A FUZZY LOGIC APPROACH TO DESIGNING A DOUBLE SAMPLING PLANS FOR ZERO INFLATED POISSON DISTRIBUTION USING IN PYTHON

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Abstract

Acceptance sampling plan by attributes is a statistical measure used in quality control in various production process. It is mainly determined for identifying whether the lot or the batch of the product is accepted or rejected based on the number of defective items in the sample. Appropriate sampling plan provides defect-free lot. There are several sampling plans are available for determine the sample size. Among the sampling plan, double sampling plan is more effective because it is always giving best result in lot selection compared with other sampling plan. In most of the practical situation, it is very hard to found the product as strictly defective or non-defective. In some situation, quality of the product can be classified several types which are expressed as good, almost good, bad not so bad and so on. This causes ambiguity deficiency in proportion value of lot or process. In mathematical tools, fuzzy set or fuzzy logic is one of the powerful modeling, which has incomplete and imprecise information. The fuzzy set theory is adopted to cope the vagueness in these linguistic expressions for the accepting sampling. In this article double sampling plans, are determined when non-conformities are fuzzy number and being modeled based on Zero-Inflated Poisson (ZIP) distribution. The Operating Characteristic (OC) function and Average Sample Number (ASN) function are evaluated both numerically and graphically in fuzzy and crisp environments.

Keywords: Acceptance double sampling plan, Fuzzy OC curve, Fuzzy average sample number, ZIP distribution, Fuzzy parameter.

1. Introduction

The present research deals with quality control issues raised by the ZIP distribution, an elaborate statistical model used in industrial operations. Traditional sample tactics, although effective for well-behaved distributions, may struggle with the zero-inflated character of certain datasets. To cope with this, we present a unique technique which integrates fuzzy logic into the construction of a double sampling plan optimised for ZIP distribution. Fuzzy logic, popular because of its adaptability in coping with imprecision and uncertainty, strengthens the plan by simulating the unclear boundary between zero and non-zero occurrences in the ZIP distribution. This study expands on the fundamental work of Zadeh and Kosko (1965) in fuzzy logic and is consistent with recent research in quality control systems. This study has a substantial impact on industry, helping to develop quality control techniques. Our objective is to provide a robust double sampling strategy that combines fuzzy logic with ZIP distribution complexities in order to enhance decision-making and consistently detect deviations from quality requirements in an industrial context where technological breakthroughs may result in occasional zero defects. ZIP distribution used in many of fields such as agriculture, epidemiology, econometrics, public health, process control, medicine, and manufacturing.

Numerous scenarios, such as Bohning, Dietz, and Schlattmann [1] use the ZIP model on dental epidemiology data to examine excess zeros in intervention effects by ZIP regression. Fuzzy probability is examined by Buckley [2], who introduced new methods of probability theory in the environment of uncertainty. Chakraborty [3] introduces a fuzzy optimization technique for single sampling plans that minimizes inspection while managing customer risk by utilizing a Poisson distribution. Duncan AJ [4] Quality Control and Industrial Statistics is a most impact guide that combines statistical methods with practical approaches to quality control in industrial circumstances.Ezzatallah and Gildeh [5] suggest a fuzzy Poisson-based acceptance double sampling strategy for the management of unpredictable defective proportions.Janani K, Vignesh A and et al.,[6] . introduce a novel fuzzy set-based tactics for identify and conserve endemic plant species in the Nilgiris Biosphere Reserve. The work effectively addresses multi-criteria decision-making by leveraging advanced fuzzy operators, which provide improved representation of ambiguity compared to previous study's. Kavithanjali, Sheik Abdullah, and Kamalanathan [7] review SQC methodologies in single and double-sampling plans, pointing at possible effects on quality.Kavithanjali and Sheik Abdullah [8] present a inovative research by integrates of fuzzy logic ZIP distribution for SSPs, managing quality control and risk in uncertainity distribution plan. Kaviyarasu and Asif T Thottathil [9] deals the application of Zero-Inflated Poisson distribution in designing optimal acceptance sampling plans for quality control in manufacturing with a focus on special type double sampling plans.LA Zadeh [10] presents fuzzy sets and the degree of membership, which lays the groundwork for the employment of the conventional theory of sets in fuzzy control.Lambert [11] shows that ZIP regression can be employed for better data analysis in manufacturing by handling excess zeros in count data. Malathi and Muthulakshmi [12] initiate an inquiry into fuzzy logic in double-sampling plans to deal with ambiguity in quality assessments. McLachlan and Peel [13] provide a detailed reference on finite mixture models, which is vital for the analysis of complex data and heterogeneous populations. Naya, Urioste, and Chang [14] employ ZIP models, demonstrating that age is a significant factor in the occurrence of black patches. Ridout, Demetrio, and Hinde [15] provide practical horticulture examples to evaluate models for excess zeros in count data.Schilling and Neubauer [16] provide a comprehensive and authoritative guide on acceptance sampling plan, offering useful insights for quality control in numerous industries. In the context of statistical process control, Xie, He, and Goh [17] establish the ZIP distribution's superiority over the Poisson distribution for over-dispersed data.

In the following sections, we will explain the methods we used, show our results, and talk about why these findings are important for experts working in quality control. We did our analysis using Python and powerful libraries like NumPy, Pandas, SciPy, and Matplotlib to help with statistics and data visualization. We believe this research will help improve methods used to solve problems caused by difficult distributions in industrial environments.

2. Methodology

2.1. Fundamental Definitions

2.1.1 Fuzzy Number: A fuzzy number $(\tilde{N})$ is a fuzzy set on the real line R, characterized by a membership function $\mu_N : R \to [0,1]$, that satisfies the following conditions:

- $(\tilde{N})$ is normal, meaning there exists some x such that $\mu_N(x) = 1$.
- $(\tilde{N})$ is convex, meaning for any $x_1, x_2 \in R$ and $\lambda \in [0,1]$, $\mu_N(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_N(x_1), \mu_N(x_2))$.
- The membership function μ_N is upper semi-continuous, meaning the set $\{x \in R \mid \mu_N(x) \geq \alpha\}$ is closed for every $\alpha \in (0,1]$.
- The support of $(\tilde{N})$, defined as $\text{Supp}(\tilde{N}) = \{x \in R \mid \mu_N(x) > 0\}$, is bounded.

2.1.2 Triangular Fuzzy Number: A triangular fuzzy number $\tilde{N}$ is defined by a triplet (a, b, c), where $a < b < c$. The membership function $\mu_{\tilde{N}}(x)$ is given by:

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x \leq b, \\ \frac{c-x}{c-b} & \text{if } b < x \leq c, \\ 0 & \text{otherwise.} \end{cases}$$

This function forms a triangular shape with $[a, c]$ as the base and the peak at $x = b$.

2.1.3 α-Cut of Fuzzy t: The α-cut of a fuzzy set $\tilde{N}$ is a crisp set of values where the membership function is at least α. It is defined as:

$$N[\alpha] = \{x \in R \mid \mu_N(x) \geq \alpha\}.$$

The fuzzy number $\tilde{N}[\alpha]$ can be represented by its lower and upper bounds as $N^L[\alpha]$ and $N^U[\alpha]$, where:

$$N^L[\alpha] = \inf\{x \in R \mid \mu_N(x) \geq \alpha\},$$
$$N^U[\alpha] = \sup\{x \in R \mid \mu_N(x) \geq \alpha\}.$$

2.1.4 ZIP Distribution: The Zero-Inflated Poisson (ZIP) distribution, define as ZIP(φ, λ), is used when there is an more number of zero counts. The probability mass function (p.m.f.) is found in Lambert [11] and Mclachlan [13] :

$$P(D = d \mid \varphi, \lambda) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda} & \text{if } d = 0, \\ (1-\varphi)\frac{e^{-\lambda}\lambda^d}{d!} & \text{if } d = 1, 2, \ldots \end{cases}$$

In this distribution:

- φ represents the probability of extra zeros.
- λ is the mean of the underlying Poisson distribution.

The ZIP distribution mean is $(1-\varphi)\lambda$, and the variance is $\lambda(1-\varphi)(1+\varphi\lambda)$.

To extend the ZIP distribution to a fuzzy setting, we replace λ with a fuzzy number $\tilde{\lambda} > 0$. The fuzzy p.m.f. can be represented as:

$$\tilde{P}(d \mid \alpha) = \begin{cases} \varphi + (1-\varphi)e^{-\lambda} & \text{if } d = 0, \\ (1-\varphi)\frac{e^{-\lambda}\lambda^d}{d!} & \text{if } d = 1, 2, \ldots \end{cases}$$

where λ belongs to the α-cut of $\tilde{\lambda}$.

2.2. Python Programming

In this present study on statistical quality control, we used Python programming to create the upper and lower bounds of the Fuzzy Operating Characteristic (OC) Band and the Fuzzy Average Sample Number (ASN) tables. Moreover, we created graphs to visualize the Fuzzy OC, the fuzzy probability of acceptance, and the average sample number curves. Python's extensive analytical abilities and versatile libraries make it straightforward to implement these statistical methods within our study methodology.

3. Operating procedure for DSP under ZIP distribution conditions

Let us consider a circumstance where we analyse the N- lot size for defects with Zero-Inflated Poisson (ZIP) distribution. These are general steps of the typical double sampling plan.
Step 1:

- Take a random sample of size n_1 and count the number of defective items (D_1).
- c_1 is the acceptance number for the first sample.
- c_2 is the acceptance number for both combined samples.

Step 2:

- Accept the lot if $D_1 \leq c_1$.
- Reject the lot if $D_1 > c_2$.
- If $c_1 < D_1 \leq c_2$, proceed to Step 3.

Step 3:

- Take a random sample from second sample n_2 and count the number of defective items (D_2).
- Add D_1 and D_2 together.
- Accept the lot if $D_1 + D_2 \leq c_2$, otherwise reject it.

Step 4:

- The random variables D_1 and D_2 follows the ZIP distribution with parameter $\lambda_1 = n_1 p$ and $\lambda_2 = n_2 p$, given a large sample size and a small probability p.
- Let P_a stand for the acceptance probability of the lot based onto the combined samples.
- $\tilde{P}_a^I$ is for the acceptance probability after the first sample and $\tilde{P}_a^{II}$ for the second sample.

Thus, the overall probability of acceptance is:

$$\tilde{P}_a = \tilde{P}_a^I \cdot \tilde{P}_a^{II}$$

Using the ZIP distribution pmf, the number of nonconforming items in the lot is given by

$$\tilde{\mathrm{P}}\left(\mathrm{D} = \mathrm{d} \mid \varphi, \lambda\right) = \tilde{\mathrm{P}}\left(\mathrm{d}\right) = \begin{cases} \varphi + (1-\varphi)\ e^{-\lambda}, & \text{When d} = 0 \\ (1-\varphi)\ \frac{e^{-\lambda}\lambda^d}{d!}, & \text{When d } = 1, 2, \ldots, 0 < \varphi < 1,\ \lambda > 0 \end{cases}$$

Given a sample size of n_1, the probability of finding no deficiencies will be

$$\tilde{P}\left(D = 0\right) = \tilde{P}_a^I\left(\alpha\right) = \varphi + (1-\varphi)\, e^{-n_1 p} \tag{1}$$

Given a sample size of n_2, the probability of finding one deficiencies will be

$$\tilde{P}\left(D = 1, D_1 + D_2 \leq 1\right) = \tilde{P}_a^{II}\left(\alpha\right) = (1-\varphi)\, \mathrm{e}^{-np}\, n_2 p \tag{2}$$

From a sample of size n_1 the probability of finding one or less defects will be

$$\tilde{P}\left(D \leq 1\right) = \varphi + (1-\varphi)\, e^{-n_1 p}\left(1 + n_1 p\right) \tag{3}$$

From a sample of size n_2 the probability of finding one or more defects will be

$$\tilde{P}_a^{II}\left(\alpha\right) = (1-\varphi)\, e^{-np}\,(0.5)(n_1 p)^2 \tag{4}$$

A DSP only accepts a lot if a sample of size n_1 has no faults and a sample of size n_2 has one defect or less. Thus, DSP's $\tilde{P}_a\left(\alpha\right)$ will be provided by

$$\tilde{P}_a\left(\alpha\right) = P_a^I\left(\alpha\right) + P_a^{II}\left(\alpha\right) \tag{5}$$

3.1. Numerical illustration 1

Consider that $\tilde{P} = (0.01, 0.02, 0.03)$, N=200, n_1=10, n_2=10, c_1= 0 ,c_2=1, $\tilde{\lambda} = n\tilde{p}$, $\varphi = 0.0001$,n=$n_1 + n_2$.

$$\tilde{P}[\alpha] = [0.01 + 0.01\alpha\,,\, 0.03 - 0.01\alpha]$$

From equation(1) the fuzzy probability of a sample of size n_1 having no faults is thus as follows:

$$\tilde{P}(D = 0) = \tilde{P}_a^I(\alpha) = \varphi + (1-\varphi)\, e^{-10p}$$

$$\tilde{P}_a^I[\alpha] = \left\{\varphi + (1-\varphi)\left(e^{-(0.3-0.1\alpha)}\right),\ \varphi + (1-\varphi)\left(e^{-(0.1+0.1\alpha)}\right)\right\}$$

From equation (2) the fuzzy probability of a sample of size n_2 having one fault is thus as follows:

$$\tilde{P}(D = 1, D_1 + D_2 \leq 1) = \tilde{P}_a^{II}(\alpha) = (1-\varphi)e^{-20p} \cdot 10p$$

$$\tilde{P}_a^{II}[\alpha] = \left\{(1-\varphi)\, e^{-(0.2+0.2\alpha)}\,(0.1 + 0.1\alpha)\,,\,(1-\varphi)\, e^{-(0.6-0.2\alpha)}\,(0.3 - 0.1\alpha)\right\}$$

From equation (5) a DSP only accepts a lot if a sample of size n_1 has no faults and a sample of n_2 has one defect or less. Thus, DSP's $\tilde{P}_a(\alpha)$ will be provided by

$$\begin{aligned}\tilde{P}_a[\alpha] &= \tilde{P}_a^I(\alpha) + \tilde{P}_a^{II}(\alpha)\\ &= \left\{\varphi + (1-\varphi)\, e^{-(0.3-0.1\alpha)} + (1-\varphi)\, e^{-(0.6-0.2\alpha)}\,(0.3 - 0.1\alpha)\,,\right.\\ &\quad \left.\varphi + (1-\varphi)\, e^{-(0.1+0.1\alpha)} + (1-\varphi)\, e^{-(0.2+0.2\alpha)}\,(0.1 + 0.1\alpha)\ \right\}\end{aligned}$$

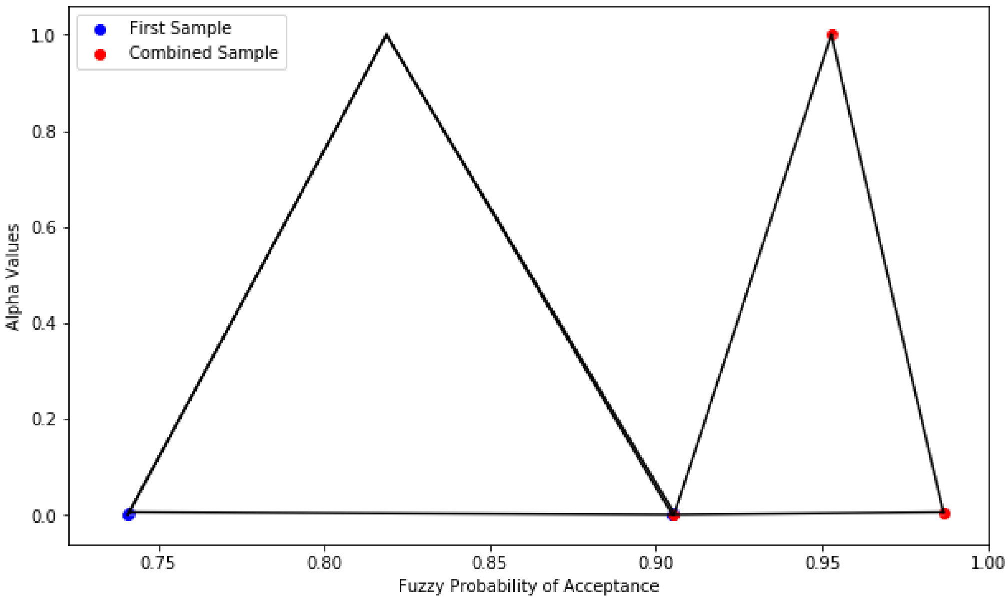

Figure 1: *Fuzzy probability of acceptance with DSPs under ZIP distribution*

Table 1: *Fuzzy probability of acceptance with DSP* $\varphi = 0.0001$, α= *0,0.005,1*

α	$\tilde{P}_a^I$	$\tilde{P}_a^{II}$	$\tilde{P}_a[\alpha]$
0.000	[0.740844, 0.904847]	[0.081865, 0.164627]	[0.905471, 0.986712]
0.005	[0.741215, 0.904395]	[0.082192, 0.164517]	[0.905732, 0.986587]
1.000	[0.818749, 0.818749]	[0.134051, 0.134051]	[0.952799, 0.952799]

As in Figure 1 and Table 1, it shows that 98 to 100 out of every 100 lots accepted in this process.

4. Fuzzy OC curve for DSPs under ZIP distribution

The fuzzy parameter is applied to construct the operating characteristic (OC) curve for the DSP. This curve demostrate the relationship between the fraction of nonconforming items, denoted as p, and the acceptance probability is $P_a(p)$. The OC curve helps differentiate between good and bad lots in the sampling plan. Producer's risk occurs when a customer rejects a product that actually meets the quality standards (i.e., the product is of good quality). On the other hand, consumer's risk happens when a customer accepts a product that fails to meet the standards (i.e., the product quality is poor). A fuzzy parameter with upper and lower bounds can be used to estimate the defective fraction. If these bounds are equal, the process is considered to be in an optimal state.

4.1. Numerical illustration 2

Consider that, $\tilde{P} = (0.01,\ 0.02,\ 0.03)$, $\varphi = 0.0001$, n_1=10, n_2=5, c_1=1, c_2=2,$n = n_1 + n_2$, b_2=0.01, b_3=0.02 following that we have, $\tilde{P}\,[\alpha] = [k + 0.01\alpha,\ 0.02 + k - 0.01\alpha]$, $0 \leq k \leq 0.98$.
Accordingly, the first sample fuzzy probability of acceptance using equation (3) is

$$\tilde{P}_a^I(\alpha) = \tilde{P}[0,1][\alpha] = [\varphi + (1-\varphi)e^{-10p}(1+10p)], f_1(p) = [\varphi + (1-\varphi)e^{-10p}(1+10p)]$$

Since is decreasing, as follows:

$$\tilde{P}_a^I[0] = \left[\varphi + (1-\varphi)\, e^{-(10k+0.2)}\ (1 + (10k+0.2))\,,\ \varphi + (1-\varphi)\, e^{-(10k)}\ (1+10k)\right]$$

and accordingly the second sample's fuzzy probability of acceptance from equation (4) is

$$\tilde{P}_a^{II}(\alpha) = \tilde{P}(D_1 = 2,\ D_2 = 0) = \left[(1-\varphi)\, e^{-15p}\ 50p^2\right], f_2\,(p) = \left[(1-\varphi)\, e^{-15p}\ 50p^2\right]$$

$$\tilde{P}_a^{II}\,(0) = \left[(1-\varphi)\, e^{-15k}\ 50\ (k^2),\ (1-\varphi)\, e^{-15(k+0.02)}\ 50(k+0.02)^2\ \right]\ 0 \leq k \leq 0.98$$

Next, we will obtain the α-cut by examining the $f_2(p)$ function in the following manner:

$$\tilde{P}_a^{II}\,(\alpha) = \begin{cases} (1-\varphi)\, e^{-15k}\ 50\ k^2,\ (1-\varphi)\, e^{-15(k+0.02)}\ 50(k+0.02)^2 & ,0 \leq k < \frac{1.7}{15} \\ (1-\varphi)\, e^{-15k}\ 50\ k^2\ ,\ 0.12 & ,\frac{1.7}{15} \leq k < 0.12 \\ (1-\varphi)\, e^{-15(k+0.02)}\ 50(k+0.02)^2,\ 0.12 & ,0.12 \leq k < \frac{2}{5} \\ (1-\varphi)\, e^{-15(k+0.02)}\ 50(k+0.02)^2\ , (1-\varphi)\, e^{-15k}\ 50\ k^2 & ,\frac{2}{5} \leq k \leq 0.98 \end{cases}$$

$$\tilde{P}_a[\alpha] = \tilde{P}_a^I\,(0) + \tilde{P}_a^{II}\,(0)$$

Table 2: *Sample values at different k levels with acceptance DSP using ZIP distribution*

k	$\tilde{P}_a^I$	$\tilde{P}_a^{II}$	$\tilde{P}_a(\alpha)$
0.00	[0.982479, 1.000000]	[0.000000, 0.014815]	[0.997294, 1.000000]
0.01	[0.963067, 0.995322]	[0.004303, 0.028690]	[0.991758, 0.999625]
0.02	[0.938454, 0.982479]	[0.014815, 0.043901]	[0.982355, 0.997294]
0.03	[0.909805, 0.963067]	[0.028690, 0.059040]	[0.968845, 0.991758]
0.04	[0.878111, 0.938454]	[0.043901, 0.073175]	[0.951286, 0.982355]
0.05	[0.844211, 0.909805]	[0.059040, 0.085726]	[0.929937, 0.968845]
0.06	[0.808811, 0.878111]	[0.073175, 0.096373]	[0.905184, 0.951286]
0.07	[0.772505, 0.844211]	[0.085726, 0.104982]	[0.877487, 0.929937]

k	$\tilde{P}_a^I$	$\tilde{P}_a^{II}$	$\tilde{P}_a(\alpha)$
0.08	[0.735785, 0.808811]	[0.096373, 0.111554]	[0.847339, 0.905184]
0.09	[0.699059, 0.772505]	[0.104982, 0.116179]	[0.815238, 0.877487]
0.10	[0.662647, 0.735785]	[0.111554, 0.118965]	[0.781683, 0.847339]
0.11	[0.626792, 0.699059]	[0.116179, 0.120058]	[0.747170, 0.815238]
0.12	[0.591660, 0.662647]	[0.118965, 0.119622]	[0.712177, 0.781683]
0.13	[0.557353, 0.626792]	[0.120058, 0.117834]	[0.677139, 0.747170]
0.14	[0.523917, 0.591660]	[0.119622, 0.114881]	[0.642457, 0.712177]
0.15	[0.491356, 0.557353]	[0.117834, 0.110947]	[0.608493, 0.677139]
0.16	[0.462891, 0.524978]	[0.116107, 0.108862]	[0.571753, 0.641086]
0.17	[0.433806, 0.493296]	[0.112817, 0.104399]	[0.538204, 0.606113]
0.18	[0.406065, 0.462891]	[0.108862, 0.099564]	[0.505629, 0.571753]
0.19	[0.379677, 0.433806]	[0.104399, 0.094479]	[0.474156, 0.538204]
0.20	[0.354635, 0.406065]	[0.099564, 0.089248]	[0.443883, 0.505629]
0.21	[0.330921, 0.379677]	[0.094479, 0.083959]	[0.414880, 0.474156]
0.22	[0.308510, 0.354635]	[0.089248, 0.078684]	[0.387195, 0.443883]
0.23	[0.287369, 0.330921]	[0.083959, 0.073486]	[0.360854, 0.414880]
0.24	[0.267458, 0.308510]	[0.078684, 0.068411]	[0.335869, 0.387195]
0.25	[0.248736, 0.287369]	[0.073486, 0.063498]	[0.312234, 0.360854]
0.26	[0.231155, 0.267458]	[0.068411, 0.058777]	[0.289932, 0.335869]
0.27	[0.214669, 0.248736]	[0.063498, 0.054268]	[0.268937, 0.312234]
0.28	[0.199228, 0.231155]	[0.058777, 0.049985]	[0.249214, 0.289932]
0.29	[0.184783, 0.214669]	[0.054268, 0.045939]	[0.230722, 0.268937]
0.30	[0.171284, 0.199228]	[0.049985, 0.042132]	[0.213416, 0.249214]
0.31	[0.158682, 0.184783]	[0.045939, 0.038565]	[0.197247, 0.230722]
0.32	[0.146928, 0.171284]	[0.042132, 0.035236]	[0.182163, 0.213416]
0.33	[0.135975, 0.158682]	[0.038565, 0.032138]	[0.168112, 0.197247]
0.34	[0.125777, 0.146928]	[0.035236, 0.029265]	[0.155041, 0.182163]
0.35	[0.116289, 0.135975]	[0.032138, 0.026607]	[0.142896, 0.168112]
0.36	[0.107469, 0.125777]	[0.029265, 0.024155]	[0.131624, 0.155041]
0.37	[0.099275, 0.116289]	[0.026607, 0.021899]	[0.121175, 0.142896]
0.38	[0.091669, 0.107469]	[0.024155, 0.019828]	[0.111497, 0.131624]
0.39	[0.084612, 0.099275]	[0.021899, 0.017933]	[0.102542, 0.121175]
0.40	[0.078069, 0.091669]	[0.019828, 0.016195]	[0.094264, 0.111497]
0.41	[0.072006, 0.084612]	[0.017930, 0.014610]	[0.086617, 0.102542]
0.42	[0.066391, 0.078069]	[0.016195,0.013167]	[0.079558, 0.094264]

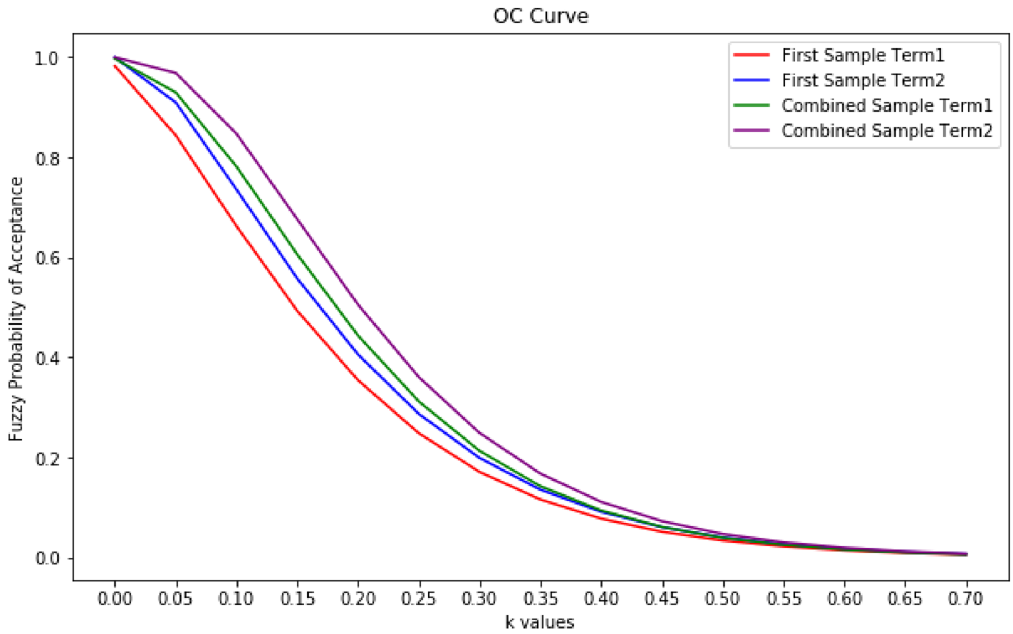

Figure 2: *FOC band with DSP using ZIP distribution* $n_1 = 10, c_1 = 1, n_2 = 5, and c_2 = 2$

The instance shown in Figure 2 is what happens when process quality drops from a perfect condition to a modest state, at which point the FOC band widens. Table 2 will be computed by simplifying and using fuzzy arithmetic.

5. Fuzzy Average Sample Number (FASN) with DSPs using ZIP distribution

The first benefit of a DSP is that it requires a small average sample size to make a sensible choice. In the context of this technique, the sample number is determined by either n_1 or n_2, which must add up to n_1+n_2. When deciding on the first sample (with fuzzy probability $\tilde{P}$ ($D_1 \leq c_1$ or $D_1 > c_2$,)) the $\tilde{P}^I$describes the fuzzy probability of drawing the first sample. If the first sample gives an uncertain result $\tilde{P}$($c_1 < D_1 \leq c_2$), a second sample with a total population of n_1+n_2 is required. This is represented by the fuzzy probability and will be referred to as $\tilde{P}^{II}$. The fuzzy mean formula is used to determine the FASN: $FASN = \tilde{\mu}_{SN}(\alpha) = \{n_1 p_1 + (n_1 + n_2)\, p_{II}\}$. $P_i \in \tilde{P}_i(\alpha), i = I, II,\ P_I + P_{II} = 1$ Thus, we obtain $FASN = \{n_1 + n_2 p_{II}\}$

The following illustration is based on FASN using illustration I
$\tilde{P} = (0.01,\ 0.02,\ 0.03)$,N=200, n_1=10, n_2=10 ,c_1= 0 ,c_2=1, $\tilde{\lambda} = np$, $\varphi = 0.0001$, $n = n_1 + n_2$

$$\tilde{P}(\alpha) = [0.01 + 0.01\alpha,\ 0.03 - 0.01\alpha]$$

$$FASN = \left\{10 + 10\left[(1-\varphi)e^{-10p}\,10p\right]\right\}$$

$$FASN = \left\{10 + (1-\varphi)\ e^{-10p}\,100p\right\}$$

$$FASN(\alpha) = \left\{10 + (1-\varphi)\ e^{-(0.1+0.1\alpha)}\,(1+\alpha)\ ,10 + (1-\varphi)\ e^{-(0.3-0.1\alpha)}\,(3-\alpha)\right\}$$

Under$\alpha = 0$ we gain $FASN(0) = 10.90, 12.22$

The figure 3 shows triangular fuzzy graph illustrates how the FASN adapts to varying degrees of ambiguity, illustrating the adaptability nature of the sampling plan under different fuzzy probability conditions.The Average Sample Number (ASN) curve in double sampling plans is used to assess sampling efficiency and resource optimisation. It helps to compare different plans and get insights into sampling performance under varied lot characteristics. ASN curves help in decision-making by indicating the predicted number of samples needed for acceptance or rejection at various quality standards. This ensures balanced inspection costs and quality control. and the sample size relies on whether or not a second sample is needed. We may depict the FASN band in terms of $\tilde{P}$, the fuzzy proportion faulty in an entering lot, using the $\tilde{P}$ structure that was developed.

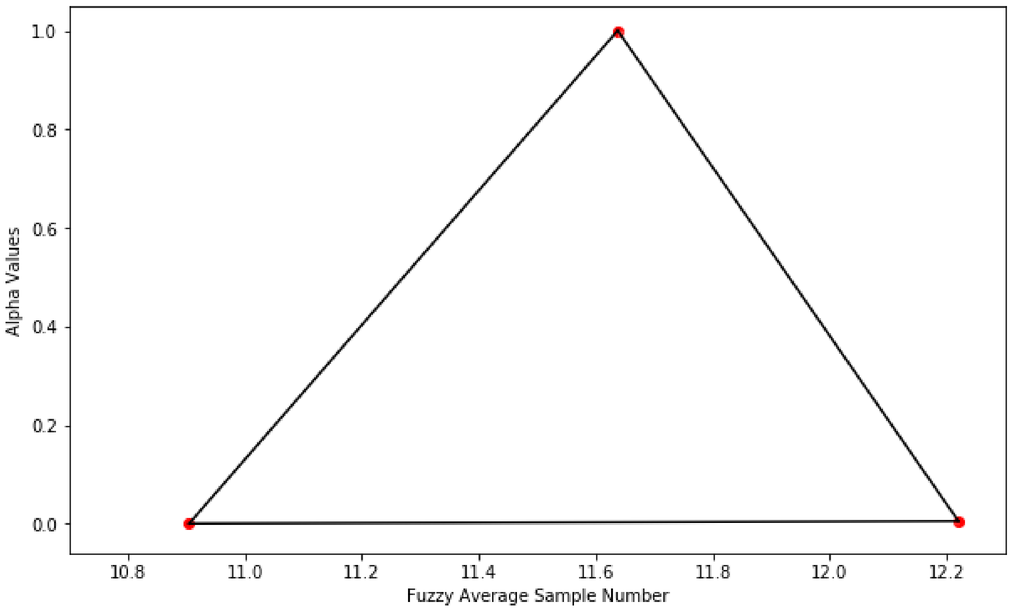

Figure 3: *FASN for a DSP with using ZIP distribution c_1 = 0, c_2 = 1, n_1 = n_2 = 10*

5.1. Numerical illustration 3

Let that $\varphi = 0.0001$, $\mathbf{c}_1$= 0, $\mathbf{c}_2$ = 1, N=200, $n_1 = n_2$ =10 and b2 = 0.01, b3 = 0.02. Then FASN is obtained as follows

$$\tilde{P}\,(\alpha) = [k + 0.01\alpha\,,\, k + 0.02 - 0.01\alpha]$$

$$FASN = \left\{10 + (1-\varphi)\,e^{-10p}\,100p\right\}$$

And α-cut of FASN is:

$$FASN\,(\alpha) = \begin{cases} FASN^{*},\, FASN^{**} & ,0 \le k < 0.08 \\ FASN^{*},\, 13.68^{**} & ,0.08 \le k < 0.09 \\ FASN^{**},\, 13.68 & ,0.09 \le k < 0.1 \\ FASN^{**},\, FASN^{*} & ,0.1 \le k < 0.98 \end{cases}$$

$$FASN^{*} = \left\{10 + (1-\varphi)\;e^{-(10k+0.1\alpha)}100\,(k+0.01\alpha)\;\right\} \text{ and}$$
$$FASN^{**} = \left\{10 + (1-\varphi)\;e^{-(10k+0.2-0.1\alpha)}100\,(k+0.02-0.01\alpha)\;\right\}$$

Figure 4 and table 3 shows the FASN band, the first sample will determine whether the batch is accepting or reject, depending on how good the process is executed. This implies that the sample number will be less, and if the process quality is average, most of the time, while selecting whether to accept or reject the lot, a second sample should be picked, increasing the sample size.

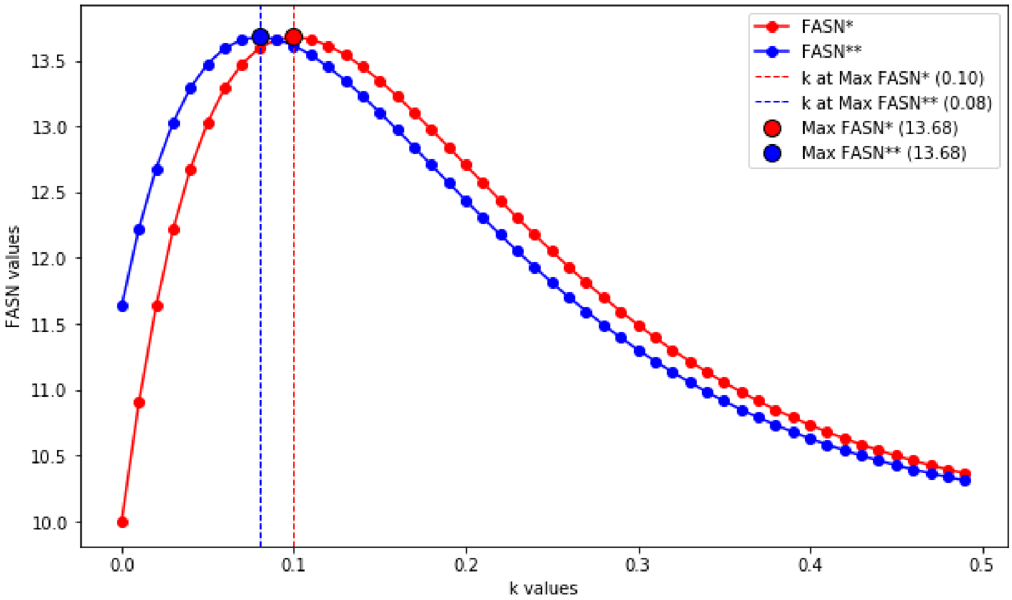

Figure 4: *FASN band for a DSP with using ZIP distribution of* $c_1 = 0, c_2 = 1, n_1 = n_2 = 10$

Table 3: *FASN for a DSP with using ZIP distribution* $c_1 = 0, c_2 = 1, n_1 = n_2 = 10$

k	FASN*	FASN**	k	FASN*	FASN**
0.00	10.0000	11.6373	0.21	12.5713	12.3057
0.01	10.9047	12.2222	0.22	12.4374	12.1770
0.02	11.6373	12.6810	0.23	12.3057	12.0519
0.03	12.2222	13.0324	0.24	12.1770	11.9309
0.04	12.6810	13.2925	0.25	12.0519	11.8144
0.05	13.0324	13.4757	0.26	11.9309	11.7025
0.06	13.2925	13.5943	0.27	11.8144	11.5955
0.07	13.4757	13.6588	0.28	11.7025	11.4935
0.08	13.5943	13.6784	0.29	11.5955	11.3964
0.09	13.6588	13.6612	0.30	11.4935	11.3043
0.10	13.6784	13.6140	0.31	11.3964	11.2170
0.11	13.6612	13.5426	0.32	11.3043	11.1346
0.12	13.6140	13.4520	0.33	11.2170	11.0568
0.13	13.5426	13.3466	0.34	11.1346	10.9836
0.14	13.4520	13.2300	0.35	11.0568	10.9147
0.15	13.3466	13.1053	0.36	10.9836	10.8500
0.16	13.2300	12.9751	0.37	10.9147	10.7894
0.17	13.1053	12.8415	0.38	10.8500	10.7326
0.18	12.9751	12.7064	0.39	10.7894	10.6794
0.19	12.8415	12.5713	0.40	10.7326	10.6298
0.20	12.7064	12.4374	0.41	10.6794	10.5834

6. Conclusion

In this study, we proposed a fuzzy ZIP distribution-based technique for developing acceptance double sampling plans (DSP) using fuzzy features. These plans are clearly defined since the results are consistent with classical plans when the proportion of damaged items is sharp. The primary parameters of the DSP, proportion defective, and sample size, are considered a triangle fuzzy number. With these parameters, the fuzzy operating characteristic and average sample number curves are generated and given in this paper. It was demonstrated that the plans OC and ASN curves resemble a band lower boundaries. Under this approach, FASN will have a lesser value depending on how perfect or inadequate the process quality.

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