PROFIT ANALYSIS OF REPAIRABLE WARM STANDBY SYSTEM

[1] Shiv Kant, [2]Shashi Kant, [3*]Mohit Yadav, [4]Arunita Chaukiyal, [5]Bindu Jamwal
•
[1]Department of Computer Science and Engineering (AI & DS), GNIOT, Greater Noida
[2]Department of Computer Science and Engineering, Sharda University, Greater Noida
[3*]Department of Mathematics, University Institute of Sciences,
Chandigarh University, Mohali, Chandigarh
[4]Department of Computer Science, Acharya Narendra Dev College, University of Delhi
[5]School of Engineering and Technology, Raffles University, Neemrana
kaushikshivkant@gmail.com, shashi.kant1@sharda.ac.in, mohit.e15793@cumail.in,
arunita@yahoo.com, bindujamwal1810@gmail.com
*Corresponding Author

Abstract

In the generation of science and technology, every company wants to increase the reliability of their products. So, they used the concept of warm standby redundancy, timely repair of the failed unit. This paper aims to explore the system of two non identical units where the primary unit is operative and the secondary unit is in warm standby mode. When the primary unit fails due to any fault then secondary unit starts working immediately. Here, times of failure of unit and times of repair of unit follow general distributions. Such types of systems are used in companies to prevent losses. The system's behaviour is calculated by using concepts of mean time to system failure, availability, busy period of the server, expected number of visits made by the server and profit values using the semi Markov process and regenerative point technique. Tables are used to explore the performance of the system.

Keywords: Warm standby, non identical units, regenerative point, semi Markov process

I. Introduction

Reliability and maintainability are the essential parameters of items and products that satisfy customers' requirements. In today's era, several approaches for performance improvement of maintainable systems have been adopted by scientists and engineers during designing them. A large amount of research work has been done on repairable systems such that Jack and Murthy [4] discovered the role of limited warranty and extended warranty for the product. Wang and Zhang [8] examined the repairable system of two non identical components under repair facility using geometric distributions. Deswal and Malik [3] evaluate the non identical units system under different working conditions by using the semi Markov process. Malik and Rathee [7] threw light on the two parallel units system under preventive maintenance and maximum operation time. Levitin et al. [6] explored the results of optimal preventive replacement of failed units in a cold standby system by using the poisson process.

Agarwal et al. [1] described the reliability and availability of water reservoir system under repair facility. Chaudhary and Sharma [2] explored the parallel non identical units system that gives priority to repair over preventive maintenance. Jia et al. [5] explored the two unit system under demand and energy storage techniques.

II. System Assumptions

There are following system assumptions:

- Initially, the system has two non identical units such that one is operative and the other is warm standby.
- When the operative unit fails then the warm standby unit starts working.
- An expert repairman is always available to repair the failed unit.
- The failed unit behaves like a new one after repair.
- Repair times and failure time follow general distribution.

III. System Notations

There are following system notations:

R	Collection of regenerative states S_i $(i = 0,1,3)$
O / Ds	Operative unit / warm standby unit of the system
λ / w	Failure rate of the unit/ rate by which the server needs refreshment
$q_{r,s}(t)/Q_{r,s}(t)$	PDF/ CDF of first passage time from r^th^ to s^th^ regenerative state or s^th^ failed state without halting in any other $S_i \in R$ in (0,1]
$M_r(t)$	Represents the probability of the system that it initially works $S_r \in R$ at a time (t) without moving through another state $S_i \in R$
$W_r(t)$	Probability that up to time (t) the server is busy at the state S_r without transit to another state $S_i \in R$ or before return to the same state through one or more non regenerative states
$\oplus / \otimes$	Laplace convolution / Laplace Stieltjes Convolution
$* / ** / '$	Laplace Transform/ Laplace Stieltjes Transform/ Function's derivative
○ / ● / ▭	Upstate/ regenerative state/ failed state

IV. State Descriptions

The individual state description is given by the table 1:

Table 1: *State Descriptions*

States	Descriptions
S_0	It is a regenerative up state with two units - A_0 and B_s .
S_1	This regenerative up state has two units such that (A_{Fur}) and (B_0).
S_2	It is a down state where one unit is failed under repair continuously from previous state (A_{FUR}) and the other is failed under repair (B_{Fur}).
S_3	It is a regenerative up state with two units such that one is operative (A_0) and the other is failed under repair (B_{Fur}).
S_4	It is a down state where one unit fails under repair continuously from previous state (A_{FUR}) and the other unit is failed under repair (B_{Fur}).

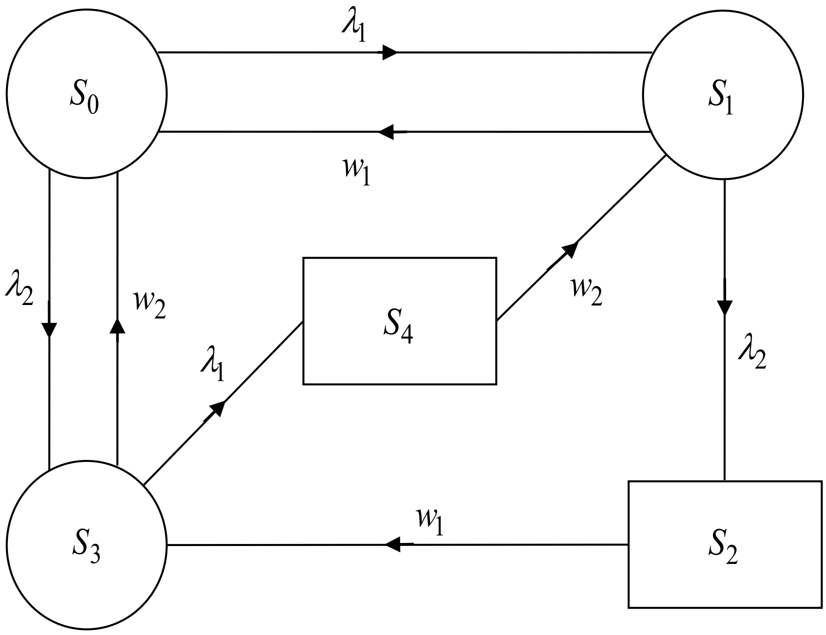

Figure 1: *State Transition Diagram*

Where, $S_0 = (A_o, B_s)$, $S_1 = (A_{Fur}, B_O)$, $S_2 = (A_{FUR}, B_{Fur})$
$S_3 = (A_O, B_{Fur})$, $S_4 = (A_{Fur}, B_{FUR})$

V. Transition Probabilities

The transition probabilities are calculated

$$p_{0,1} = \frac{\lambda_1}{\lambda_1 + \lambda_2},\ p_{0,3} = \frac{\lambda_2}{\lambda_1 + \lambda_2},\ p_{1,0} = \frac{w_1}{w_1 + \lambda_2},\ p_{1,2} = \frac{\lambda_2}{w_1 + \lambda_2}$$

$$p_{3,0} = \frac{w_2}{\lambda_1 + w_2},\ p_{3,4} = \frac{\lambda_1}{\lambda_1 + w_2},\ p_{2,3} = p_{4,1} = 1 \qquad (1)$$

It has been conclusively established that

$$p_{0,1} + p_{0,3} = 1,\ p_{1,0} + p_{1,2} = 1,\ p_{3,0} + p_{3,4} = 1 \qquad (2)$$

VI. Mean Sojourn Time

Let μ_i represents the mean sojourn time. Mathematically, time consumed by a system in a particular state is, $\mu_i = \sum_j m_{i,j} = \int_0^\infty P(T > t)\,dt$. Then

$$\mu_0 = m_{0,1} + m_{0,3} = \frac{1}{\lambda_1 + \lambda_2},\ \mu_1 = m_{1,0} + m_{1,2} = \frac{1}{w_1 + \lambda_2},\ \mu_2 = m_{2,3} = \frac{1}{w_1}$$

$$\mu_3 = m_{3,0} + m_{3,4} = \frac{1}{\lambda_1 + w_2},\ \mu_4 = m_{4,1} = \frac{1}{w_2}$$

$$\mu'_1 = m_{1,0} + m_{1,3.2} = \frac{1}{w_1 + \lambda_2},\ \mu'_3 = m_{3,0} + m_{3,1.4} = \frac{1}{\lambda_1 + w_2} \qquad (3)$$

VII. Reliability Measures Evaluations

I. Mean Time to System Failure (MTSF)

Let the cumulative distribution function of the first elapsed time be $\varphi_i(t)$ from the regenerative state S_i to the failed state of the system. Treating the failed states as an absorbing state then the repetitive interface for $\varphi_i(t)$ being

$$\varphi_0(t) = Q_{0,1}(t) \otimes \varphi_1(t) + Q_{0,3}(t) \otimes \varphi_3(t)$$

$$\varphi_1(t) = Q_{1,0}(t) \otimes \varphi_0(t) + Q_{1,2}(t)$$

$$\varphi_3(t) = Q_{3,0}(t) \otimes \varphi_0(t) + Q_{3,4}(t) \tag{4}$$

Taking LST of the relation (4) and solving for $\varphi_0^{**}(s)$ then

$$MTSF = \lim_{s \to 0} \frac{1 - \varphi_0^{**}(s)}{s} \tag{5}$$

System reliability can be obtained by using the inverse LT of equation (5). We have

$$MTSF = \frac{\mu_0 + p_{0,1}\mu_1 + p_{0,3}\mu_3}{1 - p_{0,1}p_{1,0} - p_{0,3}p_{3,0}} \tag{6}$$

II. Availability of the system

From the transition diagram, the system is available at the regenerative up states S_0, S_1 and S_2. Let $A_i(t)$ is the probability that the system is in upstate at time (t) specified that the system arrives at the regenerative state S_i at t = 0. Then the repetitive interface for $A_i(t)$ is

$$A_0(t) = M_0(t) + q_{0,1}(t) \oplus A_1(t) + q_{0,3}(t) \oplus A_3(t)$$

$$A_1(t) = M_1(t) + q_{1,0}(t) \oplus A_0(t) + q_{13.2}(t) \oplus A_3(t)$$

$$A_3(t) = M_3(t) + q_{3,0}(t) \oplus A_0(t) + q_{3,1.4}(t) \oplus A_1(t) \tag{7}$$

where, $M_0(t) = e^{-(\lambda_1+\lambda_2)t}$, $M_1(t) = e^{-(w_1+\lambda_2)t}$, $M_3(t) = e^{-(\lambda_1+w_2)t}$ (8)

Using LT of the above relation (7), there exist

$$\therefore A_0 = \lim_{s \to 0} \frac{N_A}{D_1'} = \frac{\mu_0[1 - p_{1,3}p_{3,1}] + \mu_1[p_{0,1} + p_{0,3}p_{3,1}] + \mu_3[p_{0,3} + p_{0,1}p_{1,3}]}{\mu_0[1 - p_{1,3}p_{3,1}] + \mu'_1[p_{0,1} + p_{0,3}p_{3,1}] + \mu'_3[p_{0,3} + p_{0,1}p_{1,3}]} \tag{9}$$

III. Busy Period of the Server

Let $B_i(t)$ is the probability that the repairman is busy due to the repair of the failed unit at time 't' specified that the system arrives at the regenerative state S_i at t = 0. Then the repetitive interface for $B_i(t)$ is

$$B_0(t) = q_{0,1}(t) \oplus B_1(t) + q_{0,3}(t) \oplus B_3(t)$$

$$B_1(t) = W_1(t) + q_{1,0}(t) \oplus B_0(t) + q_{1,3.2}(t) \oplus B_3(t)$$

$$B_3(t) = W_3(t) + q_{3,0}(t) \oplus B_0(t) + q_{31.4}(t) \oplus B_1(t) \tag{10}$$

where, $W_1(t) = w_1 e^{-(w_1+\lambda_2)t} + \ldots..$

$$W_3(t) = w_2 e^{-(\lambda_1+w_2)t} + \ldots.. \tag{11}$$

Using LT on relations (10) then we get

$$B_0 = \lim_{s \to 0} \frac{N_B}{D_1'} = \frac{\mu'_1 [p_{0,1} + p_{0,3} p_{3,1}] + \mu'_3 [p_{0,3} + p_{0,1} p_{1,3}]}{\mu_0 [1 - p_{1,3} p_{3,1}] + \mu'_1 [p_{0,1} + p_{0,3} p_{3,1}] + \mu'_3 [p_{0,3} + p_{0,1} p_{1,3}]} \quad (12)$$

IV. Estimated number of visits made by the server

Let $N_i(t)$ is the estimated number of visits made by the repairman for repair in (0, t] specified that the system arrives at the regenerative state S_i at t = 0. Then the repetitive interface for $N_i(t)$ is

$$N_0(t) = Q_{0,1}(t) \otimes [1 + N_1(t)] + Q_{0,3}(t) \otimes [1 + N_3(t)]$$

$$N_1(t) = Q_{1,0}(t) \otimes N_0(t) + Q_{1,3.2}(t) \otimes N_3(t)$$

$$N_3(t) = Q_{30}(t) \otimes N_0(t) + Q_{31.4}(t) \otimes N_1(t) \quad (13)$$

Using LST of the above relations (13) then we get

$$N_0 = \lim_{s \to 0} \frac{N_v}{D_1'} = \frac{1 + p_{1,3} p_{3,1}}{\begin{bmatrix} \mu_0 [1 - p_{1,3} p_{3,1}] + \mu'_1 [p_{0,1} + p_{0,3} p_{3,1}] \\ + \mu'_3 [p_{0,3} + p_{0,1} p_{1,3}] \end{bmatrix}} \quad (14)$$

V. Profit Analysis

The profit function of the system is defined by

$$P = T_0 A_0 - T_1 B_0 - T_2 N_0 \quad (15)$$

where, $T_0 = 5000$ (Revenue per unit up-time)

$T_1 = 600$ (Charge per unit for server busy period)

$T_2 = 100$ (Charge per visit made by the server)

VIII. Discussion

Let $\lambda_1 = \lambda_2 = \lambda$ and $w_1 = w_2 = w$ then the reliability measures like MTSF, availability of the system, busy period of the server, expected number of visits made by the server and profit

Table 2: *MTSF vs. Repair Rate*

w ↓	λ=0.55	λ=0.6,	λ=0.65
0.55	4.76082	4.413408	4.192308
0.6	4.929006	4.585448	4.290541
0.65	5.063985	4.728682	4.36747
0.7	5.174709	4.849785	4.429348
0.75	5.267176	4.953519	4.480198
0.8	5.345557	5.043371	4.522727
0.85	5.412844	5.121951	4.558824
0.9	5.471236	5.191257	4.589844
0.95	5.522388	5.252838	4.616788
0.1	5.567568	5.307918	4.640411

Table 3: *Availability vs. Repair Rate*

w ↓	λ=0.55	λ=0.65,	λ=0.6
0.55	0.686403	0.670466	0.657986
0.6	0.69355	0.678566	0.662905
0.65	0.699041	0.684997	0.666652
0.7	0.703393	0.690227	0.669601
0.75	0.706926	0.694563	0.671982
0.8	0.709852	0.698216	0.673945
0.85	0.712315	0.701336	0.675592
0.9	0.714416	0.704032	0.676992
0.95	0.716231	0.706384	0.678198
0.1	0.717813	0.708454	0.679247

Table 4: *Profit vs. Repair Rate*

w ↓	λ=0.55	λ=0.65,	λ=0.6
0.55	2722.902	2572.193	2533.318
0.6	2788.908	2650.579	2583.419
0.65	2838.719	2711.387	2620.861
0.7	2877.66	2759.963	2649.913
0.75	2908.947	2799.676	2673.117
0.8	2934.638	2832.757	2692.08
0.85	2956.115	2860.746	2707.868
0.9	2974.335	2884.737	2721.218
0.95	2989.99	2905.531	2732.655
0.1	3003.585	2923.73	2742.563

values are calculated. It can be seen from the tables 2, 3 and 4 that the tendency of MTSF, availability of system and profit values increase smoothly with respect to increments in repair rate(θ) whereas these values declines corresponding to increment in failure rate.

IX. Conclusion

It is calculated that the MTSF, availability and profit values of the two non identical unit system increase with respect to increments in repair rate but these reliability values decline when failure rate of unit is enhanced. The idea of repair facility is used by corporate sectors, industries, cybercafés, education, university systems, etc.

References

[1] Agrawal, A., Garg, D., Kumar, A. and Kumar, R. (2021). Performance analysis of the water treatment reverses osmosis plant. *Reliability Theory and Applications*, *16*(3): 16-25.

[2] Chaudhary, P. and Sharma, A. (2022). A two non-identical unit parallel system with priority in repair. *Reliability Theory and Applications*, *18*(3 (74)), 308-315.

[3] Deswal, S. and Malik, S. C. (2015). Reliability measures of a system of two non-identical units with priority subject to weather conditions. *Journal of Reliability and Statistical Studies*, 181-190.

[4] Jack, N., and Murthy, D. P. (2007). A flexible extended warranty and related optimal strategies. *Journal of the Operational Research Society*, 58(12), 1612–1620.

[5] Jia, H., Peng, R., Yang, L., Wu, T., Liu, D., and Li, Y. (2022). Reliability evaluation of demand based warm standby system with capacity storage.

[6] Levitin, G., Finkelstein, M., and Dai, Y. (2020). Optimal preventive replacement policy for homogeneous cold standby systems with reusable elements. *Reliability Engineering and System Safety*, *204*, 107135.

[7] Malik, S. C. and Rathee, R. (2016). Reliability modelling of a parallel system with maximum operation and repair times. *International Journal of Operational Research*, 25(1), 131-142.

[8] Wang, G. J. and Zhang, Y. L. (2007). An optimal replacement policy for repairable cold standby system with priority in use. *International Journal of Systems Science*, *38*(12), 1021-1027.