APPLICATIONS OF SIMULATIONS AND QUEUING THEORY IN SUPERMARKET

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Abstract

This paper describes the role of queuing theory in supermarket or shopping complex. Generally, a supermarket is a place where people are gathered to purchase the daily requirement products and here, a queue represents the customers/items in ascending or descending order. An interesting aspect of queuing process resides in the measures of its system's performance especially in terms of average service rate and system's utilization. Simulation is a powerful and versatile tool for modeling facilities in supermarket. So, queuing process with simulation provide the average service rate and it helps in predicting queue lengths as well as waiting durations when multiple items are manufactured and distributed using first come first serve discipline. M/M/s model and poisson process are used to explore the supermarket with server arrival rate and service rate.

Keywords: Queues availability, service discipline, simulation and production analysis.

I. Introduction

In the queuing system like post office, bus stand, supermarket, bank etc. customers arrive at service facility to receive the service. The service facility has one or more servers to attend the customers. If all servers are already busy to attend the customers then arrival of new customer join the queue until a server is free. The first come first serve pattern is used to provide the services to the customers. If a customer is served then he leaves the system and can not rejoin the queue. It is viewed that queuing theory is the subfield of

- probability theory with mathematics
- operation research with engineer
- operation management with business
- performance analysis with computer science
- performance analysis in electrical engineering

Queuing theory is the part of operation research that examined the situations related with multiple items/products in queue. It can help organizations to manage the balance between waiting time and resource utilization. The first paper on queuing theory related with theory of probabilities and telephone conversations was published by Erlang [7]. He examined the problem of determining how many telephone circuits were necessary to provide phone service that would prevent customers from waiting too long for an available circuit. In developing a solution to this problem, he began to realize that the problem of minimizing waiting time was applicable to many fields, and began developing the theory further. Baskett et al. [1] described the joint equilibrium distribution of queue sizes in a network of queues containing N service centers and R classes of customers. It is assumed that the equilibrium probabilities exist and are unique. Here, four types of service centers are considered to model central processors, data channels, terminals and routing delays. Reddy et al. [13] analyzed a bulk queue with N-policy multiple vacations and setup times.

Chakravarthy [4] discussed on disaster queue system with markovian arrivals and impatient customers. Arrivals occurring during the time the system undergoes repair are stored in a buffer of finite capacity. These customers can become impatient after waiting a random amount of time and leave the system. Jain and Jain [9] evaluate the repairable queuing system model with multiple breakdown. Igwe et al. [8] evaluate the performance of queue management in supermarkets subject to average service rate using first come first serve discipline. Zhang and Liu [15] analyzed the queue with server breakdown, working vacations and vacation interruption using the supplementary variable method.

Jhala and Bhathawala [10] analyzed applications of queuing theory in supermarket to provide the facility to the customers and conclude that single queue multi server is better in comparison to multi queue multi server. Here, The waiting time of customers waiting in the queue is reduced almost 3 times to the previous one. We also proved that expected total cost is less in case of single queue multi server as compared to multi queue multi server model. Saraswat [13] evaluated the effects of a single counter markovian queuing model with multiple inputs. Chakravarthy and Kulshrestha [3] described a queuing model with backup server in the absence of the main server to continue the process. Bura [2] examined the queuing system model with low quality of services. Generally, customers are impatient due to long waits in queue but here, it is considered that customers are not impatient due to long waits but they are impatient due to the poor quality of service. Daş et al. [6] examined the two stage stochastic model for an industrial symbiosis network under uncertain demand. It is analyzed that the profitability of these volunteer companies is critical as it affects the sustainability of these networks.

Divya and Indhira [7] explained the literature survey on queuing model with working vacation. Narmadha and Rajendran [12] examined the literature review on development of queuing networks. Here, development is a process of gradual change that takes place over many years, during which a theory slowly progress and attain a good state. Yadav et al. [14] described the applications of simulation and queuing theory in scooter industry.

One of the expected gains from studying queuing systems is to review the efficiency of the models in terms of utilization and waiting length. Hence, increasing the number of queues so customers will not have to wait longer when servers are too busy. In queuing theory, Little's theorem, states that the long term average number L of customers in a stationary system is equal to the long term average effective arrival rate λ multiplied by the average time T that a customer spends in the system. Mathematically, it is expressed as

$$L = \lambda * T.$$

This relationship has been shown to be valid for a wide class of queuing models. Consider the example of a supermarket where the customer's arrival rate (λ) doubles but the customers still spend the same amount of time (T) in the bill paying area. These facts double the number of customers (L). By the same logic, if the customer arrival rate (λ) remains the same but the customer service time doubles. These will also double the total number of customers in the system. This indicates that in order to control the three variables, managerial decisions are only required for any two of the three variables. Three fundamental relationships can be derived from Little's theorem as

- L increases if λ or T increases.
- λ increases if L increases or T decreases.
- T increases if L increases or λ decreases.

II. Assumptions

To describe the performance of the supermarket, there are following assumptions

- All similar items are grouped in a particular sequence.
- There should be multiple queues to provide the services to the customers.
- There should be some spare service channels to provide the services when any working channel failed.
- Daily items availability and selling records should be maintained.
- The average number of waiting customers and average number of available service facilities are analyzed by using simulation.
- Arrivals time follow a poisson probability distribution at an average rate of λ customers per unit of time.
- Service times are distributed exponentially with an average of p customers per unit time.
- Service rate is independent of line length.
- The average arrival rate is greater than average service rate.
- First come first serve discipline is utilized for services.

III. Notations

There are following notations

- n = number of costumers in the system
- P_n = probability of n customers in the system
- λ = Average customer arrival rate
- μ = Average number of customers served per unit time at the place of service
- $1 / \mu$ = Average service completion time
- $1 / \lambda$ = Average inter arrival time
- P_0 = Probability of no customers in the system
- s = Number of service channels
- N = Maximum number of customers allowed in the system
- L_s = Average number of customers in the system
- L_q = Average number of customers in the queue
- L = Average length of non-empty queue
- W_s = Average waiting time in the system
- W_q = Average waiting time in the queue
- P_w = Probability that an arriving customer has to wait
- For achieving a steady-state condition and analytical results to be valid, we must have $\lambda / \mu < 1$.

IV. Supermarket Analysis

A Supermarket has three salesmen at the sales counters if the service time for each customer is exponential with a mean of 5 minutes and if people arrive in the poisson fashion at the rate of 20 an hour. Now, using the M/M/s queuing model and determine the following

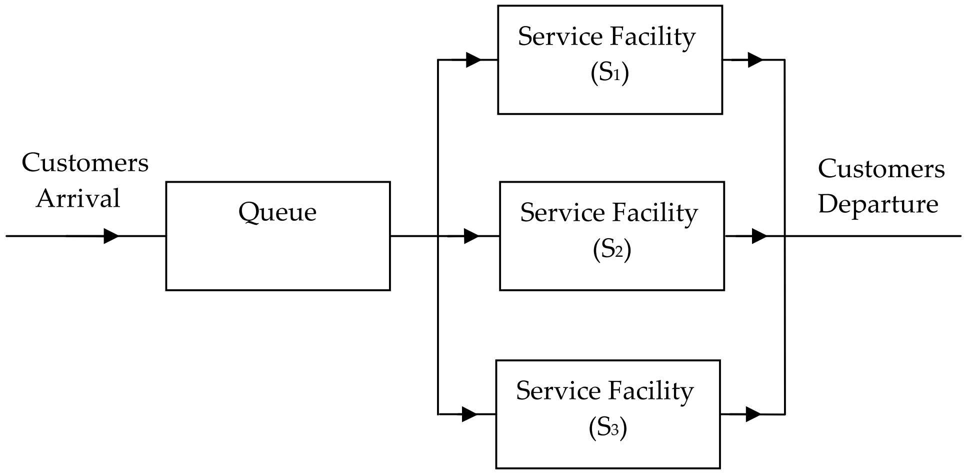

Figure 1: *Queuing System at Supermarket*

Solution: From the given data, get

- Mean service time (1/μ) = 5 minutes=1/12 hour
- Service Rate (μ)= 1/ mean service time = 1/(1/12) = 12 customers per hour
- Arrival Rate (λ) = 20 customers per hour
- Number of server (s) = 3

Now, calculate the following values

(a) Traffic Intensity (ρ)

$$\rho = \frac{\lambda}{s.\mu} = \frac{20}{3\times 12} = \frac{20}{36} = \frac{5}{9} \approx 0.55$$

(b) Probability of zero customer in the system (P_o)

$$P_o = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \cdot \frac{1}{1-\rho} \right]^{-1} \text{, where } \frac{\lambda}{\mu} = \frac{20}{12} \approx 1.66$$

Now calculate the sum:

For $n = 0$, $\frac{(1.66)^0}{0!} = 1$, For $n = 1$, $\frac{(1.66)^1}{1!} = 1.66$

For $n = 2$, $\frac{(1.66)^2}{2!} = \frac{2.77}{2} \approx 1.38$, For $n = 3$, $\frac{(1.66)^3}{3!} = \frac{4.62}{6} \approx 0.77$

Adding these values, get

Sum = $1 + 1.66 + 1.38 + +0.77 \approx 4.71$

Now, calculate $\frac{(\lambda/\mu)^s}{s!} \cdot \frac{1}{1-\rho} = \frac{(1.66)^3}{3!} \cdot \frac{1}{1-0.55}$

$$= 0.77 \cdot \frac{1}{0.45} \approx 1.73$$

Now compute $P_o = (4.81 + 1.73)^{-1} = 0.15$

(c) Average number of customers in the system (L)
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$$L = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2}{s! \times (1-\rho)}$$

Here, $\frac{\lambda}{\mu} = \frac{20}{12} \approx 1.66$

and $\frac{(\lambda/\mu)^2}{s! \times (1-\rho)} = \frac{(1.66)^2}{3! \times (1-0.55)} = \frac{2.77}{6 \times 0.45} = \frac{2.77}{2.70} = 1.02$

Thus, $L = 1.66 + 1.04 = 2.70$

(d) Average time in the system (W)

$$W = \frac{L}{\lambda} = \frac{2.7}{20} \approx 0.1354 \text{ hours} \approx 8.12 \text{ minutes}$$

(e) Average number of customers in the queue (L_q)

$$L_q = L - \frac{L}{\lambda} = 2.7 - 1.66 \approx 1.04$$

(f) Average time in the queue (W_q)

$$W = \frac{L_q}{\lambda} = \frac{1.04}{20} \approx 0.05 \text{ hours} \approx 3.1 \text{ minutes}$$

V. Discussion

In the present model, first come first serve discipline is used to provide the services to the customers. If large number of customers arrived together then there are multiple counters to provide the services to customers to reduce the length of queue in one counter. Here, number of entry gate is one and number of exit gates is three to facilitate the customers. It is analyzed that the probability of queue existence depends on the utilization of products during the day. The utilization is directly proportional to the average number of customers. So, the average number of customers increases as the utilization increases.

Thus, from the above study it is clear that utilization of system is 0.55 and probability of all idle customers is 0.15. Average number of customers in the system is less than average number of customers in the queue and average time in the system is more than average time in the queue.

VI. Conclusion

From the above study, it is clear that servers are utilized about 55% of time and it means that there is some spare capacity available. The increase in the number of servers will reduce the time of customers have to wait in line before been served. So, it will increase the efficiency of the supermarket due to the utilization of their services to the customers as and when required.

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