RT&A, No 1 (82)
Volume 20, March 2025

PERFORMANCE ANALYZATION OF ERLANG SERVICE MODEL UNDER TRIANGULAR FUZZY NUMBER BY USING THE L-R FUZZY APPROACH

Dr. V. P. Anuja

•

Department of Mathematics,
Arulmigu Palaniandavar Arts College for Women,
Palani, Tamil Nadu.

dranujakumaran24@gmail.com

Abstract

A traditional mathematical technique for analyzing line-waiting delays and overcrowding is queuing theory. It addresses the number of patrons in line as well as numerous other queue-related issues. Developing an Erlang service model in a fuzzy environment is our study's goal. This study aims to investigate the anticipated number of patients in the line as well as the queuing system's waiting time. To achieve this, we applied the L-R strategy under triangular fuzzy numbers and the alpha-cuts method. To measure various linguistic aspects in queuing systems, the fuzzy approach has been used. The findings showed that waiting times are determined using recommended techniques and that the fuzzy Erlang model is stable. Finally, we provide numerical examples to show the capabilities of the suggested method.

Keywords: Fuzzy queuing theory, α-cut method, L-R fuzzy approach, triangular fuzzy number, Erlang service model.

1. Introduction

A probabilistic method for handling queuing systems is queuing theory. Calls and Erlang initially presented queuing theory by focusing on the congestion issue in telephone exchanges and introducing the foundation for both Poisson and exponential distributions. When people wait for their turn to get services, they are essentially in a queue. Waiting is one of the processes of most troubling situations, and queuing theory addresses it. Banks, hospitals, telecoms, medical services, and other establishments frequently face queue issues. Long lines have a financial and resource cost to people. It is challenging to accommodate everyone's high needs because of the traffic. It addresses the quantity of patrons in line as well as numerous other queue-related issues.

Queueing theory is useful for creating effective queuing systems that, while lowering client wait times, also increase the number of customers that can be served. Two types of queues are distinguished: fuzzy queues and crisp queues. Using a probabilistic technique, Crisp Queue handles performance measures, and in this case, Poisson distribution is used to determine the "service time" and "inter-arrival time." In actual situations, both the service and arrival rates are informally assessed. Since the majority of information pertaining to statistics is collected in a subjective manner, the fuzzy approach describes service and arrival rates in a probabilistic manner [1, 2].

Zadeh asserts that fuzzy queues are more realistic than crisp queues. When numerous servers are involved, these queueing models work best when crisp queues are converted to fuzzy queues [3]. To create the mathematical models for customer service, queuing theory is employed. Given a probabilistic explanation of service time, the fuzzy queueing method is a more practical solution than traditional queueing theory methods. Since the fuzzy technique's boundary is specified with a limited membership degree, it differs greatly from the crisp set approach [4].

Several researchers have used Zadeh's extension concept for fuzzy queueing models [3, 5, 6, 7, 8], such as, Nagi and Lee [9] examined the α-cut method under fuzzy conditions. The fuzzy approach to diagnostic queuing theory was introduced by Umaira Zareen and Saqlain Raza [10]. The Erlang service model was used by Narayanamoorthy et al. [11] to predict the anticipated number of customers and their waiting time in the system. The single server queues under the LR approach are examined by Vijaya et al. [12] utilizing trapezoidal fuzzy numbers. Lee [13] studied the concepts of simulation and the Alpha-cut approach. Much research has been done on fuzzy queues by Prade [14], Ritha and Menon [15], Yager [16], Mukeba Kanyinda [17], and others [18, 19, 20]. Finding system performance measurements using the $\alpha-$ cuts approach is the focus of the majority of these works. In this work, fuzzy queueing models are analyzed under the L-R fuzzy approach using triangular fuzzy numbers. The L-R method is a novel approach that we use to determine how many customers in a fuzzy queue along with their waiting time.

Compared to classical queueing theory, the fuzzy queueing models are more realistic than obtaining the queue models because the service and inter-arrival times have to follow certain distributions. However, linguistic quantifiers such as speedy, gradual, or medium are often used to characterize the arrival and service patterns instead of probability distributions. In this study, the alpha-cut and L-R approaches, which are helpful in determining the function's higher and lower bounds, handle the service rate and arrival rate as triangular fuzzy numbers. There has been a lot of interest in the $M/E_C/1$ vacation systems with a single unit arrival for queuing models with single and many servers under different considerations. Researchers used the earlier findings to solve a queuing decision problem and a machine serving problem revision of queueing theory [5, 6, 21, 22, 23].

2. Preliminaries

We give some basic concepts and arithmetic operations of L-R fuzzy numbers in this section.

Definition 2.1. [18] A $\tilde{\delta}$ is said L-R fuzzy number if there exists a real numbers such as, $\delta, s > 0, t > 0$ and two positive, continuous and decreasing functions L and R such that

$$\begin{aligned} L(0) &= R(0) = 1 \\ L(1) &= 0, L(u) > 0, \lim_{u\to\infty} L(u) = 0 \\ R(1) &= 0, R(u) > 0, \lim_{u\to\infty} R(u) = 0 \end{aligned}$$

$$\lambda_{\tilde{\delta}}(u) = \begin{cases} L(\frac{\delta-u}{s}) & if \quad u \in [\delta - s, \delta] \\ R(\frac{u-\delta}{t}) & if \quad u \in [\delta, \delta + t] \\ 0 & otherwise \end{cases} \tag{1}$$

The $\tilde{\delta} =< \delta, s, t >_{LR}$. δ is called the most possible value. By the definition,$supp(\tilde{A}) = \{u \in E | \sigma_{\tilde{A}}(u) > 0\}$, then

$$supp(\tilde{\delta}) =]\delta - s, \delta] \cup [\delta, \delta + t[=]\delta - s, \delta + t[.$$

2.1. Arithmetic operations of LR fuzzy numbers

The two L-R fuzzy numbers $M =< \delta, i, j >$ and $N =< \eta, k, p >$ [24]

$$\tilde{M} + \tilde{N} =< \delta + \eta, i + k, j + p > \tag{2}$$

$$\tilde{M} - \tilde{N} = < \delta - \eta, i + p, j + k > \tag{3}$$

$$\tilde{M}.\tilde{N} \approx < \delta\eta, \delta k + \eta i - ik, \delta p + \eta j + jp > \tag{4}$$

$$\frac{\tilde{M}}{\tilde{N}} = \frac{< \delta, i, j >}{< \eta, k, p >} \approx < \frac{\delta}{\eta}, \frac{\delta p}{\eta(\eta + p)} + \frac{i}{\eta} - \frac{ip}{\eta(\eta + p)}, \frac{\delta k}{\eta(\eta - k)} + \frac{j}{\eta} + \frac{jk}{\eta(\eta - k)} > \tag{5}$$

Definition 2.2. [11] A $\tilde{F}$ is said triangular fuzzy number (TFN) then there exists a real numbers $g < h < r$ such that:

$$\eta_{\tilde{F}}(u) = \begin{cases} L(\frac{u-g}{h-g}) & if \quad g \leq u \leq h \\ R(\frac{r-u}{r-h}) & if \quad h \leq u \leq r \\ 0 & otherwise \end{cases} \tag{6}$$

By definitions, a TFN $\tilde{F} = (g/h/r)$ is LR fuzzy number. In this concepts, it can be written

$$\tilde{F} = (g/h/r) = < h, h - g, r - g >$$

for $L(x) = R(x) = max(0, 1 - u)$.

3. Mathematical Model

In a queuing system, a customer with arrival rate V and service rate Q is received by a single-server capacity. The C exponential phase makes up the fuzzy Erlang service rate $\tilde{Q}$ and the fuzzy Poisson rate $\tilde{V}$. After establishing the system capacity and calling source to infinite, customers are serviced using the the basis of FCFS [25, 26].

Here, the $\tilde{V}$ is arrival rate and service rate $\tilde{Q}$ are known and can be denoted by convex fuzzy sets. An fuzzy set $\tilde{F}$ is convex if and only if $\mu_{\tilde{F}}(\varphi x_1 + (1 - \varphi)x_2) \geq min\{\mu_{\tilde{F}}(x_1), \mu_{\tilde{F}}(x_2)\}$ where $\mu_{\tilde{F}}$ is $\phi \in [0, 1], x_1, x_2 \in X$.

Let $\mu_{\tilde{V}}(s)$, $\mu_{\tilde{Q}}(t)$ are arrival rate and service rate of membership functions respectively. We have

$$\tilde{V} = \{s, \mu_{\tilde{V}}(s)/s \in S(\tilde{V})\}$$
$$\tilde{Q} = \{t, \mu_{\tilde{Q}}(y)/t \in S(\tilde{Q})\}$$

Where $S(\tilde{\lambda})$ and $S(\tilde{\mu})$ are the supports [11]. Based on the extension principle proposed by Zadeh, the performance measure's membership function is described as

$$\mu_{\tilde{E}(\tilde{V},\tilde{Q})}(x) = \sup_{s \in S, t \in T} min\{\mu_{\tilde{V}}(s), \mu_{\tilde{Q}}(t)/x = E(s,t)\}$$

Under the steady-state condition $\rho = \frac{V}{Q} < 1$, the number of customers in the queue,

$$Lq = \left[\frac{(C+1)}{2C} . \frac{\tilde{V}^2}{\tilde{Q}(\tilde{Q} - \tilde{V})}\right]$$

The expected number of customers in the queue $\tilde{L}q$ is

$$\mu_{\tilde{L}q}(x) = \sup_{s \in S, t \in T, x < X} min\left\{\mu_{\tilde{V}}(s), \mu_{\tilde{Q}}(t)/x = \left[\frac{(C+1)}{2C} . \frac{\tilde{V}^2}{\tilde{Q}(\tilde{Q} - \tilde{V})}\right]\right\} \tag{7}$$

We can determine how long it will take for the expected number of customers in line,

$$Wq = \left[\frac{(C+1)}{2C} . \frac{\tilde{V}}{\tilde{Q}(\tilde{Q} - \tilde{V})}\right]$$

The waiting time of $\tilde{W}q$ is in the queue

$$\mu_{\tilde{W}q}(x) = \sup_{s \in S, t \in T, y < Y} min\left\{\mu_{\tilde{V}}(s), \mu_{\tilde{Q}}(t)/x = \left[\frac{(C+1)}{2C} . \frac{\tilde{V}}{\tilde{Q}(\tilde{Q} - \tilde{V})}\right]\right\} \tag{8}$$

4. The Method for Solving the Problem

An approach to constructing the $\mu_{\tilde{P}(\tilde{V},\tilde{Q})}$ is basis of deriving α-cuts of $\mu_{\tilde{P}(\tilde{V},\tilde{Q})}$. Denote α-cuts of $\tilde{V}$ and $\tilde{Q}$ as [25, 26]:

$$\tilde{V}_\alpha = [s_\alpha^L, s_\alpha^U] = [\min_{s\in S}\{s/\mu_{\tilde{V}}(s) \geq \alpha\}, \quad \max_{s\in S}\{s/\mu_{\tilde{V}}(s) \geq \alpha\}] \tag{9}$$

$$\tilde{Q}_\alpha = [t_\alpha^L, t_\alpha^U] = [\min_{t\in T}\{t/\mu_{\tilde{Q}}(t) \geq \alpha\}, \quad \max_{t\in T}\{t/\mu_{\tilde{Q}}(y) \geq \alpha\}] \tag{10}$$

Consequently, the $FM/FE_C/1$ queue can be reduced to crisp $M/E_C/1$ queues with various levels of α sets $\{\tilde{V}_\alpha < \alpha \leq 1\}$[].

By the convexity, the intervals are functions of α is

$$s_\alpha^L = min\ \mu_{\tilde{V}}^{-1}(\alpha) \text{ and } s_\alpha^U = max\ \mu_{\tilde{V}}^{-1}(\alpha)$$

$$t_\alpha^L = min\ \mu_{\tilde{Q}}^{-1}(\alpha) \text{ and } t_\alpha^U = max\ \mu_{\tilde{Q}}^{-1}(\alpha)$$

We need either $\mu_{\tilde{V}}(s) = \alpha$ and $\mu_{\tilde{Q}}(t) \geq \alpha$ (or) $\mu_{\tilde{V}}(s) \geq \alpha$ and $\mu_{\tilde{Q}}(t) = \alpha$ such that $x = \left[\frac{(C+1)}{2C}.\frac{\tilde{V}^2}{\tilde{Q}(\tilde{Q}-\tilde{V})}\right]$ to satisfy $\mu_{\tilde{Lq}}(x) = \alpha$. To find the$\mu_{\tilde{Lq}}(x)$ we have to obtain the lower value of x_α^L and the upper value of x_α^U of $\mu_{\tilde{Lq}}(x)$. Since $\mu_{\tilde{V}}(s) = \alpha$ is denoted by $s = s_\alpha^L$ (or) $s = s_\alpha^U$ this formulated as the constraint of $s = \varphi_1 s_\alpha^L + (1-\varphi_1)s_\alpha^U$, where $\varphi_1 = 0$ (or) 1. Similarly $\mu_{\tilde{Q}}(t) = \alpha$ is formulated as the constraint $t = \varphi_2 t_\alpha^L + (1-\varphi_2)t_\alpha^U$, where $\varphi_2 = 0$ (or) 1 [].

Let $(\tilde{Lq})_\alpha^L = \{(\tilde{Lq})_\alpha^{L_1}, (\tilde{Lq})_\alpha^{L_2}\}$ and $(\tilde{Lq})_\alpha^U = \{(\tilde{Lq})_\alpha^{U_1}, (\tilde{Lq})_\alpha^{U_2}\}$ respectively. where

$$(\tilde{Lq})_\alpha^{L_1} = \min_{s,t\in R} \quad s < t\left[\frac{(C+1)}{2C}.\frac{\tilde{V}^2}{\tilde{Q}(\tilde{Q}-\tilde{V})}\right] \tag{11}$$

such that $s = a_1 s_\alpha^L + (1-a_1)s_\alpha^U,\ \ t_\alpha^L \leq t \leq t_\alpha^U$ and $a_1 = 0$ (or) 1.

$$(\tilde{Lq})_\alpha^{L_2} = \min_{s,t\in R} \quad s < t\left[\frac{(C+1)}{2C}.\frac{\tilde{V}^2}{\tilde{Q}(\tilde{Q}-\tilde{V})}\right] \tag{12}$$

such that $t = a_2 t_\alpha^L + (1-a_2)t_\alpha^U,\ \ s_\alpha^L \leq s_\alpha^U$ and $a_2 = 0$ (or) 1.

Similarly, we can obtain the upper values of L_q and where $s_\alpha^L < t_\alpha^L$. Then, α-cuts of $\tilde{Lq}$ can be obtain by solving above equations.

If both $(\tilde{Lq})_\alpha^L$ and $(\tilde{Lq})_\alpha^U$ are invertible, then a left shape function $L_O(x) = ((\tilde{Lq})_\alpha^L)^{-1}$ a right shape function can be obtained. From $L_O(x)$ and $R_O(x)$, the membership function $\mu_{\tilde{Lq}}$ is constructed as

$$\mu_{\tilde{Lq}}(x) = \begin{cases} L_o(x), & (Lq)_\alpha^L = 0 \leq x \leq (Lq)_\alpha^L = 1 \\ 1, & (Lq)_\alpha^L = 1 \leq x \leq (Lq)_\alpha^U = 1 \\ R_o(x), & (Lq)_\alpha^U = 1 \leq x \leq (Lq)_\alpha^U = 0 \end{cases} \tag{13}$$

5. Numerical Example

In a hospital clinic, a doctor examines each patient who is brought in for a routine checkup. While the time spent on each part of the checkup is roughly exponentially distributed, the doctor spends an average of fifty minutes on each phase. Given that every patient undergoes a four-phase examination and that the typical patient arrives at the doctor's clinic at a rate of 20 per hour. Calculate

- How many patients are anticipated to be in line?
- How long does it typically take a patient to wait in line?
- Determine the maximum values for the anticipated patient volume and line wait time.

Solution
The classical queueing theory cannot be used to investigate this issue because the rates are given in fuzzy information. $FM/FE_C/1$ is a simple queue with fuzzy rates. Assume that te rates are fuzzy triangular numbers provided by $\tilde{V} = [10, 20, 30]$ and $\tilde{Q} = [40, 50, 60]$. The approach described in the paragraph allows us to analyze queue characteristics specified in equations since these parameters are L-R fuzzy integers.

***α*-cuts method:**

The confidence interval at α are $[10\alpha + 10, 30 - 10\alpha]$ and $[10\alpha + 40, 60 - 10\alpha]$

$$(\tilde{Lq})_{\alpha}^{L} = \left[\frac{5\alpha^2 + 10\alpha + 5}{16\alpha^2 - 136\alpha + 240}\right]$$

$(\tilde{Lq})_{\alpha}^{L}$ is invertible

$$\begin{aligned}
\alpha &= \frac{(136x - 10) \pm \sqrt{3136x^2 + 2400x}}{(32x - 10)} \\
\alpha &\geq 0, (136x - 10) \pm \sqrt{3136x^2 + 2400x} \geq 0 \\
x &= 0.0208 (or) 0.3125 \\
\alpha &\leq 1, \frac{(136x - 10) \pm \sqrt{3136x^2 + 2400x}}{(32x - 10)} \leq 1 \\
x &= 0.3125 \ (or) \ 0 \\
(\tilde{Lq})_{\alpha}^{L} &= \frac{(136x - 10) \pm \sqrt{3136x^2 + 2400x}}{(32x - 10)} \qquad 0 \leq x \leq 0.0208
\end{aligned}$$

Now,

$$(\tilde{Lq})_{\alpha}^{U} = \left[\frac{5\alpha^2 - 30\alpha + 45}{16\alpha^2 + 72\alpha + 32}\right]$$

$(\tilde{Lq})_{\alpha}^{U}$ is invertible

$$\begin{aligned}
\alpha &= \frac{-(72x + 30) \pm \sqrt{3136x^2 + 7840x}}{(32x - 10)} \\
\alpha &\geq 0, -(72x + 30) \pm \sqrt{3136x^2 + 7840x} \geq 0 \\
x &= 0.3125 (or) 1.4062 \\
\alpha &\leq 1, \frac{-(72x + 30) \pm \sqrt{3136x^2 + 7840x}}{(32x - 10)} \leq 1 \\
x &= 2.5 (or) 0.3125 \\
(\tilde{Lq})_{\alpha}^{U} &= \frac{-(72x + 30) \pm \sqrt{3136x^2 + 7840x}}{(32x - 10)} \qquad 1.4062 \leq x \leq 2.5
\end{aligned}$$

From the inverse function of $(\tilde{Lq})_\alpha^L$ and $(\tilde{Lq})_\alpha^U$ of $\tilde{Lq}$ is described as:

$$\mu_{\tilde{Lq}}(x) = \begin{cases} \frac{(136x-10)\pm\sqrt{3136x^2+7840x}}{(32x-10)}, & 0 \le x \le 0.0208 \\ 1, & 0.3125 \le x \le 0.3125 \\ \frac{-(72x+30)\pm 66\sqrt{3136x^2+7840x}}{(32x-10)}, & 1.4062 \le x \le 2.5 \end{cases}$$

Then,

$$(\tilde{Wq})_\alpha^L = \frac{(\alpha+1)}{32\alpha^2-272\alpha+480}$$

$(\tilde{Wq})_\alpha^L$ is invertible

$$\begin{aligned} \alpha &= \frac{(272x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \\ \alpha &\ge 0, (272x+1)\pm\sqrt{12544x^2+672x+1} \ge 0 \\ x &= 0.00208(or)0 \\ \alpha &\le 1, \frac{(272x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \le 1 \\ x &= 0.0022(or)0 \\ (\tilde{Wq})_\alpha^L &= \frac{(272x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \quad 0.00208 \le x \le 0.0022 \end{aligned}$$

Now,

$$(\tilde{Wq})_\alpha^U = \frac{(3-\alpha)}{32\alpha^2+144\alpha+64}$$

$(\tilde{Wq})_\alpha^U$ is invertible

$$\begin{aligned} \alpha &= \frac{-(144x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \\ \alpha &\ge 0, -(144x+1)\pm\sqrt{12544x^2+672x+1} \ge 0 \\ x &= 0(or)0.0468 \\ \alpha &\le 1, \frac{-(144x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \le 1 \\ x &= 0.0937(or)0 \\ (\tilde{Wq})_\alpha^U &= \frac{-(144x+1)\pm\sqrt{12544x^2+672x+1}}{64x} \quad 0.0468 \le x \le 0.0937 \end{aligned}$$

From the inverse function of $(\tilde{Wq})_\alpha^L$ and $(\tilde{Wq})_\alpha^U$ the waiting time of $\tilde{Wq}$ is defined as follows:

$$\mu_{\tilde{Wq}}(x) = \begin{cases} \frac{(272x+1)\pm\sqrt{12544x^2+672x+1}}{64x}, & 0.00208 \le x \le 0.0022 \\ 1, & 0 \le x \le 0 \\ \frac{-(144x+1)\pm\sqrt{12544x^2+672x+1}}{64x}, & 0.0468 \le x \le 0.0938 \end{cases}$$

We computed $\tilde{Lq}$ and $\tilde{Wq}$ for the provided values using fuzzy numbers. Substituting the values of α in the formula above yields tabular results, with a graphical depiction provided below.
Finally, the results formulation are obtained by α-cut method: the L_q in the queue is approximately between 0.0208 and 1.4062. The Wq is lies between 0.0020 and 0.0468.

Table 1: *The results obtained by α-cut approach*

α	x_α^L	x_α^U	y_α^L	y_α^U	$(\tilde{Lq})_\alpha^L$	$(\tilde{Lq})_\alpha^U$	$(\tilde{Wq})_\alpha^L$	$(\tilde{Wq})_\alpha^U$
0.0	10.0	30.0	40.0	60.0	0.0208	1.4062	0.0020	0.0468
0.1	11.1	29.9	41.1	59.9	0.0267	1.0683	0.0024	0.0368
0.2	12.2	28.8	42.2	58.8	0.0337	0.8333	0.0028	0.0297
0.3	13.3	27.7	43.3	57.7	0.0421	0.6622	0.0032	0.0245
0.4	14.4	26.6	44.4	56.6	0.0520	0.5334	0.0037	0.0205
0.5	15.5	25.5	45.5	55.5	0.0639	0.4340	0.0042	0.0173
0.6	16.6	24.4	46.6	54.4	0.0779	0.3557	0.0048	0.0148
0.7	17.7	23.3	47.7	53.3	0.0946	0.2931	0.0055	0.0127
0.8	18.8	22.2	48.8	52.2	0.1145	0.2423	0.0063	0.0110
0.9	19.9	21.1	49.9	51.1	0.1382	0.2008	0.0072	0.0095
1.0	20.0	20.0	50.0	50.0	0.1666	0.1666	0.0083	0.0083

The L-R approach:
We determine L-R representations of fuzzy numbers $\tilde{V}$ and $\tilde{Q}$, which are $\tilde{V} = <20, 10, 20>_{LR}$ and $\tilde{Q} = <50, 10, 20>_{LR}$

$$\tilde{Lq} = \left[\frac{(C+1)\tilde{V}}{2(\tilde{Q}-\tilde{V})} - \frac{(C+1)\tilde{V}}{2\tilde{Q}}\right]\frac{1}{C}$$

$$= \frac{1}{4}\left[\frac{5 <20,10,20>}{2[<50,10,20> - <20,10,20>]} - \frac{5<20,10,20>}{2<50,10,20>}\right]$$

$$(\tilde{Lq})_{LR} = <0.0625, 0.5913, 1.0982>_{LR}$$

$$\tilde{Wq} = \left[\frac{C+1}{2C} \times \frac{\tilde{V}}{\tilde{Q}(\tilde{Q}-\tilde{V})}\right]$$

$$= \left[\frac{5}{8} \times \frac{<20,10,20>}{<50,10,20>[<50,10,20> - <20,10,20>]}\right]$$

$$(\tilde{Wq})_{LR} = <0.0083, 0.0068, 0.0083>_{LR}$$

The support of $\tilde{Lq}$ varies between 0.0208 and 0.1666, indicating that the anticipated quantity of patients is uncertain. Its values can never be lower than 0.0208 or more than 0.1666. The mean value of $\tilde{Lq}$ is precisely 1, which is the maximum value that can be found in that situation. Likewise, a patient's waiting time in the queue is between 0.0020 (about one minute) and 0.0468 (approximately three minutes). It shows that there will never be a wait time in the line longer than 3 or shorter than 1. The queue's maximum waiting time is 0.0083 (around one minute). These results of L_q and W_q are shown in the figures 1 and 2.

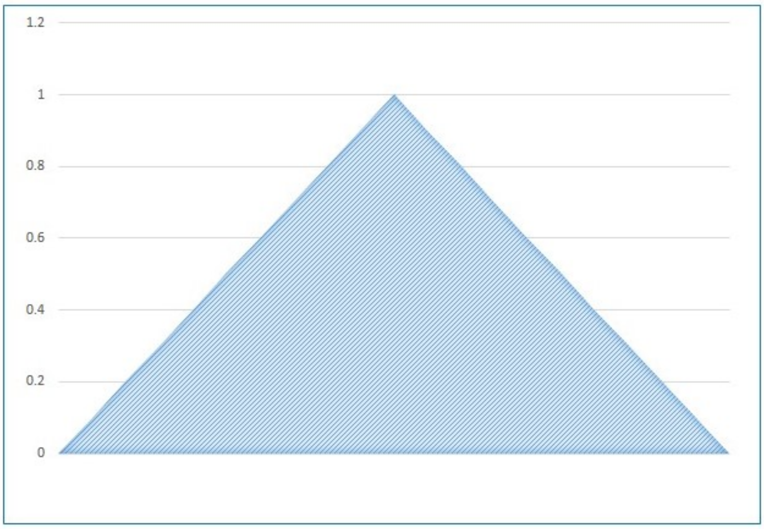

Figure 1: *The results of Lq*

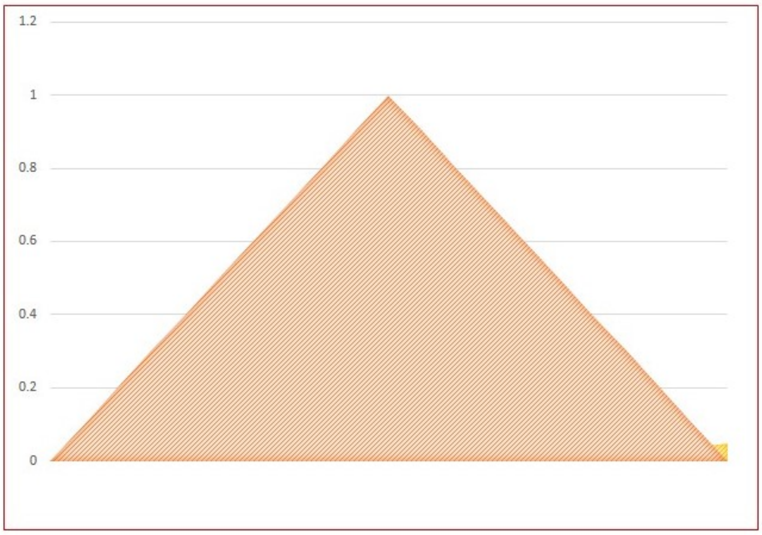

Figure 2: *The results of Wq*

6. Conclusion

The α-cut approach and the L-R method are used in this work to compute the predicted number of clients and mean waiting time of patients of the $FM/FE_C/1$ sequence. With the α-cut approach, the maximum number of patients is precisely 0.0625, while the predicted number of patients falls between 0.0208 and 1.4062. Similarly, the maximum value is precisely 0.0083, and the mean waiting time for patients falls between 0.0020 and 0.0468. The two spreads help deduce the upper and lower boundaries of the fuzzy measure. The approximation of the greatest explanatory outcomes, brevity, convenience, and flexibility are the three primary advantages of this innovative approach. Future research in this field will undoubtedly benefit from the L-R method to address several outstanding problems, such as evaluating fuzzy queueing systems' performance metrics with the Erlang service model.

References

[1] Boucherie. R.J, and Van Dijk. N.M. (2011). Queueing networks, A fundamental approach, *Springer science +Business media*. New york.

[2] Borthakur. A. (1971). A Poisson queue with general bulk service rule, *Journal of the Assam Scientific Society*. 14:162-167.

[3] Zadeh. L.A. (1978). Fuzzy Sets as a Basis for a Theory of Possibility, *Fuzzy Sets and Systems*, 1:3-28.

[4] Zimmermann. H.J. (2001). Fuzzy Set Theory and its Applications, Fourth ed., Kluwar Academic, Boston.

[5] Buckley. J.J. (1990). Elementary Queuing Theory based on Possibility Theory, *Fuzzy Set System*, 37:43-52.

[6] Buckley. J.J, Feuring. T, and Hasaki. Y. (2001). Fuzzy Queueing Theory Resisted, *International Journal of Uncertainty Fuzzy*.9:527-537.

[7] Chen. S.P. (2005). Parametric nonlinear programming approach to fuzzy queues with bulk service, *European Journal of Operational Research*. 163:434-444.

[8] Julia Rose Mary. K, Monica. P and Mythili. S. (2014). Performance measures of FM/FM/1 queueing system with n-policy, *International journal of physics and mathematical sciences*. 4(3):5-10.

[9] Nagi. D.S, and Lee. E.S. (1992). Analysis and simulation of fuzzy queue, *Fuzzy sets and systems*. 46:321-330.

[10] Umaira Zareen and Saqlain Raza (2022). Queuing Theory Diagnostic with Fuzzy Approach, *International Journal of Innovative Science and Research Technology*. 7:10.

[11] Narayanamoorthy, S., Anuja, A., Murugesan, V., and Kang, D. (2020). A distinctive analyzation of intuitionistic fuzzy queueing system using Erlang service model. *In AIP Conference Proceedings. AIP Publishing*. 2261(1).

[12] Vijaya. S., Srinivasan. N., and Suresh. M. V. (2024). Performance analysis of single sever queues with trapezoidal fuzzy number using LR method, *Journal of propulsion technology*, 5(2).

[13] Li. R.J and Lee. E.S. (1984). Analysis of fuzzy queue, *Computers and Mathematics with Applications*, 17:1143-1147.

[14] Prade. H.M. (1980). An outline of fuzzy or possibilistic models for queueing system, in : P.P. Wong, S.K. Chang(Eds)., Fuzzy sets plenum press, New York, 147-153.

[15] Ritha. W, and Sreelekha Menon. B. (2011). Analysis of fuzzy Erlang's loss queueing model:non linear programming approach, *International journal of fuzzy mathematics and systems*, 1:1-10.

[16] Yager. R.R. (1986). A characterization of the extension principle, *Fuzzy Sets and Systems*. 18:205-217.

[17] Mukeba. J.P (2016). Application of L-R method to single server fuzzy retrial queue with patient customers, *Journal of pure and applied mathematics:advances and applications*. 16(1):43-59.

[18] Mukeba Kanyinda. J.P, Mabela Makengo Matendo. R, and Ulungu Ekunda Lukata. B. (2015). Computing fuzzy queueing performance measures by L-R method, *Journal of fuzzy set valued analysis*. 1:57-67.

[19] Mukeba Kanyinda. J.P, Mabela Makengo Matendo. R, and Ulungu Ekunda Lukata. B. (2015). Fuzzy eigenvalues and fuzzy eigenvectors of fuzzy markov chain transition matrix under max-min composition, *Journal of fuzzy sets valued analysis*. 2015(1):25-35.

[20] Srinivasan. R.(1986). Fuzzy Queueing Model Using DSW algorithm, *International Journal of Advanced research in Mathematics and Applications*. 1(1):57-62.

[21] Dubois. D, and Prade. H. (1980). Fuzzy sets and systems:Theory and applications, Academic Press, New York.

[22] Gross. D, and Haris. C.M. (1998). Fundamentals of Queueing Theory, Third Ed., Wiley, New York.

[23] Hanss. M. (2005). Applied fuzzy arithmetic, An introduction with engineering applications, Springer-Verlag, Berlin Heidelberg.

[24] Luciano Stefanini, and Laerte Sorini (2009). Fuzzy arithmetic with parametric LR fuzzy numbers, *IFSA-EUSFLAT*. 600-605.
[25] Ashok Kumar. V. (2011). A membership function solution approach to fuzzy queue with Erlang service model, *Internatinal Journal of Mathematical Science and Applications*. 1:2.
[26] Erlang, A. K. (1909). The theory of probabilities and telephone conversations. Nyt. Tidsskr. Mat. Ser. B, 20, 33-39.