A PRODUCTION INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME AND PRICE RELIANT DEMAND USING FLOWER POLLINATION OPTIMIZATION

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Abstract

Effective management of production inventory for deteriorating items with dynamic demand patterns is crucial for businesses operating in today's competitive markets. This paper proposes a comprehensive model that addresses the complexities arising from the dual storage locations, item deterioration, and demand dependencies on both time and selling price. To optimize the decision variables associated with production and inventory control, we employ the Flower Pollination Optimization (FPO) algorithm, a nature-inspired meta-heuristic known for its ability to efficiently navigate complex search spaces. The two-storage production inventory model integrates the dynamics of item deterioration over time, capturing the real-world challenges faced by supply chain managers. The demand for items is modeled to be sensitive to both temporal variations and changes in selling prices, reflecting the intricate nature of market dynamics. Our approach leverages the FPO algorithm to explore and exploit the solution space, allowing for the identification of optimal or near-optimal strategies for production quantities, order quantities, and inventory levels. The FPO algorithm mimics the pollination process in nature, striking a balance between exploration and exploitation to efficiently search for solutions in a highly dynamic and nonlinear environment. The proposed model and optimization approach are validated through extensive simulations and sensitivity analyses. The results demonstrate the effectiveness of the FPO algorithm in finding robust solutions that enhance inventory management, mitigate deterioration-related losses, and adapt to varying demand scenarios. This research contributes to the field of supply chain optimization by offering a novel perspective on tackling the challenges associated with dual storage, item deterioration, and demand dependencies. The findings provide valuable insights for practitioners seeking advanced strategies for optimizing their production inventory systems in the face of evolving market conditions.

Keywords: Production Inventory Model, Deteriorating Items, Two-Storage, Shortages, Time and Selling Price Dependent Demand, Flower Pollination Optimization.

1. Introduction and related work

In the realm of supply chain management, the effective control and optimization of inventory systems play a pivotal role in ensuring the success and sustainability of businesses. As markets become increasingly dynamic and customer demands evolve, the complexities associated with managing production inventory for deteriorating items intensify. This is particularly true in scenarios where the demand for items is not only influenced by temporal variations but also by changes in selling prices. To address these challenges, we propose a two-storage production inventory model that accounts for the intricacies of dual storage locations, item deterioration, and demand dependencies on time and selling price. The management of deteriorating items presents a unique set of challenges due to the perishable nature of certain goods over time. Incorporating this deterioration factor into inventory models is crucial for avoiding unnecessary losses and ensuring that products reaching customers are of the highest quality. Moreover, the consideration of time-dependent demand recognizes the influence of various temporal factors, such as seasonality or market trends, on the overall demand pattern. Adding an additional layer of complexity, our model acknowledges the impact of selling prices on demand. Price elasticity is a well-established concept in economics, and understanding how changes in selling prices affect the demand for items is vital for making informed decisions in a competitive marketplace.To address the optimization problem inherent in this multifaceted inventory management model, we turn to the Flower Pollination Optimization (FPO) algorithm. FPO, inspired by the pollination process in flowers, offers a nature-inspired meta-heuristic that excels in navigating complex and dynamic search spaces. By mimicking the pollination behavior of flowers, the FPO algorithm strikes a balance between exploration and exploitation, making it well-suited for finding optimal or near-optimal solutions in intricate and non-linear environments.

This research aims to contribute to the field of supply chain optimization by proposing a novel approach to managing production inventory in the face of deteriorating items with time and selling price dependent demand. Through extensive simulations and sensitivity analyses, we evaluate the effectiveness of the FPO algorithm in optimizing production quantities, order quantities, and inventory levels. The outcomes of this study provide valuable insights for practitioners seeking advanced strategies to enhance their production inventory systems, adapt to changing market conditions, and minimize losses associated with item deterioration.

Supply chain management can be defined as: "Supply chain management is the coordination of production, storage, location and transport between players in the supply chain to achieve the best combination of responsiveness and efficiency for a given market. Many researchers in the inventory system have focused on a product that does not overcome spoilage. However, there are a number of things whose meaning doesn't stay the same over time. Yadav et al. [1-10] developed the deterioration of these substances plays an important role and cannot be stored for long. Yadav, et al. [11-20] studied deterioration of an object can be described as deterioration, evaporation, obsolescence and loss of use or restriction of an object, resulting in less inventory consumption than under natural conditions. When raw materials are put in stock as a stock to meet future needs, there may be a deterioration of the items in the arithmetic system which could occur for one or more reasons, etc. Storage conditions, weather or humidity. Yadav, a. al. [21-53] explore the inach generally states that management has a warehouse to store the purchased warehouse. However, for various reasons, management may buy or lend more than it can store in the warehouse and call it OW, with an extra number in a rented warehouse called RW near OW or just off it. Yadav and swami. [54-61] developed an inventory costs (including maintenance costs and depreciation costs) in RW are generally higher than OW costs due to additional costs of running, equipment maintenance, etc. Reducing inventory costs will cost-effectively utilize RW products as quickly as possible. Actual customer service is only provided by OW, and to reduce costs, RW stock is cleaned first. Such arithmetic examples are called two arithmetic examples in the shop. Yadav and Kumar [62] established the management of the supply of electronic storage devices and integration of environmental and nerve networks. Yadav, A.S. [63-65] analysize of seven supply chain management measures to improve inventory of electronic storage devices by submitting a financial burden using GA and PSO and supply chain management analysis to improve inventory and inventory of equipment using genetic computation and model design and chain inventory analysis from bi inventory and economic difficulty in transporting goods by genetic computation. Swami, et. al. [66-68] developed inventory policies for inventory and inventory needs and miscellaneous inventory costs based on allowable payments and inventory delays An example of depreciation of various types of goods and services and costs by keeping a business loan and inventory model with pricing needs low sensitive, inventory costs versus inflationary business expense loans. Gupta, et. al. [69- 70] established the objectives of the Multiple Objective Genetic Algorithm and PSO, which include the improvement of supply and deficit, inflation and a calculation model based on a genetic calculation of the scarcity and low inflation of PSO. Singh, et. al. [71, 72] studied an example with two stock depreciation on assets and inventory costs when updating particles and an example with two inventories of property damage and inventory costs in inflation and soft computer techniques. Kumar, et. al. [73-75] delayed control of alcohol supply and particle refinement and green cement supply system and inflation by particle enhancement and electronic inventory system and distribution center by genetic computations. Chauhan and Yadav [76-77] depreciation example at two stores and warehouses based on inventory using one genetic stock and one vehicle stock for demand and inflation inventory with two distribution centers using genetic stock. Pandey, et. al. [78] analysize of marble Improvement of industrial reserves based on genetic technology and improvement of multiple particles. Ahlawat, et. al. [79] studied the white wine industry in supply chain management through nerve networks. Singh, et. al. [80] examines the best policy to import damaged goods immediately and pay for conditional delays under the supervision of two warehouses. The study by Yadav et al. [81] focuses on enhancing inventory management for degrading commodities within the framework of green technology investment, accounting for factors including selling price, carbon emissions, and time-sensitive demand. It emphasizes sustainable practices in inventory models by fusing conventional economic principles with environmental factors. Using an interval number technique, Yadav, Yadav, and Bansal [82] examine a two-warehouse inventory management model for degrading commodities in order to take demand and cost uncertainty into consideration. Their analytical optimization methods demonstrate how spending money on preservation technology can save waste and increase inventory efficiency. With an emphasis on a two-warehouse system to maximize inventory levels, Yadav, Yadav, and Bansal [83] offer an inventory model that tackles the deterioration of commodities during storage. Their strategy emphasizes how crucial it is to successfully control degradation costs in order to raise total inventory efficiency.

2. Assumptions and notations

2.1. Assumptions

The following assumptions were used in the formulation of the mathematical model.

1. The unit production cost is a function of the rate of production.
2. The rate of production is considered to be decision variable.
3. The demand rate is a function of time and selling price, which is $D(t,p) = (\alpha + \beta t - \gamma t^2)p^{-\lambda}$; $\alpha > 0,\ \beta \in [0,1),\ \gamma \in [0,1)$.
4. The rate of deterioration is constant and different for both the warehouses.
5. The OW has limited capacity of W units and the RW has unlimited capacity.
6. Deterioration units can't be repaired or replaced.
7. The RW is located near the OW and thus the transportation cost between them is negligible.
8. The inventory cost (including carrying cost and deterioration cost) in RW is higher than that in OW.
9. Shortages are not allowed.
10. The holding cost is constant for both the warehouses.

2.2. Notations

Table 2 is provided a description of the notations utilised for the constructed mathematical model.

Table 1: *Notations*

Notation	Units	Description
α	Constant	Coefficient of demand function
β	Constant	Coefficient of demand function
γ	Constant	Coefficient of demand function.
θ_1	Constant	Deterioration rate in OW.
θ_2	Constant	Deterioration rate in RW.
P	–	Production rate.
p	$/Units	Selling price of product.
U	capability constraint	The owned warehouse capacity
SUC_i	$/unit	Set-up cost
C_D	$/Units	Deterioration cost.
h	$/Units	Holding cost.
TVC	$/Units	The function for total inventory cost.

3. Mathematical Model Formulation

The mathematical model for a production inventory system handling decaying products with two-storage includes time-dependent and selling price-dependent demand. Assume that I(t) reflects the inventory level at time t and that there are two storage facilities: one for immediate sales and another for buffer stock. The demand function D(p(t),t) is influenced and altered by the selling price p(t). The degradation rate affects the inventory's usefulness, therefore manufacturing costs, storage costs, and potential revenue loss due to spoiling must all be balanced. The objective is to minimize the total cost, which includes holding costs, production expenses, and lost revenue due to deterioration, subject to constraints on inventory levels, demand, and production rates. (See Fig. 1).

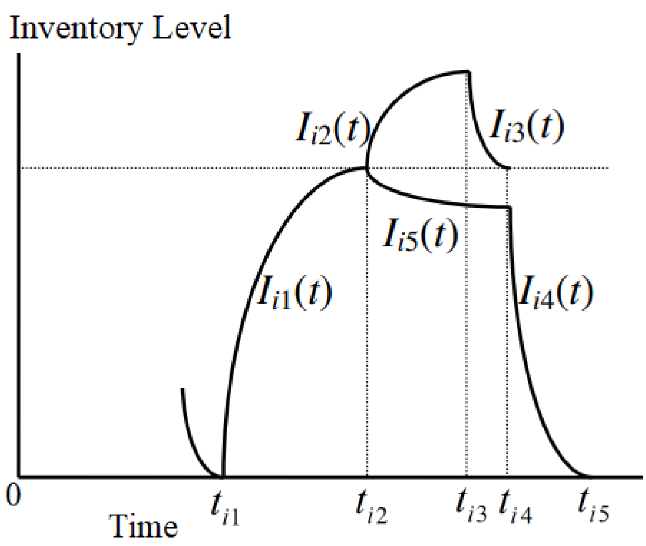

Figure 1: *A graphical depiction of the two-warehouse production inventory model.*

The inventory level is characterized by the following differential equations:

$$\frac{dI_{i1}(t)}{dt} + \theta_1 I_{i1}(t) = P - (\alpha + \beta t - \gamma t^2)p^{-\lambda}; \quad t \in [0, t_{i1}] \tag{1}$$

with the boundary condition (B.C.) $I_{i1}(0) = 0$.

$$\frac{dI_{i2}(t)}{dt} + \theta_2 I_{i2}(t) = P - (\alpha + \beta t - \gamma t^2)p^{-\lambda}; \quad t \in [t_{i1}, t_{i2}] \tag{2}$$

with the boundary conditions (B.C.) $I_{i2}(t_{i1}) = 0$.

$$\frac{dI_{i3}(t)}{dt} + \theta_2 I_{i3}(t) = -(\alpha + \beta t - \gamma t^2)p^{-\lambda}; \quad t \in [t_{i2}, t_{i3}] \tag{3}$$

with the boundary conditions (B.C.) $I_{i3}(t_{i3}) = 0$.

$$\frac{dI_{i4}(t)}{dt} + \theta_1 I_{i4}(t) = -(\alpha + \beta t - \gamma t^2)p^{-\lambda}; \quad t \in [t_{i3}, t_{i4}] \tag{4}$$

with the boundary conditions (B.C.) $I_{i4}(t_{i4}) = 0$.

$$\frac{dI_{i5}(t)}{dt} + \theta_1 I_{i5}(t) = 0; \quad t \in [t_{i1}, t_{i3}] \tag{5}$$

with the boundary conditions (B.C.) $I_{i5}(t_{i1}) = W$.

The solutions of the differential Eqs. (1) -(5) are (6) -(10), respectively:

$$I_{i1}(t) = \left\{ \frac{1}{\theta_1^3}\left(\frac{2\gamma}{P^\lambda} + P\theta_1^2 - \frac{2\alpha\theta_1^2}{P^\lambda} + \frac{\beta\theta_1}{P^\lambda}\right) - \frac{1}{\theta_1^3}\left[e^{-t\theta_1}\left(\frac{2\gamma}{P^\lambda} + P\theta_1^2 - \frac{2\alpha\theta_1^2}{P^\lambda} + \frac{\beta\theta_1}{P^\lambda}\right)\right] + \frac{\gamma t^2}{P^\lambda\theta_1} - \frac{t(2\gamma + \beta\theta_1)}{P^\lambda\theta_1^2} \right\} \tag{6}$$

$$I_{i2}(t) = \left\{ \frac{1}{\theta_2^3}\left(\frac{2\gamma}{P^\lambda} + P\theta_2^2 - \frac{2\alpha\theta_2^2}{P^\lambda} + \frac{\beta\theta_2}{P^\lambda}\right) - e^{-t\theta_2}e^{-t_{i1}\theta_2}\left[\frac{1}{\theta_2^3}\left(\frac{2\gamma}{P^\lambda} + P\theta_2^2 - \frac{2\alpha\theta_2^2}{P^\lambda} + \frac{\beta\theta_2}{P^\lambda} + \frac{\gamma t_{i1}^2}{P^\lambda\theta_2} - \frac{t_{i1}(2\gamma + \beta\theta_2)}{P^\lambda\theta_2^2}\right)\right] + \frac{\gamma t^2}{P^\lambda\theta_2} - \frac{t(2\gamma + \beta\theta_2)}{P^\lambda\theta_2^2} \right\} \tag{7}$$

$$I_{i3}(t) = \frac{(-\alpha\theta_2^2 + \beta\theta_2 + 2\gamma)}{P^\lambda\theta_2^3} + \frac{\gamma t^2}{P^\lambda\theta_2} - \frac{t(2\gamma + \beta\theta_2)}{P^\lambda\theta_2^2} - e^{\theta_2(t_{i3}-t)}\left[\frac{(-\alpha\theta_2^2 + \beta\theta_2 + 2\gamma)}{P^\lambda\theta_2^3} + \frac{\gamma t_{i3}^2}{P^\lambda\theta_2} - \frac{t_{i3}(2\gamma + \beta\theta_2)}{P^\lambda\theta_2^2}\right] \tag{8}$$

$$I_{i4}(t) = \frac{(-\alpha\theta_1^2 + \beta\theta_1 + 2\gamma)}{P^\lambda\theta_1^3} + \frac{\gamma t^2}{P^\lambda\theta_1} - \frac{t(2\gamma + \beta\theta_1)}{P^\lambda\theta_1^2} - e^{\theta_1(t_{i4}-t)}\left[\frac{(-\alpha\theta_1^2 + \beta\theta_1 + 2\gamma)}{P^\lambda\theta_1^3} + \frac{\gamma t_{i4}^2}{P^\lambda\theta_1} - \frac{t_{i4}(2\gamma + \beta\theta_1)}{P^\lambda\theta_1^2}\right] \tag{9}$$

$$I_{i5}(t) = We^{\theta_1(t_{i1}-t)} \tag{10}$$

Therefore, the relevant costs of the production inventory system are as follows.

1. **Set up costs for the cycle :**

$$SUC_i = C_{SU} \tag{11}$$

2. **Holding costs in RW for the cycle :**

$$HC_{RW} = h\left[\int_{t_{i1}}^{t_{i2}} I_{i2}(t)dt + \int_{t_{i2}}^{t_{i3}} I_{i3}(t)dt\right]$$

$$\begin{aligned} HC_{RW} = \Bigg\{ & \frac{he^{-\theta_2 t_{i2}}}{6\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i1}} - 12\gamma e^{\theta_2 t_{i2}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i1}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} + 6\beta\theta_2 e^{\theta_2 t_{i1}} \\ & - 6\beta\theta_2 e^{\theta_2 t_{i2}} + 3\beta\theta_2^3 t_{i1}^2 e^{\theta_2 t_{i1}} - 3\beta\theta_2^3 t_{i1}^2 e^{\theta_2 t_{i2}} + 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} - 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} \\ & - 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_2 t_{i1}} + 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_2 t_{i1}} - 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 6P\theta_2^2 P^{\lambda} e^{\theta_2 t_{i1}} \\ & - 6P\theta_2^2 P^{\lambda} e^{\theta_2 t_{i1}} + 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i2}} - 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i2}} - 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} + 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} \\ & - 6P\theta_2^3 P^{\lambda} t_{i1} e^{\theta_2 t_{i1}} + 6P\theta_2^3 P^{\lambda} t_{i1} e^{\theta_2 t_{i1}}\Big] + \frac{he^{-\theta_2 t_{i2}}}{6P^{\lambda}\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i2}} - 12\gamma e^{\theta_2 t_{i3}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} \\ & + 6\alpha\theta_2^2 e^{\theta_2 t_{i3}} + 6\beta\theta_2 e^{\theta_2 t_{i2}} - 6\beta\theta_2 e^{\theta_2 t_{i3}} + 3\beta\theta_2^3 t_{i2}^2 e^{\theta_2 t_{i2}} - 3\beta\theta_2^3 t_i^2 e^{\theta_2 t_{i3}} \\ & + 6\gamma\theta_2^2 t_{i2}^2 e^{\theta_2 t_{i2}} - 6\gamma\theta_2^2 t_{i3}^2 e^{\theta_2 t_{i3}} - 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i3}^3 e^{\theta_2 t_{i3}} - 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} \\ & + 12\gamma\theta_2 t_{i3}^3 e^{\theta_2 t_{i3}} + 6P\theta_2^2 P^{\lambda} e^{\theta_2 t_{i2}} - 6P\theta_2^2 P^{\lambda} e^{\theta_2 t_{i3}} + 6\alpha\theta_2^3 t_{i2} e^{\theta_2 t_{i2}} - 6\alpha\theta_2^3 t_{i3} e^{\theta_2 t_{i3}} \\ & - 6\beta\theta_2^2 t_{i2} e^{\theta_2 t_{i2}} + 6\beta\theta_2^2 t_{i3} e^{\theta_2 t_{i3}} - 6P\theta_2^3 P^{\lambda} t_{i2} e^{\theta_2 t_{i2}} + 6P\theta_2^3 P^{\lambda} t_{i3} e^{\theta_2 t_{i3}}\Big]\Bigg\} \end{aligned} \tag{12}$$

3. **Holding costs in OW for the cycle :**

$$HC_{OW} = h\left[\int_{t_0}^{t_{i1}} I_{i1}(t)dt + \int_{t_{i1}}^{t_{i3}} I_{i5}(t)dt + \int_{t_{i3}}^{t_{i4}} I_{i4}(t)dt\right]$$

$$\begin{aligned} HC_{OW} = h\Bigg\{ & \frac{he^{-\theta_1 t_{i1}}}{6P^{\lambda}\theta_1^4}\Big[12\gamma e^{\theta_1 t_{i4}} - 12\gamma e^{\theta_1 t_{i4}} - 6\alpha\theta_1^2 e^{\theta_1 t_{i3}} + 6\alpha\theta_1^2 e^{\theta_2 t_{i4}} + 6\beta\theta_1 e^{\theta_2 t_{i3}} \\ & - 6\beta\theta_1 e^{\theta_2 t_{i4}} + 3\beta\theta_1^3 t_{i3}^2 e^{\theta_1 t_{i4}} - 3\beta\theta_1^3 t^{i3} e^{\theta_1 t_{i3}} + 6\gamma\theta_1^2 t_{i3}^2 e^{\theta_1 t_{i3}} - 6\gamma\theta_1^2 t_{i4}^2 e^{\theta_2 t_{i4}} \\ & - 2\gamma\theta_1^3 t_{i3}^3 e^{\theta_1 t_{i3}} + 2\gamma\theta_1^3 t_{i4}^3 e^{\theta_1 t_{i4}} - 12\gamma\theta_1 t_{i3}^3 e^{\theta_1 t_{i3}} + 12\gamma\theta_1 t_{i4}^3 e^{\theta_1 t_{i4}} + 6P\theta_1^2 P^{\lambda} e^{\theta_1 t_{i3}} \\ & - 6P\theta_1^2 P^{\lambda} e^{\theta_1 t_{i4}} + 6\alpha\theta_1^3 t_{i3} e^{\theta_1 t_{i3}} - 6\alpha\theta_1^3 t_{i4} e^{\theta_1 t_{i4}} - 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}} + 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}} \\ & - 6P\theta_1^3 P^{\lambda} t_{i3} e^{\theta_1 t_{i3}} + 6P\theta_1^3 P^{\lambda} t_{i4} e^{\theta_1 t_{i4}}\Big] + \frac{t_{i1}}{\theta_1^3}\left(\frac{2\gamma}{P^{\lambda}} + P\theta_1^2 - \frac{2\alpha\theta_1^2}{P^{\lambda}} + \frac{\beta\theta_1}{P^{\lambda}}\right) \\ & - W\left(\frac{e^{\theta_1(t_{i1}-t_{i3})} - 1}{\theta_1}\right) + P\left(\frac{e^{\theta_1(t_{i1})} - 1}{\theta\alpha_1^2}\right) - \frac{t_{i1}^2(2\gamma + \beta\theta_1)}{2p\lambda\theta_1^2} + \frac{\gamma t_{i1}^3}{3p\lambda\theta_1} \\ & - \frac{\alpha(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^2} + \frac{\beta(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^3} + \frac{2\gamma(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^4}\Bigg\} \end{aligned} \tag{13}$$

4. **Deterioration costs in RW for the cycle :**

$$DC_{RW} = C_D\theta_2\left[\int_{t_{i1}}^{t_{i2}} I_{i2}(t)dt + \int_{t_{i2}}^{t_{i3}} I_{i3}(t)dt\right]$$

$$\begin{aligned}DC_{RW} = C_D\theta_2 e^{-\theta_2 t_{i2}}\Bigg\{ \frac{1}{6\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i1}} - 12\gamma e^{\theta_2 t_{i2}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i1}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} \\ + 6\beta\theta_2 e^{\theta_2 t_{i1}} - 6\beta\theta_2 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} - 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 3\beta\theta_2^3 t_{i2}^2 e^{\theta_2 t_{i2}} \\ - 3\beta\theta_2^3 t^{i2} e^{\theta_2 t_{i2}} + 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} - 6\gamma\theta_2^2 t_{i2}^2 e^{\theta_2 t_{i2}} - 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_1 2 t_{i2}} + 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} \\ + 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i1}} - 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i2}} + 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i1}} - 6\alpha\theta_2^3 t_{i2} e^{\theta_2 t_{i2}} - 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} \\ + 6\beta\theta_2^2 t_{i2} e^{\theta_1 t_{i2}} - 6P\theta_2^3 P^\lambda t_{i1} e^{\theta_1 t_{i1}} + 6P\theta_2^3 P^\lambda t_{i2} e^{\theta_2 t_{i42}}\Big] + \frac{1}{6P^\lambda\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i2}} \\ - 12\gamma e^{\theta_2 t_{i3}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i3}} + 6\beta\theta_2 e^{\theta_2 t_{i2}} - 6\beta\theta_2 e^{\theta_2 t_{i3}} \\ - 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i3}^3 e^{\theta_2 t_{i3}} - 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} + 12\gamma\theta_2 t_{i3}^3 e^{\theta_2 t_{i3}} \\ + 6\alpha\theta_2^3 t_{i2} e^{\theta_2 t_{i2}} - 6\alpha\theta_2^3 t_{i3} e^{\theta_2 t_{i3}} - 6\beta\theta_2^2 t_{i2} e^{\theta_2 t_{i2}} + 6\beta\theta_2^2 t_{i3} e^{\theta_2 t_{i3}}\Big]\Bigg\} \end{aligned} \quad (14)$$

5. **Deterioration costs in OW for the cycle :**

$$DC_{OW} = C_D\theta_1\left[\int_{t_0}^{t_{i1}} I_{i1}(t)dt + \int_{t_{i1}}^{t_{i3}} I_{i5}(t)dt + \int_{t_{i3}}^{t_{i4}} I_{i4}(t)dt\right]$$

$$\begin{aligned}DC_{OW} = C_D\theta_1\Bigg\{ \frac{e^{-\theta_1 t_{i3}}}{6P^\lambda\theta_1^4}\Big[12\gamma e^{\theta_1 t_{i3}} - 12\gamma e^{\theta_1 t_{i4}} - 6\alpha\theta_1^2 e^{\theta_1 t_{i3}} + 6\alpha\theta_1^2 e^{\theta_1 t_{i4}} \\ + 6\beta\theta_1 e^{\theta_1 t_{i3}} - 6\beta\theta_1 e^{\theta_1 t_{i4}} + 3\beta\theta_1^3 t_{i3}^2 e^{\theta_1 t_{i3}} - 3\beta\theta_1^3 t^{i4} e^{\theta_1 t_{i4}} + 6\gamma\theta_1^2 t_{i3}^2 e^{\theta_1 t_{i3}} \\ - 6\gamma\theta_1^2 t_{i4}^2 e^{\theta_1 t_{i4}} - 2\gamma\theta_1^3 t_{i3}^3 e^{\theta_1 t_{i3}} + 2\gamma\theta_1^3 t_{i4}^3 e^{\theta_1 t_{i4}} - 12\gamma\theta_1 t_{i3}^3 e^{\theta_1 t_{i3}} \\ + 12\gamma\theta_1 t_{i4}^3 e^{\theta_1 t_{i4}} + 6\alpha\theta_1^3 t_{i3} e^{\theta_1 t_{i3}} - 6\alpha\theta_1^3 t_{i4} e^{\theta_1 t_{i4}} - 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}} \\ + 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}}\Big] + \frac{t_{i1}}{\theta_1^3}\left(\frac{2\gamma}{P^\lambda} + P\theta_1^2 - \frac{2\alpha\theta_1^2}{P^\lambda} + \frac{\beta\theta_1}{P^\lambda}\right) - W\left(\frac{e^{\theta_1(t_{i1}-t_{i3})} - 1}{\theta_1}\right) \\ + P\left(\frac{e^{\theta_1(t_{i1})} - 1}{\theta\alpha_1^2}\right) - \frac{t_{i1}^2(2\gamma + \beta\theta_1)}{2p\lambda\theta_1^2} + \frac{\gamma t_{i1}^3}{3p\lambda\theta_1} \\ - \frac{\alpha(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^2} + \frac{\beta(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^3} + \frac{2\gamma(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^4}\Bigg\}\end{aligned} \quad (15)$$

6. **Production cost for the cycle :**

$$PC_i = n_o(P)\left[\int_0^{t_{i1}} Pdt + \int_{t_{i1}}^{t_{i2}} Pdt\right]$$

$$PC_i = P^2 n_o t_{i2} \quad (16)$$

Therefore, the present worth of total variable cost for the cycle

$$TVC = \frac{1}{T}[SUC_i + HC_{RW} + HC_{OW} + DC_{RW} + DC_{OW} + PC_i] \quad (17)$$

Note that for the detailed version of Equation (17), see Appendix A.

4. Flower Pollination Optimization Methodology

Flower Pollination Optimization (FPO) is a naturalistic approach to solving complex optimization issues by modeling flower pollination. FPO takes inspiration from the biological mechanisms of pollination, which involve the transport of pollen from flowers to pollinators and the subsequent reproduction of plants, to effectively explore and exploit the search space (see Fig. 2). In many fields, including artificial intelligence, finance, and engineering, this method excels at determining the optimal solutions to continuous and discrete optimization problems. The algorithm details of the FPO technique which were brought al. multi-purpose optimization level (Darwin, [83]) after gaining the first literature (Yang, [82]) and Investigation of Artificial Intelligence Based Optimization Algorithms. Okula, et, al., [84] are as follows:

Algorithm:

Step 1: (Installation Phase): Randomly distribute N-flower particle (potential solution variables) in solution space. Assign algorithm values, specify the transition probability parameter (go). Perform the necessary arrangements for the problem to be solved.
Step 2: Calculate the objective function value (fitness) according al. position of the flowers - particles (potential solution variables). Find out what's best.
Step 3: Repeat the following steps throughout the iterative process (eg until you reach a certain number of iterations or until you reach a desired value in the objective function): (For each particle; for each purpose function size).
Step 3.1: (Global - Local Pollination Phase): Generate a random value. If the value produced is less than the value of equation and Levy Flights (step vector: L). If the value produced is equal to or greater than the value of go, uniform distribution in the range [0, 1]. Run the local pollination process in the context.14
Step 3.2: Calculate the purpose function value (fitness) according al. updated position of flowers - particles (potential solution variables).
Step 3.3: Update the global best value (and hence the variable position) if the best objective at that time is found to be better than the function value.
Step 4: Iteration - At the end of the cycle the value (s) obtained according al. global best position is considered to be the optimum value (s).

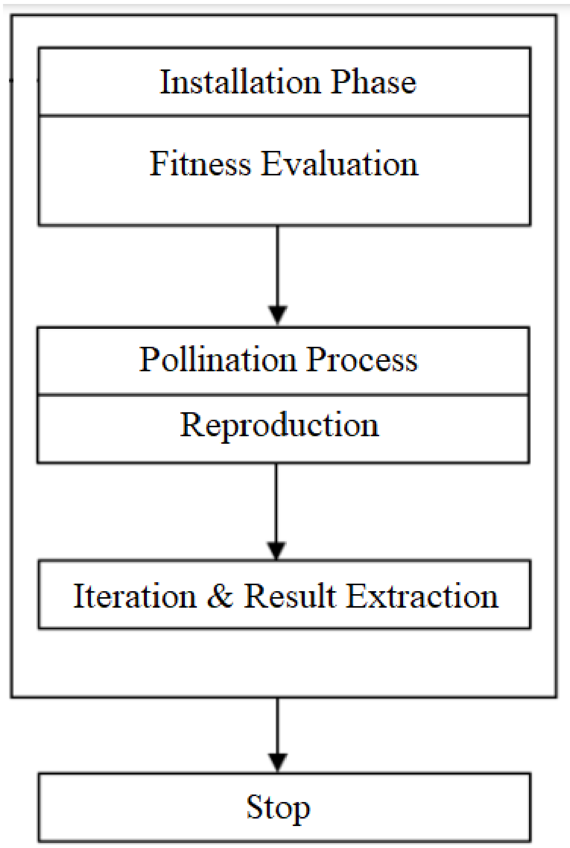

Figure 2: *Flowchart for the Flower Pollination Optimization.*

5. Graphical representation

The primary objective of the graphical representations of the suggested model is a flowchart listing the key components. To demonstrate how to manage inventory for things that are deteriorating, it begins with two storage systems. The diagram highlights the relationship between time, selling price, and demand, showing how these factors dynamically change inventory levels. The flowchart also demonstrates the Flower Pollination Algorithm approach, which shows how potential solutions are developed and improved to cut costs and boost the efficacy of inventory management. All things considered, the complex relationships revealed in the model and the optimization technique employed are well communicated by this visual aid.

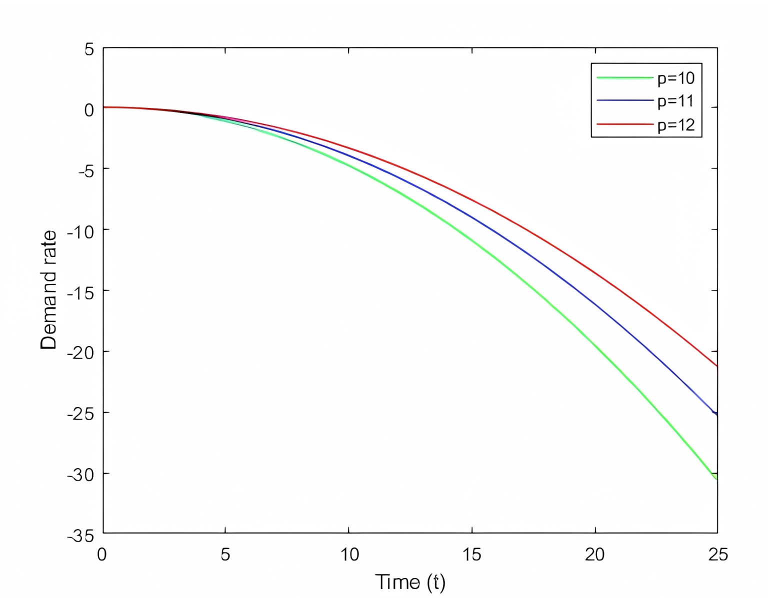

Figure 3: *Relation between Demand rate and time using different values of p.*

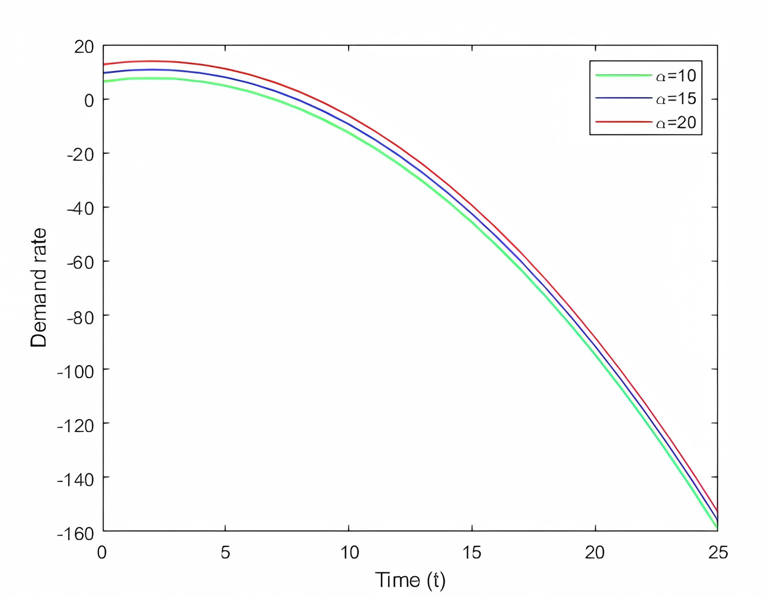

Figure 4: *Relation between Demand rate and time using different values of α.*

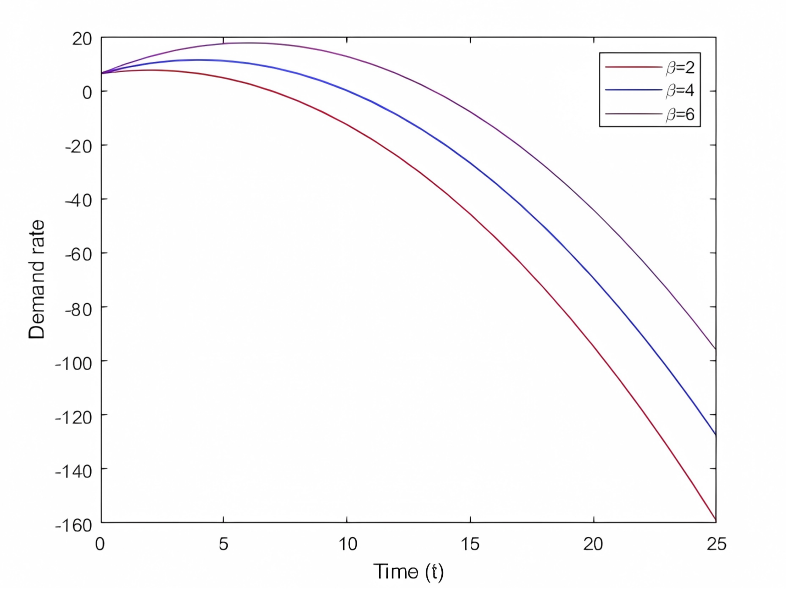

Figure 5: *Relation between Demand rate and time using different values of β.*

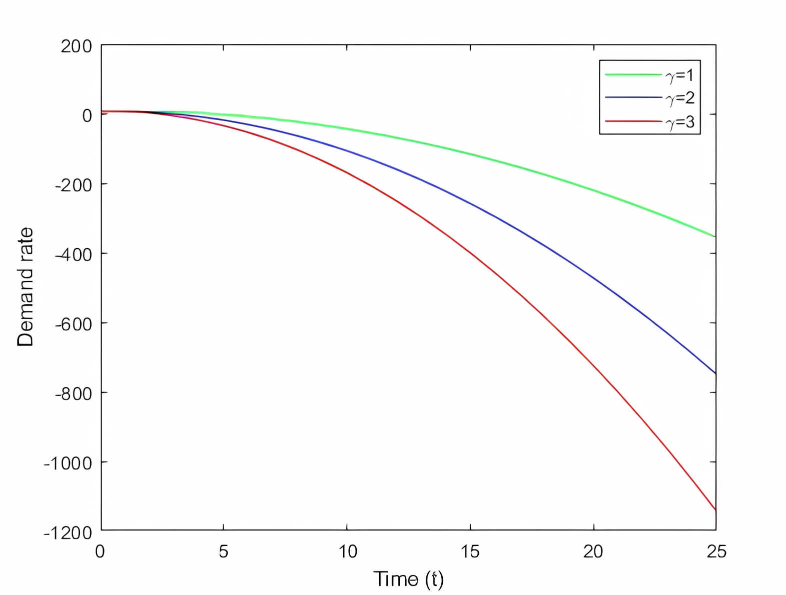

Figure 6: *Relation between Demand rate and time using different values of γ.*

Following are a few insights drawn from the graphical representation's observations.

1. If the selling price parameter p are increased, the total average inventory cost (TVC) rises due to the higher demand rate (see Fig. 3).

2. If the parameters α and β are increased, the total average inventory cost (TVC) rises quickly due to the extended running time of the warehouses. Simultaneously, both the cycle length and the product price decrease (see Fig. 4 and Fig. 5).

3. If the demand rate parameter (γ) increases, the total average inventory cost (TVC) rises rapidly due to the higher amount of product waste (see Fig. 6).

6. Conclusion

In this study, we presented a comprehensive approach to address the challenges associated with managing a two-storage production inventory model for deteriorating items with time and selling price dependent demand. The complexities of dual storage locations, item deterioration, and dynamic demand patterns were considered, reflecting the real-world intricacies faced by supply chain managers. To optimize decision variables and navigate the complex solution space, we employed the Flower Pollination Optimization (FPO) algorithm, a nature-inspired metaheuristic known for its efficacy in solving complex optimization problems.

The proposed model demonstrated its relevance by integrating the impact of item deterioration over time, allowing for a more accurate representation of inventory dynamics. The consideration of time-dependent demand and selling price dependencies further enriched the model, capturing the nuances of market fluctuations and consumer behavior. Our choice of the FPO algorithm proved effective in finding solutions that strike a balance between exploration and exploitation. By simulating the pollination process in flowers, the FPO algorithm efficiently explored the solution space, leading to robust strategies for production quantities, order quantities, and inventory levels. The adaptability of FPO to dynamic and non-linear environments was crucial in addressing the intricate nature of the proposed inventory model. Through extensive simulations and sensitivity analyses, we validated the effectiveness of our approach, showcasing its ability to enhance inventory management, mitigate deterioration-related losses, and adapt to varying demand scenarios. The findings contribute valuable insights for supply chain practitioners seeking advanced strategies to optimize their production inventory systems in the face of evolving market conditions. As we conclude, it is important to emphasize the practical implications of our research. The proposed model and optimization approach offer a forward-looking perspective on addressing the challenges in dual storage inventory systems. The integration of FPO provides a powerful tool for decision-makers to refine their inventory strategies, ultimately improving overall supply chain efficiency and resilience. While this study has provided significant contributions, there are opportunities for further research. Future investigations could explore the applicability of the proposed model in different industry contexts and evaluate the performance of other meta-heuristic algorithms for comparison. Additionally, incorporating more nuanced factors such as supply chain disruptions or sustainability considerations could enrich the model further.

In conclusion, this research contributes to advancing the field of supply chain optimization by proposing an innovative solution to a complex inventory management problem. The integration of a two-storage production inventory model with FPO optimization provides a robust framework for addressing real-world challenges and paves the way for more resilient and adaptive supply chain strategies.

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Appendix A

$$
\begin{aligned}
TVC = \frac{1}{T}\Bigg[&(C_{SU} + P^2 n_o t_{i2}) \\
&+ \Bigg\{ \frac{he^{-\theta_2 t_{i2}}}{6\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i1}} - 12\gamma e^{\theta_2 t_{i2}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i1}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} + 6\beta\theta_2 e^{\theta_2 t_{i1}} \\
&- 6\beta\theta_2 e^{\theta_2 t_{i2}} + 3\beta\theta_2^3 t_{i1}^2 e^{\theta_2 t_{i1}} - 3\beta\theta_2^3 t_{i1}^2 e^{\theta_2 t_{i2}} + 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} - 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} \\
&- 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_2 t_{i1}} + 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_2 t_{i1}} - 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i1}} \\
&- 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i1}} + 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i2}} - 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i2}} - 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} + 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} \\
&- 6P\theta_2^3 P^\lambda t_{i1} e^{\theta_2 t_{i1}} + 6P\theta_2^3 P^\lambda t_{i1} e^{\theta_2 t_{i1}}\Big] + \frac{he^{-\theta_2 t_{i2}}}{6P^\lambda\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i2}} - 12\gamma e^{\theta_2 t_{i3}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} \\
&+ 6\alpha\theta_2^2 e^{\theta_2 t_{i3}} + 6\beta\theta_2 e^{\theta_2 t_{i2}} - 6\beta\theta_2 e^{\theta_2 t_{i3}} + 3\beta\theta_2^3 t_{i2}^2 e^{\theta_2 t_{i2}} - 3\beta\theta_2^3 t_i^2 e^{\theta_2 t_{i3}} \\
&+ 6\gamma\theta_2^2 t_{i2}^2 e^{\theta_2 t_{i2}} - 6\gamma\theta_2^2 t_{i3}^2 e^{\theta_2 t_{i3}} - 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i3}^3 e^{\theta_2 t_{i3}} - 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} \\
&+ 12\gamma\theta_2 t_{i3}^3 e^{\theta_2 t_{i3}} + 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i2}} - 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i3}} + 6\alpha\theta_2^3 t_{i2} e^{\theta_2 t_{i2}} - 6\alpha\theta_2^3 t_{i3} e^{\theta_2 t_{i3}} \\
&- 6\beta\theta_2^2 t_{i2} e^{\theta_2 t_{i2}} + 6\beta\theta_2^2 t_{i3} e^{\theta_2 t_{i3}} - 6P\theta_2^3 P^\lambda t_{i2} e^{\theta_2 t_{i2}} + 6P\theta_2^3 P^\lambda t_{i3} e^{\theta_2 t_{i3}}\Big]\Bigg\} \\
&+ h\Bigg\{ \frac{he^{-\theta_1 t_{i1}}}{6P^\lambda\theta_1^4}\Big[12\gamma e^{\theta_1 t_{i4}} - 12\gamma e^{\theta_1 t_{i4}} - 6\alpha\theta_1^2 e^{\theta_1 t_{i3}} + 6\alpha\theta_1^2 e^{\theta_2 t_{i4}} + 6\beta\theta_1 e^{\theta_2 t_{i3}} \\
&- 6\beta\theta_1 e^{\theta_2 t_{i4}} + 3\beta\theta_1^3 t_{i3}^2 e^{\theta_1 t_{i4}} - 3\beta\theta_1^3 t^{i3} e^{\theta_1 t_{i3}} + 6\gamma\theta_1^2 t_{i3}^2 e^{\theta_1 t_{i3}} - 6\gamma\theta_1^2 t_{i4}^2 e^{\theta_2 t_{i4}} \\
&- 2\gamma\theta_1^3 t_{i3}^3 e^{\theta_1 t_{i3}} + 2\gamma\theta_1^3 t_{i4}^3 e^{\theta_1 t_{i4}} - 12\gamma\theta_1 t_{i3}^3 e^{\theta_1 t_{i3}} + 12\gamma\theta_1 t_{i4}^3 e^{\theta_1 t_{i4}} + 6P\theta_1^2 P^\lambda e^{\theta_1 t_{i3}} \\
&- 6P\theta_1^2 P^\lambda e^{\theta_1 t_{i4}} + 6\alpha\theta_1^3 t_{i3} e^{\theta_1 t_{i3}} - 6\alpha\theta_1^3 t_{i4} e^{\theta_1 t_{i4}} - 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}} + 6\beta\theta_1^2 t_{i3} e^{\theta_1 t_{i4}} \\
&- 6P\theta_1^3 P^\lambda t_{i3} e^{\theta_1 t_{i3}} + 6P\theta_1^3 P^\lambda t_{i4} e^{\theta_1 t_{i4}}\Big] + \frac{t_{i1}}{\theta_1^3}\left(\frac{2\gamma}{P^\lambda} + P\theta_1^2 - \frac{2\alpha\theta_1^2}{P^\lambda} + \frac{\beta\theta_1}{P^\lambda}\right) \\
&- W\left(\frac{e^{\theta_1(t_{i1}-t_{i3})} - 1}{\theta_1}\right) + P\left(\frac{e^{\theta_1(t_{i1})} - 1}{\theta\alpha_1^2}\right) - \frac{t_{i1}^2(2\gamma + \beta\theta_1)}{2p\lambda\theta_1^2} + \frac{\gamma t_{i1}^3}{3p\lambda\theta_1} \\
&- \frac{\alpha(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^2} + \frac{\beta(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^3} + \frac{2\gamma(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^4}\Bigg\} \\
&+ C_D\theta_2 e^{-\theta_2 t_{i2}}\Bigg\{\frac{1}{6\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i1}} - 12\gamma e^{\theta_2 t_{i2}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i1}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} \\
&+ 6\beta\theta_2 e^{\theta_2 t_{i1}} - 6\beta\theta_2 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} - 12\gamma\theta_2 t_{i1}^3 e^{\theta_2 t_{i1}} + 3\beta\theta_2^3 t_{i2}^2 e^{\theta_2 t_{i2}} \\
&- 3\beta\theta_2^3 t^{i2} e^{\theta_2 t_{i2}} + 6\gamma\theta_2^2 t_{i1}^2 e^{\theta_2 t_{i1}} - 6\gamma\theta_2^2 t_{i2}^2 e^{\theta_2 t_{i2}} - 2\gamma\theta_2^3 t_{i1}^3 e^{\theta_1 2 t_{i2}} + 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} \\
&+ 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i1}} - 6P\theta_2^2 P^\lambda e^{\theta_2 t_{i2}} + 6\alpha\theta_2^3 t_{i1} e^{\theta_2 t_{i1}} - 6\alpha\theta_2^3 t_{i2} e^{\theta_2 t_{i2}} - 6\beta\theta_2^2 t_{i1} e^{\theta_2 t_{i1}} \\
&+ 6\beta\theta_2^2 t_{i2} e^{\theta_1 t_{i2}} - 6P\theta_2^3 P^\lambda t_{i1} e^{\theta_1 t_{i1}} + 6P\theta_2^3 P^\lambda t_{i2} e^{\theta_2 t_{i42}}\Big] + \frac{1}{6P^\lambda\theta_2^4}\Big[12\gamma e^{\theta_2 t_{i2}} \\
&- 12\gamma e^{\theta_2 t_{i3}} - 6\alpha\theta_2^2 e^{\theta_2 t_{i2}} + 6\alpha\theta_2^2 e^{\theta_2 t_{i3}} + 6\beta\theta_2 e^{\theta_2 t_{i2}} - 6\beta\theta_2 e^{\theta_2 t_{i3}} \\
&+ 3\beta\theta_2^3 t_{i2}^2 e^{\theta_2 t_{i2}} - 3\beta\theta_2^3 t_{i3}^2 e^{\theta_2 t_{i3}} + 6\gamma\theta_2^2 t_{i2}^2 e^{\theta_2 t_{i2}} - 6\gamma\theta_2^2 t_{i3}^2 e^{\theta_2 t_{i3}} \\
&- 2\gamma\theta_2^3 t_{i2}^3 e^{\theta_2 t_{i2}} + 2\gamma\theta_2^3 t_{i3}^3 e^{\theta_2 t_{i3}} - 12\gamma\theta_2 t_{i2}^3 e^{\theta_2 t_{i2}} + 12\gamma\theta_2 t_{i3}^3 e^{\theta_2 t_{i3}} \\
&+ P\left(\frac{e^{\theta_1(t_{i1})} - 1}{\theta\alpha_1^2}\right) - \frac{t_{i1}^2(2\gamma + \beta\theta_1)}{2p\lambda\theta_1^2} + \frac{\gamma t_{i1}^3}{3p\lambda\theta_1} - \frac{\alpha(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^2} \\
&+ \frac{\beta(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^3} + \frac{2\gamma(e^{-\theta_1 t_{i1}} - 1)}{p\lambda\theta_1^4}\Bigg\} + P^2 n_o t_{i2}\Bigg] \qquad (18)
\end{aligned}
$$