 1)$.

Here, $W_i(t)$ = probability that the system stays in state i while repairing rather than transferring to any other state.

The expected time in which the repairman is busy for the repair of the feeder 1, 2 and 3, that are connected from transformer 1, respectively, in steady state, is given by

$Bd_{f_{11}} = \lim_{s\to 0} s\, Bd_{f_{11}}^*(s) = \lim_{s\to 0} s\, \frac{N_{f_{11}}^b(s)}{D_1(s)} = \frac{N_{f_{11}}^b}{D_1}$; (15)

$$Bd_{f_{12}} = \lim_{s\to 0} s\, Bd^*_{f_{12}}(s) = \lim_{s\to 0} s\, \frac{N^b_{f_{12}}(s)}{D_1(s)} = \frac{N^b_{f_{12}}}{D_1}; \tag{16}$$

$$Bd_{f_{13}} = \lim_{s\to 0} s\, Bd^*_{f_{13}}(s) = \lim_{s\to 0} s\, \frac{N^b_{f_{13}}(s)}{D_1(s)} = \frac{N^b_{f_{13}}}{D_1}; \tag{17}$$

where, $N^b_{f_{11}}(s)$, $N^b_{f_{12}}(s)$ & $N^b_{f_{13}}(s)$ are obtained as above.

Continuing in the same way, the expected time the repairman is busy for the repair of the feeder 1, 2 and 3, that are connected from transformer 2, respectively, in steady state, is given by

$$Bd_{f_{21}} = \lim_{s\to 0} s\, Bd^*_{f_{21}}(s) = \lim_{s\to 0} s\, \frac{N^b_{f_{21}}(s)}{D_1(s)} = \frac{N^b_{f_{21}}}{D_1}; \tag{18}$$

$$Bd_{f_{22}} = \lim_{s\to 0} s\, Bd^*_{f_{22}}(s) = \lim_{s\to 0} s\, \frac{N^b_{f_{22}}(s)}{D_1(s)} = \frac{N^b_{f_{22}}}{D_1}; \tag{19}$$

$$Bd_{f_{23}} = \lim_{s\to 0} s\, Bd^*_{f_{23}}(s) = \lim_{s\to 0} s\, \frac{N^b_{f_{23}}(s)}{D_1(s)} = \frac{N^b_{f_{23}}}{D_1}; \tag{20}$$

where, $N^b_{f_{21}}(s)$, $N^b_{f_{22}}(s)$ & $N^b_{f_{23}}(s)$ are obtained as above.

III. Expected Number of Repair

The expected time of repairs of transformer 1 and transformer 2, respectively, in steady state is given by

$$Nd_{t_1} = \lim_{s\to 0} s\, Nd^*_{t_1}(s) = \lim_{s\to 0} s\, \frac{N^n_{t_1}(s)}{D_1(s)} = \frac{N^n_{t_1}}{D_1}; \tag{21}$$

$$Nd_{t_2} = \lim_{s\to 0} s\, Nd^*_{t_2}(s) = \lim_{s\to 0} s\, \frac{N^n_{t_2}(s)}{D_1(s)} = \frac{N^n_{t_2}}{D_1}; \tag{22}$$

where,

$N^n_{t_1}(s) = q_{01}^*(s)\, q_{10}^*(s)(q_{93}^*(s)q_{10,6}^*(s) - q_{94}^*(s) - q_{95}^*(s) - q_{10,6}^*(s) - q_{10,7}^*(s) - q_{10,8}^*(s) - q_{93}^*(s) + q_{93}^*(s)q_{10,7}^*(s) + q_{94}^*(s)q_{10,6}^*(s) + q_{93}^*(s)q_{10,8}^*(s) + q_{94}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,6}^*(s) + q_{94}^*(s)q_{10,8}^*(s) + q_{95}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,8}^*(s) + 1);$

$N^n_{t_2}(s) = q_{02}^*(s)\, q_{20}^*(s)(q_{93}^*(s)q_{10,6}^*(s) - q_{94}^*(s) - q_{95}^*(s) - q_{10,6}^*(s) - q_{10,7}^*(s) - q_{10,8}^*(s) - q_{93}^*(s) + q_{93}^*(s)q_{10,7}^*(s) + q_{94}^*(s)q_{10,6}^*(s) + q_{93}^*(s)q_{10,8}^*(s) + q_{94}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,6}^*(s) + q_{94}^*(s)q_{10,8}^*(s) + q_{95}^*(s)q_{10,7}^*(s) + q_{95}^*(s)q_{10,8}^*(s) + 1);$

The expected number of repairs of the feeder 1, 2 and 3, that are connected from transformer 1, respectively, in steady state, is given by,

$$Nd_{f_{11}} = \lim_{s\to 0} s\, Nd^*_{f_{11}}(s) = \lim_{s\to 0} s\, \frac{N^n_{f_{11}}(s)}{D_1(s)} = \frac{N^n_{f_{11}}}{D_1}; \tag{23}$$

$$Nd_{f_{12}} = \lim_{s\to 0} s\, Nd^*_{f_{12}}(s) = \lim_{s\to 0} s\, \frac{N^n_{f_{12}}(s)}{D_1(s)} = \frac{N^n_{f_{12}}}{D_1}; \tag{24}$$

$$Nd_{f_{13}} = \lim_{s\to 0} s\, Nd^*_{f_{13}}(s) = \lim_{s\to 0} s\, \frac{N^n_{f_{13}}(s)}{D_1(s)} = \frac{N^n_{f_{13}}}{D_1}; \tag{25}$$

where,

$$\begin{aligned}
N^n_{f_{11}}(s) = &-q_{8,10}^*(s)(q_{08}^*(s)q_{93}^*(s) - q_{02}^*(s)q_{28}^*(s) - q_{08}^*(s) + q_{08}^*(s)q_{94}^*(s) + q_{08}^*(s)q_{95}^*(s) \\
&- q_{06}^*(s)q_{10,8}^*(s) + q_{08}^*(s)q_{10,6}^*(s) - q_{07}^*(s)q_{10,8}^*(s) + q_{08}^*(s)q_{10,7}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{93}^*(s) \\
&+ q_{02}^*(s)q_{28}^*(s)q_{94}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{95}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{10,8}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{10,6}^*(s) \\
&- q_{02}^*(s)q_{27}^*(s)q_{10,8}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{10,7}^*(s) + q_{06}^*(s)q_{93}^*(s)q_{10,8}^*(s) - {}_{08}^*(s)q_{93}^*(s)q_{10,6}^*(s) \\
&+ q_{06}^*(s)q_{94}^*(s)q_{10,8}^*(s) + q_{07}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{08}^*(s)q_{93}^*(s)q_{10,7}^*(s) \\
&- q_{08}^*(s)q_{94}^*(s)q_{10,6}^*(s) + q_{06}^*(s)q_{95}^*(s)q_{10,8}^*(s) + q_{07}^*(s)q_{94}^*(s)q_{10,8}^*(s) \\
&- q_{08}^*(s)q_{94}^*(s)q_{10,7}^*(s) - q_{08}^*(s)q_{95}^*(s)q_{10,6}^*(s) + q_{07}^*(s)q_{95}^*(s)q_{10,8}^*(s) - q_{08}^*(s)q_{95}^*(s)q_{10,7}^*(s) \\
&+ q_{02}^*(s)q_{26}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{93}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{94}^*(s)q_{10,8}^*(s) \\
&+ q_{02}^*(s)q_{27}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{93}^*(s)q_{10,7}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{94}^*(s)q_{10,6}^*(s) \\
&+ q_{02}^*(s)q_{26}^*(s)q_{95}^*(s)q_{10,8}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{94}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{94}^*(s)q_{10,7}^*(s) \\
&- q_{02}^*(s)q_{28}^*(s)q_{95}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{95}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{95}^*(s)q_{10,7}^*(s));
\end{aligned}$$

$$\begin{aligned}
N^n_{f_{12}}(s) = &- q_{6,10}^*(s)\,(q_{06}^*(s)q_{93}^*(s) - q_{02}^*(s)q_{26}^*(s) - q_{06}^*(s) + q_{06}^*(s)q_{94}^*(s) + q_{06}^*(s)q_{95}^*(s) \\
&+ q_{06}^*(s)q_{10,7}^*(s) - q_{07}^*(s)q_{10,6}^*(s) + q_{06}^*(s)q_{10,8}^*(s) - q_{08}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{93}^*(s) \\
&+ q_{02}^*(s)q_{26}^*(s)q_{94}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{95}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{10,7}^*(s) - q_{02}^*(s)q_{27}^*(s)q_{10,6}^*(s) \\
&+ q_{02}^*(s)q_{26}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{10,6}^*(s) - q_{06}^*(s)q_{93}^*(s)q_{10,7}^*(s)
\end{aligned}$$

$+ q_{07}^*(s)q_{93}^*(s)q_{10,6}^*(s) - q_{06}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{06}^*(s)q_{94}^*(s)q_{10,7}^*(s)$
$+ q_{07}^*(s)q_{94}^*(s)q_{10,6}^*(s) + q_{08}^*(s)q_{93}^*(s)q_{10,6}^*(s) - q_{06}^*(s)q_{94}^*(s)q_{10,8}^*(s)$
$- q_{06}^*(s)q_{95}^*(s)q_{10,7}^*(s) + q_{07}^*(s)q_{95}^*(s)q_{10,6}^*(s) + q_{08}^*(s)q_{94}^*(s)q_{10,6}^*(s)$
$- q_{06}^*(s)q_{95}^*(s)q_{10,8}^*(s) + q_{08}^*(s)q_{95}^*(s)q_{10,6}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{93}^*(s)q_{10,7}^*(s)$
$+ q_{02}^*(s)q_{27}^*(s)q_{93}^*(s)q_{10,6}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{94}^*(s)q_{10,7}^*(s)$
$+ q_{02}^*(s)q_{27}^*(s)q_{94}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{93}^*(s)q_{10,6}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{94}^*(s)q_{10,8}^*(s)$
$- q_{02}^*(s)q_{26}^*(s)q_{95}^*(s)q_{10,7}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{95}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{94}^*(s)q_{10,6}^*(s)$
$- q_{02}^*(s)q_{26}^*(s)q_{95}^*(s)q_{10,8}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{95}^*(s)q_{10,6}^*(s));$

$N_{f_{13}}^{n}(s) = - q_{7,10}^*(s)\, (q_{07}^*(s)q_{93}^*(s) - q_{02}^*(s)q_{27}^*(s) - q_{07}^*(s) + q_{07}^*(s)q_{94}^*(s) + q_{07}^*(s)q_{95}^*(s)$
$- q_{06}^*(s)q_{10,7}^*(s) + q_{07}^*(s)q_{10,6}^*(s) + q_{07}^*(s)q_{10,8}^*(s) - q_{08}^*(s)q_{10,7}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{93}^*(s)$
$+ q_{02}^*(s)q_{27}^*(s)q_{94}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{95}^*(s) - q_{02}^*(s)q_{26}^*(s)q_{10,7}^*(s) + q_{02}^*(s)q_{27}^*(s)q_{10,6}^*(s)$
$+ q_{02}^*(s)q_{27}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{28}^*(s)q_{10,7}^*(s) + q_{06}^*(s)q_{93}^*(s)q_{10,7}^*(s)$
$- q_{07}^*(s)q_{93}^*(s)q_{10,6}^*(s) + q_{06}^*(s)q_{94}^*(s)q_{10,7}^*(s) - q_{07}^*(s)q_{94}^*(s)q_{10,6}^*(s)$
$+ q_{06}^*(s)q_{95}^*(s)q_{10,7}^*(s) - q_{07}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{07}^*(s)q_{95}^*(s)q_{10,6}^*(s)$
$+ q_{08}^*(s)q_{93}^*(s)q_{10,7}^*(s) - q_{07}^*(s)q_{94}^*(s)q_{10,8}^*(s) + q_{08}^*(s)q_{94}^*(s)q_{10,7}^*(s) - q_{07}^*(s)q_{95}^*(s)q_{10,8}^*(s)$
$+ q_{08}^*(s)q_{95}^*(s)q_{10,7}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{93}^*(s)q_{10,7}^*(s) - q_{02}^*(s)q_{27}^*(s)q_{93}^*(s)q_{10,6}^*(s)$
$+ q_{02}^*(s)q_{26}^*(s)q_{94}^*(s)q_{10,7}^*(s) - q_{02}^*(s)q_{27}^*(s)q_{94}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{26}^*(s)q_{95}^*(s)q_{10,7}^*(s)$
$- q_{02}^*(s)q_{27}^*(s)q_{93}^*(s)q_{10,8}^*(s) - q_{02}^*(s)q_{27}^*(s)q_{95}^*(s)q_{10,6}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{93}^*(s)q_{10,7}^*(s)$
$- q_{02}^*(s)q_{27}^*(s)q_{94}^*(s)q_{10,8}^*(s) + q_{02}^*(s)q_{28}^*(s)q_{94}^*(s)q_{10,7}^*(s) - q_{02}^*(s)q_{27}^*(s)q_{95}^*(s)q_{10,8}^*(s)$
$+ q_{02}^*(s)q_{28}^*(s)q_{95}^*(s)q_{10,7}^*(s)).$

The expected number of repairs of the feeder 1, 2 and 3, that are connected from transformer 2, respectively, in steady state, is given by

$$Nd_{f_{21}} = \lim_{s\to 0} s\, Nd_{f_{21}}^*(s) = \lim_{s\to 0} s\, \frac{N_{f_{21}}^{n}(s)}{D_1(s)} = \frac{N_{f_{21}}^{n}}{D_1}\ ; \quad (26)$$

$$Nd_{f_{22}} = \lim_{s\to 0} s\, Nd_{f_{22}}^*(s) = \lim_{s\to 0} s\, \frac{N_{f_{22}}^{n}(s)}{D_1(s)} = \frac{N_{f_{22}}^{n}}{D_1}\ ; \quad (27)$$

$$Nd_{f_{23}} = \lim_{s\to 0} s\, Nd_{f_{23}}^*(s) = \lim_{s\to 0} s\, \frac{N_{f_{23}}^{n}(s)}{D_1(s)} = \frac{N_{f_{23}}^{n}}{D_1}\ ; \quad (28)$$

where, $N_{f_{21}}^{n}(s)$, $N_{f_{22}}^{n}(s)$ & $N_{f_{23}}^{n}(s)$. Here, $D_1(s)$ is as obtained in equation (13).

VII. Profit Equation

The profit equation of the system is as follows:

$$P_d = R_d Ad_0 - C_{t_1}(Bd_{t_1} + Nd_{t_1}) - C_{t_2}(Bd_{t_2} + Nd_{t_2}) - C_{f_{11}}(Bd_{f_{11}} + Nd_{f_{11}}) - C_{f_{12}}(Bd_{f_{12}} + Nd_{f_{12}}) - C_{f_{13}}(Bd_{f_{13}} + Nd_{f_{13}}) - C_{f_{21}}(Bd_{f_{21}} + Nd_{f_{21}}) - C_{f_{22}}(Bd_{f_{22}} + Nd_{f_{22}}) - C_{f_{23}}(Bd_{f_{23}} + Nd_{f_{23}})$$

where,

R_d = Revenue generated by substation

C_{t_1}/C_{t_2} = Cost per unit time for engaging the repairman and cost for repair due to failure in transformer 1/ transformer 2.

$C_{f_{11}}(C_{f_{21}})/C_{f_{12}}(C_{f_{22}})/C_{f_{13}}(C_{f_{23}})$ = Cost per unit time for engaging the repairman and cost for repair due to failure in feeder 1/ 2/ 3 that are connected to transformer 1 (transformer 2).

VIII. Numerical Results and Discussion

The results obtained from substituting values mentioned in Section 5 (Data Summary) that is calculated from electrical distribution substation for the above obtained measures in Section 7 are mentioned as below:

Availability of electrical distribution substation, Ad_0 = 0.4709

Busy period for repair of Feeder 1 that is connected from Transformer 1, $Bd_{f_{11}}$ = $4.4019*10^{-5}$

Busy period for repair of Feeder 2 that is connected from Transformer 1, $Bd_{f_{12}}$ = $9.1030*10^{-5}$

Busy period for repair of Feeder 3 that is connected from Transformer 1, $Bd_{f_{13}}$ = 3.6454*10^{-5}
Busy period for repair of Feeder 1 that is connected from Transformer 2, $Bd_{f_{21}}$ = 1.8535*10^{-10}
Busy period for repair of Feeder 2 that is connected from Transformer 2, $Bd_{f_{22}}$ = 3.7501*10^{-6}
Busy period for repair of Feeder 3 that is connected from Transformer 2, $Bd_{f_{23}}$ = 3.8778*10^{-6}
Expected number of repairs of Feeder 1 connected from Transformer 1, $Nd_{f_{11}}$ = 2.8722*10^{-5}
Expected number of repairs of Feeder 2 connected from Transformer 1, $Nd_{f_{12}}$ = 5.7440*10^{-5}
Expected number of repairs of Feeder 3 connected from Transformer 1, $Nd_{f_{13}}$ = 1.9153*10^{-5}
Expected number of repairs of Feeder 1 connected from Transformer 2, $Nd_{f_{21}}$ = 2.5566*10^{-10}
Expected number of repairs of Feeder 2 connected from Transformer 2, $Nd_{f_{22}}$ = 4.7874*10^{-6}
Expected number of repairs of Feeder 3 connected from Transformer 2, $Nd_{f_{23}}$ = 2.3938*10^{-6}

I. Impact of Various Rates and Cost Functions on Profit Function

The change in Profit (P_d) with varying values of failure rate ($\lambda_{f_{11}}$) and revenue (R_d) is shown in Figure 3. The surface suggests a relatively flat gradient, with a slight increase in P_d as both $\lambda_{f_{11}}$ and R_d increases. It shows that

- The profit falls with increase in failure rate.
- The higher values of revenue contribute to a rise in profit.

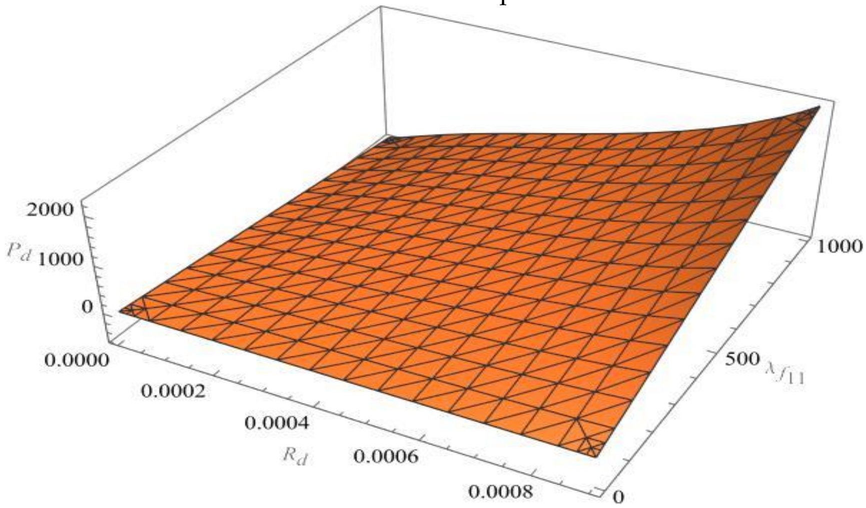

Figure 3: *Profit (P_d) with varying values of failure rate ($\lambda_{f_{11}}$) and revenue (R_d)*

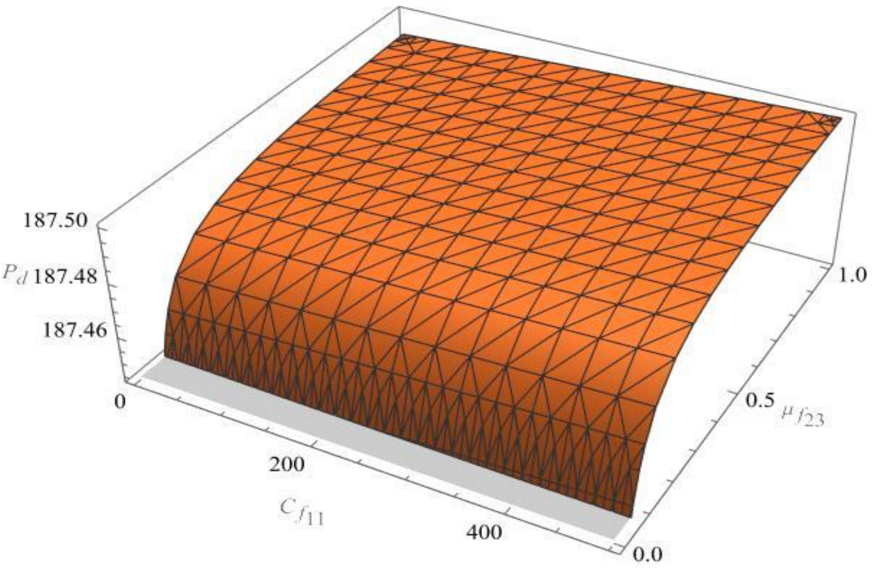

Figure 4: *Profit (P_d) with varying values of of repair rate ($\mu_{f_{23}}$) and cost ($C_{f_{11}}$)*

The change in Profit (P_d) with varying values of repair rate ($\mu_{f_{23}}$) and cost ($C_{f_{11}}$) is shown in Figure 4. The surface reveals that as $C_{f_{11}}$ increases, P_d shows a slight upward trend, while $\mu_{f_{23}}$ appears to have more substantial impact. It is also observed that when $\mu_{f_{23}}$ is near 0, P_d is lower and as $\mu_{f_{23}}$ increases towards 1, P_d significantly rises. This suggests a strong positive correlation between $\mu_{f_{23}}$ and P_d, with $C_{f_{11}}$ also contributing positively but to a lesser extent.

II. Sensitivity Analysis

Sensitivity analysis is a powerful tool for assessing the robustness and reliability of models or decisions in the face of changing conditions or uncertainties in input parameters. It provides a structured approach to understanding how sensitive outcomes are to changes in key factors, thereby aiding in risk management and decision optimization. It is a technique to understand the changes in certain variables like availability and profit of a system. Relative sensitivity analysis is a normalized form of sensitivity analysis that focuses on comparing the relative impact of changes in different input variables or parameters on the output of a model or system. It is particularly useful in scenarios where the magnitude of changes in input variables varies widely. The sensitivity and relative sensitivity analysis of availabilities with different parameters involved are shown in Table 2.

Table 2: *Sensitivity and Relative Sensitivity Analysis of Availabilities*

Parameter (x)	Sensitivity Analysis $\frac{dA_0}{dx}$	Relative Sensitivity Analysis $\frac{dA_0}{dx} * \frac{x}{A_0}$
λ_{t_1}	$-1.6482*10^3$	-0.0402
λ_{t_2}	$-1.2812*10^3$	-0.0520
$\lambda_{f_{11}}$	$-2.1252*10^4$	-1.3789
$\lambda_{f_{12}}$	$-2.1252*10^4$	-2.5853
$\lambda_{f_{13}}$	$-2.1252*10^4$	-1.2065
$\lambda_{f_{21}}$	$-1.3847*10^4$	-0.3369
$\lambda_{f_{22}}$	$-1.3847*10^4$	-0.4492
$\lambda_{f_{23}}$	$-1.3847*10^4$	-0.6738
$\mu_{f_{21}}$	$3.9248*10^{-6}$	$1.1496*10^{-5}$
$\mu_{f_{22}}$	$5.0140*10^{-6}$	$1.3593*10^{-5}$
$\mu_{f_{23}}$	$3.6972*10^{-5}$	$4.8468*10^{-5}$

Thus, the order in which the parameters (failure/repair rates) impact the availability is $\lambda_{f_{12}} > \lambda_{f_{11}} > \lambda_{f_{13}} > \lambda_{f_{23}} > \lambda_{f_{22}} > \lambda_{f_{21}} > \lambda_{t_2} > \lambda_{t_1} > \mu_{f_{23}} > \mu_{f_{22}} > \mu_{f_{21}}$.

Figure 5 is a 3D surface plot illustrating the relationship between three variables: $C_{f_{23}}$ on the x-axis, R_d on the y-axis, and change in profit on the z-axis. The surface shows a consistent increase in P_d as both $C_{f_{23}}$ and R_d increase, indicating a positive correlation among these variables. The smooth and upward-sloping nature of the surface indicates a steady and predictable relationship. This kind of graphical outcome is useful for optimizing and redirecting system performance based on these key variables.

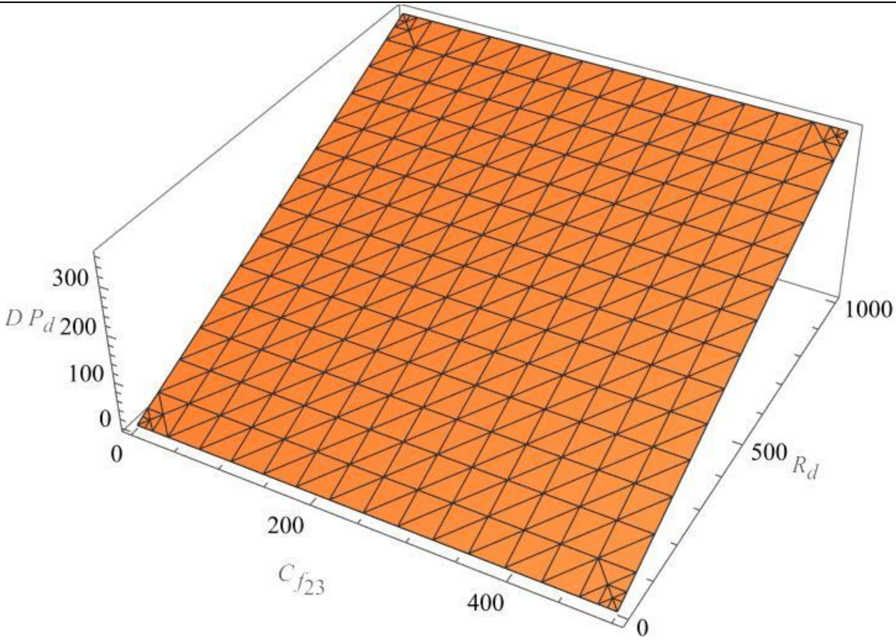

Figure 5: *Sensitivity Analysis of Profit (P_d) with varying values of revenue (R_d) and cost ($C_{f_{23}}$)*

IX. Conclusion

The analysis of the electrical distribution substation reveals critical insights into the reliability, profitability, and sensitivity of the system. By utilizing real maintenance data, the study evaluates the performance of the substation's transformers and outgoing feeders. Key reliability indices such as availability, repair durations, and expected repair frequencies are calculated using Markov processes and regenerative point techniques. The findings highlight that while the substation maintains a high level of reliability, certain failure types and repair times significantly impact overall availability and profitability. It must be noted that the availability of the system is 0.4709 which is quite low and need to be addressed by adopting a robust maintenance strategy. The profit analysis, supported by graphical representations, underscores the economic viability of the substation operations. Furthermore, the sensitivity analysis provides a comprehensive understanding of how variations in failure and repair rates affect the system's performance. This research not only contributes valuable knowledge for optimizing maintenance strategies but also offers a robust framework for assessing the economic and operational viability of electrical distribution substations. Future research should focus on integrating advanced predictive maintenance technologies and exploring alternative economic models to further enhance system reliability and profitability.

X. Acknowledgment

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