SOME APPLICATIONS OF EXPONENTIATED LOG-UNIFORM DISTRIBUTION

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Abstract

In this paper we introduced Exponentiated Log - Uniform distribution as a generalisation of the Log - Uniform distribution and its properties are studied. We provide graphical representations of its density function, cumulative distribution function, hazard rate function, and survival function. And derive various statistical properties such as moments, mean deviations, and quantile function of the new distribution. We also obtain the probability density functions of the order statistics of the Exponentiated Log-Uniform Distribution.To estimate the parameters of the distribution and the stress strength parameters, we use the maximum likelihood method, and validate the estimates of the model parameters through a simulation study. Our findings reveal that the Exponentiated Log-Uniform Distribution exhibits the least bias and that the values of the mean square error decrease as the sample size increases, indicating the effectiveness of this distribution in modeling real-world data. We applied the Exponentiated Log-Uniform distribution to a real data set and compared it with Exponentiated Quasi Akash Distribution and Exponentiated Weibull Distribution. It was found that the new distribution was a better fit than the other distributions based on the values of the AIC, CAIC, BIC, HQIC, the Kolmogorov-Smirnov (K-S) goodness-of-fit statistic and the p-values.

Keywords: Log - Uniform distribution, Exponentiated Log - Uniform distribution, Stress strength Reliability

1. Introduction

Statisticians are constantly pushing the boundaries of statistical analysis by exploring new families of distributions. Their goal is to increase the flexibility of existing distributions, which allows for the creation of compound distributions that can better analyze a wide range of data sets.The Log-Uniform distribution is a continuous probability distribution where the logarithm of a random variable is uniformly distributed [9].

One of the most attractive features of the Log-Uniform distribution is its scale-invariance property. This means that the distribution remains the same, regardless of the units of measurement used.The exponentiated family has become a fundamental concept in mathematical statistics and machine learning.

Ahmed [2] introduced a new version of the Exponentiated Burr X distribution, Exponentiated Transmuted Exponential distribution was proposed by Al-Kadim [3] which is more adaptable than other distributions, Alzaghal [4] derived Exponentiated TX family of distributions. The Beta Exponentiated Weibull distribution was derived by Cordeiro [5]. Ahmad Dar [6] introduced Exponentiated Quasi Akash distribution. Gupta and Kundu [8] proposed Exponentiated Exponential family. Hassan [10] derived a new generalization of Ishita distribution and obtained properties of the distribution along with applications of the proposed model. The Exponential distribution and its applications was proposed by Jowett [11]. The Exponentiated Kumaraswamy distribution and its log-transform was studied by Lemonte [15]. Exponentiated weibull distribution is studied by Pal and Ali [16]. Ramires [17] introduced Exponentiated Uniform distribution: An interesting alternative to truncated models.

If X is a random variable with a Log Uniform distribution, then Log(X) is Uniformly distributed. A random variable X is said to have a Log-Uniform distribution if its probability density function is given by:

$$f(x) = \begin{cases} \frac{1}{x\left[ln(b)-ln(a)\right]}; & \text{if }, a \leq x \leq b, 0 < a < b, a, b \in R \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where a and b are the parameters of the distribution and they are location parameters that define the minimum and maximum values of the distribution on the original scale and ln is the natural Log function (the logarithm to base e).

The Cumulative distribution function of the distribution is given by:

$$F(x) = \begin{cases} \frac{ln(x)-ln(a)}{ln(b)-ln(a)}; & \text{if }, a \leq x \leq b, 0 < a < b, a, b \in R \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

The exponentiated family of distributions is a set of distributions that can be derived by raising a positive real number to the cumulative distribution function (cdf) of a parent distribution.

The Cumulative distribution function of the family is given by:

$$F(x) = (H(x))^{\beta}; \quad \beta > 0 \tag{3}$$

where $H(x)$ is the distribution function of a random variable X.
The corresponding probability density function (pdf) is given by:

$$f(x) = \beta h(x)(H(x))^{\beta-1} \tag{4}$$

where $H(x)$ and $h(x)$ are cdf and pdf of the parent distribution and $\beta > 0$ is a shape parameter.

2. Exponentiated Log-uniform Distribution

On introducing Log-Uniform distribution to Exponentiated family of distributions we obtain a new distribution called the Exponentiated Log-Uniform distribution.

The distribution function has the form:

$$F(x) = (H(x))^{\beta}; \quad \beta > 0 \tag{5}$$

where $H(x)$ is the cdf of the base distribution. Therefore the cdf of Exponentiated Log-uniform Distribution is given by

$$F(x) = \begin{cases} \left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\right)^{\beta}, & \text{if } \beta > 0, a \leq x \leq b, 0 < a < b, a, b \in R \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where a and b are the parameters of the distribution and they are location parameters and β is a shape parameter.

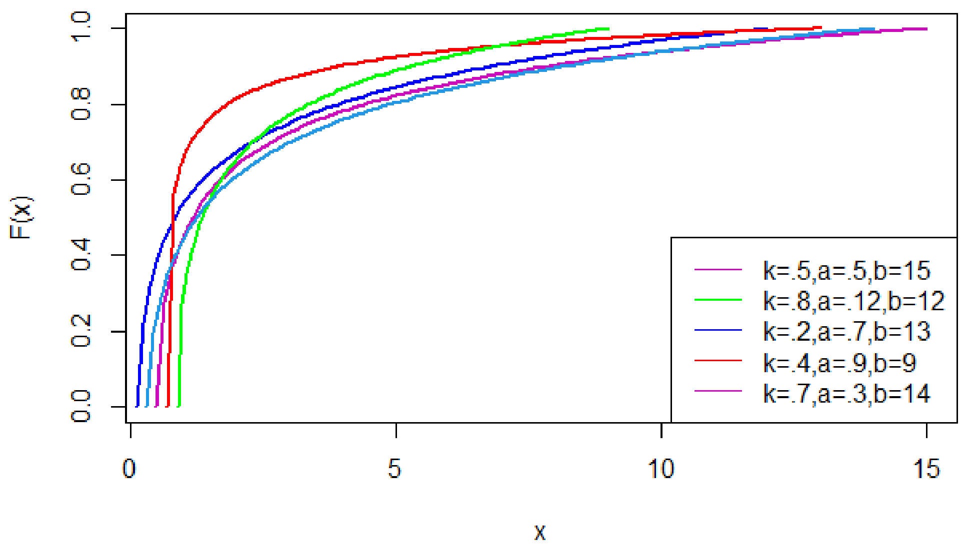

Figure 1: *Plot of cdf of Exponentiated Log-Uniform distribution*

The probability density function of the distribution is given by:

$$f(x) = \begin{cases} \beta \frac{1}{x ln(\frac{b}{a})} \left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})} \right)^{\beta - 1}, & \text{if } \beta > 0, a \leq x \leq b, 0 < a < b, a, b \in R \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

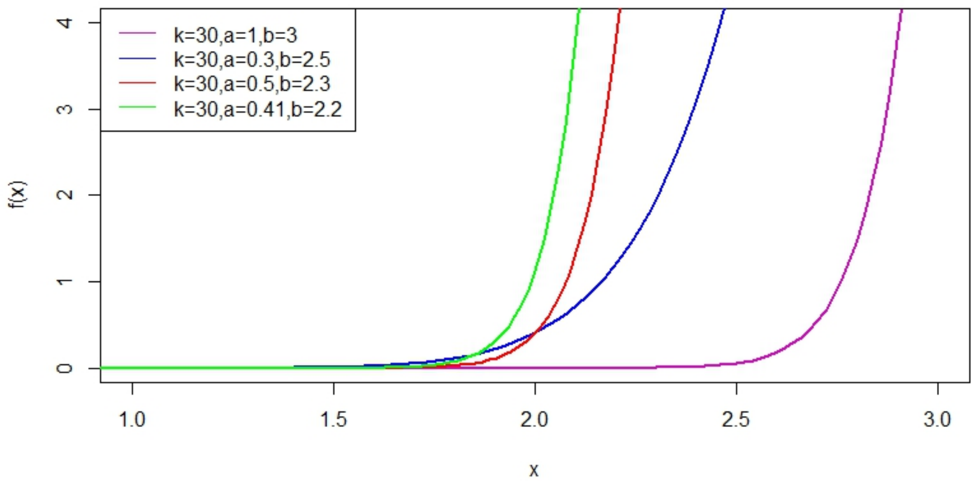

Figure 2: *Plot of pdf of Exponentiated Log-Uniform distribution*

The Survival function of Exponentiated Log-Uniform distribution is,

$$S(x) = \frac{\left(ln(\frac{b}{a})\right)^{\beta} - \left(ln(\frac{x}{a})\right)^{\beta}}{\left(ln(\frac{b}{a})\right)^{\beta}}; \beta > 0, a \le x \le b, 0 < a < b, b \in R \tag{8}$$

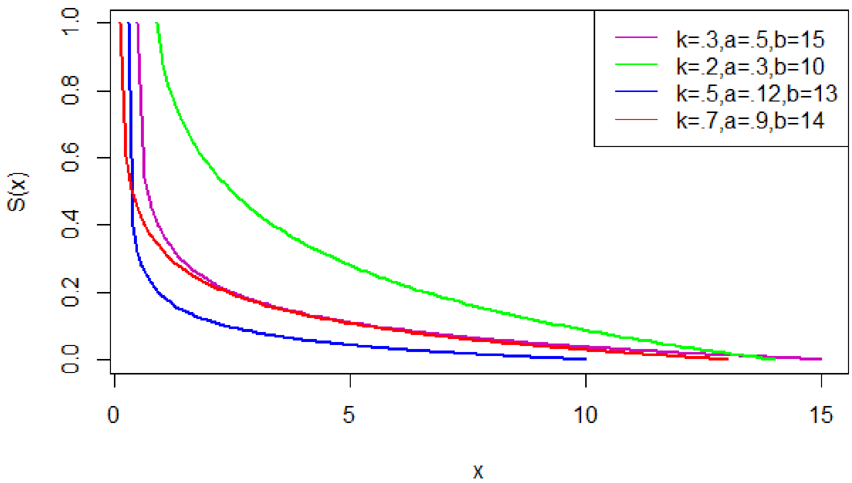

Figure 3: *Plot of survival function of Exponentiated Log-Uniform distribution*

The failure rate function of Exponentiated Log-Uniform distribution is,

$$h(x) = \frac{\beta(ln(\frac{x}{a}))^{\beta-1}}{x((ln\frac{b}{a})^{\beta} - (ln(\frac{x}{a}))^{\beta})}; \beta > 0, a \le x \le b, 0 < a < b, b \in R \tag{9}$$

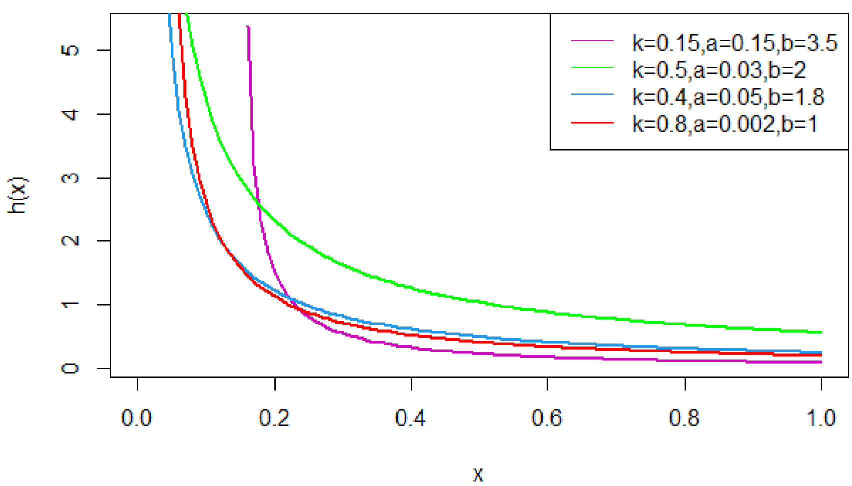

Figure 4: *Plot of hazard function of Exponentiated Log-Uniform distribution*

- Special Case:
  If we put $\beta = 1$, then Exponentiated Log-Uniform distribution reduces to Log-Uniform distribution.

2.1. Statistical Properties

Some structural properties of the Exponentiated Log-Uniform distribution have been evaluated in this section.The properties like Moments, Quantile function, Mean Deviation and Order Statistics are considered.

2.1.1 Moments

A distribution can have several moments, and shape of the distribution is determined by its moments. The first moment is the mean of the distribution. Suppose X is a random variable following Exponentiated Log-Uniform distribution with parameters a,b, β and then the r^{th} moment for a given probability distribution is given by:

$$\mu_r^{'} = E(X^r) = \int_a^b x^r f(x,a,b,\beta)dx \tag{10}$$

$$\mu_r^{'} = \int_a^b x^r \beta \left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\right)^{\beta-1} \left(\frac{1}{xln(\frac{b}{a})}\right)dx \tag{11}$$

$$= \frac{\beta}{(ln(\frac{b}{a}))^{\beta}} \int_a^b x^{r-1} \left(ln(\frac{x}{a})\right)^{\beta-1} dx$$

$$E(X^r) = \mu_r^{'} = a^r \left(\frac{\beta\Gamma\beta}{(-rln(\frac{b}{a}))^{\beta}} - \frac{\beta\Gamma(\beta, -rln(\frac{b}{a}))}{(-rln(\frac{b}{a}))^{\beta}}\right) \tag{12}$$

- On substituting r=1 on equation 12 results in mean of the distribution.

2.1.2 Quantile Function

The quantile function is important as it can be used to generate random numbers and to find quartiles, median, measures of skewness and kurtosis.
The Quantile function is given by,

$$x = Q(p) = F^{-1}(p), 0 < p < 1 \tag{13}$$

The Quantile function of Exponentiated Log-Uniform distribution is obtained by inverting distribution function.

$$p = \frac{\left(ln(\frac{x}{a})\right)^{\beta}}{\left(ln(\frac{b}{a})\right)^{\beta}}$$

Thus the quantile function is given by,

$$x = Q(p) = ae^{p^{\frac{1}{\beta}}\left(ln(\frac{b}{a})\right)} \tag{14}$$

- The second quartile (Median) of exponentiated log-uniform distribution is obtained by putting $p = \frac{1}{2}$ in 14

$$Q_2 = m = ae^{\left((\frac{1}{2})^{\frac{1}{\beta}} ln(\frac{b}{a})\right)} \tag{15}$$

- The r^{th} moment is defined as

$$\mu_r^{'} = E(X^r) = \int_a^b x^r f(x, a, b, \beta) dx$$

Which can be written in terms of the quantile function as

$$mu_r^{'} = \int_0^1 (Q(u))^r du$$

2.1.3 Mean Deviation

The mean deviation is a measure of amount of scatter in a random variable. Let X follows Exponentiated Log-Uniform distribution with mean μ and meadian M.

- Mean Deviation from the Mean is given by,

$$\delta_1(x) = \int_a^b |x - \mu| f(x) dx \; = 2\mu(F(\mu) - 1) + 2T(\mu) \tag{16}$$

where μ is the mean of the distribution and

$$T(\mu) = \int_\mu^b x f(x) dx \tag{17}$$

$$T(\mu) = \int_\mu^b x\beta \frac{1}{xln(\frac{b}{a})} \Big(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\Big)^{\beta-1} dx$$

$$T(\mu) = \frac{a\beta}{(ln(\frac{b}{a}))^\beta} \left(\frac{(ln(\frac{\mu}{a}))^\beta \Gamma(\beta, (-ln(\frac{\mu}{a})))}{(-ln(\frac{\mu}{a}))^\beta} - \frac{(\ln(\frac{b}{a}))^\beta \Gamma(\beta, -ln(\frac{b}{a}))}{(-ln(\frac{b}{a}))^\beta} \right) \tag{18}$$

- Similarly,the Mean Deviation about Median is,

$$\delta_2(x) = \int_a^b |x - M| f(x) dx \; = 2T(M) - \mu \tag{19}$$

where M is the median of the distribution and μ is the mean of the distribution and

$$T(M) = \int_M^b x f(x) dx \tag{20}$$

$$T(M) = \frac{a\beta}{(ln(\frac{b}{a}))^\beta} \left(\frac{(ln(\frac{M}{a}))^\beta \Gamma(\beta, (-ln(\frac{M}{a})))}{(-ln(\frac{M}{a}))^\beta} - \frac{(\ln(\frac{b}{a}))^\beta \Gamma(\beta, -ln(\frac{b}{a}))}{(-ln(\frac{b}{a}))^\beta} \right) \tag{21}$$

The mean deviation about mean is obtained by substituting the mean, cdf and $T(\mu)$ in (16). The mean deviation about median is obtained by substituting the mean, cdf and $T(M)$ in (19).

2.1.4 Order Statistics

Let $X_{(1)}$,$X_{(2)}$,$X_{(3)}$,...,$X_{(n)}$ denote the order statistics of a random sample X_1,X_2,X_3,...,X_n drawn from the continuous distribution with pdf $f_X(x)$ and cdf $F_X(x)$, then the pdf of r^{th} order statistics $X_{(r)}$ is given by,

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} f(x) [F(x)]^{(r-1)} [1 - F(x)]^{(n-r)} \tag{22}$$

Using the equations (6) and (7) the probability density function of r^{th} order statistics $X_{(r)}$ of Exponentiated Log-Uniform distribution is given by,

$$f_{X_{(r)}}(x,\beta,a,b) = \frac{n!}{(r-1)!(n-r)!}\beta\left(\frac{1}{xln(\frac{b}{a})}\right)\left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\right)^{\beta-1}\left(\left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\right)^{\beta}\right)^{(r-1)}\left(1-\left(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\right)^{\beta}\right)^{(n-r)} \tag{23}$$

Then the probability density function of first order statistics $X_{(1)}$ of Exponentiated Log-Uniform distribution is given by,

$$f_{X_{(1)}}((x,\beta,a,b) = n\beta\left(\frac{1}{x\ln(\frac{b}{a})}\right)\left(\frac{\ln(\frac{x}{a})}{\ln(\frac{b}{a})}\right)^{(\beta-1)}\left(1-\left(\frac{\ln(\frac{x}{a})}{\ln(\frac{b}{a})}\right)^{\beta}\right)^{(n-1)} \tag{24}$$

and the probability density function of n^{th} order $X_{(n)}$ of Exponentiated Log-Uniform distribution is given as:

$$f_{x_{(n)}}((x,\beta,a,b) = n\beta\left(\frac{1}{x\ln(\frac{b}{a})}\right)\left(\frac{\ln(\frac{x}{a})}{\ln(\frac{b}{a})}\right)^{(\beta-1)}\left(\left(\frac{\ln(\frac{x}{a})}{\ln(\frac{b}{a})}\right)^{\beta}\right)^{(n-1)} \tag{25}$$

2.2. Maximum Likelihood Estimation

In this section, we discuss the method of maximum likelihood(ML) for the estimation of the unknown parameters a, b, β of Exponentiated Log-Uniform distribution. Let X_1,X_2,X_3,...,X_n be the random sample of size n drawn from Exponentiated Log-Uniform distribution, the likelihood function is given by,

$$L(x_i;\beta,a,b) = \prod_{i=1}^{n}\beta\Big(\frac{1}{x_i ln(\frac{b}{a})}\Big)\Big(\frac{ln(\frac{x_i}{a})}{ln(\frac{b}{a})}\Big)^{\beta-1}$$

The log-likelihood function is given by,

$$lnL(x_i;\beta,a,b) = nln\beta - n\beta(ln(ln(\frac{b}{a}))) + (\beta-1)\sum_{i=1}^{n}(ln(ln(\frac{x_i}{a})) - \sum_{i=1}^{n}ln(x_i) \tag{26}$$

$$\frac{\partial lnL}{\partial a} = \frac{n\beta}{aln(\frac{b}{a})} - \frac{\beta-1}{a\sum_{i=1}^{n}ln(\frac{x_i}{a})} = 0 \tag{27}$$

$$\frac{\partial lnL}{\partial b} = \frac{-n\beta}{bln(\frac{b}{a})} = 0 \tag{28}$$

$$\frac{\partial lnL}{\partial \beta} = \frac{n}{\beta} - nln(ln(\frac{b}{a})) + \sum_{i=1}^{n}ln(ln(\frac{x_i}{a})) = 0 \tag{29}$$

Solving this system of equations, in a, b, β gives the MLEs of a, b, β as $\hat{a}$,$\hat{b}$,$\hat{\beta}$.

2.3. Stress Strength Reliability

In this section, the procedure of estimating reliability of $R = P(X_2 < X_1)$ model is considered. The expression $R = P(X_2 < X_1)$ measures the reliability of a component in terms of probability. The random variables X_1 representing the stress experienced by the component does not exceed X_2 which represents the strength of the component. If stress exceeds strength, the component would fail and vice-versa.
In order to estimate the stress-strength parameter, considering two random variables X and Y with Exponentiated Log-Uniform (β_1, a, b) and Exponentiated Log-Uniform (β_2, a, b) distributions

respectively. We assume that X and Y are independent random variables and the stress-strength parameter is obtained in the form,

$$R = P(Y < X) = \int_{X<Y} f(x,y)dxdy = \int_0^\infty f(x;\beta_1,a,b)F(x;\beta_2,a,b)dx \tag{30}$$

where $f(x,y)$ is the joint probability density function of random variables X and Y, having Exponentiated Log-Uniform distribution.so that

$$R = \int_a^b \beta_1 \frac{1}{xln(\frac{b}{a})} \Big(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\Big)^{\beta_1 - 1} \Big(\frac{ln(\frac{x}{a})}{ln(\frac{b}{a})}\Big)^{\beta_2} dx$$

On simplification we get,

$$R = \frac{\beta_1}{\beta_1 + \beta_2} \tag{31}$$

To compute the maximum likelihood estimate of R, we need to compute the maximum likelihood estimate of β_1 and β_2. Suppose $X_1, X_2, ..., X_n$ is random sample of size n from the Exponentiated Log-Uniform (β_1, a, b) and $Y_1, Y_2, ..., Y_m$ is an independent random sample of size m from Exponentiated Log-Uniform (β_2, a, b). The likelihood function of the combined random sample can be obtained as follow:

$$L = \prod_{i=1}^n f(x_i;\beta_1,a,b) \prod_{i=1}^m f(y_i;\beta_2,a,b) \tag{32}$$

$$L = \prod_{i=1}^n \beta_1 \Big(\frac{1}{x_i ln(\frac{b}{a})}\Big)\Big(\frac{ln(\frac{x_i}{a})}{ln(\frac{b}{a})}\Big)^{\beta_1 - 1} \prod_{i=1}^m \beta_2 \Big(\frac{1}{y_i ln(\frac{b}{a})}\Big)\Big(\frac{ln(\frac{y_i}{a})}{ln(\frac{b}{a})}\Big)^{\beta_2 - 1} \tag{33}$$

The log-likelihood function is,

$$\begin{aligned} lnL = nln\beta_1 - n\beta_1(ln(ln(\frac{b}{a}))) + (\beta_1 - 1)\sum_{i=1}^n (ln(ln(\frac{x_i}{a})) - \sum_{i=1}^n ln(x_i) \\ + mln\beta_2 - m\beta_2(ln(ln(\frac{b}{a}))) + (\beta_2 - 1)\sum_{i=1}^m (ln(ln(\frac{y_i}{a})) - \sum_{i=1}^m ln(y_i) \end{aligned} \tag{34}$$

The maximum likelihood estimate (MLE) of β_1 and β_2 can be obtained as the solution of,

$$\frac{\partial lnL}{\partial \beta_1} = \frac{n}{\beta_1} - nln(ln(\frac{b}{a})) + \sum_{i=1}^n ln(ln(\frac{x_i}{a})) = 0 \tag{35}$$

$$\frac{\partial lnL}{\partial \beta_2} = \frac{m}{\beta_2} - mln(ln(\frac{b}{a})) + \sum_{i=1}^m ln(ln(\frac{y_i}{a})) = 0 \tag{36}$$

From the equation (36) and (37),we obtain,

$$\hat{\beta_1} = \frac{n}{nln(ln(\frac{b}{a})) - \sum_{i=1}^n ln(ln(\frac{x_i}{a}))} \tag{37}$$

$$\hat{\beta_2} = \frac{m}{mln(ln(\frac{b}{a})) - \sum_{i=1}^m ln(ln(\frac{y_i}{a}))} \tag{38}$$

The corresponding ML estimate of R is computed from (32) by replacing β_1 and β_2 by their ML estimates.

$$\hat{R} = \frac{\hat{\beta_1}}{\hat{\beta_1} + \hat{\beta_2}} \tag{39}$$

3. Simulation Study and Data Analysis

3.1. Simulation Study

Simulation studies are an important tool for statistical research. Here we take distinct combinations of parameters a, b, β with sample size n. Bias and the mean square error(MSE) of the parameter estimates are obtained using the following equations.
If $\hat{\theta} = T(X)$ is an estimator, then the bias of $\hat{\theta}$ is the difference between its expectation and the 'true' value:

$$Bias(\hat{\theta}) = E_{\theta}(\hat{\theta}) - \theta \tag{40}$$

An estimator T(X) is unbiased for θ if $E_{\theta}(T(X)) = \theta$ for all θ, otherwise it is biased.
The MSE of an estimator $\hat{\theta}$ is

$$MSE = E_{\theta}\left[(\hat{\theta} - \theta)^2\right] \tag{41}$$

The simulation is done by using different true parameter values. The chosen true parameter values are as follows:

- $a = 0.12$, $b = 12$, $\beta = 0.5$

  As the n increases, MSE decreases for the selected parameter values given in table 1. Moreover, the bias is close to zero as the sample size increases. Thus, as the sample size increases the estimates tend to be closer to the true parameter values.

Table 1: *Simulation study at $a = 0.12$, $b = 12$,$\beta = 0.5$*

n	Parameter	Estimate	Bias	MSE
30	a	0.1354	0.154	0.00238
	b	8.9662	3.0338	9.2039
	β	0.4904	0.5096	0.2596
75	a	0.12038	0.0824	0.00133
	b	12.009	0.962	0.0565
	β	0.4209	0.0790	0.0842
100	a	0.12005	0.04732	0.000276
	b	11.438	0.5520	0.0315
	β	0.3717	0.012827	0.0624
500	a	0.1200081	0.02019	0.0001108
	b	11.867	0.1328	0.01762
	β	0.4572	0.004270	0.0128
1000	a	0.120006	0.01650	0.00007403
	b	11.961	0.03891	0.001513
	β	0.49096	0.00904	0.00550

Comparing the performance of the estimators in Table 1, we can verify that the bias and MSE values decreases as the sample size increases.

3.2. Data Analysis

Here we consider the data set represents the strength of glass of the aircraft window reported by Fuller et al. (1994) and we fit these data to Exponentiated Log-Uniform distribution and compare the results with the Exponentiated Quasi Akash Distribution and Exponentiated Weibull Distribution.

Table 2: *Strength of glass of the aircraft window*

18.83	20.80	21.657	23.03	23.23	24.05	24.321	25.50
25.52	25.80	26.69	26.77	26.78	27.05	27.67	29.90
31.11	33.20	33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29	45.381	

Table 3: *AIC, CAIC, BIC,and HQIC statistics of the fitted model in data set*

Distribution	AIC	CAIC	BIC	HQIC
Exponentiated Log-Uniform Distribution	196.3994	197.2883	200.7014	197.8017
Exponentiated Quasi Akash Distribution	214.165	215.054	218.467	215.567
Exponentiated Weibull Distribution	214.0852	214.9741	218.3871	215.4875

From the table 3, it has been observed that the Exponentiated log-uniform Distribution possesses the lesser AIC, CAIC, BIC,and HQIC values as compared to Exponentiated Quasi Akash distribution and Exponentiated Weibull distribution. To check the model goodness of fit we had considered the Kolmogorov-Smirnov (K-S) test (goodness-of-fit) statistics for the strength of glass of the aircraft window data. Since the p-value of fitted model is highest than the other distributions we have considered. Therefore the results indicate, that the Exponentiated Log-Uniform distribution performed better than other distributions.

4. Summary and Conclusions

In this study, we introduce a new distribution, called the Exponentiated Log-Uniform Distribution, which is generated using an exponentiated technique based on the two-parameter Log-Uniform distribution as the base distribution. We provide graphical representations of its density function, cumulative distribution function, hazard rate function, and survival function. And derive various statistical properties such as moments, mean deviations, and quantile function of the new distribution. We also obtain the probability density functions of the order statistics of the Exponentiated Log-Uniform Distribution.

To estimate the parameters of the distribution and the stress strength parameters, we use the maximum likelihood method, and validate the estimates of the model parameters through a simulation study. Our findings reveal that the Exponentiated Log-Uniform Distribution exhibits the least bias and that the values of the mean square error decrease as the sample size increases, indicating the effectiveness of this distribution in modeling real-world data.

Furthermore, we apply the Exponentiated Log-Uniform Distribution to a real data set and compare it with Exponentiated Quasi Akash distribution and Exponentiated Weibull distribution. Our results indicate that the new distribution outperforms these models based on various criteria such as the Akaike information criterion (AIC), the corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC), Hannan information criterion (HQIC), the Kolmogorov-Smirnov (K-S) goodness-of-fit statistic, and the p-values obtained for the models.

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