STRATIFIED RANDOM SAMPLING WITH RISK APPROACH

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Abstract

In stratified random sampling, the sample size allocation is a problem which is tackled by many scientists and survey practitioners. Generally the proportional allocation, Neyman allocation and cost based allocation, are used to conduct sample surveys for gathering information from each strata. One can think of risk imposed on the life of investigators which is yet not considered while sample size allocation to risky strata. In this paper, the risk indicators stratum-wise are defined using police station records and hospital records. Such indicators are used for the determination of sample size allocation. For optimization, the Lagrange multiplier technique is used with two constants whose values need to be determined. An algorithm is proposed and analysed for such using simulation. The outcome of analysis provides that sample size allocation is directly proportional to the strata size and variability but inversely proportional to the square root of risk indicators of the stratum (with varying values of constants). This paper opens a new approach for the consideration of risk based sample size allocation and estimation in the setup of stratified sampling.

Keywords: Simple Random Sampling (SRSWOR), Stratified Random Sampling, Stratum, Sample survey, Lagrange Multiplier, Allocation to Strata, Risk data, Risk Indicators, Optimization, Variance of sample estimate.

1. Introduction

Sample surveys play important role in exploring the hidden characteristics of the whole population without complete enumeration. The sample survey methodologies are used in areas and regions where epidemic occurs. A survey is usually conducted to know about the patients health conditions, rate of spread of disease, estimation of average number of deaths due to disease etc. Where natural disaster happened, to collect facts about casualties, about root causes of natural disaster are further possible field based studies.

Stratification is a sampling technique used in surveys in order to improve upon the precision of the sample estimate. Several authors have developed optimal techniques to allocate sample sizes to stratum (for the single variable under study) using Lagrange multiplier optimization technique. There may be other situations such as war, naxalite movement, dense forest where issue risk is involved on the field officers, who are involved in data collection for the conduct of sample survey. The risk may be on life, infection due to disease or being unhealthy for short duration. In this study, the problem of risk occurrence during the data collection, if so exit, is considered in context to sample size allocation to each strata.

In literature, many studies exist, where the authors have considered the stratum size, variability and stratum cost of the data collection while surveying the population. Yadav and Verma

[5] studied the exponential ratio-type estimators under the linear cost function in the set up of stratified random sampling. Focus of study is on the estimation of population parameter with the help of collected data using proposed method. A linear function is used to determine the relation among sample sizes of each stratum. Yadav et al.[6] worked on the behaviour of ratio-product-cum-exponential-cum-logarithmic type estimators with one auxiliary variable in the stratified random sampling setup and analyzed such with the linear cost function using numerical illustrations. Ghosh[1] suggested a new method of allocation of sample sizes for stratum. In this, the author has taken the average of the optimum allocation for the different characters individually. Khan et al.[2] used compromised allocation in multivariate stratified sampling for an integral solution. Varshney [4] worked on an optimum allocation of sample sizes in the presence of non-response factor under the multivariate stratified double sampling setup. In such, authors have considered sample size allocation problem in stratified random sampling for single character as well as for multiple characters with varying cost functions.

Koyuncu and Kalidar[8] suggested a new family of estimators for stratified random sampling utilizing the information of the coefficient of kurtosis of the population and obtained efficient conditions between the adopted and proposed families. Theoretical finding are supported by the numerical examples with original data. Singh et al.[9] addressed the problem of various types of estimation of the main variable parameter in the presence of non-response and measurement error both incorporating the information of two auxiliary variables. In that, authors derived the optimum strata weights using the suitable calibration technique. Bhushan et al.[10] developed efficient classes of estimators in stratified sampling for combined ratio and separate ratio type estimators. Such estimators are theoretically justified and compared over the conventional estimator, classical ratio estimator and classical regression estimator using the simulation study. Tiwari et al.[11] proposed a general class of estimators for estimating the population mean of study variable using the support variable based correlated information. Members of such proposed class are identified and compared in terms of efficiency. Kadilar and Chingi [16] derived some ratio-type estimators and discussed their properties in the setup of stratified sampling. Aamir et al. [13] suggested a generalised class of exponential-type estimators for population mean by taking the two auxiliary variables for estimating the unknown means with the case of sub-sampling and non-response. In such, authors derived the conditions under which proposed estimators are more efficient as composed to other estimators. Cekim and Kalidar [12] suggested some estimators for estimating the population variances in stratified sampling, in the form of in-function type estimators. Ahnad et al.[17] proposed an improved family of estimators for estimating the population distribution function. The main aim of such contribution was to develop an enhanced family of log ratio-exponential based estimation procedure under stratified sampling. Zaman and Kalidar [15] suggested exponential ratio and product type estimators the mean by considering the two phase sampling setup in stratified sampling.

This paper considers the risk factor exposed on the life of survey workers over different strata. A sample size allocation keeping the method is discussed considering a risk function with computation of allocations variance optimal.

1.1. Risk in Survey Sampling

While data collection, using stratified sampling, some strata may have higher risk on the life of surveyor while others may have a little. For example, a strata of a population is affected by the nuxalite movement, next strata bears high rate of murders and killings, third one is affected by the dangerous epidemics(like malaria, dengue, COVID-19). The strata-wise risk on the life of investigators could be pre-estimated using police record and hospital records (for last one/five years) as below:

Strata I	(1) Deaths due to murder and mass killing=α_{11} (2) Deaths due to communal riots= α_{12} (3) Deaths due to epidemics and community diseases= α_{13}
Strata II	(1) Deaths due to murder and mass killing=α_{21} (2) Deaths due to communal riots= α_{22} (3) Deaths due to epidemics and community diseases= α_{23}

The minimum risk may be assumed as r_0 which includes the normal risk of natural death during the survey work.

1.2. Symbols used for analysis

Let a population of finite size N, divided into L stratum. Each stratum is of size N_i where $N_1 + N_2 + ... + N_L = N$ holds and samples are taken from each strata of size n_i such that $n = (n_1 + n_2 + ... + n_L)$, where n denotes total size of sample.
Notations used for population parameters are:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{L}\sum_{j=1}^{N_i} Y_{ij}, \qquad S^2 = \frac{1}{N-1}\sum_{i=1}^{L}\sum_{j=1}^{N_i}(Y_{ij} - \bar{Y})^2 \tag{1.1}$$

$$\bar{Y}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} Y_{ij}, \qquad S_i^2 = \frac{1}{N_i - 1}\sum_{j=1}^{L}(Y_{ij} - \bar{Y}_i)^2 \tag{1.2}$$

where Y_{ij} is the j^{th} observation of the i^{th} strata in a population of size N.
Let a sample of size n is drawn by the SRSWOR sampling scheme keeping n_i from each stratum, then sample related notations are:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{L}\sum_{j=1}^{n_i} y_{ij}, \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{L}\sum_{j=1}^{n_i}(y_{ij} - \bar{y})^2 \tag{1.3}$$

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}, \qquad s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2 \tag{1.4}$$

1.3. Risk Indicators

Define risk indicators r_i as:

$$r_i = (\frac{T_i}{N_i}), \text{ for } i^{th} \text{ strata} \tag{1.5}$$

$$\text{At } i = 1, \qquad r_1(Risk) = \frac{T_1}{N_1}, \ for\ strata\ I \tag{1.6}$$

$$\text{At } i = 2, \qquad r_2(Risk) = \frac{T_2}{N_2}, \ for\ strata\ II \tag{1.7}$$

(1.8)

where,

$$Total : T_1 = (\alpha_{11} + \alpha_{12} + \alpha_{13}) \tag{1.9}$$

$$Total\ strata\ I\ Population = N_1 \tag{1.10}$$

$$Total : T_2 = (\alpha_{21} + \alpha_{22} + \alpha_{23}) \tag{1.11}$$

$$Total\ strata\ I\ Population = N_2 \tag{1.12}$$

(1.13)

These indicators are crude measures of the intensity of risk imposed on the life of field investigators who collect primary data through sample survey in a stratified population.

1.4. Motivation

The proportional allocation is stratum size based and Neyman allocation is size + variability based for i^{th} stratum. There is one more method which is cost based allocation per stratum but involvement of stratum risk is yet not considered by any author. In order to utilize the information contained in the risk indicators r_i, the problem of sample size determination is attempted in this paper.

2. Mean Estimation approach in Stratified Sampling

The usual mean estimator under the stratified sampling is:

$$\bar{y}_{st} = \sum_{i=1}^{L} W_i \bar{y}_i \tag{2.1}$$

The variance for stratified random sampling is:

$$V(\bar{y}_{st}) = \sum_{i=1}^{L} W_i^2 \left\{ \frac{1}{n_i} - \frac{1}{N_i} \right\} S_i^2 \tag{2.2}$$

where W_i represents the weight of each stratum on the basis of its size i.e. $W_i = \left(\frac{N_i}{N}\right)$.

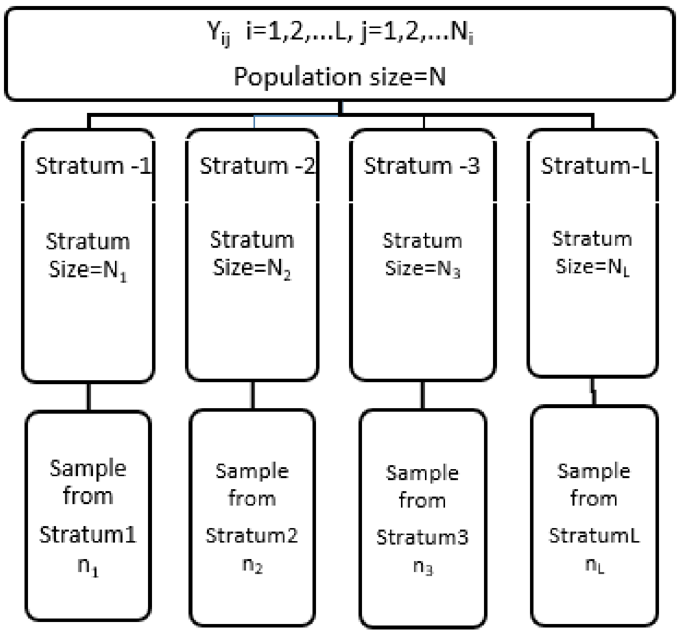

Figure 1: *Sampling structure for L stratum based stratified sampling*

3. Linear risk function

Consider the linear risk function for the stratified sampling:

$$r = r_0 + \sum_{i=1}^{L} n_i r_i \tag{3.1}$$

where,
r_0: Minimum pre fixed risk exposed on life of investigators.
r_i: Risk per unit in a stratum.
r^*: Total risk exposed on investigator while survey of entire population including natural death.
The objective of this paper is to determine the sample size from each stratum using the linear risk function, keeping variance minimum. This objective can be achieved by optimizing following,

$$Minimize\ V(\bar{y}_{st}) \tag{3.2}$$

subject to the conditions,

$$\sum_{i=1}^{L} r_i n_i = r^* \tag{3.3}$$

$$\sum_{i=1}^{L} n_i = n \tag{3.4}$$

For solution using the Lagrange multiplier technique defined and optimize the following function ϕ

$$\phi = V(\bar{y}_{st}) + \lambda_1 \left(\sum_{i=1}^{L} n_i - n \right) + \lambda_2 \left(\sum_{i=1}^{L} r_i n_i - r^* \right) \tag{3.5}$$

where λ_1, λ_2 are constants called Lagrange multipliers. Differentiating ϕ with respect to n_i, λ_1, λ_2 and equating to zero, one can get,

$$n_i = \frac{W_i S_i}{\sqrt{\lambda_1 + \lambda_2 r_i}} \tag{3.6}$$

Summing (3.6) on both sides,

$$n = \sum_{i=1}^{L} n_i = \sum_{i=1}^{L} \left[\frac{W_i S_i}{\sqrt{\lambda_1 + \lambda_2 r_i}} \right] \tag{3.7}$$

From (3.6) and (3.7), one get insights,

$$n_i \propto N_i \tag{3.8}$$

$$n_i \propto S_i \tag{3.9}$$

$$n_i \propto \frac{1}{\sqrt{\lambda_1 + \lambda_2 r_i}} \tag{3.10}$$

where r_i is risk related to i^{th} strata.

4. Computational Algorithm for Optimal Variance along with choice of λ_1 and λ_2

Step I : For given N, n calculate initial values N_i, S_i, $\bar{Y}_i$ and $(N_i S_i)$ and r_i of the population

Step II : Find $V_{(}\bar{y}_{st})$ using Neyman allocation, which is based on $n_i \propto N_i$ and $n_i \propto S_i$ with expression $n_i = \left\{ \frac{n W_i S_i}{\sum W_i S_i} \right\}$. Find variance $V(\bar{y}_{st})$ using proportional allocation which is based on criteria $n_i \propto N_i$ only with expression $n_i = n W_i$

Step III : Find the risk r_i and use risk function $r^* = \sum r_i n_i$.

Step IV : Set

$$\phi = V(\bar{y_{st}}) + \lambda_1 \left(\sum_{i=1}^{L} n_i - n \right) + \lambda_2 \left(\sum_{i=1}^{L} r_i n_i - r^* \right) \tag{4.1}$$

where λ_1, λ_2 are constants to determine under risk assumption.

Step V : For risk based allocation of sample size n_i,

$$n_i \propto N_i \tag{4.2}$$

$$n_i \propto S_i \tag{4.3}$$

$$n_i \propto \frac{1}{\sqrt{\lambda_1 + \lambda_2 r_i}} \tag{4.4}$$

Step VI : Use simulation procedure to find values of λ_1 and λ_2 to optimize variance $V(\bar{y}_{st})$

(a) Fix the values of λ_1,

(b) Vary λ_2 on x-axis of the graph and plot graph for variance, along with n_1 *and* n_1,

(c) Continue the process of creating graphs for different values of λ_1,

(d) When variance line becomes parallel to x-axis then stop the simulation process.

  (i) Choose that input-data set $n_1, n_2, \lambda_1, \lambda_2$ (producing parallel line)

  (ii) Use values to get optimal solution.

5. Empirical Study

Consider following data of size N= 244 from 6^{th} Minor Irrigation Census - Village Schedule - Assam[7]. The crime data obtained from police station and hospitals as under(assumed data for a year):

Strata I :

(a) Deaths due to bullet firing = 8

(b) Deaths due to riots = 11

(c) Deaths due to epidemic = 6
Total = 25
Total strata size= 127

Strata II :

(a) Deaths due to bullet firing = 11

(b) Deaths due to riots = 15

(c) Deaths due to epidemic = 10
Total = 36
Total strata size= 135

The basic data and basic computation is as under:

Table 1: *Data for Strata (Source, please see [7])*

i	N_i	W_i	$\bar{Y}_i$	S_i^2	r_i
1	127	0.5205	703.74	883.83	19%
2	135	0.48	413	644.922	26%

Table 2: *The proportional allocation provides*

n_1	n_2	n	$V(\bar{y}_{st})_{prop}$
72	108	180	804.5

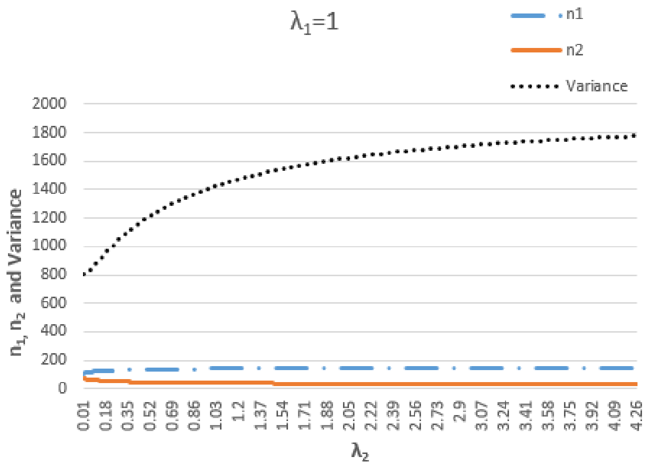

Figure 2: *Variation when λ_1=1 fixed*

Fig.(2) reveals that, for fixed value of $\lambda_1 = 1$, the variance of V($\bar{y}_{st}$) has growing trend and under risk consideration . It is observed that λ_2 increases for fixed λ_1.Moreover, V($\bar{y}_{st}$) fluctuates between between 800 to 1800. There is miled increase in n_1 for increasing λ_2.

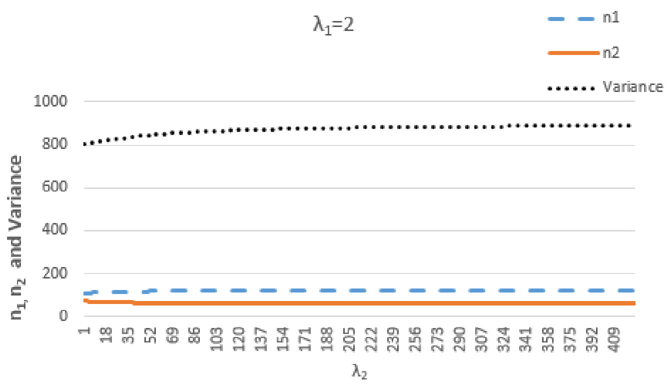

Figure 3: *Variation when λ_1=2 fixed*

Fig(3) is an indicator of the analysis of V($\bar{y}_{st}$), as the value of λ_2 increases for fixed value of $\lambda_1 = 2$, the value of V($\bar{y}_{st}$) lies between 800 to 1000.

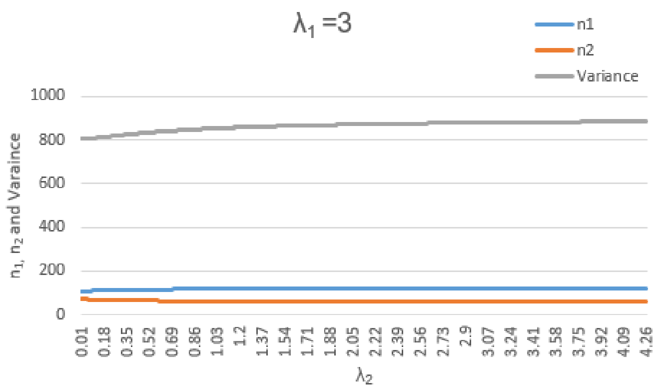

Figure 4: *Variation when λ_1=3 fixed*

Fig.(4) opens starting avenue for decrease in V($\bar{y}_{st}$) as the value of λ_1 is increases, the V($\bar{y}_{st}$) reduces.

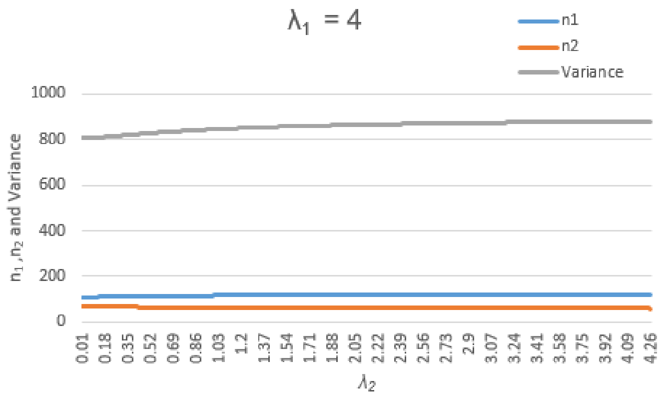

Figure 5: *Variation when λ_1=4 fixed*

Fig.(5) shows that the V($\bar{y}_{st}$) line is tending to become parallel to the x-axis(on higher λ_2 values).

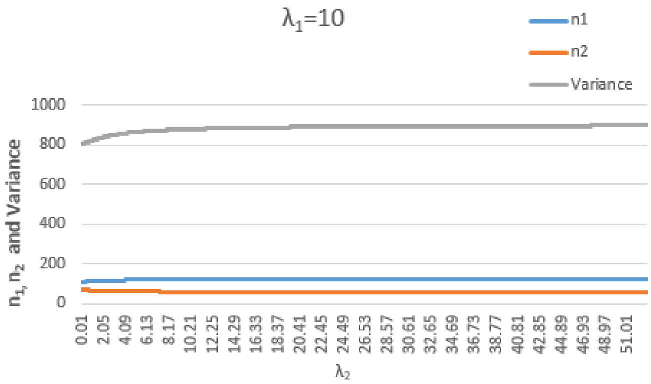

Figure 6: *Variation when λ_1=10 fixed*

Fig.(6) represents the similar pattern as observed in Fig.(5) to get $V(\bar{y}_{st})$.

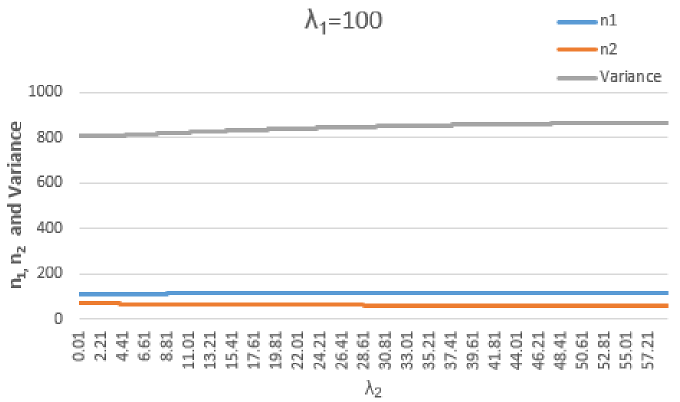

Figure 7: *Variation when λ_1=100 fixed*

Fig.(7) highlights that for higher values of λ_1, the relation between V($\bar{y}_{st}$) over the incrementing values of λ_2 is almost parallel to x-axis. Such indicates for V($\bar{y}_{st}$) being almost independent to the variation of λ_2.

Table 3: *The Neyman allocation provides*

n_1	n_2	n	$V(\bar{y}_{st})_{Ney}$
71	119	180	803.5

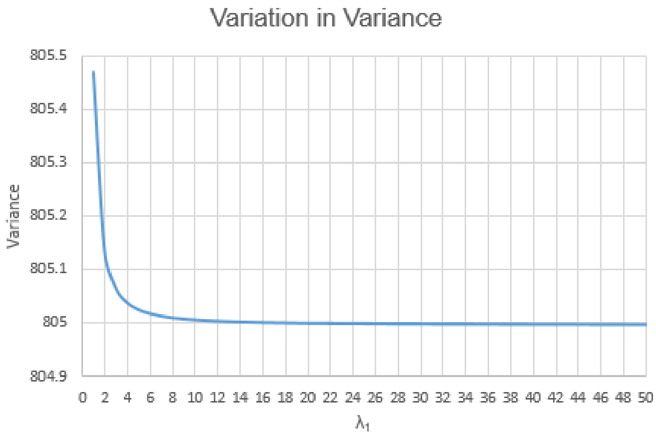

Figure 8: *Variation with respect to parameters*

Fig.(8) depicts the relation between λ_1 and V($\bar{y}_{st}$). The value of V($\bar{y}_{st}$) is gradually decreasing as the values of λ_1 increases from 1 to 19. After $\lambda_2 = 19$ (approximately), there is no significant change in V($\bar{y}_{st}$).

6. Comparison and Discussion

On comparing the different types of allocations (Table 5) it is evident that allocations are very close to each other and providing the optimal variance. The approach aimed at to utilize the

Table 4: *The risk based allocation provides*

n_1	n_2	n	$V(\bar{y}_{st})_{risk}$
72	118	180	805.26

Table 5: *Different allocation methods provides*

Proportional allocation provides	n_1	n_2	n	$V(\bar{y}_{st})_{prop}$
	72	108	180	804.5
Neyman allocation provides	n_1	n_2	n	$V(\bar{y}_{st})_{prop}$
	71	119	180	803.5
Risk based allocation	n_1	n_2	n	$V(\bar{y}_{st})_{risk}$
	72	108	180	805.26

crime record information of police station and hospital records of the strata during sample survey. Such can be useful to determine the sample size allocation n_i from the i^{th} strata $(i = 1, 2, 3, ..., L)$, so that $n = \sum_{i=1}^{L} n_i$ remains intact. Risk indicators are suggested and defined using the crime record and hospital records. The Lagrange multiplier technique provides two constants λ_1 and λ_2 whose values need to be computed using the available data. As evident in graphical pattern from [fig(2) to fig(8)], the increasing values of λ_1 provides the solution for best choice of n_1 & n_2 at the situation when variance remain stable (independent of increasing λ_2). An appendix added at the end provides choice of λ_1, λ_2 and n_1 , n_2. At λ_1=300, λ_2=0.01 one gets n_1=108, n_2=72 with lowest variance 804.99 as displayed in the table(7) of appendix. When λ_1 increases then λ_2 decreases to attain same level of optimality. For any arbitrary choice of λ_1 the table(7) provides the value of λ_2 for quick selection. In general, $21 \leq \lambda_1 \leq 40$ and $0.1 \leq \lambda_2 \leq 0.30$ is the recommended rapid selection of λ-values.

7. Conclusion

This paper presents a new idea of using the regional (strata) risk on the life of survey investigators with the help of risk indicators. In literature, when stratified sampling is used, the problem of sample size allocation appears that it could be resolved as per population strata size or as per population strata variability. The proportional allocation is based on population strata sizes while the Neyman allocation is based on size and variability both. Such allocations do not consider the risk factor imposed on the life of investigator. If risk is high for a particular strata then smaller sample size is required from that strata. The proposed risk based sample size allocation is like $n_i \propto N_i$,$n_i \propto S_i$ and $n_i \propto \frac{1}{\sqrt{\lambda_1+\lambda_2 r_i}}$ incorporating two constants λ_1 & λ_2. An algorithm is proposed in this paper showing how to compute λ_1 and λ_2 constants with minimizing the population variability factor of the mean estimate. If $1 \leq \lambda_1 \leq 10$ then it is suggested to choose $\lambda_2 = \frac{\lambda_1}{200}$ as per table 7. Similarly when $10 \leq \lambda_1 \leq 20$ then recommended to choose $\lambda_2 = \frac{\lambda_1}{100}$ as per table 7, shown in appendix. Various graphs from (fig.(2) to fig.(8)) reveal that when variance line becomes parallel to x-axis for set of values $(\lambda_1, \lambda_2, n_1, n_2)$, such provide the optimal solution for lowest variability due to the risk based sample size allocation. In general, one can work with risk based allocations choosing 21$\leq \lambda_1 \leq 40$ and 0.1$\leq \lambda_2 \leq 0.3$ (table 7) to get nearly optimal result. The crime data of all police stations and health data from hospitals can be utilized for risk computation and accordingly can be used in risk based sample size allocation. The table 7 attached in appendix helps in rapid selection of λ_2 for an arbitrary choice of λ_1

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Appendix

Table 6: λ_1 *varies but* λ_2 *fixed*

λ_1	λ_2	n_1	n_2	n	$V(\bar{y}_{st})$	λ_1	λ_2	n_1	n_2	n	$V(\bar{y}_{st})$
1	0.01	109	71	180	805.469	30	0.01	108	72	180	804.998
2	0.01	108	72	180	805.134	32	0.01	108	72	180	804.997
3	0.01	108	72	180	805.063	33	0.01	108	72	180	804.997
4	0.01	108	72	180	805.037	34	0.01	108	72	180	804.997
5	0.01	108	72	180	805.024	35	0.01	108	72	180	804.997
6	0.01	108	72	180	805.017	36	0.01	108	72	180	804.997
7	0.01	108	72	180	805.012	37	0.01	108	72	180	804.997
8	0.01	108	72	180	805.009	38	0.01	108	72	180	804.997
9	0.01	108	72	180	805.007	39	0.01	108	72	180	804.997
10	0.01	108	72	180	805.005	40	0.01	108	72	180	804.997
11	0.01	108	72	180	805.004	41	0.01	108	72	180	804.997
12	0.01	108	72	180	805.003	42	0.01	108	72	180	804.997
13	0.01	108	72	180	805.002	43	0.01	108	72	180	804.997
14	0.01	108	72	180	805.001	44	0.01	108	72	180	804.997
15	0.01	108	72	180	805.001	45	0.01	108	72	180	804.997
16	0.01	108	72	180	805.000	46	0.01	108	72	180	804.997
17	0.01	108	72	180	805.000	47	0.01	108	72	180	804.997
18	0.01	108	72	180	805.000	48	0.01	108	72	180	804.997
19	0.01	108	72	180	804.999	49	0.01	108	72	180	804.997
20	0.01	108	72	180	804.999	50	0.01	108	72	180	804.997
21	0.01	108	72	180	804.999	20	0.01	108	72	180	804.999
22	0.01	108	72	180	804.999	25	0.01	108	72	180	804.998
23	0.01	108	72	180	804.998	50	0.01	108	72	180	804.997
24	0.01	108	72	180	804.998	100	0.01	108	72	180	804.996
25	0.01	108	72	180	804.998	150	0.01	108	72	180	804.996
26	0.01	108	72	180	804.998	200	0.01	108	72	180	804.996
27	0.01	108	72	180	804.998	250	0.01	108	72	180	804.996
28	0.01	108	72	180	804.998	300	0.01	108	72	180	804.996
29	0.01	108	72	180	804.998	1000	0.01	108	72	180	804.996

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Table 7: λ_1 *and* λ_2 *are varying*

λ_1	λ_2	n_1	n_2	n	$V(\bar{y}_{st})$
1.00	0.50	118.94	61.06	180.00	863.32
2.00	0.49	116.89	63.11	180.00	843.62
3.00	0.48	115.43	64.57	180.00	832.17
4.00	0.47	114.34	65.66	180.00	824.95
5.00	0.46	113.48	66.52	180.00	820.12
6.00	0.45	112.79	67.21	180.00	816.75
7.00	0.44	112.23	67.77	180.00	814.30
8.00	0.43	111.76	68.24	180.00	812.49
9.00	0.42	111.36	68.64	180.00	811.11
10.00	0.41	111.01	68.99	180.00	810.03
11.00	0.40	110.71	69.29	180.00	809.19
12.00	0.39	110.45	69.55	180.00	808.51
13.00	0.38	110.22	69.78	180.00	807.96
14.00	0.37	110.02	69.98	180.00	807.51
15.00	0.36	109.83	70.17	180.00	807.14
16.00	0.35	109.67	70.33	180.00	806.84
17.00	0.34	109.51	70.49	180.00	806.58
18.00	0.33	109.38	70.62	180.00	806.36
19.00	0.32	109.25	70.75	180.00	806.17
20.00	0.31	109.14	70.86	180.00	806.02
21.00	0.30	109.03	70.97	180.00	805.88
22.00	0.29	108.93	71.07	180.00	805.77
23.00	0.28	108.84	71.16	180.00	805.67
24.00	0.27	108.76	71.24	180.00	805.58
25.00	0.26	108.68	71.32	180.00	805.50
26.00	0.25	108.61	71.39	180.00	805.44
27.00	0.24	108.54	71.46	180.00	805.38
28.00	0.23	108.48	71.52	180.00	805.33
29.00	0.22	108.42	71.58	180.00	805.29
30.00	0.21	108.36	71.64	180.00	805.25
31.00	0.20	108.31	71.69	180.00	805.21
32.00	0.19	108.25	71.75	180.00	805.18
33.00	0.18	108.21	71.79	180.00	805.16
34.00	0.17	108.16	71.84	180.00	805.13
35.00	0.16	108.12	71.88	180.00	805.11
36.00	0.15	108.08	71.92	180.00	805.10
37.00	0.14	108.04	71.96	180.00	805.08
38.00	0.13	108.00	72.00	180.00	805.07
39.00	0.12	107.97	72.03	180.00	805.05
40.00	0.11	107.93	72.07	180.00	805.04