STUDY OF DECİSİON-MAKİNG MODELS BASED ON Z NUMBERS WHEN SOLVİNG DECİSİON-MAKİNG PROBLEMS UNDER CONDİTİONS OF İNCOMPLETE İNFORMATİON

Kifayat Mammadova, Matanat Hasanquliyeva, Ibrahim Abasov

•

Azerbaijan State Oil and Industy University
ka.mamedova@yandex.ru
metahesenquliyeva@gmail.com
abasov_i99@mail.ru

Abstract

In this article, decision-making models based on Z numbers were studied during the decision-making process under conditions of incomplete information, and a number of scientific and practical issues related to their application to solving the given problem were reviewed. As a result of the research, it became clear that it is not easy to form rational decision-making ability and make optimal decisions in conditions of uncertain, fuzzy or incomplete information based on existing approaches, and sometimes it is inaccurate. Thus, solving real decision-making problems, decision analysis, and systematic analysis in economics, ecology, and other fields are characterized by vagueness and partial reliability of certain data. In this regard, the Z numbers proposed by Professor Zade have a strong effect in expressing the uncertainty due to the unique structure.

This paper proposes a fuzzy approach for decision making based on Z-information based on exact computation on Z-numbers when solving decision-making problems under conditions of incomplete information. The approach is intended for hierarchical imprecise models. In the article, the models considered in relation to the research and application of decision-making models based on Z numbers are based on the processing of uncertain information and the use of word computing technology in decision-making.

Keywords: fuzzy information, fuzzy sets, Z numbers, probability distribution, decision making model

I. Introduction

Fuzzy logic theory deals with the processing of information that is vague, imprecise, fuzzy, partially true, or has no sharp boundaries arising from perception and intelligence. Fuzzy logic can be applied to linguistic evaluation, decision making, and information analysis with linguistic evaluation. Fuzzy logic, including fuzzy sets, linguistic variables, fuzzy rules, fuzzy mathematics, fuzzy database queries, computational theory of intelligence, and linguistic evaluations for computing with words, are very useful in determining the degree of uncertainty of variables.

In the researched problem, if the uncertainty arising during information processing is due to randomness, lack of information, the theory of probability, and if it is due to the inaccuracy of information, the theory of fuzzy logic is applied as a powerful tool.

The presence of uncertainty and inaccuracy in the vast majority of information in decision-making models requires greater caution and high intellectuality. Thus, it is difficult to form a rational decision-making ability based on imprecise, fuzzy or incomplete information [1].

Currently, one of the most effective and widely analyzed research problems is the problem of multi-criteria decision making. As uncertainty, impreciseness and complexity exist in the studied environment, fuzzy sets [2] will enter as one of the widely used key issues in the decision making process.

The purpose of the research in [2] is the study of representation models of uncertain information and its application to management. In this study, models of description and processing of fuzzy, imprecise information were analyzed. Issues of qualitative analysis of the researched information were considered. For this purpose, models based on objective and subjective probability theory, interval type, 1st, 2nd type and generalized fuzzy models, hybrid models were analyzed when solving decision-making problems under conditions of incomplete information, and the problems of processing uncertain information were considered.

Real decision-making problems in decision analysis, systematic analysis, economics, ecology and other fields are characterized by the vagueness and partial reliability of certain data. In recent times, the solution of a number of scientific and practical issues related to the research and application of decision-making models based on Z numbers has been considered. The considered models enable the processing of uncertain information and the use of word computing technology in decision-making. The concept proposed by Professor Zade - Z-numbers has become a center of research in fuzzy theory. Unlike previous fuzzy sets, Z numbers have a stronger ability to express uncertainty due to their unique structure [3,4].

The article [5] describes the methods of forming the uncertainty caused by the loss of information during the solution of a number of problems using Z numbers. To achieve this goal, the article examines how to convert real numbers and fuzzy numbers into Z numbers, and develops a new system of Z-linear equations.

Two processes of Z numbers ranking methods are presented in models that can efficiently use uncertain decision-making data [6]. Decision making is based on the proposed Z numbers. In this work, first Z numbers were converted into fuzzy numbers, and then ranking method using sigmoid function and sign method were used to record fuzzy numbers. In the next step, the method is extended to related Z numbers. This method was used to prioritize items and solve some patterns.

II. Models based on Z number theory

The Z number represents both the uncertain variable and its reliability. The Z number is used in decision-making, risk assessment, etc. A multi-criteria fuzzy decision-making method with stronger potential is proposed by its application in fields. Here, the assessment of each alternative by evaluators for each criterion, as well as the reliability of this assessment, is described as a Z number [3-6].

Decisions should be made based on accurate information. For decisions to be useful and effective, information must be reliable. Therefore, it is more appropriate to use the number Z. The Z number is a fuzzy number that describes the reliability of the information. The number Z is represented by the pair Z=(A,B) with two components. The first component A is the admissible constraint on the acceptance of the real-valued variable X into the fuzzy set. The second component, B, is a measure of the reliability (certainty) of the first component. Typically, A and B are described in natural language. Calculation with Z numbers is considered more important [5-7]. For example, "İnci is excellent" (Very High, Probable). The first part, "Very High" is the assessment limit of Pearl's knowledge, that is, the value of A, and the second part, "Probable" is R, which is a measure of probability that tells how true this information is.

Counting with Z numbers belongs to the field of counting with words (CW or CWW). This article introduces the concept of Z number and methods of calculation with Z numbers. The Z-number concept has great potential in many applications, economics, decision analysis, risk assessment, and predictive problems based on characteristic rules of inexact functions and relationships.

Zade introduced the concept of Z-numbers to describe vague information in a generalized form. A Z-number is a pair of fuzzy numbers $(\tilde{A}, \tilde{B})$. Here, $\tilde{A}$ is the value of a variable, and $\tilde{B}$ represents an idea closely related to a certain concept, such as certainty, confidence, reliability, degree of accuracy, and probability. Let's look at an example: Which number is more accurate for the highest age limit among people in the country: (about 100, true) or (about 90, very true). The main field of application of the proposed number theory Z will be word computing technology.

In modern activity models, for example, socio-economic, technical-medical models, it is very difficult to solve decision-making problems without taking into account the intuitions of the decision-maker. It is known that in classical decision-making models, probability values are required to be accurately described. In most real-world cases, this is impossible to achieve. In reality, especially in economics, information is subjective and imprecise. There are many approaches that describe the imprecision of relevant probabilistic information: classical numbers (level 0, for example, 1.9-17); interval numbers (level 1, for example, temperature from 20 to 25); random number (level 2, for example, male height is normally distributed around 1m 85cm); fuzzy number (level 2, eg its moral weight is "too great"); finally, Z-numbers (level 3, for example, the price of crude oil on the world market will be "high", the belief is "great"). One of these approaches is used for hierarchical imprecise models [3-7]. These models are used to account for second-order uncertainty that describes real problems. According to this approach, the opinion of the expert about the imprecise probabilities is estimated imprecisely.

III. Methods of solving the decision-making problem based on Z-numbers

It should be noted that in recent decision-making models, optimal decisions are made based on Z-numbers. Because Z is a fuzzy number that describes the reliability of information. Therefore, it is considered more appropriate to use the Z number. Lutfi Zade proposed some operations using the expansion principle for computation with Z-numbers.Diskret ədədlər üzrə arifmetik əməliyyatların ümumi strukturu.

For example, $Z_1 = (\tilde{A}_1, \tilde{B}_1)$ and $Z_2 = (\tilde{A}_2, \tilde{B}_2)$ are discrete Z-numbers that describe information about the characteristics of the random and variable. It is possible to perform various analytical calculations on Z-numbers as well as on fuzzy numbers [9-11].

$$Z_{12} = Z_1 * Z_2, * \in \{+,-,\cdot,/\} \tag{1}$$

First, $Z_{12}^+ = Z_1^+ * Z_2^+$ is calculated:

$$Z_1^+ * Z_2^+ = (\tilde{A}_1 * \tilde{A}_2, R_1 * R_2) \tag{2}$$

Here we describe the discrete probability distributions with R1 and R2:

$$\begin{aligned} p_{R_1} &= p_{R_1}(x_{11}) \backslash x_{11} + p_{R_1}(x_{12}) \backslash x_{12} + \ldots + p_{R_1}(x_{1n}) \backslash x_{1n}, \\ p_{R_2} &= p_{R_2}(x_{21}) \backslash x_{21} + p_{R_2}(x_{22}) \backslash x_{22} + \ldots + p_{R_2}(x_{2n}) \backslash x_{2n} \end{aligned} \tag{3}$$

Restrictions on probability distributions (4):

$$\sum_{k=1}^{n} p_{R_1}(x_{1k}) = 1, \quad \sum_{k=1}^{n} p_{R_2}(x_{2k}) = 1 \tag{4}$$

and (5) eligibility conditions

$$\sum_{k=1}^{n} x_{1k}\, p_{R_1}(x_{1k}) = \frac{\sum_{k=1}^{n} x_{1k} \mu_{\tilde{A}_1}(x_{1k})}{\sum_{k=1}^{n} \mu_{\tilde{A}_1}(x_{1k})}, \quad \sum_{k=1}^{n} x_{2k}\, p_{R_2}(x_{2k}) = \frac{\sum_{k=1}^{n} x_{2k} \mu_{\tilde{A}_2}(x_{2k})}{\sum_{k=1}^{n} \mu_{\tilde{A}_2}(x_{2k})} \tag{5}$$

is given. Here, the composition $p_{12} = p_1 \circ p_2$ is the convolution of $R_1 * R_2$, $* \in \{+,-,\cdot,/\}$.

The 'true' probability distributions p_{R_1} and p_{R_2} for $Z_1 = (\tilde{A}_1, \tilde{B}_1)$ və $Z_2 = (\tilde{A}_2, \tilde{B}_2)$ are not precisely known. Known information is given in the form of fuzzy constraints. That is,

$$\sum_{k=1}^{n} \mu_{\tilde{A}_1}(x_{1k}) p_{R_1}(x_{1k}) \ \tilde{B}_1 - dir, \qquad \sum_{k=1}^{n} \mu_{\tilde{A}_2}(x_{2k}) p_{R_2}(x_{2k}) \ \tilde{B}_2 - dir$$

This is described in terms of membership functions:

$$\mu_{\tilde{B}_1}\left(\sum_{k=1}^{n} \mu_{\tilde{A}_1}(x_{1k}) p_{R_1}(x_{1k})\right), \mu_{\tilde{B}_2}\left(\sum_{k=1}^{n} \mu_{\tilde{A}_2}(x_{2k}) p_{R_2}(x_{2k})\right) \tag{6}$$

It means that there are fuzzy sets of probability distributions p_{R_1} and p_{R_2} with membership functions.

$$\mu_{p_{R_1}}(p_{R_1}) = \mu_{\tilde{B}_1}\left(\sum_{k=1}^{n} \mu_{\tilde{A}_1}(x_{1k}) p_{R_1}(x_{1k})\right),$$
$$\mu_{p_{R_2}}(p_{R_2}) = \mu_{\tilde{B}_2}\left(\sum_{k=1}^{n} \mu_{\tilde{A}_2}(x_{2k}) p_{R_2}(x_{2k})\right) \tag{7}$$

Then, since $\tilde{B}_1$ and $\tilde{B}_2$ are discrete, the values of

$$\mu_{\tilde{B}_j}(b_{jl}),$$
$$b_{jl} \in \operatorname{supp} \tilde{B}_j, \quad j = 1,2; l = 1,\ldots,n \tag{8}$$

μ can be found by solving a series of linear programming problems:

$$\sum_{k=1}^{n} \mu_{\tilde{A}_j}(x_k) p_j(x_k) \to b_{jl}$$
$$\left.\begin{aligned} &\sum_{k=1}^{n_j} p_j(x_{jk}) = 1 \\ &p_j(x_{jk}) \geq 0 \end{aligned}\right\} \tag{9}$$

We recognize that the "true" probability distributions p_1 and p_2 are not exact. There are fuzzy constraints μ_{p1} and μ_{p2} of p_1 and p_2 generated only by B_1 and B_2.

During the solution of the linear programming problem, we calculate the membership degrees $\mu_{pj}(x_j)$, j=1,2. Consider finding the membership degrees μ_{p1} and μ_{p2} of the distributions p_1 and p_2. It is calculated according to the formula

$$\mu_{p1}(p_1) = \mu_{B1}(\textstyle\sum_{k=1}^{n1} \mu_{A1}(x_{1k}) p_1(x_{1k}))$$

using the known values of A_1 and p_1.

For the solution of the p_j th problem, each $l = 1,\ldots,n$ index is denoted by $p_{jl} = p_j$. The membership degree of p is calculated as (10):

$$\mu_{p_j}(p_j) = \mu_{\tilde{B}_j}\left(\sum_{k=1}^{n} \mu_{\tilde{A}_j}(u_k) p_j(u_k)\right), j = 1,2. \tag{10}$$

The probability distributions p_{1l} and p_{2l} bring the fuzzy set to the fuzzy set $p_{12s},\ s=1,\dots,l^2$, and their membership functions are described as (11):

$$\mu_{p_{12}}(p_{12}) = \max_{p_1,p_2}[\mu_{p_1}(p_1) \wedge \mu_{p_2}(p_2)],\ p_{12} = p_1 \circ p_2 \tag{11}$$

Here the sign “ $\wedge$ ” is a *min* operator.

According to (12) in the next step

$$A_{12} = A_1 * A_2 \tag{12}$$

the fuzzy price is calculated:

$$P(\tilde{A}_{12}) = \sum_w p_{12}(w)\mu_{\tilde{A}_{12}}(w) \tag{13}$$

In the last step,

$$P(\tilde{A}_{12})\ P(\tilde{A}_{12}) = b_{12} \tag{14}$$

we calculate the probability measure b_{12}, where p_{12} is known. However, we only know the fuzzy constraint described by the probability function.

Thus, $P(\tilde{A}_{12})$ is a B12 fuzzy set with $\mu_{\tilde{B}_{12}}$ membership function:

$$\mu_{\tilde{B}_{12}}(b_{12s}) = \sup(\mu_{p_{12s}}(p_{12s})),\ b_{12s} = \sum_k p_{12s}(x_k)\mu_{\tilde{A}_{12}}(x_k) \tag{15}$$

Consequently, Z_{12} is calculated as:

$$Z_{12} = Z_1 * Z_2,\ * \in \{+,-,\cdot,/\}\ \ Z_{12} = (\tilde{A}_{12}, \tilde{B}_{12})$$

Figure 1. shows the probability measure indicating that the knowledge assessment threshold is correct on a 10-point scale (A).

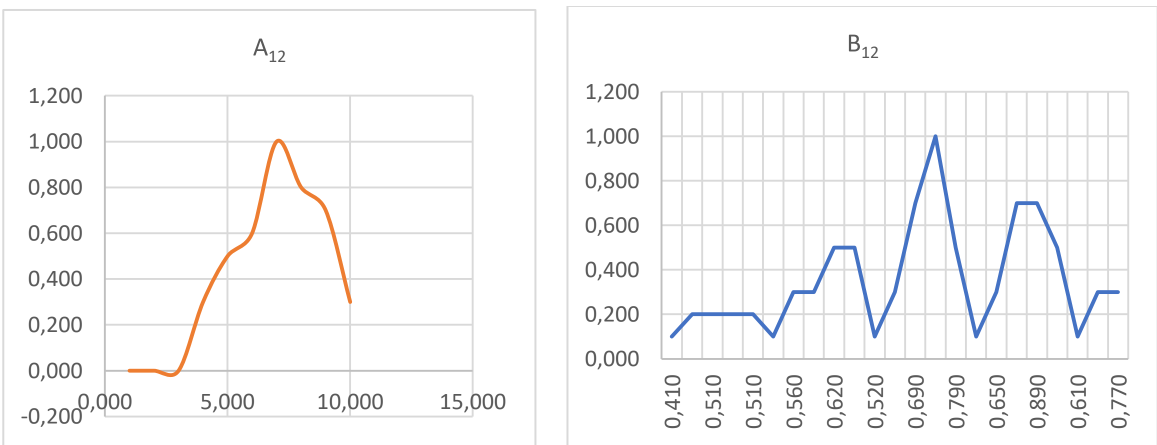

Figure 1: *Results of summing discrete Z-numbers*

IV. Result

This article presents an approach for Z-information based decision making based on exact computation on Z-numbers under conditions of incomplete information. This approach is based on finding a probability measure by applying arithmetic operations on discrete Z-numbers that describe information about the properties of random variables X_1 and X_2. This approach is widely used in solving management issues in the field of economics.

References

[1] Qardaşova L.A. Decision making in the fuzzy environment in oil refinery management systems.Baku, 2012, 327 p.

[2] Mammadova K.A. Determining the risk level for food safety in a fuzzy environment using Z-numbers. // News of Azerbaijan Engineering Academy/International scientific and technical journal. 2022, vol 14, № 1, s. 117-124.

[3] Zadeh L.A. From computing with numbers to computing with words - from manipulation of measurements to manipulation of perceptions. // IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 1999, 45(1):105–119

[4] Zadeh L.A. A Note on Z-numbers // Information Sciences, 2010, v.181, pp. 2923–2932

[5] Motamedi Z.P., Allahviranloo T. Z-Numbers for Uncertainty Formulation. Progress in Intelligent Decision Science. January 2021, pp.975-985

[6] Somayeh E., Allahviranloo T. Two new methods for ranking of Z-numbers based on sigmoid function and sign method. // International Journal of Intelligent Systems 33(7). March 2018. DOI: 10.1002/int.21987

[7] Aliev R.A., Alizadeh A.V., Huseynov O.H. The arithmetic of discrete z-numbers // Information Sciences, 2015, v.290, no 1, pp. 134-155

[8] Aliev R.A., Alizadeh A.V., Huseynov O.H., Jabbarova K.I. Z-number based Linear Programming // International Journal Of Intelligent Systems, 2015, v.30, pp. 563–589

[9] Aliev R.A., Akif V.A., Shirinova U.K. Fuzzy Chaos Approach to fuzzy linear programming problem / Proceedings of the 2nd International Conference on Soft Computing, Computing with Words and Perceptions in System Analysis, Decision and Control, Turkey, Antalya, 2003, pp.287-295

[10] Mammadova K.A., Sultanova A.B., Aliyeva Y.N., Huseynova A.N. Problem solving based on z-numbers. 14th International Conference on Applications of Fuzzy Systems, Soft Computing, and Artificial Intelligence Tools – ICAFS-2020, pp.555-564

[11] Mammadova K.A., Aliyeva Y. N., Huseynova A. N. Determining the Level of Risk for Food Safety in a Fuzzy Environment with the Help of Z Numbers. Aerosol Science and Engineering. https://doi.org/10.1007/s41810-022-00136-7 . Springer