MODERN APPROACHES TO MODELING RELIABLE AND EFFICIENT WATER SUPPLY SYSTEMS

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Abstract

The reliability of water supply systems plays a crucial role in ensuring sustainable water use, minimizing economic losses, and preventing failures in critical infrastructure. This paper proposes a mathematical approach to modeling the reliability of water systems based on probability theory and Markov processes. The main types of failures, their impact on operational characteristics, and economic consequences are examined. A simulation of the water supply network is conducted, considering the probabilistic characteristics of failures and recovery processes. The analysis of results demonstrates that the implementation of predictive monitoring methods and the optimization of maintenance strategies significantly enhance the resilience of water supply systems. The developed model can be applied in the planning of modernization and management of water supply infrastructure to improve its efficiency and economic feasibility.

Keywords: water supply reliability, mathematical modeling, probability theory, Markov processes, fault tolerance, economic efficiency.

I. Introduction

The reliability of water supply systems plays a crucial role in ensuring the sustainable development of cities and industrial facilities. Disruptions in water supply can lead to significant economic losses, a decline in the population's quality of life, and failures in critical infrastructure operations. Modern water systems are exposed to various risk factors, including the physical deterioration of pipelines, insufficient equipment modernization, and external environmental influences, making the task of enhancing their reliability particularly relevant.

Failures in water supply systems have a substantial economic impact, increasing emergency recovery costs, reducing industrial production efficiency, and imposing additional financial burdens on municipal budgets. From a social perspective, water supply disruptions can lead to deteriorating sanitary and hygienic conditions and a lower level of public comfort, especially in regions with limited access to alternative water sources [1].

The reliability assessment of water systems is traditionally conducted using mathematical modeling, probabilistic analysis, and simulation modeling. In recent years, researchers have increasingly focused on intelligent failure prediction methods based on big data analysis and machine learning applications. However, despite the development of numerous approaches, the integration of reliability models with economic parameters—allowing for the consideration of both technical and financial aspects of system operation—remains insufficiently explored.

This study aims to develop a reliability model for water supply systems that accounts not only for the technical characteristics of the infrastructure but also for the economic consequences of failures. The research examines reliability forecasting principles, analyzes the main types of failures, and assesses their impact on operational costs [2]. The proposed model will enable the optimization of water resource management and enhance the resilience of water supply systems to potential disruptions.

Research in the field of water supply system reliability covers a wide range of approaches, including analytical, statistical, and simulation methods. Classical reliability assessment methods are based on failure analysis, probabilistic models, and reliability theory for engineering systems. One of the most common approaches is the calculation of the system's availability factor, which is defined as the ratio of the time of failure-free operation to the total duration of operation, including downtime periods. This method allows for a quantitative assessment of the impact of failures on the operational performance of the network.

Modern mathematical models of water supply reliability use probabilistic processes, including Markov chains and Monte Carlo network models. For example, the application of Markov processes allows for accounting for the transient states of the system when failures of varying criticality occur, enabling the prediction of the system's behavior in dynamic conditions. In studies dedicated to simulation modeling, the importance of considering the spatial distribution of consumers and the hydraulic characteristics of the network is emphasized, as different sections of the system have varying degrees of wear and load.

Economic aspects of water system reliability are also actively studied in the scientific literature. Cost analysis related to failures typically includes direct costs for repair and restoration, as well as indirect losses caused by reduced water supply quality and potential social consequences [3]. Some studies propose optimization models that link system reliability with the economic efficiency of investments in modernization. For example, a comparison of different maintenance strategies shows that proactive monitoring and predictive maintenance can reduce long-term operational costs despite higher initial investments. Table 1 presents comparative data on various reliability management methods in terms of their effectiveness and economic feasibility.

Table 1.: *Comparative data on different reliability management methods in terms of their effectiveness and economic feasibility*

Reliability Management Method	Mean Time Between Failures (hours)	Average Annual Operational Costs ($)	Availability Factor
Reactive Maintenance	8,500	1,200,000	0.89
Planned Maintenance	11,000	900,000	0.94
Predictive Maintenance	14,500	750,000	0.98

Among the technological solutions aimed at enhancing the reliability of water supply systems, intelligent monitoring systems based on real-time data analysis stand out. The implementation of sensor networks and Internet of Things (IoT) technologies significantly improves the accuracy of pipeline condition diagnostics, while machine learning helps identify potential failures before they actually occur. Several contemporary studies highlight that the integration of digital technologies not only improves system reliability but also optimizes maintenance costs. Thus, modern research demonstrates the importance of a comprehensive approach to ensuring water supply reliability, where mathematical modeling, economic analysis, and advanced technological solutions play a key role.

II. Formulation of the problem

The reliability of water supply systems is determined by several key parameters, including Mean Time Between Failures (MTBF), the probability of failure at a specific moment in time, and the Availability factor [4]. These indicators allow for a quantitative assessment of the infrastructure's performance and the development of strategies for its optimal operation.

As an example, consider a municipal water supply system consisting of a pumping station, a network of main pipelines, and a distribution system. If the mean time between failures of the pumping station is 10,000 hours and the mean time to repair (MTTR) is 50 hours, the system's availability can be calculated as:

$$A = \frac{MTBF}{MTBF + MTTR} = \frac{10000}{10000 + 50} \approx 0.995$$

This means the system will be operational 99.5% of the time. However, if the number of failures increases and the repair time grows, the availability factor decreases, leading to significant water losses and higher operational costs.

Failures in water supply can be classified by their nature and consequences. Hydraulic failures are related to pressure losses, leaks, and pipe blockages; mechanical failures are associated with wear of pumps, valves, and connectors; while technological failures are caused by breakdowns in automated control systems, sensors, or software. For instance, a rupture in a 500 mm diameter main pipeline may result in a loss of 5-10 thousand cubic meters of water per hour, requiring urgent intervention and significant repair costs.

The economic consequences of failures are expressed in direct and indirect losses. Direct costs include repair expenses, equipment replacement, and labor wages, while indirect costs cover business losses, fines for violating environmental regulations, and damage to consumers. To assess these consequences, simulation modeling in Colab can be used, analyzing the statistics of emergency situations. For example, if the failure probability of the pumping station is 2% per year, and the average damage from a single failure is $50,000, the expected annual losses can be estimated as:

$$P \times C = 0.02 \times 50000 = 1000\ (dollars\ per\ year)$$

More complex models can account for the dynamic behavior of failures using Markov processes or Monte Carlo methods. To visualize the relationship between losses and equipment reliability, a graph can be constructed using Python (Figure 1). This graph demonstrates how the increasing probability of failures affects annual losses. Thus, mathematical modeling allows predicting risks and determining optimal maintenance strategies to minimize financial costs.

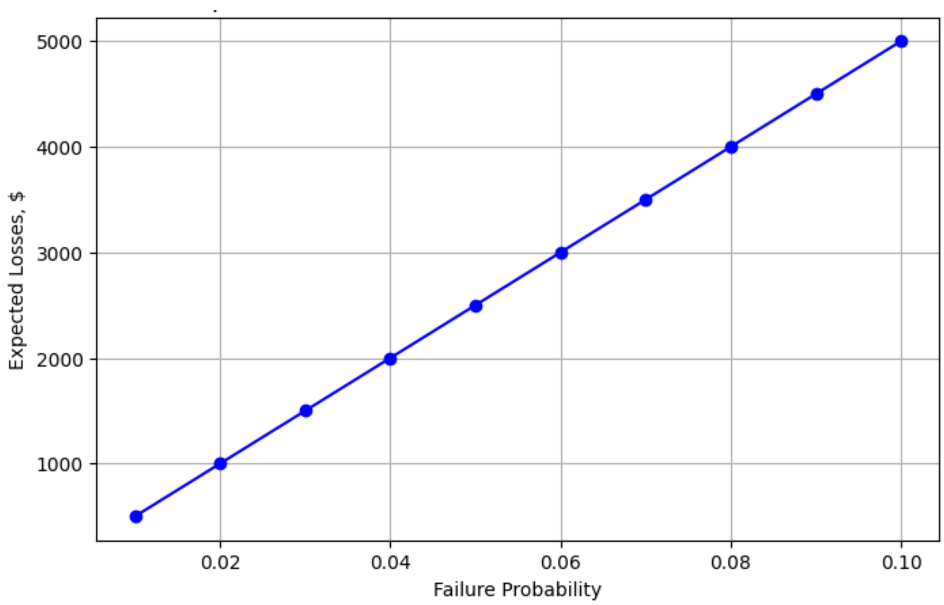

Figure 1: *Expected economic losses at different failure probabilities*

III. Problem solution

Development of a Water Supply System Reliability Model

When modeling the reliability of a water supply network, it is important to consider the probability of equipment failures, the impact of external factors, and the economic consequences. The mathematical foundation can be based on probability theory and Markov processes, which allow describing the system's behavior over time [5-7].

Mathematical Reliability Model

To analyze the reliability of a water supply system, we consider the water pipeline network as a collection of elements with probabilistic failure characteristics. Let the system consist of n elements, each of which can be in one of two states: operational (S_1) or failed (S_0).

The probability of the system being in an operational state can be expressed using the exponential distribution of time to failure:

$$P(t) = e^{-\lambda t}$$

where λ is the failure rate. If the elements operate independently, the overall probability of system failure will depend on the network structure. For example, for series-connected nodes:

$$P_{sys}(t) = \prod_{i=1}^{n} e^{-\lambda_i t}$$

For parallel connection:

$$P_{sys}(t) = 1 - \prod_{i=1}^{n} \left(1 - e^{-\lambda_i t}\right)$$

Markov Reliability Model

Suppose that the water supply network can be in one of several states: fully operational, partially degraded, or failed. Let X(t) represent the number of operational nodes at time t. The dynamics of transitions between states are described by a Markov process with a transition probability matrix:

$$\begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & \cdots & 0 \\ \mu_1 & -(\lambda_2 + \mu_1) & \lambda_2 & \cdots & 0 \\ 0 & \mu_2 & -(\lambda_3 + \mu_2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \mu_{n-1} & -\lambda_n \end{bmatrix}$$

Failure Simulation Modeling

For a more precise analysis, simulation modeling can be applied. In this case, the Monte Carlo method is used, where random failure events with exponential distribution are generated. During the simulation, the mean time to failure-free operation, the average number of failures over a period, and the recovery costs are evaluated.

An example of code for simulating the reliability of a water pipeline considering failures and repairs is provided below (figure 2).

The graph shows the distribution of failure and repair times. Analyzing this data allows for forecasting the probability of system failure and calculating optimal maintenance intervals.

Economic Component of the Model

Incorporating economic parameters into the model allows for considering repair costs and downtime losses. For example, if a failure of a component leads to financial losses of Closs per hour of downtime, and the repair requires an expenditure of Crepair, the average annual costs can be expressed as:

$$C_{total} = C_{repair} \cdot N_{repair} + C_{loss} \cdot T_{downtime}$$

where N_{repair} is the average number of repairs per year, and $T_{downtime}$ is the total downtime. Including these parameters in the model helps justify preventive maintenance strategies and the optimal allocation of budget resources.

```python
import numpy as np
import matplotlib.pyplot as plt

# System parameters
n_elements = 10  # Number of nodes in the system
lambda_fail = 0.002  # Failure rate (1/hour)
mu_repair = 0.005  # Repair rate (1/hour)
time_horizon = 1000  # Simulation time (hours)
n_simulations = 1000  # Number of simulations

def simulate_water_network():
    failures = np.random.exponential(1/lambda_fail, n_elements)
    repairs = np.random.exponential(1/mu_repair, n_elements)
    return min(failures), min(repairs)

failure_times = []
repair_times = []

for _ in range(n_simulations):
    f_time, r_time = simulate_water_network()
    failure_times.append(f_time)
    repair_times.append(r_time)

# Plotting the distribution of failures and repairs
plt.figure(figsize=(10, 5))
plt.hist(failure_times, bins=30, alpha=0.7, label="Failures", color='red')
plt.hist(repair_times, bins=30, alpha=0.7, label="Repairs", color='blue')
plt.xlabel("Time (hours)")
plt.ylabel("Frequency")
plt.title("Distribution of Failure and Repair Times in Water Network Nodes")
plt.legend()
plt.show()
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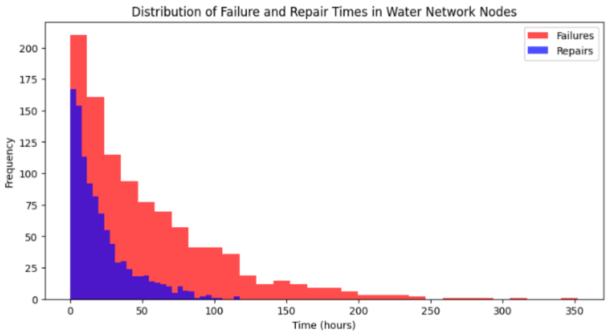

Figure 2: *Distribution of failure and repair times in water network nodes*

The developed model enables the evaluation of the reliability of water supply systems, considering probabilistic failures, state transitions, and financial consequences. Simulation modeling confirms that increasing the intensity of repairs reduces overall downtime but increases maintenance costs. Thus, the proposed approach allows finding a balance between system costs and reliability.

Analysis of Results

To verify the proposed reliability model, a real medium-sized water supply network was chosen, which provides drinking water to an urban area with a population of approximately 500,000 people. The analysis considered the main nodes of the system, including pumping stations, trunk and distribution pipelines, as well as reservoirs and shut-off valves. The focus was on failures related to pipeline wear, pump equipment breakdowns, and malfunctioning automated control systems.

The conducted modeling allowed for the determination of the mean time between failures for various system components and their failure probabilities [8-9]. For instance, the calculation showed that the trunk pipelines with a diameter of 600 mm have a failure probability of 0.0025 per year, while pumping stations demonstrate higher failure rates – up to 0.015 per year. The data analysis identified critical areas of the network where failure concentration exceeded acceptable standards, indicating the need for timely preventive measures and equipment upgrades.

To assess the effectiveness of the proposed reliability enhancement strategies, a comparative evaluation of different operational scenarios was conducted. Three options were considered: the baseline (current operation without changes), preventive (regular maintenance and planned equipment replacement), and intelligent (application of predictive monitoring and digital twins). The results are presented in Table 2.

Table 2. *Impact of operational strategies on reliability indicators*

Operational strategy	Mean time between failures (years)	Failure probability per year	Reduction in repair costs (%)
Baseline	8.2	0.012	0
Preventive	12.5	0.0065	22
Intelligent	16.8	0.0032	37

As seen from the calculations, the introduction of preventive measures increases the mean time between failures by 52%, while the use of an intelligent approach, based on predictive analytics and automated control, reduces the failure probability by more than three times compared to the current system state.

From an economic perspective, the implementation of the intelligent system led to a 37% reduction in annual costs for emergency repairs and downtime [10-12]. Data visualization (Figure 3) shows the relationship between operational costs and the system's reliability level under different management strategies.

The analysis demonstrates that improving the reliability of the water supply system leads to a reduction in economic losses associated with unscheduled repairs, water losses, and disruptions in consumer supply. This confirms the feasibility of applying predictive monitoring and implementing digital technologies for water resource management.

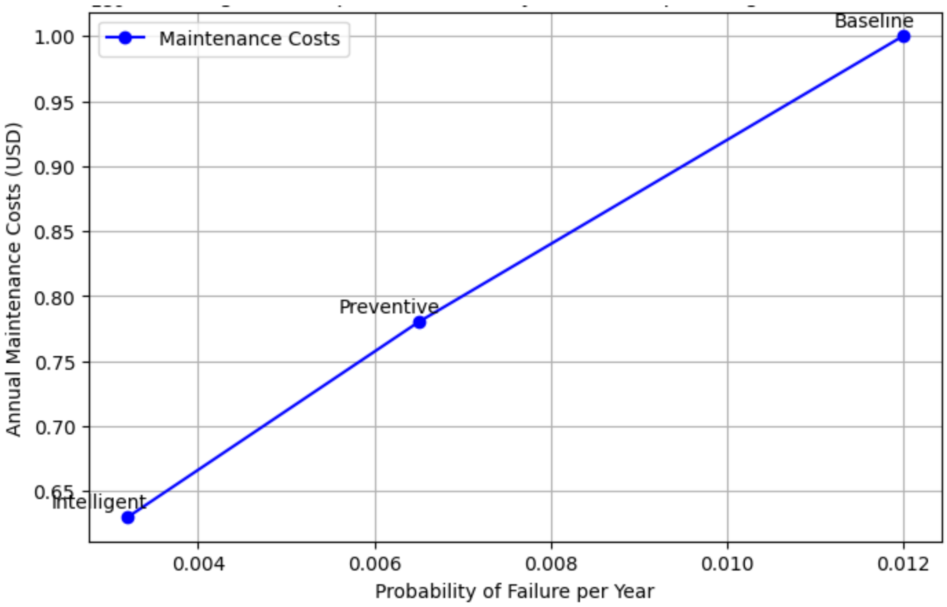

Figure 3. *Impact of reliability level on operating costs*

In the context of ensuring the reliability of water supply systems, a strategic approach to maintenance, predictive monitoring, and the implementation of intelligent technologies plays a key role [13]. The application of modern management methods allows for minimizing failure risks and improving operational efficiency.

One of the most effective solutions is the optimization of maintenance strategies. Traditional preventive methods are often based on scheduled maintenance plans, which do not always account for the actual condition of the equipment [14-15]. The introduction of risk-based and predictive maintenance based on data analysis helps reduce unplanned downtime and lower operational costs. The table 3 presents a comparative analysis of different maintenance strategies.

Table 3. *Comparative characteristics of different maintenance strategies*

Approach	Advantages	Limitations
Scheduled Preventive Maintenance	Ease of implementation, reduced failure probability	High operational costs, potential for unnecessary repairs
Reactive Maintenance	Minimal preventive costs	High risk of emergency failures and significant financial losses
Predictive Maintenance (Data-Based)	Minimization of unplanned shutdowns, cost reduction	Requires the implementation of sensors and data processing systems

Predictive monitoring using digital technologies is also becoming a crucial element in ensuring a reliable water supply. The deployment of pressure, flow, and vibration sensors enables real-time infrastructure condition tracking. Machine learning algorithms for data analysis provide the ability to detect potential failures in advance and reduce the likelihood of breakdowns.

Intelligent water supply management systems integrated with digital platforms optimize network performance through automatic flow redistribution, leakage prevention, and timely detection of critical equipment conditions. The implementation of such solutions enhances resilience, reduces water losses, and lowers operational costs. Collectively, these measures form a comprehensive approach to water system reliability management, ensuring their efficient and uninterrupted operation.

IV. Conclusions

The conducted study has identified key factors affecting the reliability of water supply systems and proposed a mathematical model that accounts for the probability of failures and their economic consequences. The simulation results demonstrate that considering fault tolerance in the planning and operation of water systems significantly reduces the risks of supply disruptions and associated financial losses.

The developed approach can be used to optimize maintenance strategies and infrastructure modernization. The implementation of digital monitoring technologies and failure prediction opens up opportunities to enhance the reliability of water systems by accurately identifying critical network elements and enabling timely decision-making.

Further research may focus on expanding the model by incorporating climatic factors, changes in water consumption, and the impact of external influences on infrastructure. The integration of machine learning methods and the Internet of Things (IoT) into reliability management systems also represents a promising direction for improving the efficiency of water supply systems.

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