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OPTIMIZING INVENTORY OF DETERIORATING PRODUCTS WITH PRICE-DEPENDENT DEMAND USING QUANTUM-BEHAVED AGTO VARIANTS

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Abstract

Preservation of a product is an important issue in the inventory control system. It prevents the deterioration effect of the products while these are stored in the warehouse/showroom. Considering deterioration effect of the product and preservation technology, an inventory model of non-instantaneous deteriorating items is developed with the demand dependent on the selling price of the product. Two different preservation rates are considered. Shortages are allowed partially with two different backlogging rates. Due to consideration of three-parameter Weibull distributed deterioration and preservation facility, the corresponding optimization problems are highly nonlinear. So, these problems cannot be solved analytically due to nonlinearity. To overcome this situation, different variants of quantum-behaved Artificial Gorilla Troops Optimizer (AGTO) are used. To illustrate and validate the proposed model, a numerical example is considered and solved for each case, and compared the results with the different variants of AGTO algorithms. Finally, a sensitivity analysis is performed to study the effect of changes of different parameters of the model on the optimal policy.

Keywords:AGTO;Quantum-Behaved;Deteriorating Products;Price-Dependent Demand.

1. Introduction

Preserving product quality during storage is a critical issue in inventory management, especially for items susceptible to deterioration [1]. When setting up supplies, systems with two-level assembly and unpredictable lead timing are taken into account. Most likely, the demand for the finished product and its deadline are known. As soon as all necessary components arrive, each level's assembly process begins [2]. Shortages are permitted, but only partially, and are handled with two separate backlogging rates, allowing flexibility in managing stockouts [3]. The product has a set shelf life and is perishable. Expenses include fixed ordering and inventory holding expenses[4]. The deterioration process of the items follows a three-parameter Weibull distribution, capturing the complexity of product degradation over time [5]. Managing stochastic multi-state production and distribution systems can make it difficult to figure out how much safety stock to have on hand. [6]. Due to the inclusion of this deterioration model and the preservation measures, the resulting optimization problem becomes highly nonlinear, making it challenging to solve using traditional analytical methods. To tackle this, various versions of the Quantum-behaved AGTO are employed, which are metaheuristic algorithms inspired by gorilla troops' natural behaviours [7]. These AGTO variants are particularly well-suited for handling the nonlinear, multi-parameter nature of the inventory problem. A numerical example is provided to demonstrate and validate the effectiveness of the model, with results compared across different AGTO variants [8] . Additionally, a sensitivity analysis is conducted to understand the impact of changing key parameters, such as demand rate, deterioration rate, and preservation efficiency, on the optimal inventory policy. This analysis provides valuable insights into how different factors influence the overall cost and performance of the inventory system, guiding decision-makers in applying the best preservation strategies to maximize profitability and minimize product waste [9]. This study focuses on developing an advanced inventory model for non-instantaneously deteriorating items, considering both the deterioration effects and preservation technologies to minimize losses during storage in warehouses or showrooms. In this model, product demand is driven by its selling price, reflecting real-market dynamics. The inventory system also incorporates two distinct preservation rates, addressing different levels of protection against product degradation.

2. Assumptions and Notation

The following assumptions and notation are considered to develop the model:

1. The inventory system contains only one item with an infinite time horizon.
2. The item is a non-instantaneously deteriorating item. Deterioration occurs after time $h = \gamma$ with the rate $\theta(h)$, which follows a three-parameter Weibull distribution. That is,
$$\theta(h) = \frac{s(h)}{1 - S(h)} = \alpha\beta(h-\gamma)^{\beta-1},$$
where α, β, and γ (> 0) are the parameters of the Weibull distribution, with $s(h)$ and $S(h)$ as the probability density function and distribution function, respectively.
3. To reduce the deterioration effect, preservation technology is used in the inventory control system. Let $n(\zeta) < 0$ be the preservation technology function, which is an increasing function with $n''(\zeta) < 0$. Here, we consider $n(\zeta)$ as:
$$n(\zeta) = \frac{y_1\zeta}{1 + y_1\zeta}, \quad y_1 > 0,$$
$$n(\zeta) = 1 - e^{-y_1\zeta}, \quad y_1 > 0.$$
4. Lead time is constant and known.
5. Shortages are considered partially with a rate $Z(H-h)$, where h represents the length of waiting time for the customers. In this model, two partial backlogging rates are considered:
$$Z(H-h) = \frac{1}{1 + \delta(H-h)},$$
$$Z(H-h) = e^{-\delta(H-h)}.$$
6. The demand of the item is dependent linearly on the selling price, i.e.,
$$X(l) = y - zl, \quad l < \frac{y}{z}, \ y,z > 0.$$

Notation	Description
W_l	Purchase cost per unit item (in \$)
l	Selling price per unit item (decision variable) (in \$), $l > W_l$
$X(l) = y - zl$	The demand function, $l < \frac{y}{z}$, $y, z > 0$
W_0	Replenishment cost per order (in \$)
W_t	Holding cost per unit item per unit time (in \$)
W_g	Backordering cost per unit item per unit time (in \$)
W_{pg}	Lost sale cost per unit item per unit time (in \$)
γ	Starting time of deterioration (in month)
h_1	Time of zero ending inventory (decision variable) (in month)
H	Cycle length (decision variable) (in month)
I	Initial ordering quantities (unit)
J	Maximum shortage level (unit)
$Q(h)$	Inventory level at time h (unit)
$\theta(h)$	Deterioration rate per unit time
$Z(H-h)$	Backorder rate at time h, $h_1 < h \leq H$ (in month)
δ	Backlogging parameter
ζ	Preservation cost per unit item per unit time (decision variable) (in \$)
$B(l, \zeta, h_1, H)$	Average profit function (in \$)

3. Model formulation

Let us suppose that a retailer places an order of $(I + J)$ units of a product at time h = 0. After that, the deterioration starts at time $h = \gamma$ and inventory level reaches to zero at time $h = h_1$ due to the combined effect of demand and deterioration [10]. Then partially backlogged shortages are allowed with the backlogging rates. $Z(H-h) = \frac{1}{1+\delta(H-h)}$ and $Z(H-h) = e^{-\delta(H-h)}$: During the time period $[0, \gamma]$, there is no deterioration and after that deterioration starts at $h = \gamma$ and continues upto the time $h = h_1$. To reduce the rate of deterioration, we have considered preservation facility with the rates $n(\zeta) = \frac{y_1 \zeta}{1+y_1 \zeta}$ and $n(\zeta) = 1 - e^{-y_1 \zeta}$ where $z_1 > 0$. Hence during the time interval $[0, H]$; the inventory levels are governed by the differential equations as follows:

$$\frac{\mathrm{d}Q(h)}{\mathrm{d}h} = -X(l), \quad 0 < h \leq \gamma \tag{1}$$

$$\frac{\mathrm{d}Q(h)}{\mathrm{d}h} = -X(l) - \theta(h)[1 - n(\zeta)]Q(h), \quad \gamma < h \leq h_1 \tag{2}$$

$$\frac{\mathrm{d}Q(h)}{\mathrm{d}h} = -X(l)Z(H-h), \quad h_1 < h \leq H \tag{3}$$

With the boundary conditions $Q(0) = I$ and $Q(H) = -J$. Also $Q(h)$ is continuous at $h = \gamma$ and $h = h_1$.

$$Q(h) = -X(l)h + I, \quad 0 < h \leq \gamma \tag{4}$$

$$Q(h) = -X(l)e^{-(h-\gamma)^{\beta}(1-n(\zeta))} \int_{h}^{h_1} e^{\alpha(h-\gamma)^{\beta}(1-n(\zeta))} \, \mathrm{d}h, \quad \gamma < h \leq h_1 \tag{5}$$

$$Q(h) = X(l) \int_{h_1}^{h} Z(H-h) \, \mathrm{d}h, \quad h_1 < h \leq H \tag{6}$$

Now applying the continuity condition of $Q(h)$ at $h = \gamma$ and h_1, we have:

$$I = X(l)\gamma + X(l) \int_{h_1}^{j} e^{\alpha(h-\gamma)^{\beta}(1-n(\zeta))} \, \mathrm{d}h \tag{7}$$

And

$$J = X(l) \int_{h_1}^{H} Z(H-h) \, \mathrm{dh} \tag{8}$$

Total sales revenue $(\mathrm{GJ}) = \mathrm{l}X(l)h_1 + \mathrm{l}J$

Total purchase cost $(\mathrm{LW}) = W_l(I+J)$

$$= W_l X(l)\gamma + W_l X(l)\int_{h_1}^{j} e^{\alpha(h-\gamma)^{\beta}(1-n(\zeta))}\,\mathrm{d}h + W_l X(l)\int_{h_1}^{H} Z(H-h)\,\mathrm{d}h \tag{9}$$

The total inventory holding cost

$$\mathrm{TW} = W_t\int_0^{\gamma} Q(h)\,\mathrm{d}h + W_t\int_{\gamma}^{h_1} Q(h)\,\mathrm{d}h \tag{10}$$

$$= W_t I_{\gamma} - \frac{W_t X(l)\gamma^2}{2} + W_t X(l)\int_{\gamma}^{h_1} e^{-\alpha(h-\gamma)^{\beta}\{1-n(\zeta)\}} \times \int_{h}^{h_1} e^{\alpha(u-\gamma)^{\beta}}\mathrm{dudh} \tag{11}$$

The total shortage cost and lost sale cost are

$$W_g X(l)\int_{h_1}^{H}\int_{h_1}^{h} Z(H-h)\,\mathrm{d}u\,\mathrm{d}h$$

And

$$W_{\mathrm{pg}}X(l)(H-h_1) - W_{\mathrm{pg}}J,$$

Respectively, The preservation technology cost is given by ζH. Therefore, the average profit per cycle is given by...

$$B\left(l,\zeta,h_1,H\right) = \frac{1}{H}[\text{Sales revenue} - \text{Purchase cost} - \text{Holding cost} - \text{Shortage cost} - \text{Lost sale cost} - \text{Ordering cost} - \text{Preservation technology cost}] \tag{12}$$

$$i.e., B(l,\zeta,h_1,H) = \frac{1}{H}\left[\begin{array}{c} \mathrm{lX}(l)h_1 + \mathrm{lJ} - \left\{W_l X(l)\gamma + W_l X(l)\int_{\gamma}^{h1} e^{\alpha(h-\gamma)^{\beta}\{1-n(\zeta)\}\mathrm{dh}} + W_l J\right\} \\ - \left\{W_t I fl - \frac{W_t X(l)\gamma^2}{2} + W_h X(l)\int_{\gamma}^{h1} e^{-\alpha(h-\gamma)^{\{1-n(\zeta)\}}}\mathrm{dudh}\right\} \\ -W_g X(l)\int_{h1}^{H}\int_{h1}^{h} Z(H-h)\mathrm{dhdh} - W_{\mathrm{pg}}\left\{X(l)\left(H-h_1\right) - J\right\} - \imath H - W_0 \end{array}\right] \tag{13}$$

Based on the partial backlogging rate $Z\left(H-h\right)$ and also the preservation facility rate $n\left(\zeta\right)$, four possible cases may arise:

Case 1: $Z\left(H-h\right) = \frac{1}{1+\delta(H-h)}$ and $n(\zeta) = \frac{y_1\zeta}{1+y_1\zeta},\ y_1 > 0$

Case 2: $Z\left(H-h\right) = \frac{1}{1+\delta(H-h)}$ and $n(\zeta) = 1 - e^{-y_1\zeta},\ y_1 > 0$

Case 3: $Z\left(H-h\right) = e^{-\delta(H-h)}$ and $n(\zeta) = \frac{y_1\zeta}{1+y_1\zeta},\ y_1 > 0$

Case 4: $Z\left(H-h\right) = e^{-\delta(H-h)}$ and $n(\zeta) = 1 - e^{-y_1\zeta},\ y_1 > 0$

Now we have discussed each case separately.

Case 1: $Z\left(H-h\right) = \frac{1}{1+\delta(H-h)}$ and $n(\zeta) = \frac{y1\imath}{1+y1\imath}, y1 > 0$. In this case,

$$I = X(l)\gamma + X(l)\int_{j}^{h_1} e^{\frac{\alpha(h-\gamma)^{\beta}}{1+\zeta}}\,dh \tag{14}$$

And

$$J = \frac{X(l)}{\delta} \log(l + \delta(H - h_1)) \tag{15}$$

The total purchase cost is given by

$$\text{LW} = W_l(I + J)$$

$$= W_l X(l)\gamma + W_l X(l) \int_j^{h_1} e^{\frac{\alpha(h-\gamma)^\beta}{1+\zeta}} dh + W_l \times \frac{X(l)}{\delta} \log|1 + \delta(H - h_1)| \tag{16}$$

The total inventory holding cost is

$$\text{TW} = W_t \int_0^\gamma Q(h)\, dh + W_t \int_\gamma^{h_1} Q(h)\, dh$$

$$= W_t I_\gamma - \frac{W_t X(l)\gamma^2}{2} + W_t X(l) \int_\gamma^{h_1} e^{\frac{\alpha(h-\gamma)^\beta}{1+\zeta}} \int_h^{h_1} e^{\frac{\alpha(U-\gamma)^\beta}{e^{1+\zeta}}} du\, dh \tag{17}$$

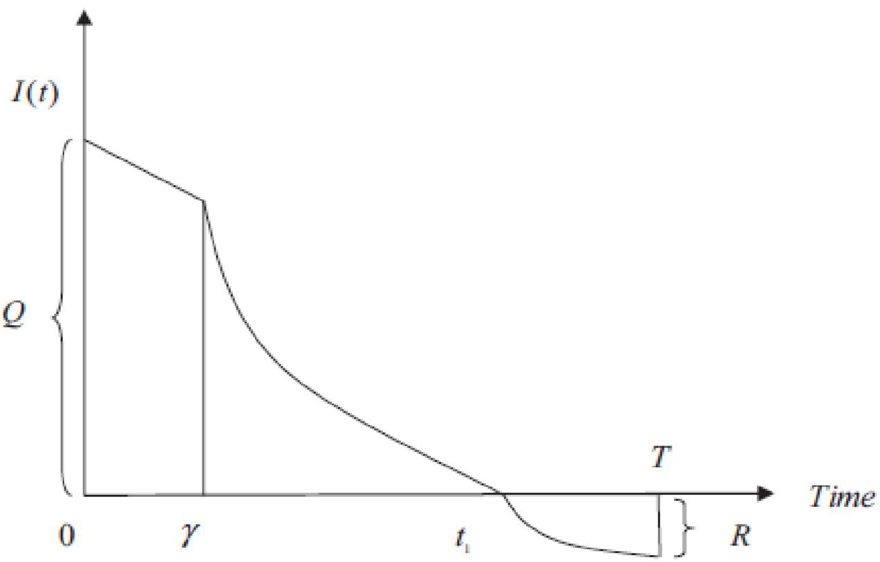

Figure 1: *Pictorial representation of inventory level during the cycle*

$$I = X(l)\gamma + X(l) \int_\gamma^{h_1} e^{\alpha(h-\gamma)^\beta e^{-y_1}\zeta} dh \tag{18}$$

And

$$J = \frac{X(l)}{\delta} \log(1 + \delta(H - h_1)) \tag{19}$$

Now, the total average profit per cycle is given by

$$
\begin{aligned}
B(l,\zeta,h_1,H) = \frac{1}{H}\Bigg[& lX(l)h_1 + lJ - \Big\{ W_l X(l)\gamma + W_l X(l)\int_{\gamma}^{h_1} e^{\alpha(h-\gamma)^{\beta}} e^{-y_1\zeta}\,dh \\
& + W_l \frac{X(l)}{\delta}\log(1+\delta(H-h_1))\Big\} \\
& - \Big\{ W_t I_{\gamma} - \frac{W_t X(l)\gamma^2}{2} + W_t X(l)\int_{\gamma}^{h_1} e^{\alpha(h-\gamma)^{\beta}} e^{-y_1\zeta}\,du\,dh \Big\} \\
& - \frac{W_g}{\delta}\Big(X(l)(H-h_1) - J \Big) \\
& - W_{\text{pg}}\Big(X(l)(H-h_1) - J \Big) - \zeta H - W_0 \Bigg]
\end{aligned} \tag{20}
$$

Shortage cost = $\frac{W_g}{\delta}\,[X(l)\,(H-h1)-J]$
Lost sale cost = $C_{\text{pg}}\,[X(l)\,(H-h1)-J]$
Therefore, the average profit per cycle is given by
In this case, the corresponding optimization problem is as follows:

$$
\begin{aligned}
B(l,\zeta,h_1,H) = \frac{1}{H}\Bigg[& lX(l)h_1 + lJ - \Big\{ W_l X(l)\int_{\gamma}^{h_1} e^{\frac{\alpha(h-\gamma)^{\beta}}{1+\zeta}}\,dh + W_l\frac{X(l)}{\delta}\log\left(1+\delta(H-h_1)\right) \Big\} \\
& - \Big\{ W_t I_{\gamma} - \frac{W_t X(l)\gamma^2}{2} + W_t X(l)\int_{\gamma}^{h_1} e^{-\frac{\alpha(h-\gamma)^{\beta}}{1+\zeta}} \int_{h}^{h_1} e^{\frac{\alpha(u-\gamma)^{\beta}}{1+\zeta}}\,du\,dh \Big\} \\
& - \frac{W_g}{\delta}\left(X(l)(H-h_1)-J\right) - W_{\text{pg}}\left(X(l)(H-h_1)-J\right) - \zeta H - W_0 \Bigg]
\end{aligned} \tag{21}
$$

The corresponding optimization problem is as follows:
Maximize $B(l,\zeta,h_1,H)$
Subject to:

$$
l > \frac{y}{z}, \zeta > 0, H > h_1 > 0
$$

Case 2: $Z(H-h) = \frac{1}{1+\delta(H-h)}$ and $n(\zeta) = 1 - e^{-y_1\zeta}, \quad y_1 > 0$
In this case, the corresponding optimization problem is as follows:
Maximize $B(l,\zeta,h_1,H)$
Subject to:

$$
l > \frac{y}{z}, \zeta > 0, H > h_1 > 0 \tag{22}
$$

Case 3: When $Z\,(H-h) = e^{-\delta(H-h)}$ and $n(\zeta) = \frac{y_1\zeta}{1+y_1\zeta}, \quad y_1 > 0$ When

$$
I = X(l)\gamma + X(l)\int_{\gamma}^{h_1} e^{\frac{\alpha(h-\gamma)^{\beta}}{1+\zeta}}\,dh \tag{23}
$$

And

$$
J = \frac{X(l)}{\delta}\left[1 - e^{\delta(H-h_1)}\right] \tag{24}
$$

The total purchase cost (Purcos$t$) $= W_l(I+J)$

$$
= W_l X(l)\gamma + W_l X(l)\int_{\gamma}^{h1} e^{\frac{\alpha(h-\gamma)\beta}{1+\zeta}}\,\mathrm{dh} + W_l\frac{X(l)}{\delta}\left[1 - e^{-\delta(H-h1)}\right] \tag{25}
$$

Shortage cost = $\frac{W_g}{\delta} X(l)(H-h1)e^{-\delta(H-h1)} - \frac{W_g}{\delta} J$ Lost sale cost = $W_{\text{pg}}\{X(l)(H-h1)-J\}$ Here, the average profit per cycle is given by

$$B(l,\zeta,h_1,H)=\frac{1}{H}\Bigg[lX(l)h_1+lJ-\left\{W_lX(l)\gamma+W_lX(l)\int_\gamma^{h_1}e^{\frac{\alpha(h-\gamma)^\beta}{1+\zeta}}dh+W_lJ\right\}$$
$$-\left\{W_tI_\gamma-\frac{W_tX(l)\gamma^2}{2}+W_tX(l)\int_\gamma^{h_1}e^{\frac{\alpha(h-\gamma)^\beta}{1+\zeta}}\int_h^{h_1}e^{\frac{\alpha(u-\gamma)^\beta}{1+\zeta}}du\,dh\right\}$$
$$-\frac{W_g}{\delta}X(l)(H-h_1)e^{-\delta(H-h_1)}+\frac{W_g}{\delta}J-W_{pg}X(l)(H-h_1)-\zeta H-W_0\Bigg] \tag{26}$$

The corresponding optimization problem is as follows:
Maximize
$B(l,\zeta,h1.H)$
subject to $l > \frac{y}{z}, \zeta > 0, H > h1 > 0$
Case 4: When $Z(H-h) = e^{-\delta(H-h)}$ and $n(\zeta) = 1-e^{-y1\iota}, y1 > 0$ In this case,

$$I = X(l)\gamma + X(l)\int_\gamma^{h1} e^{\alpha(h-\gamma)^\beta e^{-y_1\zeta}}\text{dh} \tag{27}$$

And $J = \frac{X(l)}{\delta}\left[1-e^{-\delta(H-h1)}\right]$
The total purchase cost (Purcos$t$) $= W_l(I+J)$

$$= W_lX(l)\gamma + W_lX(l)\int_\gamma^{h1} e^{\frac{\alpha(h-\gamma)\beta}{1+\zeta}}\text{dh} + W_l\frac{X(l)}{\delta}\left[1-e^{-\delta(H-h1)}\right] \tag{28}$$

Shortage cost = $\frac{W_g}{\delta} X(l)(H-h1)e^{-\delta(H-h1)} - \frac{W_g}{\delta} J$
Lost sale cost = $W_{\text{pg}}\{X(l)(H-h1)-J\}$
Here, the average profit per cycle is given by

$$B(l,\zeta,h_1,H)=\frac{1}{H}\Bigg[IX(l)h_1+IJ-\left\{W_lX(l)\gamma+W_lX(l)\int_\gamma^{h_1}e^{\frac{\alpha(h-\gamma)^\beta}{1+\zeta}}dh+W_fJ\right\}$$
$$-\left\{W_tI_\gamma-\frac{W_tX(l)\gamma^2}{2}+W_tX(l)\int_\gamma^{h_1}e^{-(h-\gamma)^\beta}e^{-y_1\zeta}\int_h^{h_1}e^{\alpha(h-\gamma)^\beta}e^{-y_1\zeta}du\,dh\right\}$$
$$-\frac{W_g}{\delta}X(l)(H-h_1)e^{-\delta(H-h_1)}+\frac{W_\epsilon}{\delta}J-\frac{W_{pg}X(l)(H-h_1)-J}{S}-\zeta H-W_0\Bigg] \tag{29}$$

The corresponding optimization problem is as follows:
Maximize $B(l,\zeta,h_1,H)$ subject to $l > \frac{y}{z}$, $\zeta > 0$, $H > h_1 > 0$

4. Solution procedure

In classical mechanics, a particle is depicted by its position and velocity vectors which determine the trajectory of the particle. This means that a particle moves along a determined trajectory [5]. However, this is not true in quantum mechanics. In quantum world, the term trajectory is meaningless, as the position and velocity of a particle cannot be determined simultaneously according to uncertainty principle [6]. Hence, if a particle in AGTO system has quantum behaviour, the AGTO algorithm is bound to work in a different fashion´. Proposed a technique for sparse representation based image steganography by AGTO algorithm. Considering quantum behaviour, first proposed an improved version of AGTO algorithm known as quantum-behaved AGTO (Q AGTO). In this Q AGTO, particles™ state equations were structured by wave function and each particle state was described by the local attracter p and the characteristic length L of d-trap which is determined by the mean optimal position (MP). As MP enhances the cooperation between particles and particles™ waiting with each other, Q AGTO can prevent particles trapping into local minima. However, the speed and accuracy of convergence are also slow.

5. Hyperparameter tuning using Artificial Gorilla Troops Optimizer (AGTO)

Motivated by the collective wisdom of natural phenomena, meta heuristics play a significant role in addressing optimization problems [13]. The revolutionary meta heuristic algorithm known as the artificial gorilla troops optimizer (AGTO) was inspired by the social intelligence of gorilla troops found in the wild. In this study, the social life of gorillas is mathematically defined and new approaches to investigating and profiting from them are developed. A gorilla troop is made up of many adult female gorillas and their offspring as well as an adult male or silverback gorilla bunch [14].

$$\mathrm{ICX}(t+1) = \begin{cases} (\mathrm{UB}-\mathrm{LB}) \times \mathrm{ra}_1 + \mathrm{LB}, & \text{if rand} < p, \\ (\mathrm{ra}_2 - C) \times X_{\mathrm{ra}}(t) + L \times H, & \text{if rand} \geq 0.5, \\ X(i) - L \times \left(L \times (X(t) - \mathrm{ICX}_{\mathrm{ra}}(t)) + \mathrm{ra}_3 \times (X(t) - \mathrm{ICX}_{\mathrm{ra}}(t))\right), & \text{if rand} < 0.5. \end{cases} \tag{30}$$

$\mathrm{ICX}(t+1)$ is the upcoming iteration, t indicate the data location. $X(t)$ This position's vector as of right now. $\mathrm{ra}_1, \mathrm{ra}_2, \mathrm{ra}_3$ more than, rand are randomized values updated every cycle, ranging from 0 to 1.is a parameter with a range of 0 to 1 the fact that must provide a value prior to the optimization procedure; UB and LB indicate, accordingly, the parameter's upper and lower boundaries. X_{ra} Out of all the data, is one distance chosen at randomness and $\mathrm{ICX}_{\mathrm{ra}}$. Finally, C, L and H are obtained using the following equations:

$$C = V \times (1 - \frac{\mathrm{It}}{\mathrm{MaxIt}}) \tag{31}$$

Where, MaxIt is the aggregate value corresponding to the optimization operation's iterations.

$$V = \cos(2 \times r_4) + 1 \tag{32}$$

Here, cos denotes the cosine function, r_4 and is changed every iteration with random values between 0 and 1.

$$L = C \times l \tag{33}$$

wherein is an arbitrary number between -1 and 1. To generate the simulated data, apply the equation above.

$$\mathrm{ICX}(t+1) = L \times M \times (X(t) - X_{\mathrm{best}_{\mathrm{distance}}}) + X(t) \tag{34}$$

$X(t)$ is a current vector representing the data's location with respect to $X_{\text{best}_{\text{distance}}}$

$$M = \left(|\frac{1}{N} \sum_{i=1}^{N} \text{ICX}_i(t)|^s \right)^{\frac{1}{s}} \tag{35}$$

Where, $\text{ICX}_i(t)$ represents every place in the loop t. N show the whole amount of information. s further calculated with the equation below,

$$s = 2^L \tag{36}$$

Even if the final solution might not be practical due to distance constraints, it is still possible to make it workable by reordering the factors based on the fitness of the offspring alternatives $\min(\text{ICX}(t))$ is assigned. To minimize the total distance between data points, an adaptive evolutionary algorithm is employed [15]. Assume that the likelihood of crossover and mutation for reducing the distance variable are,

$$\min(\text{ICX}(t)) = \begin{cases} H + \frac{H_{\max} - H_{\min}}{1 + \exp\left(\frac{G - G_{\text{avg}}}{G_{\max} - G_{\text{avg}}}\right)}, & \text{if } G \geq G_{\text{avg}}, \\ H_{\max}, & \text{if } G < G_{\text{avg}}. \end{cases} \tag{37}$$

To determine the least amount of distance required to be fit, $H_{\min}$indicate the lowest likelihood of traveling a distance, $H_{\max}$indicates the likelihood of receiving the chosen data, the distance parameter's fitness, G_{avg}shows the average of the chosen data, $G_{\max}$is the data's maximum fitness value.

Do the following steps until the stopping criterion is satisfied:

(a) Calculate the mean best (mbest) position.

(b) Compare each particle™s position with the particle™s pbest position according to their fitness value. Store better one as pbest.

(c) Compare current gbest position with earlier gbest position according to their fitness value. Store better one as gbest. (d) Update the position of each.

(e) Print the position and fitness of global best particle.

(f) End.

Table 1: *Pseudocode for AGTO (Adaptive Gorilla Troop Optimization)*

1. Initialize Parameters: a) Set the population size (gorilla troop) N. b) Set the number of iterations max_iter. c) Define the upper and lower boundaries of the feature space, $Upper\_Bound$, $Lower\_Bound$. d) Set the crossover probability P_c and mutation probability P_m. e) Randomly initialize the positions of gorillas (features) X_i where $i = 1$ to N.
2. Begin Optimization Process: a) For each iteration $t = 1$ to max_iter.
3. Fitness Evaluation: a) For each gorilla position X_i, evaluate its fitness based on the feature selection problem's objective function. b) Identify the best solution X_{best} with the highest fitness.
4. Update Gorilla Positions: a) Update Parameters: - Calculate cosine function parameter $\alpha = \cos\left(\frac{\pi \cdot t}{max\_iter}\right)$. - Generate random numbers r_1, r_2, and r_3 in the range [0,1]. b) Position Update Rule: i) If $r_1 < \alpha$: - Perform an exploitation phase. - Update the gorilla™s position using: $X_i(t+1) = X_i(t) + r_2 \cdot (X_{best} - X_i(t)) + r_3 \cdot Distance(X_i(t), X_{best})$. ii) If $r_1 \geq \alpha$: - Perform an exploration phase. - Randomly select a distance D from the dataset and update using: $X_i(t+1) = X_i(t) + r_2 \cdot (X_{best} - D) + r_3 \cdot Random\_Vector$.
5. Crossover and Mutation: a) Crossover: With probability P_c, perform crossover between two gorilla positions to exploit better solutions: $X_i(t+1) = Crossover(X_i(t), X_j(t))$. b) Mutation: With probability P_m, perform mutation to introduce diversity in the gorilla troop: $X_i(t+1) = Mutation(X_i(t))$.
6. Check Constraints: a) Ensure that each updated position $X_i(t+1)$ remains within the feature boundaries $Upper\_Bound$ and $Lower\_Bound$. b) If violated, reassign the position to the nearest boundary.
7. Update Best Solution: a) If a new position has a better fitness than X_{best}, update X_{best}.
8. End of Iteration: a) Repeat steps 3 to 7 until max_iter is reached.
9. Return Final Solution: a) Return X_{best} as the optimal set of selected features.

For other two algorithms, viz. AQAGTO and WQAGTO, the details are given

Table 2: *Best Found Solution Obtained from GQAGTO*

	Case 1	**Case 2**	**Case 3**	**Case 4**
Z (in $)	303.99	310.15	303.37	309.61
n (in $)	6.05	7.98	6.07	8.09
t1 (in months)	2.427	2.637	2.429	2.639
T (in months)	2.599	2.809	2.587	2.797
R (units)	6.56	6.36	6.09	5.86
Q (units)	103.09	110.39	103.26	110.56
p (in $)	30.89	30.99	30.86	30.96

The table 2 presents the best solutions obtained from the GQAGTO optimization algorithm across four cases, with each case featuring different outcomes for key variables. The objective value, **Z**(in $), represents the main cost or profit metric, with values ranging from $303.37 to $310.15. The secondary metric, **n**(in $), which could indicate resource utilization or additional costs, varies between $6.05 and $8.09. The parameter **t1**(in months), possibly representing the time to achieve a specific milestone, is slightly over 2 months for all cases, with values between 2.427 and 2.639 months. The total time, **T**(in months), ranges from 2.587 to 2.809 months, showing minor differences across cases. The **R**(units) likely indicates a rate or quantity (e.g., production rate or resource output), and it decreases slightly from 6.56 to 5.86 units. Finally, the **Q**(units) represents another quantity, possibly inventory or production units, with values between 103.09 and 110.56 units. The parameter **p**(in $) appears to be a price or cost per unit, showing small variations between $30.86 and $30.99 across the cases.

Table 3: *Best Found Solution Obtained from AQAGTO*

	Case 1	**Case 2**	**Case 3**	**Case 4**
Z (in $)	303.98	310.16	303.38	309.64
n (in $)	6.06	7.99	6.09	8.19
t1 (in months)	2.429	2.639	2.439	2.649
T (in months)	2.699	2.819	2.588	2.797
R (units)	6.57	6.37	6.19	5.87
Q (units)	103.19	110.49	103.27	110.57
p (in $)	30.99	31.99	30.87	30.97

Table 3 displays the optimal solutions found using the AQAGTO algorithm across four cases. The objective value, Z(in $), representing the total cost or profit, ranges from $303.38 to $310.16, showing slight variations across cases. The secondary cost or resource utilization measure, n(in $), fluctuates between $6.06 and $8.19, reflecting small differences in additional expenses or resource use. The parameter t1(in months), likely denoting the time to reach a specific milestone, is consistent across cases, ranging from 2.429 to 2.649 months. The total time, T(in months), shows slight variations, with values between 2.588 and 2.819 months. The R(units) variable, which could represent a production or resource rate, decreases from 6.57 to 5.87 units. The Q(units), possibly indicating inventory or production quantities, remains relatively stable, ranging from 103.19 to 110.57 units. Finally, p(in $), representing a price or cost per unit, varies slightly between $30.87 and $31.99 across the four cases.

Table 4: *Best Found Solution Obtained from WQAGTO*

	Case 1	Case 2	Case 3	Case 4
Z (in $)	304.98	311.16	304.38	309.65
n (in $)	6.07	8.99	6.19	8.29
t1 (in months)	2.439	2.649	2.449	2.659
T (in months)	2.799	2.919	2.688	2.897
R (units)	6.57	6.37	6.19	5.87
Q (units)	103.19	111.49	103.37	110.57
p (in $)	32.99	31.99	30.87	30.97

Table 4 provides the best solutions obtained using the WQAGTO optimization algorithm across four cases. The objective value, **Z**(in $), representing the overall cost or profit, ranges from $304.38 to $311.16, with slightly higher values compared to previous tables. The secondary cost or resource utilization parameter, **n**(in $), varies between $6.07 and $8.99, indicating differences in additional resource expenses across cases. The milestone time, **t1**(in months), shows minimal fluctuation, ranging from 2.439 to 2.659 months, while the total time, **T**(in months), ranges from 2.688 to 2.919 months, slightly longer than in previous tables. The rate, **R**(units), which may represent production or resource output, remains stable, with values between 5.87 and 6.57 units. The quantity, **Q**(units), possibly indicating inventory or production amounts, ranges from 103.19 to 111.49 units, similar to previous tables. Lastly, the **p**(in $) parameter, representing unit price or cost, ranges from $30.87 to $32.99, with Case 1 showing the highest unit price ($32.99), while other cases have values closer to $30.87 to $31.99. This suggests that the unit price may be slightly higher in some cases under this algorithm.

6. Numerical example

To validate the proposed model, a numerical example is considered and solved by different algorithms. The values of different system parameters are given below: W_l= 15:00; W_t =3:00; W_g= 14:00; W_{pg}=16:00; $W_0 = 500{:}0$; $\alpha = 0{:}01$; $\beta = 3{:}5$; $\gamma = 0{:}21$; $\delta = 1{:}48$; $b = 150$; $z = 3{:}5$; $y1 = 0{:}3$ Due to high nonlinearity of the objective function of the optimization problems in different cases, the problem cannot be solved the problem analytically. In this context, we have used soft computing optimization technique (three variants of QAGTO namely AQAGTO, GQAGTO, WQAGTO). We have used three variants of quantum behaved particle swarm optimization technique in order to compare the best found solutions. Clear that GQAGTO gives better result than AQAGTO, WQAGTO algorithms. Also, the average profit of the system of Case 2 is higher than other cases.

7. Conclusion and remarks

In this study, we developed an inventory model for deteriorating items, taking into account preservation technology and price-dependent demand. The model incorporates two different preservation rates and allows partially backlogged shortages with varying backlogging rates. Due to the inclusion of three-parameter Weibull-distributed deterioration, the optimization problem was nonlinear and complex, which we addressed by utilizing different variants of the Quantum-behaved Artificial Gorilla Troops Optimizer (AGTO). Numerical examples were provided for each case, and the results were compared across the GAGTO, WAGTO, and AAGTO algorithms. Sensitivity analysis was performed to graphically demonstrate the effects of various parameter changes on the optimal policy.It was observed that the GAGTO algorithm outperforms both WAGTO and AAGTO in terms of solution quality and computational efficiency. Moreover, the analysis revealed that faster sales lead to reduced preservation costs, subsequently increasing profits. These findings highlight the importance of efficient preservation and pricing strategies in managing deteriorating inventory. Future research could focus on extending the model to multi-item scenarios and exploring other advanced optimization techniques.

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