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A FAILURE DISTRIBUTION FOR RELIABILITY PREDICTION OF MECHATRONIC COMPONENTS AND HUMAN-MACHINE SYSTEM

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Abstract

Modern machines and equipment's have a complex mechatronic structure consisting of various components, and their reliability depends on a large number of random factors that arise during design, production and operation, which are often impossible to predict. Each element of the modern machines is characterized by different performance criteria and corresponding failures. Various statistical models of failure distribution are widely used to quantify the reliability of machines and devices. The choice of a statistical model and its parameters is important for a proper assessment of reliability. The chosen statistical model should reflect the actual distribution of failures fairly correctly. In presented article is proposed a new failure distribution for reliability prediction of mechatronic components of modern machines and human-machine systems. A large number of sudden failures of modern complex technical facilities containing electronic and mechatronic structural elements seriously affect its λ-characteristic.

Various studies have already shown that the failure behavior of complex systems cannot always be characterized by the "bathtub curve". This is especially true for modern complex machines, which, among other things, consist of numerous electronic components for which no wear and fatigue failures are assumed. For this reason, an alternative service life distribution for the description failure behavior of modern mechatronic components and human-machine systems is proposed. This is about the failure curves, which are initially characterized by a low or high failure rate and then tend to a constant failure rate.

To determine the reliability indexes are provided analytical formulas. Methods for estimating the parameters of this distribution are presented based on failure statistic. To determine distribution parameters, statistical data on failures of the technical system are sufficient only in the first period of its operation. This is one of the main advantages of the presented distribution. On the example of practical cases, the hypothesis of compliance of the proposed theoretical distribution to the actual statistical data on failures of various mechatronic systems and human-machine system was tested.

Keywords: failure distribution, reliability indexes, failure rate, mechatronic components, human-machine system

I. Introduction

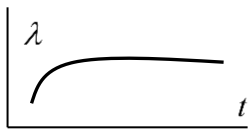
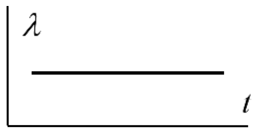
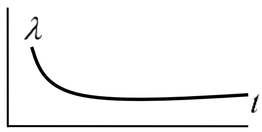
Modern machines and equipments have a complex mechatronic structure consisting of various components, and their reliability depends on a large number of random factors that arise during design, production and operation, which are often impossible to predict. Each element of the modern machines is characterized by different performance criteria and corresponding failures. Various statistical models of failure distribution are widely used to quantify the reliability of machines and devices. The choice of a statistical model and its parameters is important for a proper assessment of reliability. The chosen statistical model should reflect the actual distribution of failures fairly correctly. Currently, dozens of different statistical models are used to assess the reliability of machines and their structural elements [1-6]. Each of the existing distribution has certain advantages and disadvantages. When choosing a statistical failure model for any technical system, it is necessary to take into account the operating conditions, the type and nature of failures, the design features of the unit, and other factors. [1] it was proposed to use the Weibul distribution to assess the reliability of transmission mechanisms. However, the need for different price perception of the Weibul distribution parameters at different stages of operation creates certain difficulties in calculations.

The conducted research shows that the laws of dependence of failure rate on time in modern mechatronic systems and human-machine systems depend on many factors. A large number of sudden failures of modern complex technical facilities containing electronic and mechatronic structural elements seriously affect its λ-characteristic.

II. Problem statement

The investigations show that the failure behavior according to Table 1, which are characteristic of complex machines and mechatronic components, occur most frequently in today's mechanical engineering [7]. Therefore, the description of these failure behavior by statistical distributions is of interest. The failure behavior according to the pattern "B" can of course be well described with the exponential distribution. The other two failure behavior, can in principle be described with the Weibull distribution, but with different parameters for the two ranges [1].

Table 1: *Failure behavior for random failures*

	Failure behavior		Overall Characteristic
Random failures	A		complex machines with high-stress tests after commissioning
	B		Well-designed complex machines
	C		Electronic components and complex components after repair

For this reason, an alternative service life distribution for the description of the three above-mentioned failure behavior types of modern mechatronic components and human-machine systems is proposed in the present work. The failure rate is described in the following form:

$$\lambda(t) = \lambda\left[1 + (\alpha - 1)e^{-\beta t}\right], \qquad (1)$$

where λ - the failure rate in the range of random failures; α - the shape parameter characterizes the value of the failure rate at the beginning of the first range:

$$\alpha = \frac{\lambda_0}{\lambda}. \qquad (2)$$

The parameter β describes the duration of the characteristic early failure time t_f (Fig.1). It has been determined by calculations that by α=2 and λ=0,25 results $\beta \approx 4/t_f$. As follows from the figure, by α=1 results in exponential distribution that determines the failure behavior according to the pattern "B" (Tab. 1) describes. With the presented distribution, the failure behavior according to the patterns "A" (for $0<\alpha<1$) and "C" (for $\alpha>1$) can be described very well.

If, in the case of a failure behavior, the failures begin only from a point in time t_0, such a course can be described with a corresponding four-parameter distribution (Fig. 2). For the failure rate in this case, the following applies

$$\lambda(t) = \lambda\left[1 + (\alpha - 1)e^{-\beta(t-t_0)}\right]. \qquad (3)$$

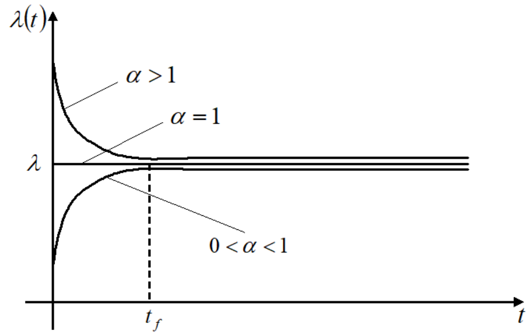


Figure 1: *Course of the failure rate in the three-parameter distribution*

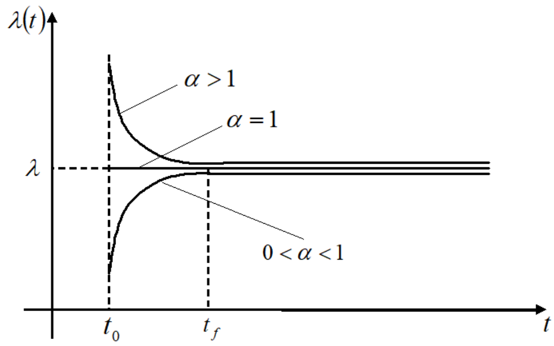


Figure 2: *Course of the failure rate in the four-parameter distribution*

III. Analytical determination of the remaining failure functions

By integration according to [1], the analytical formulas for the remaining failure functions of the three-parameter and four-parameter distribution (survival probability R (t), failure probability F (t) and density function f (t)) can be determined. Table 2 provides formulas for determining the failure functions of a three-parameter distribution.

Table 2: *Analytical formulas for the failure functions of the three-parameter distribution*

Failure functions	Failure rate	$\lambda(t) = \lambda\left[1 + (\alpha - 1)e^{-\beta t}\right]$
	Survival probability	$R(t) = \exp\left(-\int_0^t \lambda(t)dt\right) = \exp\left[\frac{\lambda}{\beta}(\alpha - 1)\left(e^{-\beta t} - 1\right) - \lambda t\right]$
	Failure probability	$F(t) = 1 - R(t) = 1 - \exp\left[\frac{\lambda}{\beta}(\alpha - 1)\left(e^{-\beta t} - 1\right) - \lambda t\right]$
	Failure density function	$f(t) = \lambda(t) \cdot R(t) = \lambda\left[1 + (\alpha - 1)e^{-\beta t}\right] \cdot \exp\left[\frac{\lambda}{\beta}(\alpha - 1)\left(e^{-\beta t} - 1\right) - \lambda t\right]$

In Figure 3, the graphical curves of the distribution failure functions (survival probability R(t) and failure density function f(t)) for different shape parameters α are shown. As can be seen from the figure, different density functions can be generated with the presented lifetime distribution in contrast to the exponential distribution. Depending on the shape parameter α, the density function changes significantly. For $\alpha \geq 1$, there is a decreasing density function, as with the exponential distribution. For $\alpha < 1$, the density function for decreasing α-values starts at lower failure densities, then reaches a steep maximum and finally falls flat.

Other statistical measures of the present distribution (mathematical expectation, variance, etc.) can only be calculated using numerical methods, since the resulting integral expressions cannot be solved analytically. A significant advantage of the illustrated distribution is that the parameters of the distribution can be determined by an incomplete test during the period of early failures. In this way, the testing effort of lifetime tests can be significantly reduced.

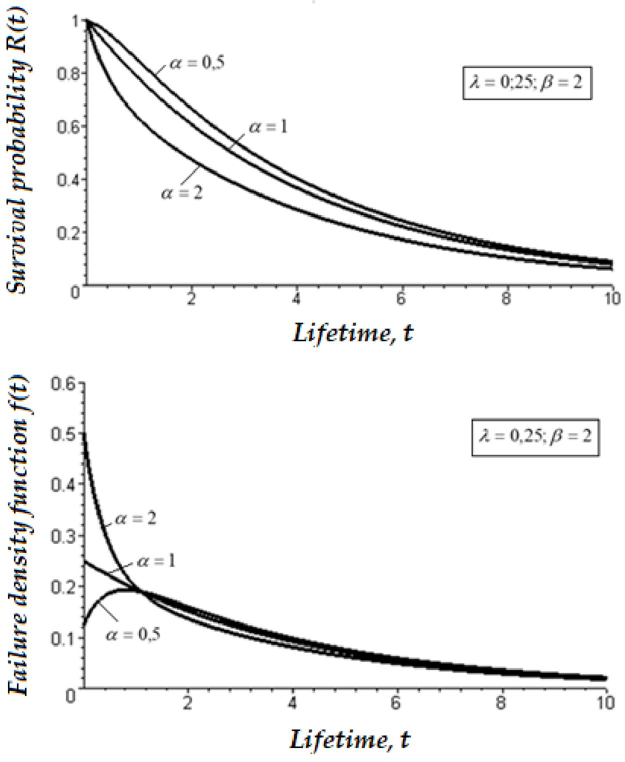


Figure 3: *Distribution failure functions*

IV. Determination of the distribution parameters

The parameters of the distribution can be determined by lifetime tests or failure statistics for an order statistics for m test specimens of the size n. For the test specimens i under consideration of the size n, let the failure times, ordered according to their size be $t_{1,i}$, $t_{2,i}$, $t_{3,i}$, ..., $t_{k,i}$.... A value of the failure rate corresponds to each failure times. According to the definition of the failure rate, these can be determined in the following form[1]:

$$\lambda_{1,i} = \frac{1}{n \cdot t_{1,i}}, \quad \lambda_{2,i} = \frac{1}{(n-1)(t_{2,i}-t_{1,i})}, \dots, \lambda_{k,i} = \frac{1}{(n-k+1)(t_{k,i}-t_{k-1,i})}, \dots \tag{4}$$

For the entire order statistics, the average values of the failure times and failure rates can be determined according to the respective rank to:

$$t_1 = \frac{1}{m}\sum_{i=1}^{m} t_{1,i},\ t_2 = \frac{1}{m}\sum_{i=1}^{m} t_{2,i},\ \dots,\ t_k = \frac{1}{m}\sum_{i=1}^{m} t_{k,i}, \dots \tag{5}$$

$$\lambda_1 = \frac{1}{m}\sum_{i=1}^{m} \lambda_{1,i},\ \lambda_2 = \frac{1}{m}\sum_{i=1}^{m} \lambda_{2,i}, \dots,\ \lambda_k = \frac{1}{m}\sum_{i=1}^{m} \lambda_{k,i}, \dots \tag{6}$$

By entering the obtained pairs of values in the graph (Fig. 4), it is possible to assess in a first approximation whether the investigated failure behavior corresponds to the present distribution. The failure times t_f is called the characteristic early failure time when the difference $\lambda(t_f) - \lambda(t_{f+1})$ is very small. Thus, we can approximately assume that the parameter $\lambda \approx \lambda(t_{f+1})$, and the failure rate tends to this value. The shape parameter α can be determined approximately in the following form:

$$\alpha = \frac{\lambda_0}{\lambda} \approx \frac{\lambda(t_1)}{\lambda(t_{f+1})}. \tag{7}$$

In this case, it is assumed that the value of the failure rate at the beginning of the first range λ_0 is equal to $\lambda(t_1)$.

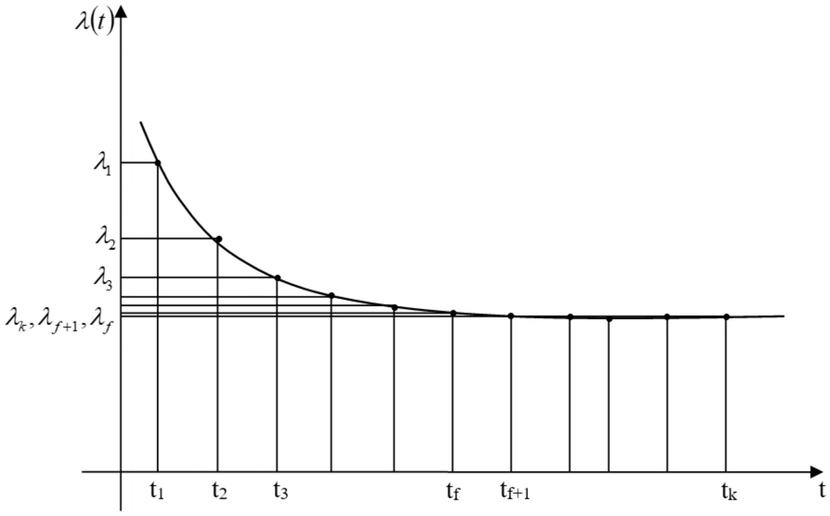


Figure 4: *The graph for determining the distribution parameters*

For the determination of the parameter β, formula (1) for the failure time $t = t_f$ is represented in the form:

$$\lambda(t_f) - \lambda = \lambda(\alpha - 1)e^{-\beta \cdot t_f}. \tag{8}$$

By logarithmizing, one obtains:

$$\ln\left[\lambda(t_f) - \lambda\right] = \ln[\lambda(\alpha - 1)] - \beta \cdot t_f. \tag{9}$$

It follows from this

$$\beta = \frac{1}{t_f}\ln\frac{\lambda(\alpha-1)}{\lambda(t_f)-\lambda}. \tag{10}$$

Suppose that for α >1, the condition $\lambda(t_f)-\lambda=e^{-4}\cdot\lambda\approx0{,}018\ \lambda$ pays off at the time t_f. Then, based on the expression (10), we get:

$$\beta = \frac{1}{t_f}\left(4 + \ln(\alpha - 1)\right). \tag{11}$$

By analogy, one can write for the case α < 1:

$$\beta = \frac{1}{t_f}\left(4 + \ln(1 - \alpha)\right). \tag{12}$$

The parameters of the distribution can also be determined by three pairs of values (t_L,λ_L), (t_M,λ_M), (t_N,λ_N) (Figure 5). For this, the following system of equations must be solved:

$$\begin{cases} \lambda\left[1 + (\alpha - 1)e^{-\beta\cdot t_L}\right] = \lambda_L \\ \lambda\left[1 + (\alpha - 1)e^{-\beta\cdot t_M}\right] = \lambda_M \\ \lambda\left[1 + (\alpha - 1)e^{-\beta\cdot t_N}\right] = \lambda_N \end{cases} \tag{13}$$

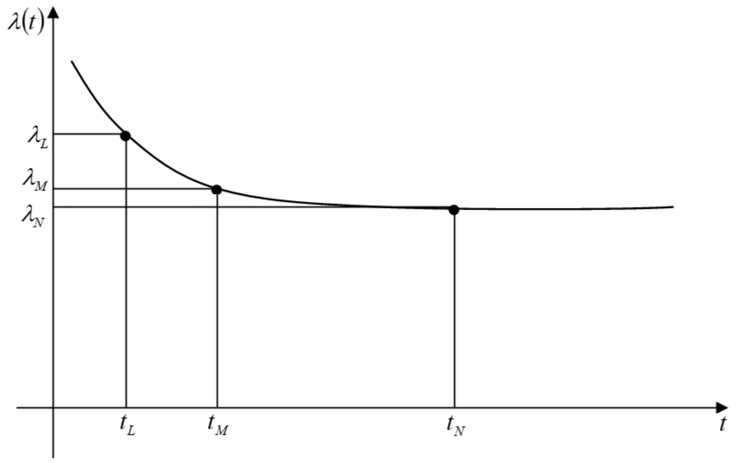


Figure 5: *Determination of distribution parameters based on three test results*

V. Application and evaluation

In order to evaluate the possibility of applying the proposed distribution, it is necessary to consider some practical cases.

In [8], the results of tests for the service lifetime of submersible electric motors of oil field equipment PED-32 and PED-45 were presented. The sample size was n=197 failures during 1548 days of testing. The class number was determined by the formula of Sturgess and after rounding it was taken K=8. Thus, the class length was taken h=193,5 days. Statistical data on failures were presented in Table 3. As can be seen from the table, the actual values of failure rates correspond to the case "A" of the proposed distribution (Tab. 1), since in the first period these values gradually increase, and then remain almost constant during normal operation. Only in the last period, the failure rate increases sharply. This is due to the fact that by the end of the test period, the number of serviceable electric motors remains very small. This has little effect on the reliability of the electric motors in question during the first period and normal operation. Therefore, this difference can be ignored.

Based on the calculations of the distribution parameters using the formulas (7) and (12), the following results were obtained: λ=0,003 day^{-1}; α=0,3; β=0,012 day^{-1}.

Figure 5 shows a histogram of the failure rate of a submersible electric motor of oilfield equipment based on test results and a curve of changes in the values of this parameter according to the present distribution.

The test of the hypothesis about the conformity of the present theoretical distribution to the actual statistical data on failures was carried out according to the χ^2-Pearson criteria. According to [9], the measure of the discrepancy between the theoretical and empirical probability density functions can be determined by the expression below:

$$\chi^2 = \sum_{i=1}^{8} \frac{(n_i - np_i)^2}{np_i}. \quad (14)$$

Here p_i is the theoretical probability of a random variable falling into each of the intervales.

Table 3: *Statistical data on failures of submersible electric motors*

N-r of classes	Lifetime intervals, in days	Number of failures, n_i	Number of operable motors in the middle of the interval, N_i	Actual survival probability $R_i=N_i/N$	Actual failure rate $\lambda_i = \frac{n_i}{N_i \cdot \Delta t}$
1	0 – 193,5	58	168	0,85	0,0018
2	193,5- 387	62	108	0,55	0,003
3	387- 580,5	34	60	0,31	0,003
4	580,5- 774	17	34	0,17	0,0026
5	774- 967,5	12	20	0,1	0,0031
6	967,5- 1161	6	11	0,06	0,0028
7	1161- 1354,5	4	6	0,03	0,0034
8	1354,5- 1548	4	2	0,01	0,01
Total		197			

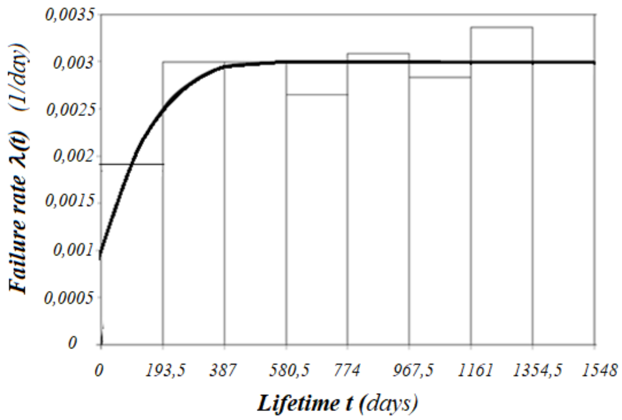


Figure 6: *Histogram of the failure rate of the submersible electric motor according to the test results and the curve of the values change according to the proposed distribution.*

The results of calculation based on the χ^2-Pearson test were presented in Table 4. The critical value χ_α^2 for the significance level α=0,80 and degrees of freedom m=K-1=8-1=7 is χ_α^2=3,82 [9]. Since the calculated χ^2=3,18 is less than χ_α^2 at a high level of significance, we can safety conclude that the proposed distribution is well suited for describing the actual distribution of failures of the tested submersible electric motors of oilfield equipment.

Table 4: *The results of calculation based on the χ^2 test for the summersible electric motors*

Nr.	Lifetime, t, days	Theoretical survival probability, R(t)	Theoretical failure frequency $p_i=R(t_{i-1})-R(t_i)$	Number of failures, n_i	$\frac{(n_i - n \cdot p_i)^2}{n \cdot p_i}$
1	0	1,0			
2	193,5	0,655	0,345	58	1,461
3	387	0,372	0,283	62	0,70
4	580,5	0,209	0,163	34	0,111
5	774	0,117	0,092	17	0,070
6	967,5	0,065	0,052	12	0,301
7	1161	0,037	0,028	6	0,042
8	1354,5	0,02	0,017	4	0,126
9	1548	0,005	0,015	4	0,369
Total				n=197	3,18

The following case from practice for the application of the present distribution realates to traffic-related mortality. Statistical data on traffic-related mortality in Germany in 2002 show the dependency of the human failure rate on age as shown in Figure 7 [10]. Since the right to drive a car is allowed from the age 18, here you can take t_0=18 year.

As can be seen from Figure 7, the traffic-related mortality at $t=t_0$=18 year, is equal to λ_0=0,00032 dead/(year·person). Over time, due to increased experience of drivers, the mortality decreases and takes a constant minimum value – λ=0,000052 dead/(year·person). The parameter α is determined based on the formula (7): α=6,15. For α >1, from the condition $\lambda(t_f)-\lambda=e^{-4}\cdot\lambda\approx 0{,}018\ \lambda$ pays the time t_f. Since $\lambda(t_f)\approx 1{,}018\ \lambda$=0,000053 dead/(year·person), according to the graph in Fig. 7, we get $t_f\approx$33 year.

Using the formula (12), the following value for β was obtained: β=0,171 day^{-1}.

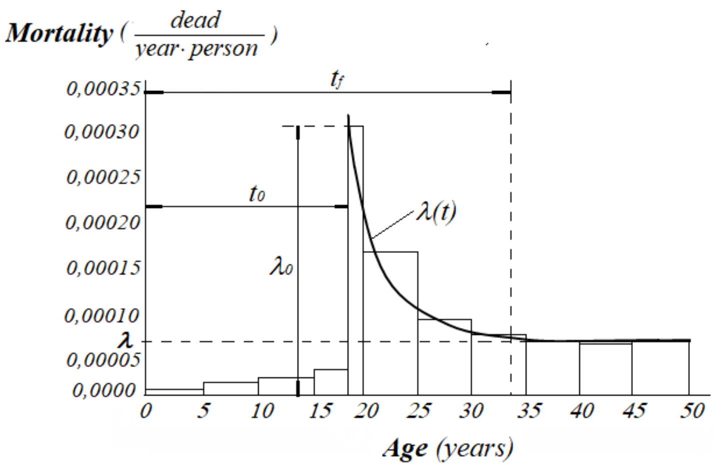


Figure 7: *Traffic-related mortality in Germany in 2002*

After determining all distribution parameters, it is possible to calculate other indexes of reliability of a man-machine system using present distribution. A graph of the human survival probability for this case is presented in Figure 8.

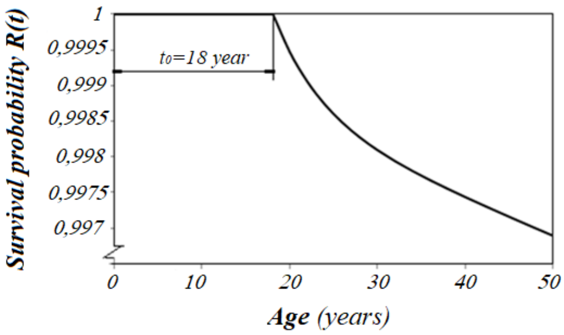


Figure 8: *Traffic-related human survival probability on age*

VI. Conclusions

The presented new theoretical distribution can be applied to describe the failures of various modern mechatronic components and human-machine systems. Using the presented distribution, reliability prediction in many cases can produce more effective results. This model can also be applied to assess the reliability of the human-machine system. To determine distribution parameters, statistical data on failures of the technical system are sufficient only in the first period of its operation. This is one of the main advantages of the presented distribution. Using the example of practical cases, a hypothesis was tested on the conformity of the present distribution to the actual statistical data on failures of various technical systems. Based on the obtained positive results, we can conclude that the proposed distribution can be successfully applied to describe the actual distribution of failures and assess the reliability of various modern mechatronic machines and human-machine system.

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