CONSTRUCTION OF GAMMA ZERO-INFLATED POISSON DOUBLE SAMPLING PLANS

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Abstract

In a well-supervised production framework, non-conformities occur seldom, resulting in a more number of zeros in the count of non-conformities. The zero-inflated Poisson (ZIP) distribution is a suitable model for handling zero inflation. Double sampling plan (DSP) is a precise quality inspection method where a decision on the approval or rejection of a lot is made after reviewing two samples, providing stronger conclusions than single sampling plan (SSP). In practice, decision-making for submitted lots requires a consistent assessment of both within-lot and between-lot variations, which can be addressed using Bayesian methodology. A Bayesian approach integrates prior knowledge and provides more information for making decisions about the approval or rejection of a lot. This article focuses on the designing of Bayesian DSPs; employing a Gamma prior to the parameter in the Poisson component of ZIP distribution the operating characteristic (OC) function is derived. Examples are provided to assess Gamma-ZIP (GZIP) DSPs. The significance of GZIP DSPs over conventional ZIP DSPs is also presented.

Keywords: Sampling inspection by attributes, Double sampling plan, Prior distribution, Zero-inflated Poisson distribution, Average quality level, Limiting quality level, Operating characteristics function.

I. Introduction

Sampling inspection is a method employed to assess the quality of items by examining a sample rather than inspecting every individual item. This approach is widely applied in manufacturing, service industries, and other sectors where full inspection would be too expensive or time-consuming. Acceptance sampling is a strategy that helps determine whether entire batches can be approved or declined based on sample inspection. Sampling inspection is categorized into two main types: sampling inspection by attributes and variables, both of which help to assess the quality standard of a batch.

In a SSP, the judgment to approve or decline a batch is based on inspecting only one sample. However, there are situations where a single sample may not provide sufficient information for a conclusive decision. In such cases, a DSP is implemented, where the decision is made based on the inspection of two samples. DSP functions as an extension of SSP, offering more reliable decision-making in quality control. Designing DSP parameters offers enhanced decision-making accuracy and provides better protection to both producer and consumer.

A Bayesian approach to acceptance sampling integrates prior knowledge with observed data to improve decision-making. When items are manufactured in lots, quality variations can occur due to within-lot and between-lot variation. Conventional acceptance sampling often assumes that between-lot variation is less significant than within-lot variation, leading to the assumption that the fraction of nonconforming items in a lot remains constant. In reality, decisions about submitted lots should consider both within-lot and between-lot variations. In such cases, Bayesian methods can be employed to design effective sampling plans based on predictive distributions.

The ZIP distribution is particularly effective in situations where non-conformities are rare. It is suitable for processes where there is a high occurrence of zero non-conformities, though occasional non-conformities are still possible. Loganathan and Shalini [5, 6] pioneered the determination of ZIP SSPs. Later, Uma and Ramya [17], Rao and Aslam [13], and Fu-Kwun and Sharew [3] discussed the construction of sampling plans in different perspectives. The Bayesian approach to developing ZIP SSPs has been explored by Suresh and Latha [15], Vijayaraghavan *et al.*,[19], Shalini *et al.*, [11], Palanisamy and Latha [7, 8] and Kaviyarasu and Sivakumar [4].

The designing of ZIP DSPs has been addressed by Shalini and Sheik [12], Pramote and Wimonmas [9], Wimonmas and Pramote [20]. The integration of Bayesian principles into the designing of DSPs have been discussed by Vijayaraghavan and Sakthivel [18], Balamurali *et al.*,[1], and Suresh and Usha [16].

According to the literature, there has been no research conducted on developing GZIP DSPs. This article focuses on the determination of GZIP DSPs. The OC function of the GZIP DSP is derived in Section 2. Designing GZIP DSPs is discussed in Section 3. Numerical examples and the significance of GZIP DSPs over the conventional ZIP DSPs are given in Section 4. Results are summarized in the concluding section.

II. OC function of GZIP DSPs

DSPs offer more flexibility than SSPs by reducing the risk of making premature decisions. This approach is widely used in industries where cost and time efficiency are critical, providing a balance between minimizing inspection efforts and ensuring product quality.

A DSP is structured around five specific parameters: n_1, n_2, c_1, c_2 and c_3, where $c_1 < c_2$ and $c_2 \leq c_3$ (the third acceptance number). When c_2 is taken in equal to c_3 (*i.e.*, $c_2 = c_3$), the DSP is described through its parameters n_1, n_2, $c_1$and c_2, which represent the sizes of the first and second samples and the first and second acceptance numbers, respectively (Duncan [2] and Stephens [14]).

Upon inspecting all items in the sample, the number of nonconformities (d_1) is established. If $d_1 \leq c_1$, the lot is approved; if $d_1 > c_2$, it is declined. When $c_1 < d_1 \leq c_2$, the initial sample fails, a second sample of size n_2 is taken and the nonconformities (d_2) are counted. The cumulative count, $D = d_1 + d_2$, is compared to c_2: the lot is approved if $D \leq c_2$ and declined if $D > c_2$. This two-stage process enhances the reliability of quality assessment, allowing for more effective decision-making regarding lot approval or rejection based on observed nonconformities (Schilling and Neubauer [10]).

The effectiveness of a sampling plan can be evaluated through its OC function. In various industrial environments, careful monitoring of production processes often results in the frequent occurrence of zero non-conformities. In such scenarios, the ZIP distribution is the fitting probability distribution for nonconformities. The probability mass function (*pmf*) of the ZIP distribution is defined as follows

$$P\,(X = d|\varphi, \lambda) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda}, & when\ d = 0 \\ (1-\varphi)\dfrac{e^{-\lambda}\lambda^{d}}{d!}, & when\ d = 1,2,3,\ldots \end{cases} \tag{1}$$

In this model, φ and λ are parameters, where φ $(0 < \varphi < 1)$ denotes the mixing proportion, which is assumed to be known.

When the variation in lot quality is significant from lot-to-lot, it indicates an unstable production process. In such cases, the process parameter λ is assumed to vary randomly between lots and follows Gamma (*a, m*) distribution. This gamma distribution is the natural conjugate prior to $\lambda = np$ in the Poisson component of the ZIP distribution.

Shalini et al. [11] derived the probability distribution of d under the conditions of a ZIP distribution and a gamma prior distribution for λ, where the shape parameter of the gamma distribution is m.

$$f\,(d|\varphi,n,p,m) = \begin{cases} \varphi + (1-\varphi)\left(\frac{m}{np+m}\right)^m, & when\ d = 0 \\ (1-\varphi)\binom{m+d-1}{m-1}\left(\frac{np}{np+m}\right)^d\left(\frac{m}{np+m}\right)^m, & when\ d = 1,2,3,\ldots \end{cases} \quad (2)$$

The OC function of the GZIP DSP can be described as:

$$P_a(p) = F(d_1 \le c_1|n_1) + \sum_{d_1=c_1+1}^{c_2} f\big(d_1|n_1,p,\varphi,m\big)F(d_2 \le c_3 - d_1|n_2) \quad (3)$$

As proposed by Vijayaraghavan and Sakthivel [18], the prior knowledge about the production process must be used to estimate the value of m. The moment estimator $\hat{m} = \frac{s_1^2}{s_2}$ can be used for m, where $s_1 = \frac{1}{k}\sum_{i=1}^{k}\hat{\lambda}_i = \bar{\lambda}$ and $s_2 = \frac{1}{k}\sum_{i=1}^{k}\big(\hat{\lambda}_i - \bar{\lambda}\big)^2$; $\hat{\lambda}_i$, i = 1, 2, 3, … , k.

III. Designing GZIP DSPs

GZIP DSPs are designed by determining the optimum parameters n_1, n_2, c_1 and c_2 based on the prescribed points $(p_1, 1\text{-}\alpha)$ and (p_2, β) on the OC curve so that the determined GZIP DSPs provide adequate protection to both producer and consumer.

The plan must satisfy the following requirements:

(i) $P_a(p_1) \ge 1 - \alpha$

(ii) $P_a(p_2) \le \beta$

The values of the plan parameters n_1, n_2, c_1 and c_2 can be derived for each set of φ, m, p_1, α, p_2 and β by applying the unity value approach. The values np_1 and np_2 satisfying respectively equations *(i)* and *(ii)* are termed as unity values (Schilling and Neubauer [10]). The plan parameters can be arranged in tables for different combinations of $(p_1, \alpha, p_2, \beta)$. The use of an operating ratio $R = \frac{np_2}{np_1}$, reduces the number of tables.

The plan parameters are determined for specific sets of values of φ, m, p_1, α, p_2 and β under the condition of GZIP distribution. The unity values are computed for various values combinations of $(\varphi, m, c, P_a(p))$ by solving the OC function of GZIP DSPs for each combination of c_1 and c_2 with $n_1 = n_2 = n$. The values taken for m are 5 and 10 and for $Pa\ (p)$ are 0.99, 0.90, 0.50, 0.20 and 0.10. The value considered for c_1 and c_2 in these combinations are 1(1)9 and 2(1)10 respectively. The values taken for φ are 0.03 and 0.07 and are given in Table 1. The operating ratio values calculated corresponding to (α = 0.05, β = 0.10), (α = 0.10, β = 0.20), φ = 0.03 and 0.07, m = 5 and 10, c_1 = 1(1)9 and c_2 = 2(1)10 are listed in Table 2.

For specified strength $(p_1, \alpha, p_2, \beta)$ and values of φ and m these tables can be used to determine the plan parameters by implementing the following procedure:

First, we compute the operating ratio $R = \frac{p_2}{p_1}$.Next select the unity value np_1 and the acceptance numbers (c_1, c_2) from Table 2 corresponding to the value of φ, m, α, β associated with an operating ratio closest to R. Then, determine the unity values np_1 from Table 1 and calculate the sample size n

as $\frac{np_1}{p_1}$. Thus, the acceptance numbers and the sample size determined together with φ and m are the parameters of the desired plan.

Table 1: *Unity Value of GZIP DSPs*

φ	m	c_1	c_2	Pa (p)					
				0.99	0.95	0.9	0.5	0.2	0.1
0.03	5	1	2	0.3488	0.6455	0.8704	2.2795	4.2836	6.2789
		1	3	0.5684	0.9343	1.1976	2.7349	4.7742	6.7438
		1	4	0.791	1.2209	1.5219	3.2119	5.3513	7.3541
		1	5	1.0182	1.5102	1.8489	3.7034	5.9797	8.0607
		1	6	1.2504	1.8033	2.1795	4.2049	6.6399	8.8299
		1	7	1.4872	2.1002	2.5138	4.7136	7.3208	9.6402
		1	8	1.728	2.4004	2.8512	5.2275	8.0156	10.4782
		1	9	1.9723	2.7034	3.1912	5.745	8.7201	11.3353
		1	10	2.2194	3.0087	3.5333	6.2654	9.4313	12.2058
		2	3	0.6147	1.0308	1.3368	3.2234	5.9143	8.5762
		2	4	0.8351	1.3024	1.6345	3.5798	6.2375	8.8573
		2	5	1.0569	1.5775	1.9398	3.9877	6.6701	9.2693
		2	6	1.2833	1.8588	2.2535	4.4292	7.1839	9.7977
		2	7	1.515	2.1461	2.5744	4.8934	7.755	10.4186
		2	8	1.7515	2.4385	2.9012	5.3734	8.3665	11.1092
		2	9	1.9921	2.7352	3.2328	5.8649	9.0067	11.8514
		2	10	2.2362	3.0355	3.5682	6.3648	9.6678	12.6316
		3	4	0.9132	1.4502	1.8397	4.2225	7.5954	10.8922
		3	5	1.1254	1.6984	2.1037	4.4942	7.8127	11.082
		3	6	1.3418	1.9581	2.3863	4.8329	8.1275	11.3654
		3	7	1.5646	2.2284	2.6835	5.2197	8.5307	11.7496
		3	8	1.7936	2.5073	2.9918	5.6407	9.006	12.2295
		3	9	2.028	2.7932	3.3088	6.0863	9.5374	12.7925
		3	10	2.2669	3.0847	3.6324	6.55	10.1113	13.4239
		4	5	1.2365	1.8964	2.3715	5.2571	9.2958	13.2063
		4	6	1.4357	2.1188	2.6012	5.4625	9.4506	13.3504
		4	7	1.6438	2.3606	2.8589	5.7376	9.6809	13.5583
		4	8	1.8607	2.6176	3.1373	6.0692	9.9911	13.8426
		4	9	2.0853	2.8862	3.4308	6.4439	10.3759	14.2093
		4	10	2.3161	3.1639	3.7359	6.8511	10.8249	14.6568
		5	6	1.5793	2.364	2.9258	6.3131	11.0028	15.5146
		5	7	1.7633	2.5604	3.123	6.4695	11.121	15.6345
		5	8	1.961	2.7829	3.3551	6.6905	11.2948	15.7986
		5	9	2.1704	3.0249	3.613	6.9699	11.5339	16.0185
		5	10	2.3891	3.2817	3.8902	7.298	11.8413	16.3033
		6	7	1.9377	2.8482	3.4973	7.3819	12.7115	17.8174
		6	8	2.1057	3.02	3.6652	7.5033	12.808	17.9234
		6	9	2.2918	3.2226	3.872	7.6804	12.9446	18.0614
		6	10	2.4923	3.4487	4.1088	7.9135	13.1325	18.2403
		7	8	2.3088	3.3455	4.0817	8.4583	14.4201	20.1156

φ	m	c_1	c_2	$P_a(p)$					
				0.99	0.95	0.9	0.5	0.2	0.1
		7	9	2.4608	3.4947	4.224	8.555	14.5031	20.2132
		7	10	2.6346	3.6778	4.4068	8.6978	14.6156	20.3348
0.03	5	8	9	2.6901	3.8529	4.676	9.5394	16.128	22.4102
		8	10	2.8268	3.9821	4.7965	9.6186	16.2024	22.5023
		9	10	3.0796	4.3683	5.2775	10.6234	17.8349	24.7018
		1	2	0.3675	0.6684	0.8907	2.192	3.8343	5.2986
		1	3	0.6025	0.9714	1.2293	2.6331	4.2825	5.709
		1	4	0.843	1.2746	1.568	3.0994	4.8159	6.2568
		1	5	1.0908	1.5836	1.9123	3.5825	5.3982	6.8918
		1	6	1.3466	1.8991	2.2628	4.0768	6.0096	7.5801
		1	7	1.6097	2.2207	2.6189	4.5787	6.6386	8.3008
		1	8	1.879	2.5474	2.9796	5.0858	7.2784	9.0415
		1	9	2.1537	2.8783	3.3442	5.5963	7.9249	9.7945
		1	10	2.4329	3.2129	3.7119	6.1093	8.5754	10.5551
		2	3	0.6554	1.0761	1.3762	3.0882	5.2284	7.1066
		2	4	0.8908	1.3588	1.6808	3.4237	5.5103	7.3413
		2	5	1.1303	1.649	1.9979	3.817	5.9025	7.703
		2	6	1.378	1.9494	2.3277	4.2483	6.3765	8.1781
		2	7	1.6342	2.2591	2.6681	4.7053	6.9071	8.7405
		2	8	1.8981	2.5768	3.017	5.18	7.4764	9.3656
		2	9	2.1685	2.9009	3.3727	5.6669	8.0719	10.0347
		2	10	2.4444	3.2302	3.7337	6.1625	8.6853	10.7341
		3	4	0.9843	1.5265	1.9068	4.0436	6.6651	8.9182
		3	5	1.209	1.7809	2.1715	4.2873	6.8423	9.0682
0.03	10	3	6	1.442	2.0528	2.4618	4.6048	7.115	9.3052
		3	7	1.6857	2.3403	2.7723	4.977	7.4781	9.6411
		3	8	1.9393	2.6406	3.0984	5.3883	7.9157	10.0719
		3	9	2.2014	2.9512	3.4365	5.8276	8.41	10.5838
		3	10	2.4706	3.27	3.7839	6.287	8.9462	11.1598
		4	5	1.3465	2.0131	2.4756	5.0383	8.1145	10.7165
		4	6	1.5546	2.2357	2.6989	5.2109	8.2333	10.8275
		4	7	1.7771	2.4852	2.9588	5.4576	8.4208	10.9938
		4	8	2.0134	2.7563	3.2463	5.7678	8.6878	11.2317
		4	9	2.2617	3.0441	3.5545	6.1278	9.0324	11.5508
		4	10	2.5197	3.3449	3.8786	6.5254	9.4443	11.9512
		5	6	1.7368	2.5297	3.0754	6.0563	9.5646	12.5006
		5	7	1.925	2.7199	3.2593	6.1786	9.6524	12.5929
		5	8	2.1339	2.9448	3.487	6.3653	9.7858	12.7213
		5	9	2.3604	3.1967	3.7488	6.616	9.98	12.8984
		5	10	2.601	3.4693	4.0364	6.9226	10.243	13.1364
		6	7	2.1506	3.0708	3.6995	7.0875	11.0118	14.2731
		6	8	2.3177	3.2302	3.8481	7.1763	11.083	14.3555
		6	9	2.511	3.4293	4.0438	7.3159	11.1839	14.4633
		6	10	2.7254	3.6602	4.2784	7.5139	11.328	14.6044

φ	m	c_1	c_2	$P_a(p)$					
				0.99	0.95	0.9	0.5	0.2	0.1
0.03	10	7	8	2.5841	3.6314	4.3422	8.126	12.4553	16.0363
		7	9	2.7301	3.7629	4.4607	8.1931	12.5175	16.1132
		7	10	2.9065	3.9365	4.6261	8.2982	12.5992	16.2091
		8	9	3.0339	4.2073	4.9987	9.1684	13.8953	17.7925
		8	10	3.1598	4.3147	5.0928	9.2219	13.9523	17.8657
		9	10	3.4971	4.795	5.6656	10.213	15.3323	19.543
0.07	5	1	2	0.3554	0.66	0.8925	2.3944	4.8306	8.5599
		1	3	0.5786	0.9547	1.2272	2.8707	5.3748	9.1613
		1	4	0.8047	1.2465	1.5583	3.3685	6.013	9.946
		1	5	1.0351	1.5407	1.8917	3.8806	6.7086	10.8654
		1	6	1.2702	1.8385	2.2284	4.4028	7.4406	11.8792
		1	7	1.5099	2.1399	2.5686	4.9321	8.1966	12.9589
		1	8	1.7534	2.4444	2.9117	5.4664	8.9689	14.085
		1	9	2.0003	2.7516	3.2573	6.0045	9.7525	15.2443
		1	10	2.2501	3.0611	3.605	6.5453	10.5441	16.4278
		2	3	0.6249	1.0517	1.3677	3.376	6.631	11.531
		2	4	0.8491	1.3291	1.6727	3.7531	7.0123	11.9703
		2	5	1.0741	1.6092	1.9844	4.1799	7.5013	12.556
		2	6	1.3036	1.895	2.3041	4.64	8.0733	13.2756
		2	7	1.5381	2.1866	2.6307	5.1228	8.7063	14.1084
		2	8	1.7772	2.4833	2.963	5.6217	9.3835	15.0324
		2	9	2.0205	2.7842	3.3	6.1323	10.093	16.0281
		2	10	2.2671	3.0885	3.6408	6.6514	10.8262	17.0794
		3	4	0.9273	1.4775	1.8794	4.4115	8.4709	14.4839
		3	5	1.1431	1.7314	2.1507	4.7059	8.756	14.8475
		3	6	1.3627	1.9959	2.4394	5.0632	9.1321	15.32
		3	7	1.5883	2.2705	2.7422	5.4671	9.5924	15.9023
		3	8	1.8199	2.5535	3.0558	5.905	10.1245	16.589
		3	9	2.0569	2.8434	3.3779	6.3677	10.7143	17.3698
		3	10	2.2983	3.1388	3.7067	6.8489	11.3496	18.2324
		4	5	1.2543	1.9301	2.4198	5.4807	10.3229	17.4184
		4	6	1.4573	2.1584	2.6573	5.712	10.5536	17.742
		4	7	1.6684	2.4048	2.9209	6.0069	10.8534	18.1493
		4	8	1.888	2.6658	3.2045	6.3553	11.226	18.6439
		4	9	2.115	2.9383	3.503	6.7457	11.6683	19.2263
		4	10	2.3483	3.2196	3.8129	7.1685	12.173	19.8938
		5	6	1.6007	2.4037	2.9824	6.5697	12.1774	20.339
		5	7	1.7887	2.6065	3.1881	6.7554	12.3767	20.6388
		5	8	1.9894	2.8335	3.4261	6.9985	12.6274	21.0059
		5	9	2.2013	3.0794	3.6891	7.2957	12.9365	21.4438
		5	10	2.4223	3.3398	3.9708	7.6391	13.3066	21.9543
		6	7	1.9627	2.8938	3.5619	7.6702	14.0314	23.2495
		6	8	2.1349	3.0725	3.7391	7.824	14.2118	23.5338
		6	9	2.324	3.2798	3.9521	8.0262	14.4309	23.8743

φ	m	c_1	c_2	$P_a(p)$					
				0.99	0.95	0.9	0.5	0.2	0.1
		6	10	2.5269	3.5097	4.1939	8.2787	14.6957	24.2737
		7	8	2.3372	3.3968	4.154	8.7776	15.8839	26.1528
		7	9	2.4938	3.5536	4.3066	8.9092	16.0525	26.426
0.07	5	7	10	2.6706	3.7416	4.4961	9.08	16.2509	26.7478
		8	9	2.7219	3.9098	4.7557	9.889	17.7348	29.0506
		8	10	2.8635	4.0472	4.8875	10.0052	17.8956	29.3156
		9	10	3.1147	4.4305	5.3644	11.0028	19.5843	31.9442
		1	2	0.3744	0.6832	0.9129	2.2942	4.2581	6.8443
		1	3	0.6134	0.9921	1.2588	2.7535	4.7481	7.3537
		1	4	0.8574	1.3007	1.6042	3.2377	5.3299	8.0376
		1	5	1.1086	1.6146	1.9546	3.7384	5.9662	8.8447
		1	6	1.3675	1.9346	2.3108	4.2501	6.6352	9.734
		1	7	1.6334	2.2605	2.6724	4.7692	7.3243	10.6776
		1	8	1.9055	2.5913	3.0384	5.2933	8.0258	11.6572
		1	9	2.1828	2.9262	3.4081	5.8206	8.7349	12.6611
		1	10	2.4645	3.2646	3.7808	6.35	9.4486	13.6816
		2	3	0.6663	1.0975	1.4071	3.2217	5.7651	9.0165
		2	4	0.9057	1.386	1.7188	3.5751	6.0952	9.3774
		2	5	1.1485	1.681	2.042	3.9842	6.5325	9.8823
		2	6	1.3992	1.9858	2.3772	4.4308	7.0528	10.5184
		2	7	1.6583	2.2997	2.7228	4.903	7.6328	11.2631
		2	8	1.9249	2.6213	3.0767	5.393	8.2547	12.0916
		2	9	2.1978	2.9492	3.4373	5.8954	8.9057	12.983
		2	10	2.4762	3.2823	3.8031	6.4065	9.5767	13.9204
		3	4	0.9993	1.5547	1.9465	4.2066	7.3015	11.1529
0.07	10	3	5	1.228	1.8147	2.2183	4.4706	7.5386	11.4466
		3	6	1.4641	2.091	2.514	4.8033	7.8632	11.8479
		3	7	1.7105	2.3824	2.8293	5.1888	8.2725	12.3593
		3	8	1.9666	2.6864	3.1601	5.6132	8.7546	12.9752
		3	9	2.2312	3.0006	3.5027	6.0657	9.2948	13.6839
		3	10	2.5028	3.3229	3.8546	6.5388	9.8795	14.4708
		4	5	1.3657	2.0478	2.5239	5.2283	8.8418	13.2582
		4	6	1.5777	2.2761	2.7546	5.4251	9.0295	13.5176
		4	7	1.803	2.5298	3.0196	5.6884	9.2796	13.8605
		4	8	2.0419	2.8044	3.3113	6.0113	9.6	14.2917
		4	9	2.2924	3.0954	3.6237	6.3826	9.9908	14.8122
		4	10	2.5527	3.3994	3.9518	6.7914	10.4459	15.4192
		5	6	1.7601	2.5708	3.132	6.2712	10.378	15.3406
		5	7	1.9523	2.7671	3.324	6.4218	10.5396	15.5798
		5	8	2.1639	2.9959	3.5567	6.6279	10.7447	15.8875
		5	9	2.3926	3.2509	3.8223	6.8922	11.0024	16.2675
		5	10	2.6353	3.5264	4.1135	7.2093	11.3191	16.722
		6	7	2.1779	3.118	3.7639	7.3255	11.9085	17.4062
		6	8	2.3492	3.2841	3.9216	7.4462	12.0559	17.6321

φ	m	c_1	c_2	$P_a(p)$					
				0.99	0.95	0.9	0.5	0.2	0.1
0.07	10	6	9	2.545	3.4872	4.1225	7.609	12.2343	17.9166
		6	10	2.7615	3.7209	4.3607	7.8226	12.4509	18.2623
		7	8	2.6153	3.6844	4.4139	8.3859	13.4337	19.4595
		7	9	2.7658	3.8236	4.5429	8.4879	13.5728	19.6757
		7	10	2.9447	4.0012	4.714	8.6197	13.7352	19.9435
		8	9	3.0688	4.2658	5.0773	9.4496	14.9545	21.5035
		8	10	3.1997	4.3818	5.1832	9.5399	15.0883	21.7122
		9	10	3.5356	4.8587	5.7507	10.515	16.4715	23.5402

Table 2: *Operating Ratio of GZIP DSPs*

φ	m	c_1	c_2	R	
				$\alpha = 0.05, \beta = 0.10$	$\alpha = 0.10, \beta = 0.20$
0.03	5	1	2	9.7272	0.2222
		1	3	7.218	4.9214
		1	4	6.0235	3.9865
		1	5	5.3375	3.5162
		1	6	4.8965	3.2342
		1	7	4.5901	3.0465
		1	8	4.3652	2.9122
		1	9	4.193	2.8113
		1	10	4.0568	2.7325
		2	3	8.3199	2.6693
		2	4	6.8008	4.4242
		2	5	5.8759	3.8162
		2	6	5.271	3.4386
		2	7	4.8547	3.1879
		2	8	4.5558	3.0124
		2	9	4.3329	2.8838
		2	10	4.1613	2.786
		3	4	7.5108	2.7094
		3	5	6.525	4.1286
		3	6	5.8043	3.7138
		3	7	5.2727	3.4059
		3	8	4.8776	3.1789
		3	9	4.5799	3.0102
		3	10	4.3518	2.8824
		4	5	6.9639	2.7836
		4	6	6.3009	3.9198
		4	7	5.7436	3.6332
		4	8	5.2883	3.3862
		4	9	4.9232	3.1846
		4	10	4.6325	3.0243
		5	6	6.5629	2.8975
		5	7	6.1063	3.7606

φ	m	c_1	c_2	R	
				$\alpha = 0.05, \beta = 0.10$	$\alpha = 0.10, \beta = 0.20$
		5	8	5.677	3.561
		5	9	5.2955	3.3665
		5	10	4.9679	3.1923
		6	7	6.2557	3.0439
		6	8	5.9349	3.6347
		6	9	5.6046	3.4945
0.03	5	6	10	5.289	3.3431
		7	8	6.0127	3.1962
		7	9	5.784	3.5329
		7	10	5.5291	3.4335
		8	9	5.8164	3.3166
		8	10	5.6509	3.4491
		9	10	5.6548	3.378
		1	2	7.9273	3.3794
		1	3	5.8771	4.3048
		1	4	4.9088	3.4837
		1	5	4.352	3.0714
		1	6	3.9914	2.8229
		1	7	3.7379	2.6558
		1	8	3.5493	2.5349
		1	9	3.4029	2.4427
		1	10	3.2852	2.3697
		2	3	6.604	2.3102
		2	4	5.4028	3.7992
		2	5	4.6713	3.2784
		2	6	4.1952	2.9544
		2	7	3.869	2.7394
		2	8	3.6346	2.5888
0.03	10	2	9	3.4592	2.4781
		2	10	3.323	2.3933
		3	4	5.8423	2.3262
		3	5	5.0919	3.4954
		3	6	4.5329	3.151
		3	7	4.1196	2.8902
		3	8	3.8142	2.6974
		3	9	3.5863	2.5548
		3	10	3.4128	2.4473
		4	5	5.3234	2.3643
		4	6	4.843	3.2778
		4	7	4.4237	3.0506
		4	8	4.0749	2.846
		4	9	3.7945	2.6762
		4	10	3.573	2.5411
		5	6	4.9415	2.435

φ	m	c_1	c_2	R	
				$\alpha = 0.05, \beta = 0.10$	$\alpha = 0.10, \beta = 0.20$
0.03	10	5	7	4.6299	3.11
		5	8	4.3199	2.9615
		5	9	4.0349	2.8064
		5	10	3.7865	2.6622
		6	7	4.648	2.5377
		6	8	4.4442	2.9766
		6	9	4.2176	2.8801
		6	10	3.9901	2.7657
		7	8	4.416	2.6477
		7	9	4.2821	2.8684
		7	10	4.1176	2.8062
		8	9	4.229	2.7235
		8	10	4.1407	2.7798
		9	10	4.0757	2.7396
0.07	5	1	2	12.9695	2.7062
		1	3	9.596	5.4124
		1	4	7.9791	4.3797
		1	5	7.0522	3.8587
		1	6	6.4614	3.5463
		1	7	6.0558	3.339
		1	8	5.7622	3.1911
		1	9	5.5402	3.0803
		1	10	5.3666	2.994
		2	3	10.9642	2.9249
		2	4	9.0063	4.8483
		2	5	7.8026	4.1922
		2	6	7.0056	3.7801
		2	7	6.4522	3.5039
		2	8	6.0534	3.3095
		2	9	5.7568	3.1669
		2	10	5.53	3.0585
		3	4	9.803	2.9736
		3	5	8.5754	4.5072
		3	6	7.6757	4.0712
		3	7	7.0039	3.7436
		3	8	6.4966	3.4981
		3	9	6.1088	3.3132
		3	10	5.8087	3.1719
		4	5	9.0246	3.0619
		4	6	8.22	4.266
		4	7	7.5471	3.9716
		4	8	6.9937	3.7158
		4	9	6.5433	3.5032
		4	10	6.179	3.3309

φ	m	c_1	c_2	R	
				$\alpha = 0.05, \beta = 0.10$	$\alpha = 0.10, \beta = 0.20$
		5	6	8.4615	3.1926
		5	7	7.9182	4.0831
		5	8	7.4134	3.8822
		5	9	6.9636	3.6856
		5	10	6.5735	3.5067
		6	7	8.0342	3.3511
		6	8	7.6595	3.9393
0.07	5	6	9	7.2792	3.8009
		6	10	6.9162	3.6515
		7	8	7.6992	3.5041
		7	9	7.4364	3.8238
		7	10	7.1488	3.7274
		8	9	7.4302	3.6144
		8	10	7.2434	3.7292
		9	10	7.2101	3.6615
		1	2	10.018	3.6508
		1	3	7.4123	4.6644
		1	4	6.1794	3.7719
		1	5	5.478	3.3225
		1	6	5.0315	3.0524
		1	7	4.7236	2.8714
		1	8	4.4986	2.7407
		1	9	4.3268	2.6415
		1	10	4.1909	2.563
		2	3	8.2155	2.4991
		2	4	6.7658	4.0972
		2	5	5.8788	3.5462
		2	6	5.2968	3.1991
		2	7	4.8976	2.9669
0.07	10	2	8	4.6128	2.8033
		2	9	4.4022	2.683
		2	10	4.2411	2.5909
		3	4	7.1737	2.5181
		3	5	6.3077	3.7511
		3	6	5.6661	3.3984
		3	7	5.1878	3.1278
		3	8	4.83	2.9239
		3	9	4.5604	2.7704
		3	10	4.3549	2.6536
		4	5	6.4744	2.563
		4	6	5.9389	3.5032
		4	7	5.4789	3.278
		4	8	5.0962	3.0731
		4	9	4.7852	2.8992

φ	m	c_1	c_2	R	
				α = 0.05, β =0.10	α = 0.10, β =0.20
0.07	10	4	10	4.5359	2.7571
		5	6	5.9672	2.6433
		5	7	5.6304	3.3135
		5	8	5.3031	3.1708
		5	9	5.004	3.021
		5	10	4.7419	2.8785
		6	7	5.5825	2.7517
		6	8	5.3689	3.1639
		6	9	5.1378	3.0742
		6	10	4.908	2.9677
		7	8	5.2816	2.8553
		7	9	5.1459	3.0435
		7	10	4.9844	2.9877
		8	9	5.0409	2.9137
		8	10	4.9551	2.9454
		9	10	4.845	2.911

IV. Numerical Examples

This section outlines the process for choosing GZIP DSPs for a defined strength, along with numerical examples.

When solving the OC function of the GZIP DSPs conditions, the unity values for various combinations of (φ, m, c, $P_a(p)$) are calculated by considering each combination of c_1 and c_2 with the condition $n_1 = n_2 = n$. Then, the plan parameters for specific values of φ, m, p_1, α, p_2 and β are determined GZIP DSPs.

I. Example 1

Suppose that $\varphi = 0.07$, $m = 10$, and the strength of the plan is specified as $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.06$, $\beta = 0.10$, the operating ratio R corresponding to these specifications is computed as 6. The acceptance number can be determined from Table 2 as $c_1 = 5$ and $c_2 = 6$ with an R value of 5.9672, which is close to 6. The unity values corresponding to the φ, m, p_1, α, $p_2$and β parameters are obtained from Table 1 as $np_1 = 2.5708$.

Since $\mathrm{n} = \frac{np_1}{p_1} = \frac{2.5708}{0.01} \approx 257$. Based on these calculations, the optimum DSP is $n_1 = 257$, $c_1 = 5$, $n_2 = 257$ and $c_2 = 6$.

II. Example 2

Assuming $\varphi = 0.03$, $m = 10$, and the plan's strength is set at $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.06$, $\beta = 0.10$, the operating ratio R can be calculated as 6. The acceptance number can be determined from Table 2 as $c_1 = 1$ and $c_2 = 3$ with an R value of 5.8771, which is close to 6. Consequently, the unity values for φ, m, p_1, α, $p_2$and β from Table 1 are $np_1 = 0.9714$.

Since, $\mathrm{n} = \frac{np_1}{p_1} = \frac{0.9714}{0.01} \approx 97$. the optimal GZIP DSPs for the specified specifications is $n_1 = 97$, $c_1 = 1$, $n_2 = 97$, $c_2 = 3$, $\varphi = 0.03$ and $m = 10$.

The ZIP DSP for the specified strength is $n_1 = 102$, $c_1 = 1$, $n_2 = 102$, $c_2 = 3$, $\varphi = 0.03$.

This indicates that GZIP DSP requires smaller sample compared to ZIP DSP.

III. Significance of GZIP DSPs over non-Bayesian ZIP DSPs.

The producer's risk for the ZIP DSP is 4.63% when the lot fraction non-conforming is $p = 0.009$. On the other hand, the producer's risk for the GZIP DSPs for the given values of $m = 5$ and 10 are 3.32% and 3.56%, respectively. When the lot fraction non-conforming is $p = 0.059$, the consumer's risk for the ZIP DSP is 10.35%. In contrast, the consumer's risk for the GZIP DSPs for the specified values of $m = 5$ and 10 is 10.36% and 9.94%, respectively. For the ZIP DSP, the combined producer and consumer risk is 14.98%. On the other hand, the GZIP DSP has a total risk for $m = 5$ and 10 are 13.68% and 13.50% respectively.

Table 3: *Values of OC function of DSP ZIP and GZIP DSPs for ($p_1 = 0.005$, $\alpha = 0.05$, $p_2 = 0.05$, $\beta = 0.10$)*

Model	Parameters					Lot Fraction non-conforming (p)			Producer's Risk (%)	Consumer's Risk (%)	Total Risk (%)
	φ	m	c_1	c_2	n	0	0.009	0.059			
ZIP DSP	0.03	-	1	2	75	1	0.9537	0.1035	4.63	10.35	14.98
GZIP DSP	0.03	5	7	8	335	1	0.9668	0.1036	3.32	10.36	13.68
		10	1	3	97	1	0.9643	0.0994	3.56	9.94	13.50

V. Conclusion

The fundamental assumption in the theory of sampling inspection procedures by attributes is that the fraction of nonconforming items in a lot remains constant. However, in real-world scenarios, lots produced from a process often exhibit quality variations due to random fluctuations, leading to a random variation in the fraction of nonconforming units across lots. In such situations, Bayesian acceptance sampling plans (BASP), which incorporate prior information about process variability when making decisions on submitted lots, can offer an advantage over conventional plans. In this paper, Bayesian double sampling plans by attributes are determined for the two specified points on the OC curve based on gamma-ZIP distribution. The GZIP DSPs requires fewer sample units for inspection compared to non-Bayesian ZIP DSPs. As a result, GZIP DSPs effectively lower both producer's and consumer's risks, offering better protection for both parties by minimizing the chances of rejecting good-quality lots and accepting poor-quality ones. By implementing GZIP DSPs, optimal sample sizes and acceptance numbers are achieved, reducing overall risk and delivering benefits such as enhanced customer satisfaction, increased productivity, and sustained market competitiveness.

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