ALTERNATE QUADRA-SUBMERGING POLAR FUZZY GRAPH AND ITS DECISION MAKING ANALYSIS

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Abstract

In this article, the two extreme values [-1,1] is proposed with it's uncertain submerging values [-0.5,0.5] as the Alternate Quadra Submerging Polar (AQSP) Fuzzy Graph. The AQSP Fuzzy graph COVID-19 vaccines survey model has been analyzed to find the highest and the lowest membership and the non-membership value of the five influencing factors effectively. The notion of the AQSP fuzziness has been considered from the various points of view, in the specification of variables with the multiple input of single output rule. The self-reporting nature of the collected survey data of the COVID - 19 Booster shots acceptance and the non-acceptance values between [-1,0] and [0,1] converges precisely with the level of fixation [-0.5.0] and [0,0.5] alternatively by using the uncertain values in decision making process of the human behaviours in mathematical Analysis.

Keywords: Alternate Quadra - Submerging Polar Fuzzy Graph, AQSP Fuzzy Sub-merging Polar Relations, AQSP Fuzzy Submerging level of Fixation and AQSP Fuzzy Regular, Totally Regular, Strong and Complete graphs.

1. Introduction

The origin expression of fuzzification was first introduced by L.A. Zadeh [30] in his well-known concept 'Fuzzy sets'. The fuzzy arena is rising exponentially in the arena of fuzzy graph which is extended by A. Rosenfeld [22] who introduced the 'fuzzy graph' in 1975. He originated the fuzzy relation by considering fuzzy sets and functions as a structure of fuzzy graph. And in 1973 Kauffman [13] presented the first definition of fuzzy graph. Different types of fuzzy graph analogues and concepts were hosted by many fuzzy mathematicians. Fuzzy set theory and operations on fuzzy graph with several properties are introduced and defined by Klir[14] and Yuan[29] in and M.S.Sunitha.[26] Many important perceptions on fuzzy graphs were presented by Moderson and NagoorGani A,[17] Nair. Akram[1] has proposed fuzzy graphs concepts. J.N. Moderson[16] presented the fuzzy line graph.

In this analysis of AQSP fuzzy graph COVID 19 vaccine survey, we find the membership value of conflict feelings and frustration which is vague and uncertain is measured precisely. To highlight human beings, alternate conflict feelings, attitudinal behavior with an alternate equal association of membership and non - membership principles are polarized in alternate quadrant. The concept, level of fixation is defined as [-0.5,0] and [0, 0.5] will precise the many submerging level of uncertain human behavior with an Alternate Quadra values as the level of presumption along with the level of membership values in the given fuzzy graph which is denoted as{ [0,0],[0,1],[1,1],[1,0] } and { [0,0],[-1,0],[-1,-1],[0,-1] } of fuzzy sets. This mid-sub merging alternate quadra - values can be used to describe the increasing or decreasing level towards destination of certain values, which is consistent with mid-submerging uncertain fuzzified values in AQSP Fuzzy graphs are{ [0,0],[0,0.5],[0.5,0.5],[0.5,0] } and { [0,0],[-0.5,0],[-0.5,-0.5],[0,-0.5] }.The submerging alternate quadrant sets with equal opposite polarized reaction which implement the destination of the mind to decide whether to accept or not accept the Booster shot in future. The fixed level of confidence in AQSP fuzzy graph, indicates all possible preferential membership and non-membership values of reaching certain level of precise value which is reliable.

2. Preliminaries

2.1. Fuzzy Graph [22]

Let ϑ is a non-empty set. A fuzzy graph $G : (\vartheta, \sigma, \mu)$ matching to the crisp graph $G^* : (\sigma^*, \mu^*)$ is a non-empty set ϑ together with pair of functions $\sigma : \vartheta \to [0,1]$ where σ is a fuzzy subset of ϑ and $\mu : \vartheta \times \vartheta \to [0,1]$, μ is a symmetric fuzzy relation on σ, for all values of $x, y \in \vartheta$, such that $\mu(x,y) \le \min(\sigma(x), \sigma(y)) \forall x, y \in \vartheta$

2.2. Partial Fuzzy Sub Graph [23]

The fuzzy graph $\varphi = (\vartheta_1, \alpha, \beta)$ is called a partial fuzzy subgraph of $G : (\vartheta, \sigma, \mu)$ $if\ \alpha \le \sigma\ and$ $\beta \le \mu$. In precise mod, $\varphi = (\vartheta_1, \alpha, \beta)$ is called a fuzzy subgraph of $G : (\vartheta, \sigma, \mu)$ persuaded by by ϑ_1 if $\vartheta_1 \subseteq \vartheta$ and $\alpha(x) = \sigma(x)\ \forall\ x \in \vartheta_1$ with $\beta(x,y) = \mu(x, y)\ \forall\ x, y \in \vartheta_1$. The fuzzy graph $G : (\vartheta, \sigma, \mu)$ is trivial if $|\varphi| = 1$.

2.3. Alternate Quadra Sub - merging Polar(AQSP) Fuzzy Graph [5]

An Alternate Quadra - Submerging Polar (AQSP) Fuzzy Graph $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a fuzzy graph with crisp graph $G^* = (\sigma^*_{AQSP}, \mu^*_{AQSP})$ is given as $V = (\sigma^P(x), \sigma^N(x), \rho^P(x), \rho^N(x))$ which is the membership value of vertices along with the uncertain membership value of edges is given as, $E = V \times V = (\mu^P(x,y), \mu^N(x,y), \gamma^P(x,y), \gamma^N(x,y))$.

Here the vertex set V is defined with the given condition in a unique method which is an alternate contrast submerging polarized uncertain transformation.Here $\sigma^P = V \to [0,1]$, $\sigma^N = V \to [-1,0]$, $\rho^P = d\ |\ 0.5, \sigma^P\ (x)\ |$ and $\rho^N = -d\ |-0.5,\ \sigma^N\ (x)|$. Here (-0.5, 0.5) is the fixation of uncertain alternate contrast polarized submerging transformation into certain consistent preferable position. And the edge set E satisfies the following sufficient conditions.

$$(i)\ \ \mu^P(x,y) \le \min(\sigma^P(x), \sigma^P(y)), \quad (ii)\ \mu^N(x,y) \ge \max(\sigma^N(x), \sigma^N(y))$$

$$(iii)\ \gamma^P(x,y) \le \min(\rho^P(x), \rho^P(y)) \quad (iv)\ \gamma^N(x,y) \ge \max(\rho^N(x), \rho^N(y)),$$

$\forall (x,y) \in E$. By definition, $\mu^P = V \times V \to [0,1] \times [1,0]$, $\mu^N = V \times V \to [-1,0] \times [0,-1]$ and the submerging mappings, $\gamma^P = V \times V \to [0, 0.5] \times [0.5, 0]$, $\gamma^N = V \times V \to [-0.5, 0] \times [0, -0.5]$, which denotes the impact of the alternate quadrant polarized fuzzy mapping. The maximum of submerging presumption to be at the level of confidence $[0, 0.5] \subseteq [0,1]$ and the minimum of submerging presumption level of confidence is $[-0.5, 0] \subseteq [-1, 0]$ extension of the graph with its membership and non - membership values portrait the unique level of submerging destination in an AQSP fuzzy graph.

Also it must satisfy the condition, $-1 \le \sigma^P(x) + \sigma^N(x) \le 1$ *and* $|\rho^P(x) + \rho^N(x)| \le 1$ with constrains $0 \le \sigma^P(x) + \sigma^N(x) + |\rho^P(x) + \rho^N(x)| \le 2$ such that the uncertain status of submerging presumption, transform into its precise consistent level with fixation mid - value 0.5, which implies that level of confidence 0.5 in an AQSP as the valuable membership of its position which is real and valid in the fuzzification. The example of AQSP fuzzy graph is given in the Figure.1.

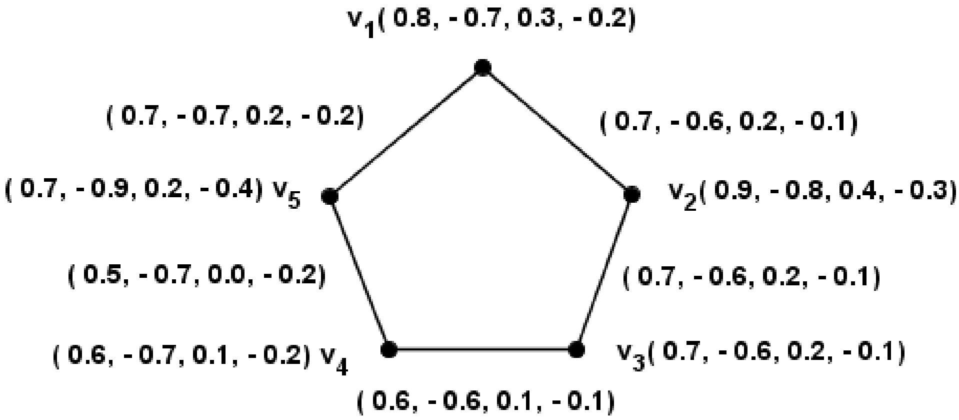

Figure 1: *AQSP Fuzzy Graph*

2.4. Complement of Fuzzy Graph [17]

Let $G = (\vartheta,\ \sigma, \mu)$ be a fuzzy graph corresponding to the crisp graph. The complement of uncertain graph G is defined as $\bar{G}:\ \bar{\sigma}, \bar{\mu}$ where, $\bar{\sigma} = \sigma,\ \ \bar{\mu}\ = \mu$ and the definition is given as $\bar{\mu}\ (x,y) = \sigma(x) \wedge \sigma(y) -\ \mu(x,y)\ \forall\ x,y \in \sigma$.

2.5. Complete Fuzzy Graph [17]

Let $G = (\sigma, \mu)$ is called as complete fuzzy graph with underlying crisp graph $G^* = (\sigma^*, \mu^*)$ such as $\mu(x,y) = \min(\sigma(x), \sigma(y))\ \forall\ x,y\ \in V$ with given fuzzy node and edge sets.

2.6. Strong Fuzzy Graph [17]

Let fuzzy graph is a pair of functions $G : (\sigma, \mu)$ where σ is a uncertain fuzzy subset of X, μ is a symmetrical uncertain fuzzy relation on σ, where $\sigma : X \to [0,1]$ and $\mu : X \times X \to [0,1]$ such that, $\mu(x,y) = \min(\sigma(x), \sigma(y)),\quad x,y\ \in E$ with satisfying membership values constrain in edge set is called as strong fuzzy graph.

2.7. Intersection of two fuzzy graphs [18]

Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs with $V_1 \cap V_2 = \emptyset$ and then $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ such that $G^* = G_1^* \cap G_2^* = (V_1 \cap V_2, E_1 \cap E_2)$ be the intersection of crisp graph of G_1^* and G_2^*. Then the intersection of $G_1 \cap G_2$ is defined as $(\sigma_1 \cap \sigma_2, \mu_1 \cap \mu_2)$ of the crisp graph G_1^* and G_2^* ,

$(i)\ (\sigma_1 \cap \sigma_2)(x) \quad = \sigma_1(x) \quad if\ x \in\ V_1 \cap \bar{V_2}$

$(ii)\ (\sigma_1 \cap \sigma_2)(x) \quad = \sigma_2(x) \quad if\ x \in\ V_2 \cap \bar{V_1}$

$(iii)\ \ (\mu_1 \cap \mu_2)(x,y)\ =\ \mu_1(x,y)\ if\ x,y \in\ E_1 \cap \bar{E_2}$

$(iv)\ \ (\mu_1 \cap\ \mu_2)(x,y)\ =\ \mu_2(x,y)\ if\ x,y \in\ E_2 \cap \bar{E_1}$.

3. Classifications of AQSP Fuzzy Graphs

3.1. Order of AQSP Fuzzy graph

Let $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ be an AQSP Fuzzy graph with vertex set V, then the order of G is defined as, $O\left(G_{AQSP}\right) = (\sum_{x\in V}\ \sigma^P(x), \sum_{x\in V} \sigma^N(x), \sum_{x\in V} \rho^P(x), \sum_{x\in V} \rho^P(x))$.

3.2. Size of AQSP Fuzzy graph

Let $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ be an AQSP Fuzzy graph with edge set E, then the size of G is defined as $S\left(G_{AQSP}\right) = (\sum_{(x,y)\in V} \mu^P(x,y), \sum_{(x,y)\in V} \mu^N(x,y), \sum_{(x,y)\in V} \gamma^P(x,y), \sum_{(x,y)\in V} \gamma^N(x,y))$.

3.3. Degree of AQSP Fuzzy graph

Let $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ be an AQSP Fuzzy graph ,then the degree of a vertex 'x' is defined with an example presented in Figure.2,
$deg_G(x) = (\sum_{x\neq y}\ \mu^P\ (x,y)), (\sum_{x\neq y}\ \mu^N\ (x,y)), (\sum_{x\neq y}\ \gamma^P\ (x,y)), (\sum_{x\neq y}\ \gamma^N\ (x,y))$.

3.4. AQSP Partial Fuzzy Sub graph

The fuzzy graph $S : (\rho, \vartheta)$ is called a partial AQSP fuzzy sub graph of the AQSP fuzzy graph, $G = \left(\sigma_{AQSP}\ ,\ \mu_{AQSP}\right)$ if $\rho \subseteq \sigma$ and $\gamma \subseteq \mu$ if vertex set $\rho\ (x) \leq\ \sigma\ (x)\ \forall\ x\ \in V$ and the edge set of membership value is, $\gamma^P\ (x,y)\ \leq\ \mu^P\ (x,y)$ then the non - membership value such as, $\gamma^N\ (x,y)\ \geq\ \mu^N\ (x,y)$ for all values of $(x,y)\ \in E \subseteq v \times v$.

In a unique way an AQSP membership values are defined and compared with the fixation (-0.5, 0.5) level of confidence. If the sufficient condition of fuzzy sub graph is defined with the membership and non - membership values such as,

$\rho\,(x) =\ \sigma\,(x)\ \forall\ x\ \in\ \rho^*$ and $\gamma\,(x,y) =\ \mu\ (x,y)\ \forall\ (x,y)\ \in\ \gamma^*$ satisfied then it is obvious $S : (\rho, \gamma)$is a fuzzy sub graph of $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ and it is denoted as $|(\sigma, \rho)| = 1$. Figure.2 represents The AQSP Partial fuzzy graph.

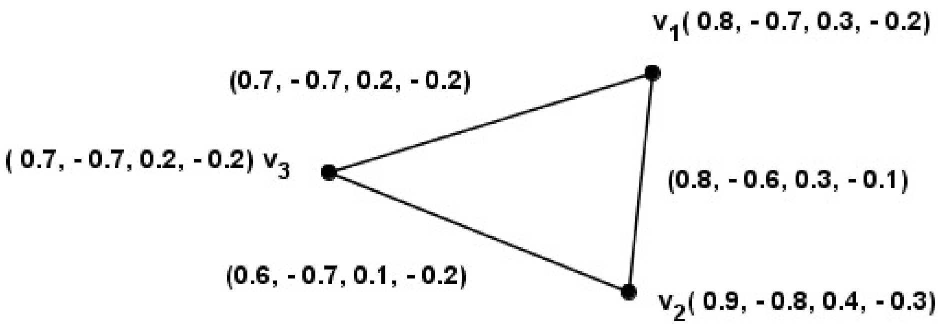

Figure 2: *AQSP Partial Fuzzy Sub graph* $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$

4. Regular and Totally Regular AQSP fuzzy graph

4.1. Regular AQSP Fuzzy graphs

Let $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ be the given AQSP fuzzy graph. If $d_G\,(v) = k\ \forall\ v\ \in V$ if each vertex has same degree of k- elements then G is called as the regular AQSP Fuzzy graph, is of degree k (or) a k - regular AQSP fuzzy graph. An AQSP fuzzy Regular graph is shown in Figure.3. Here $d_G\,(v_1) = \ d_G\,(v_2) = \ d_G\,(v_3) = \ \{\ 1.4,\ -1.2,\ 0.4,\ -0.2\}$.

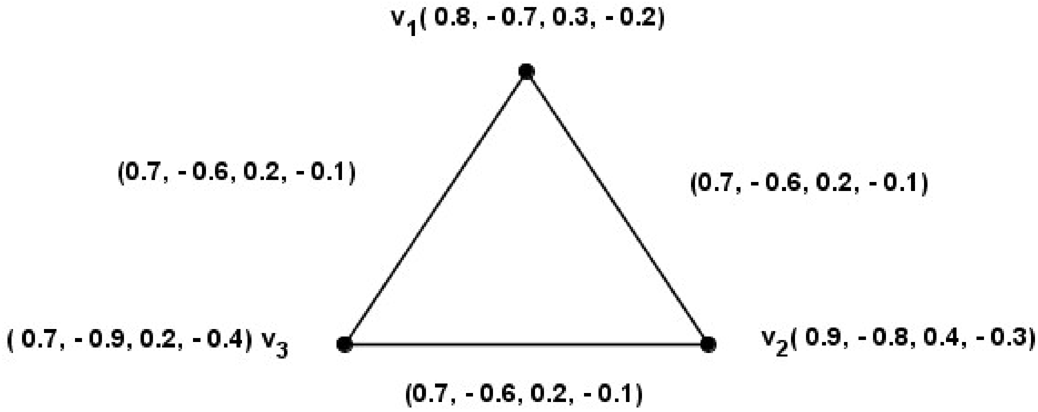

Figure 3: *Regular AQSP Fuzzy graph* $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$

4.2. Totally Regular AQSP Fuzzy graphs

Let $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$be an AQSP fuzzy graph. Then the AQSP fuzzy graph total degree of each vertex $x \in V$ and $T\,[d_G\,(x)]$ is denoted as with the constraints.

$(\sum_{(x,y)\in E}(\mu^p(x,y) + \sigma^P(x))$ + $\sum_{(x,y)\in E}(\mu^p(x,y) + \sigma^N(x))$
+ $\sum_{(x,y)\in E}(\rho^p(x,y) + \rho^P(x))$ + $\sum_{(x,y)\in E}(\rho^p(x,y) + \rho^N(x)))$.

Here each vertex of $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ has the same total degree 'k' then the AQSP is said to be a totally regular AQSP fuzzy graph with degree 'k'. The following illustration presents in Figure.4, implement that the AQSP is totally regular fuzzy graph of $G^* = \left(\sigma^*_{AQSP}\ ,\ \ \mu^*_{AQSP}\right)$.

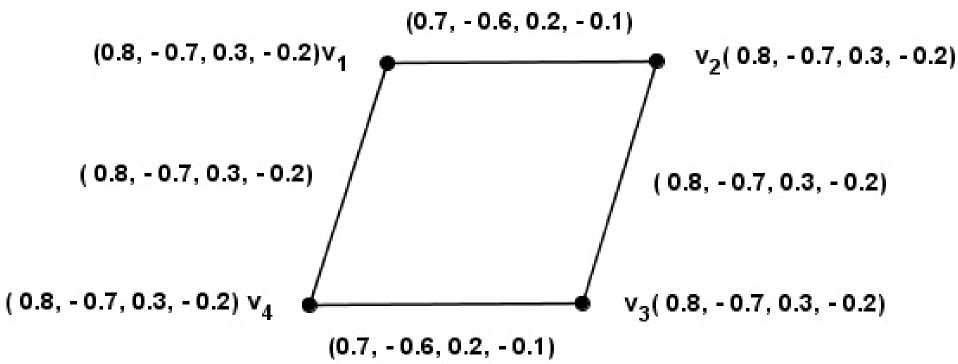

Figure 4: *Totally Regular AQSP Fuzzy graph* $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$

4.3. Example of Totally Regular AQSP Fuzzy graphs

Let $v_1 = v_2 = v_3 = v_4 = (0.8, -0.7, 0.3, -0.2)$

$v_1v_2 = v_3v_4 = (0.7, -0.6, 0.2, -0.1)$ and $v_2v_3 = v_1v_4 = (0.8, -0.7, 0.3, -0.2)$, is totally regular and regular AQSP. The AQSP fuzzy graph in Figure.3 with vertex and edge set, shows that it is regular and regular AQSP fuzzy graph .

$v_1 = v_2 = v_3 = v_4 = (0.8, -0.7, 0.3, -0.2)$

$v_1v_2 = v_3v_4 = (0.7, -0.6, 0.2, -0.1)$ and

$v_2v_3 = v_1v_4 = (0.8, -0.7, 0.3, -0.2)$,

But the example in Figure.4 shows that it's only regular AQSP fuzzy graph. And it is not totally regular AQSP fuzzy graph .

4.4. Theorem of Totally Regular AQSP Fuzzy graphs

Let $G = (\sigma_{AQSP}, \mu_{AQSP})$ be an AQSP fuzzy graph with underlying crisp graph $G^* = (\sigma^*_{AQSP}, \mu^*_{AQSP})$ and $V = (\sigma^P(a), \sigma^N(a), \rho^P(a), \rho^N(a))$ is a constant function if and only if the following conditions are equivalent.

(i) $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a regular AQSP fuzzy graph.

(ii) $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a totally regular AQSP fuzzy graph.

Proof.

Let $G = (\sigma_{AQSP}, \mu_{AQSP})$ be a regular AQSP fuzzy graph. Suppose V is a constant function.

Then $deg_G(a) = c \; \forall \; a \in V$ where c is a constant.

Here $\sigma^P(a) = c_1$, $\sigma^N(a) = c_2$, $\rho^P(a) = c_3$ and $\rho^N(a) = c_4 \; \forall \; a \in V$.

(i) $\implies (ii)$

Consider, $G = (\sigma_{AQSP}, \mu_{AQSP})$ be a regular AQSP Fuzzy graph, then ,

$deg_G\sigma^P(a) = k_1,\; deg_G\sigma^N(a) = k_2,$

$deg_G\rho^P(a) = k_3 \; and \; deg_G\rho^N(a) = k_4 \; \forall \; a \in V.$

Now,

$tdeg_G(\sigma^P(a)) = deg_G(\sigma^P(a) + \sigma^P(a)),\;\; tdeg_G(\sigma^N(a)) = deg_G(\sigma^N(a) + \sigma^N(a),$

$tdeg_G(\rho^P(a)) = deg_G(\rho^P(a) + \rho^P(a))$, and

$tdeg_G(\rho^N(a)) = deg_G(\rho^N(a) + \rho^N(a) \forall \; a \in V$.

Hence ,

$tdeg_G(\sigma^P(a)) = k_1 + c_1,$

$tdeg_G(\sigma^N(a)) = k_2 + c_2,$

$tdeg_G(\rho^P(a)) = k_3 + c_3$ and

$tdeg_G(\rho^P(a)) = k_3 + c_3 \; \forall \; a \in V.$

Thus , $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a totally regular AQSP fuzzy graph.

(ii) $\implies$ (i)

Suppose that $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a totally regular AQSP fuzzy graph,

$tdeg_G(\sigma^P(a)) = t_1,$

$tdeg_G(\sigma^N(a)) = t_2,$

$tdeg_G(\rho^P(a)) = t_3,$ and

$tdeg_G(\rho^N(a)) = t_4 \; \forall \; a \in V$. Otherwise,

$deg_G(\sigma^P(a) + \sigma^P(a) = t_1,\;\; deg_G(\sigma^N(a) + \sigma^N(a) = t_2,$

$deg_G(\rho^P(a) + \rho^P(a) = t_3$ and $deg_G(\rho^N(a) + \rho^N(a) = t_4 \; \forall \; a \in V.$

The other notion of the result such as,

$deg_G(\sigma^P(a)) + c_1 = t_1$,

$deg_G(\sigma^N(a)) + c_2 = t_2$,

$deg_G(\rho^P(a)) + c_3 = t_3$ and

$deg_G(\rho^N(a)) + c_4 = t_4 \; \forall \; a \in V$ or,

$deg_G(\sigma^P(a) = t_1 - c_1, deg_G(\sigma^N(a) = t_2 - c_2,$

$deg_G(\rho^P(a) = t_3 - c_3$ and $deg_G(\rho^N(a) = t_4 - c_4 \; \forall \; a \in V.$

Theis implies that $G = (\sigma_{AQSP}, \mu_{AQSP})$ is a regular AQSP Fuzzy graph.

Hence it is obvious that (i) and (ii) are equivalent.

5. Strong AQSP fuzzy graph

A fuzzy graph $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ is said to be a strong AQSP fuzzy graph if it satisfies the following conditions.

$(i)\ \mu^P\ (x,y) = \min\left(\sigma^P\ (x)\ ,\ \sigma^P\ (y)\ \right),$

$(ii)\mu^N\ (x,y) = \max\left(\sigma^N\ (x)\ ,\ \sigma^N\ (y)\ \right)$

$(iii)\gamma^P\ (x,y) = \min\left(\rho^P\ (x)\ ,\ \rho^P\ (y)\ \right)\ ,$

$(iv)\gamma^N\ (x,y) = \max\left(\rho^N\ (x)\ ,\ \rho^N\ (y)\ \right)$ for $x,y \in \mu_{AQSP}$.

Figure.5. is an illustration of the Strong AQSP Fuzzy graph.

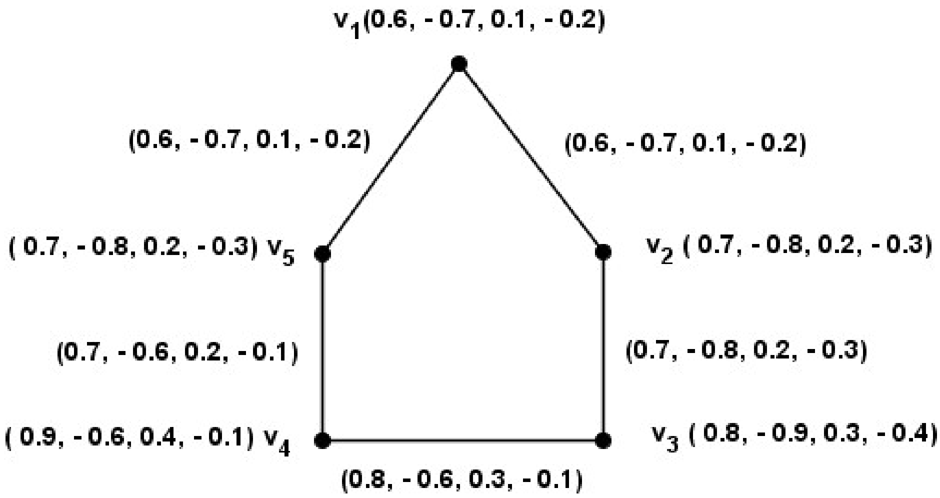

Figure 5: *Strong AQSP Fuzzy graph* $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$

5.1. Theorem of the Strong AQSP Fuzzy graph

If $G_1 \times G_2$ is strong AQSP fuzzy graphs then at least $G_1 = (\sigma_{AQSP}, \mu_{AQSP})$ or $G_2 = (\sigma_{AQSP}, \mu_{AQSP})$ must be strong.

Proof. Consider that $G_1 = (\sigma_{AQSP}, \mu_{AQSP})$ and $G_2 = (\sigma_{AQSP}, \mu_{AQSP})$ are not strong AQSP fuzzy graphs. Then there exists $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$ such that, for membership and non - membership values of AQSP fuzzy graph is given with the conditions,

$\mu_1^p(u_1, v_1)\ <\ \wedge\left((\sigma_1^P)(u_1), (\sigma_1^P)(v_1)\right),\ \ \mu_2^p(u_1, v_1)\ <\ \wedge\left((\sigma_2^P)(u_1), (\sigma_2^P)(v_1)\right)$

$\mu_1^N(u_1, v_1)\ >\ \vee\left((\sigma_1^N)(u_1), (\sigma_1^N)(v_1)\right),\ \mu_2^N(u_1, v_1)\ >\ \vee\left((\sigma_2^N)(u_1), (\sigma_2^N)(v_1)\right)$

$\forall\ (u_1, v_1) \in E_1\ , \forall\ (u_2, v_2) \in E_2$. And for submerging membership and non - membership values of AQSP fuzzy graph is,

$\gamma_1^p(u_1, v_1)\ <\ \wedge\left((\rho_1^P)(u_1), (\rho_1^P)(v_1)\right),\ \ \ \gamma_2^p(u_1, v_1)\ <\ \wedge\left((\rho_2^P)(u_1), (\rho_2^P)(v_1)\right)$

$\gamma_1^N(u_1, v_1)\ >\ \vee\left((\rho_1^N)(u_1), (\rho_1^N)(v_1)\right),\ \gamma_2^N(u_1, v_1)\ >\ \vee\left((\rho_2^N)(u_1), (\rho_2^N)(v_1)\right)$

Consider membership and non - membership values of AQSP fuzzy graph,

$$\mu_2^P(u_2, v_2)\ \leq \left(\mu_1^p(u_1, v_1)\right)\ <\ \wedge\left(\sigma_1^P(u_1), \sigma_1^P(v_1)\right)\ \leq\ \sigma_1^P(u_1) \tag{1}$$

$$\mu_2^N(u_2, v_2)\ \geq \left(\mu_1^N(u_1, v_1)\right)\ >\ \vee\left(\sigma_1^P(u_1), \sigma_1^N(v_1)\right)\ \geq\ \sigma_1^N(u_1) \tag{2}$$

Consider submerging membership values of AQSP fuzzy graph,

$$\gamma_2^P(u_2, v_2)\ \leq \left(\mu_1^p(u_1, v_1)\right)\ <\ \wedge\left(\rho_1^P(u_1), \rho_1^P(v_1)\right)\ \leq\ \rho_1^P(u_1) \tag{3}$$

$$\gamma_2^N(u_2,v_2) \geq (\mu_1^N(u_1,v_1)) > \vee(\rho_1^P(u_1),\rho_1^N(v_1)) \geq \rho_1^N(u_1) \quad (4)$$

Let us assume that,
$E = \{(u_1,v_1),(u_1,v_2)/u_1 \in V_1, u_2, v_2 \in E_2\} \cup \{(u_1,w),(v_1,w)/w \in V_2, u_1, v_1 \in E_1\}$

Consider $(u,u_2),(u,v_2) \in E$ we have,
$(\mu_1^P \times \mu_2^P)((u_1,u_2),(u_1,v_2)) = \wedge(\sigma_1^P(u_1),\mu_2^P(u_2,v_2))$
$(\mu_1^P \times \mu_2^P)((u_1,u_2),(u_1,v_2)) < \wedge(\sigma_1^P(u_1),\sigma_2^P(u_2),\sigma_2^P(v_2))$ and
$(\sigma_1^P \times \sigma_2^P)(u_1,u_2) = \wedge(\sigma_1^P(u_1),\sigma_2^P(u_2))$, $(\sigma_1^P \times \sigma_2^P)(u_1,v_2) = \wedge(\sigma_1^P(u_1),\sigma_2^P(v_2))$
Therefore,
$\wedge((\sigma_1^P \times \sigma_2^P)(u_1,u_2),(\sigma_1^P \times \sigma_2^P)(u_1,v_2)) = \wedge(\sigma_1^P(u_1),\sigma_2^P(u_2),\sigma_2^P(v_2))$ Hence, the membership and non - membership value of AQSP fuzzy graph is,
(i) $(\mu_1^P \times \mu_2^P)((u_1,u_2),(u_1,v_2)) < \wedge((\sigma_1^P \times \sigma_2^P)(u_1,u_2),(\sigma_1^P \times \sigma_2^P)(u_1,v_2))$
(ii) $(\mu_1^N \times \mu_2^N)((u_1,u_2),(u_1,v_2)) > \vee((\sigma_1^N \times \sigma_2^N)(u_1,u_2),(\sigma_1^N \times \sigma_2^N)(u_1,v_2))$ Similarly we get for the submerging membership / non - membership value of AQSP fuzzy graph is,
(iii) $(\gamma_1^P \times \gamma_2^P)((u_1,u_2),(u_1,v_2)) < \wedge((\rho_1^P \times \rho_2^P)(u_1,u_2),(\rho_1^P \times \rho_2^P)(u_1,v_2))$
(iv) $(\gamma_1^N \times \gamma_2^N)((u_1,u_2),(u_1,v_2)) > \vee((\rho_1^N \times \rho_2^N)(u_1,u_2),(\rho_1^N \times \rho_2^N)(u_1,v_2))$

Therefore, $G_1 \times G_2$ is not strong AQSP fuzzy graph, it is a contradiction. Hence $G_1 \times G_2$ is strong AQSP fuzzy graph, then at least G_1 or G_2 must be strong AQSP fuzzy graph.

6. Complement of a Strong AQSP fuzzy graph

Complement of a strong AQSP fuzzy graph $G = (\sigma_{AQSP}, \mu_{AQSP})$ of $G^* = (\sigma_{AQSP}, \mu_{AQSP})$ is a strong AQSP fuzzy graph $\overline{G} = (\overline{\sigma}_{AQSP}, \overline{\mu}_{AQSP})$ on $\overline{G^*} = (\overline{\sigma^*}_{AQSP}, \overline{\mu^*}_{AQSP})$, $\overline{\sigma}_{AQSP} = (\overline{\sigma}^P(x), \overline{\sigma}^N(x), \overline{\rho}^P(x), \overline{\rho}^N(x))$ and $\overline{\mu}_{AQSP} = ((\overline{\mu}^P(x), (\overline{\mu}^N(x), (\overline{\gamma}^P(x), (\overline{\gamma}^N(x))$ are defined by with the conditions,

(i) $\overline{V} = V$
(ii) $\overline{\sigma}^P(x) = \sigma^P(x)$, $\overline{\sigma}^N(x) = \sigma^P(x)$,
$\overline{\rho}^P(x) = \rho^P(x)$, $\overline{\rho}^N(x) = \rho^N(x) \quad \forall\, x \in V$,
(iii)

$$\overline{\mu}^P(x,y) = \begin{cases} 0 & if \ \ \mu^P(x,y) > 0, \\ \wedge(\mu^P(x),\mu^P(y)) & if \ \ \mu^P(x,y) = 0, \end{cases}$$

$$\overline{\mu}^N(x,y) = \begin{cases} 0 & if \ \ \mu^N(x,y) > 0, \\ \vee(\mu^N(x),\mu^N(y)) & if \ \ \mu^N(x,y) = 0, \end{cases}$$

$$\overline{\gamma}^P(x,y) = \begin{cases} 0 & if \ \ \gamma^P(x,y) > 0, \\ \wedge(\gamma^P(x),\gamma^P(y)) & if \ \ \gamma^P(x,y) = 0, \end{cases}$$

$$\overline{\gamma}^N(x,y) = \begin{cases} 0 & if \ \ \gamma^N(x,y) > 0, \\ \vee(\gamma^N(x),\mu^N(y)) & if \ \ \gamma^N(x,y) = 0, \end{cases}$$

6.1. Theorem of the AQSP Fuzzy Bijective Map

Let $G_1 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ and $G_2 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ be AQSP fuzzy graphs. Then the $G_1 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ = $G_2 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ if and only if $G_1 = (\sigma_{AQSP}\ ,\ \mu_{AQSP}) \approx \overline{G_2} = (\overline{\sigma}_{AQSP}\ ,\ \overline{\mu}_{AQSP})$.

Proof. Consider that $G_1 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ and $G_2 = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ isomorphic, There exists a bijective map $\phi : v_1 \to v_2$ satisfying the AQSP fuzzy graph with submerging membership and non - membership values. $\sigma_1^P(x) = \sigma_2^P(x)$, $\sigma_1^N(x) = \sigma_2^N(x)$,

$\rho_1^P(x) = \rho_2^P(x)$, $\rho_1^N(x) = \rho_2^N(x)\ \forall\ x \in V$.
$\mu_1^P(x,y) = \mu_2^P(\phi(x),\phi(y)),\ \mu_1^N(x,y) = \mu_2^N(\phi(x),\phi(y)),$
$\gamma_1^P(x,y) = \gamma_2^P(\phi(x),\phi(y)),\ \gamma_1^N(x,y) = \gamma_2^N(\phi(x),\phi(y))\ \ \forall\ x,y \in E.$
By definition of complement of AQSP fuzzy graph we have,
$\overline{\mu_1}^P(x,y) = \wedge(\sigma_1^P(x),\sigma_1^P(y)) = \wedge(\sigma_1\phi(x),\sigma_1\phi(y)) = \overline{\mu_1}^P(\phi(x),\phi(y)),$
$\overline{\mu_1}^N(x,y) = \vee(\sigma_1^N(x),\sigma_1^N(y)) = \vee(\sigma_1\phi(x),\sigma_1\phi(y)) = \overline{\mu_1}^N(\phi(x),\phi(y)),$
$\overline{\gamma_1}^P(x,y) = \wedge(\rho_1^P(x),\rho_1^P(y)) = \wedge(\rho_1\phi(x),\rho_1\phi(y)) = \overline{\gamma_1}^P(\phi(x),\phi(y)),$
$\overline{\gamma_1}^N(x,y) = \vee(\rho_1^N(x),\rho_1^N(y)) = \vee(\rho_1\phi(x),\rho_1\phi(y)) = \overline{\gamma_1}^N(\phi(x),\phi(y))$
$\forall$ x , y $\in E_1$. Hence, $G_1 = (\sigma_{AQSP}\ ,\ \mu_{AQSP}) \approx \overline{G_2} = (\overline{\sigma}_{AQSP}\ ,\ \overline{\mu}_{AQSP})$.
The converse is true is shown in Figure. 11 as an example.

7. Complete AQSP fuzzy graph

An AQSP fuzzy graph $G = (\sigma_{AQSP}\ ,\ \mu_{AQSP})$ is said to be a complete AQSP fuzzy graph if the necessary and sufficient conditions are satisfied. Figure.6. represents the complete AQSP fuzzy graph in the following.
$(i)\ \mu^P(x,y) = \min(\sigma^P(x)\ ,\ \sigma^P(y)\ ),\ (ii)\mu^N(x,y) = \max(\sigma^N(x)\ ,\ \sigma^N(y)\ )$
$(iii)\gamma^P(x,y) = \min(\rho^P(x)\ ,\ \rho^P(y)\ ),\ (iv)\gamma^N(x,y) = \max(\rho^N(x)\ ,\ \rho^N(y)\ ).$
$\forall\ (x,y) \in \mu^*_{AQSP}$.

7.1. Theorem

Let $G_1 = (\sigma_{AQSP},\mu_{AQSP})$ and $G_2 = (\sigma_{AQSP},\mu_{AQSP})$ be complete AQSP Fuzzy graphs, then $G_1 \cap G_2$ is also a complete AQSP Fuzzy graph.

Proof. If $(x_1,y_1),(x_2,y_2) \in E$, since $G_1 = (\sigma_{AQSP},\mu_{AQSP})$ and
$G_2 = (\sigma_{AQSP},\mu_{AQSP})$ are complete AQSP Fuzzy graphs, then ,
for the membership value of AQSP fuzzy graph is,
$(\mu_1^P \cap \mu_2^P)((x_1,y_1),(x_2,y_2)) = \mu_1^P(x_1,x_2) \wedge \mu_2^P(y_1,y_2),$
$= \sigma_1^P(x_1) \wedge \sigma_2^P(x_2) \wedge \sigma_2^P(y_1) \wedge \sigma_2^P(y_2),$
$= \sigma_1^P(x_1) \wedge \sigma_2^P(y_1) \wedge \sigma_1^P(x_2) \wedge \sigma_2^P(y_2),$
$= \sigma_1^P \cap \sigma_2^P(x_1,y_1) \wedge \sigma_1^P \cap \sigma_2^P(x_2,y_2).$
Similarly for the non - membership values of AQSP fuzzy graph is ,
$(\mu_1^N \cap \mu_2^N)((x_1,y_1),(x_2,y_2)) = \mu_1^N(x_1,x_2) \wedge \mu_2^N(y_1,y_2),$
$= \sigma_1^N(x_1) \vee \sigma_2^N(x_2) \vee \sigma_2^N(y_1) \vee \sigma_2^N(y_2),$
$= \sigma_1^N(x_1) \vee \sigma_2^P(y_1) \vee \sigma_1^N(x_2) \vee \sigma_2^N(y_2),$
$= \sigma_1^N \cap \sigma_2^N(x_1,y_1) \vee \sigma_1^N \cap \sigma_2^N(x_2,y_2).$
For the submerging AQSP fuzzy graph membership value is,
$(\gamma_1^P \cap \gamma_2^P)((x_1,y_1),(x_2,y_2)) = \gamma_1^P(x_1,x_2) \wedge \gamma_2^P(y_1,y_2),$
$= \rho_1^P(x_1) \wedge \rho_2^P(x_2) \wedge \rho_2^P(y_1) \wedge \rho_2^P(y_2),$
$= \rho_1^P(x_1) \wedge \rho_2^P(y_1) \wedge \rho_1^P(x_2) \wedge \rho_2^P(y_2),$
$= \rho_1^P \cap \rho_2^P(x_1,y_1) \wedge \rho_1^P \cap \rho_2^P(x_2,y_2).$
For the submerging non - membership values of AQSP fuzzy graph is,
$(\gamma_1^N \cap \gamma_2^N)((x_1,y_1),(x_2,y_2)) = \gamma_1^N(x_1,x_2) \wedge \gamma_2^N(y_1,y_2),$
$= \rho_1^N(x_1) \wedge \rho_2^N(x_2) \vee \rho_2^N(y_1) \vee \rho_2^P(y_2),$
$= \rho_1^N(x_1) \vee \rho_2^N(y_1) \vee \rho_1^N(x_2) \vee \rho_2^N(y_2),$
$= \rho_1^N \cap \rho_2^N(x_1,y_1) \vee \rho_1^N \cap \rho_2^N(x_2,y_2).$
Hence, $G_1 \cap G_2$ is complete AQSP Fuzzy graph is proved.

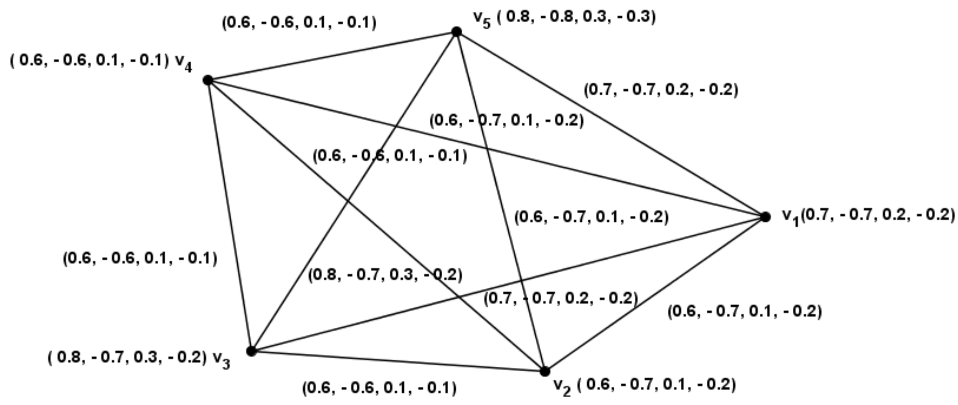

Figure 6: *Complete AQSP Fuzzy Graph*

8. AQSP Fuzzy Graph COVID - 19 Vaccines Survey Analysis from different college Teaching and Non - Teaching staff and Students

In this application of AQSP Fuzzy graph, the determination of accepting/non - accepting Booster shot in future is a Decision - making problem in the experience of COVID - 19 Vaccine, which is taken by willingness/compulsion of each person in pandemic time. If a person 'A' wants to decide in taking booster shot after taking one or two doses during pandemic time was influenced by five influencing factors (IF) which we considered for survey vertices such as, v_1 = Government, v_2 = Health care Authorities, v_3 = Family Members, v_4 = Personal Interest, v_5 = Social Media. Each influencing factor has some common alternate conflict fuzzification attitude, that we considered as $s_1(\sigma,\rho)$ =Gender, $s_2\ (\sigma,\rho)$ =Age, $s_3(\sigma,\rho)$ = Number of Doses, $s_{4(}\ \sigma,\rho)$ = Weightage of IF, $s_5\ (\sigma,\rho)$ = Effects of Vaccines. Here the Influencing factor cannot be measured specifically, since it depends on the decision maker with attitudinal submerging conflict feelings which we considered as edges, v_1v_2 = Positive and Negative effects, v_1v_3 = Reduction of Risk, v_1v_4 = Compulsion and Willingness, v_1v_5 = Immunity, v_2v_3 = Necessity of Booster, v_2v_4 = Behavioral feelings, v_2v_5 = Psychological proses, v_3v_4 = Conflict thoughts, v_3v_5 = Acceptance and non - acceptance of Booster Shotter and v_4v_5 = Essential age group to receive Booster Shotter.

The edge is drawn between the apexes if they have at minimum of communal and attitudinal conflict feelings of accepting / not accepting of third dose booster shot. Thus, each node has multiple attributes compressing the feelings of uncertainty.

Table 1: *Tabular representation of Impact on FIVE Influencing Factors with Vertices of AQSP Fuzzy Graph.*

Vertices	Influencing Factors (IF)	Recurrences	AQSP fuzzy values
v_1	Government	(36%)	(1.0,-0.8,0.5,-0.3)
v_2	Health care Authorities	(11.4%)	(0.9,-0.8,0.4,-0.3)
v_3	Family Members	(15.6%)	(1.0,-0.9,0.5,-0.4)
v_4	Personal Interest	(36.2%)	(1.0,-1.0,0.5,-0.5)
v_5	social media	(0.8%)	(0.5,-0.7,0.0,-0.2)

$(i)\ (\mu,\gamma)^P s_i \ \leq\ (\ (\sigma,\rho)\,(x)\ \wedge\ (\sigma,\rho)\,(y)),\forall\ (x,y)\ \epsilon\ \mu^P_{(x,y)}.$
$(ii)\ (\mu,\gamma)^N s_i \ \geq\ (\ (\sigma,\rho)\,(x)\ \vee\ (\sigma,\rho)\,(y)),\ \forall\ (x,y)\ \epsilon\ \mu^N_{(x,y)}.$
We get the score values and functions of the AQSP Fuzzy Graph using the constrain,

$$\frac{1}{n}\ \left(\frac{1}{S^P_{AQSP}}\sum\theta^P_x - \frac{1}{S^N_{AQSP}}\sum\theta^N_x\ \right).$$

Table 2: *Tabular representation of AQSP Survey Analysis of Attitudinal Conflict Feelings .*

Vertices	Sequences	Recurrences	AQSP fuzzy values
Gender	Female	(85.7%)	$(0.9, -0.6, 0.4, -0.1)$
	Male	(14.3 %)	
Age	< 40	(89.1%)	$(1.0, -0.8, 0.5, -0.3)$
	> 40	(10.4 %)	
No. of doses	Two	(89.7%)	$(0.9, -0.5, 0.4, 0.0)$
	One	(4.8 %)	
	Booster	(8%)	
	None	(0.6 %)	
Weightage of (IF)	Agree	(60.5%)	$(0.9, -0.8, 0.4, -0.3)$
	Not Agree	(40 %)	
Effects of Vaccines	High	(69.9%)	$(0.7, -0.6, 0.2, -0.1)$
	Low	(30 %)	
Immunity level	High	(71.8 %)	$(1.0, -0.8, 0.5, -0.3)$
	Low	(35.8 %)	
Necessity of Booster	Strongly Agree	(52.7 %)	$(0.8, -0.5, 0.3, 0.0)$
	Strongly Disagree	(47.2 %)	
Behavioral feelings	Desire	(52.3 %)	$(0.8, -0.5, 0.3, 0.0)$
	Non desire	(47.6 %)	
Psychological Conflict feelings	Positive	(65.3%)	$(0.5, -0.7, 0.0, -0.2)$
	Negative	(34.6 %)	
Submerging thought process	Less Immunity	(48.7%)	$(1.0, -0.8, 0.5, -0.3)$
	More Immunity	(51.3 %)	
Acceptance of Booster	Agree	(37.8%)	$(0.5, -0.7, 0.0, -0.2)$
	Not Agree	(62.2 %)	
Merits/Demerits(Booster)	Merits	(72%)	$(1.0, -0.6, 0.5, -0.1)$
	Demarits	(28 %)	
Neccessity of Booster	Important	(53.3%)	$(0.8, -0.6, 0.3, -0.1)$
	Not Important	(46.7 %)	
Essential Age(Booster)	< 40	(56.9 %)	$(0.9, -0.7, 0.4, -0.2)$
	> 40	(43.1 %)	

Table 3: *Tabular representation of Combinataric Factors in Boost Shotter survey analysis in AQSP Fuzzy graphs.*

Variables	$(\sigma,\rho)s_i$	v_1	v_2	v_3	v_4	v_5
Gender	$s_1(\sigma,\rho)$ $(0.9,-0.6,0.4,-0.1)$	0.575	0.550	0.600	0.625	0.425
Age	$s_2(\sigma,\rho)$ $(1.0,-0.8,0.5,-0.3)$	0.600	0.575	0.675	0.700	0.500
No. of Doses	$s_3(\sigma,\rho)$ $(0.9,-0.5,0.4,0.0)$	0.550	0.525	0.575	0.600	0.525
Weightage of IF	$s_4(\sigma,\rho)$ $(0.9,-0.8,0.4,-0.3)$	0.625	0.600	0.650	0.675	0.475
Effects of Vaccines	$s_5(\sigma,\rho)$ $(0.7,-0.6,0.2,-0.1)$	0.525	0.500	0.550	0.575	0.375
Average Score	$s_i(\sigma,\rho)$	0.585	0.560	0.550	0.575	0.375

9. AQSP fuzzy graphs Multi input with single output Method

Influencing Factor represents the vertex set of AQSP fuzzy graph = (IF)
Attitudinal Feelings is the edge set of AQSP fuzzy graph = (AF)
(i)Total score weightage (IF) = $\sum(IF_i\ (\bar{A}_i)) + (IF + \frac{(\sigma,\rho)s_1}{10})$, $\alpha_{cut} = S_1$

(ii)Total score weightage (AF)= $\sum(AF_i\ (\bar{A}_i)) + (AF + \frac{(\mu,\gamma)s_i}{10})$, $\alpha_{cut} = IF_5$

$$A\,(\sigma,\rho) \to B\,(\mu,\gamma) = I_{AQSP}\ ((\sigma,\rho)\ ,\ (\mu,\gamma)\ ) = \begin{cases} 1 & (\sigma,\rho) \leq (\mu,\gamma) \\ (\mu,\gamma) & (\sigma,\rho) \geq (\mu,\gamma) \end{cases}$$

$$A\,(\sigma,\rho) \;=\; [0.9,\ 0.7,\ 0.8,\ 1.0,\ 0.6]\ ,$$

$$B\,(\mu,\gamma) = [0.8,\ 0.9,\ 1.0,\ 1.0,\ 0.7,\ 0.8,\ 0.6,\ 1.0,\ 0.6,\ 0.6]$$

$$R_{AQSP}[A\,(\sigma,\rho)\ ,\ B\,(\mu,\gamma)]\ = (\sigma,\rho) \to (\mu,\gamma)$$

$$R_{AQSP} \;=\; \begin{bmatrix} 0.9 \\ 0.7 \\ 0.8 \\ 1.0 \\ 0.6 \end{bmatrix} \begin{bmatrix} 0.8, & 0.9, & 1.0, & 1.0, & 0.7, & 0.8, & 0.6, & 1.0, & 0.6, & 0.6 \end{bmatrix}$$

$$R_{AQSP} \;=\; \begin{bmatrix} 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 0.8 & 0.6 & 1.0 & 0.6 & 0.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 1.0 & 0.6 & 0.6 \\ 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 1.0 & 0.6 & 1.0 & 0.6 & 0.6 \\ 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 0.8 & 0.6 & 1.0 & 0.6 & 0.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$A'\,(\sigma,\rho) = \quad [0.7,\ 0.6,\ 0.8,\ 1.0,\ 0.5],\quad B'\,(\mu,\gamma) = \quad A'\ T_M{}^{\circ}R_{AQSP}$$

$$[0.7,\ 0.6,\ 0.8,\ 1.0,\ 0.5]\quad T_M\circ\quad \begin{bmatrix} 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 0.8 & 0.6 & 1.0 & 0.6 & 0.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 1.0 & 0.6 & 0.6 \\ 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 1.0 & 0.6 & 1.0 & 0.6 & 0.6 \\ 0.8 & 1.0 & 1.0 & 1.0 & 0.7 & 0.8 & 0.6 & 1.0 & 0.6 & 0.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$B'(\mu,\gamma) = \vee(A'(x) \wedge R(A,B))$

$$B'(\mu,\gamma)= \begin{bmatrix} \vee(0.7,0.6,0.8,0.8,0.5) & \vee(0.7,0.6,0.8,1.0,0.5) \\ \vee(0.7,0.6,0.8,1.0,0.5) & \vee(0.7,0.6,0.8,1.0,0.5) \\ \vee(0.7,0.6,0.7,0.7,0.5) & \vee(0.7,0.6,0.8,0.8,0.5) \\ \vee(0.6,0.6,0.6,0.6,0.5) & \vee(0.7,0.6,0.8,1.0,0.5) \\ \vee(0.6,1.0,0.6,0.6,0.5) & \vee(0.6,1.0,0.6,0.6,0.5) \end{bmatrix}$$

$B'(\mu,\gamma) = [0.8,\ 1.0,\ 1.0,\ 1.0,\ 0.7,\ 0.8,\ 0.6,\ 1.0,\ 0.6,\ 0.6]$

$B'(\mu,\gamma) = [0.6\ \ 1.0]$, Using AQSP Fuzzy graph MISO method, we get the result of the following,

Highest Influencing Factor = v_4 (Personal Interest)

Lowest Influencing Factor = v_5 (social media)

Acceptance/ Non acceptance of Booster shot AQSP fuzzy membership and non - membership value = [0.6 , 1.0]

10. Conclusion

Fuzzy graph remains an essential mathematical tool to solve the complex uncertain problems. Its applications are renowned in the evolving fields of Engineering Mathematics, Control Engineering, Real Analysis, Topology, Operations Research, Optimization, Data Science and Computer Science. Specially, Fuzzy graph modules provides a more generalized notion to resolve the problems with vagueness. Based on the study and analysis of the various fuzzy graph modules, a new fuzzy graph Alternate Quadra Submerging Polar Fuzzy Graph (AQSP) has been proposed. This novel method enables to precisely identify the membership, the non-membership values that improves and drives the decision-making analysis in the uncertain situations. Specifically, the application of the AQSP fuzzy graph is based on the behavioral transformation of the human beings due to the various influencing factors. It has been proved that the potential association between the influencing factors and the attitudinal human feelings lead to take a concrete decision of either the acceptance or the non-acceptance of the booster shot. In future, the AQSP fuzzy graph method can be further explored for the possibilities of its applications in the interdisciplinary fields of Artificial intelligence, Approximate reasoning, and Machine learning process to be precise in the identification of the membership or non-membership values due to the uncertain thought process of the human behaviors. Furthermore, the AQSP Fuzzy soft graph and the Matrix representations of the AQSP fuzzy graphs can be further analyzed and explored for the identification and the evaluation of the reliable values in the uncertain situations.

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Conflict of interest

The authors declared that they have no conflict of interest regarding the publication of the research article.

Contributions

The authors worked equally regarding the publication of the research article.

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