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A PRODUCTION INVENTORY MODEL WITH TIME-DEPENDENT DEMAND, PRODUCTION AND DETERIORATION OVER A FINITE PLANNING HORIZON WITH TWO STORAGES

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Abstract

The complexities of time-dependent demand, production rates, and deterioration over a limited planning horizon are taken into consideration in our comprehensive production inventory model, which has two distinct storage facilities. In our approach, these elements work together to provide a unified framework that maximizes inventory management strategies while staying within realistic bounds. Specifically, considering the effects of both short- and long-term deterioration, we look into how various demand trends and production capacity affect stock levels and storage decisions. Organizations can lower the risk of rotting and enable dynamic modifications to production schedules by employing a dual-storage method to assess inventory allocation in greater detail. Our model makes use of advanced optimization techniques to offer useful insights into how to meet fluctuating demand while controlling the expenses of manufacturing, storage, and inventory. We demonstrate the model's efficacy and adaptability through numerical simulations and sensitivity analyses, offering managers a valuable instrument to enhance operational efficiency in scenarios including time-varying variables. This research improves the field by offering a strong solution framework for inventory management in complex scenarios with dual storage considerations, paving the way for more reliable and effective production strategies.

Keywords: Production Inventory Optimization; Time-Dependent Demand; Deterioration Management; Dual Storage Systems; Finite Planning Horizon.

1. Introduction and Review of Existing Research

One of the important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory under goes decay or deterioration. Most of the researchers in inventory system were directed towards non-deteriorating products. However, there are certain substances, whose utility do not remain same with the passage of time. Deterioration of these items plays an important role and items cannot be stored for a long time. Deterioration of an item may be defined as decay, evaporation, obsolescence, loss of utility or marginal value of an item that results in the decreasing usefulness of an inventory from the original condition. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be the deterioration of items in the inventory system, which may occur due to one or many factors i.e. storage conditions, weather conditions or due to humidity.

Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as alcohol, gasoline, etc. undergo physical depletion over time through the process of evaporation. Electronic goods, photographic film, grain, chemicals, pharmaceuticals etc. deteriorate through a gradual loss of potential or utility with the passage of time. Thus deterioration of physical goods in stock is very realistic feature. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored. In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accumulate more practical features of the real inventory systems.

The analysis of deteriorating inventory began with Ghare and Schrader [4], who established the classical no-shortage inventory model with a constant rate of decay. However, it has been empirically observed that failure and life expectancy of many items can be expressed in terms of Weibull distribution. This empirical observation has prompted researchers to represent the time to deterioration of a product by a Weibull distribution. Covert and Philip [1] extended Ghare and Schrader™s [4] model and obtain an economic order quantity model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. Philip [12] presented an EOQ model for items with Weibull distribution deterioration rate. An order level inventory model for a system with constant rate of deterioration was presented by Shah and Jaiswal [14]. Roychowdhury and Chaudhuri [13] formulated an order level inventory model for deteriorating items with finite rate of replenishment. Hollier and Mak [8] developed inventory replenishment policies for deteriorating item with demand rate decreases negative exponentially and constant rate of deterioration. Datta and Pal [3] investigated an order level inventory model with power demand pattern with a special form of Weibull function for deterioration rate, considering deterministic demand as well as probabilistic demand. An EOQ model for deteriorating items with a linear trend in demand was formulated by Goswami and Chaudhuri [7]. Inventory models for perishable items with stock dependent selling rate were suggested by Padmanabhan and Vrat [11]. The selling rate was assumed to be a function of current inventory level and rate of deterioration was taken to be constant with complete, partial backlogging and without backlogging. Su et al. [15] formulated a deterministic production inventory model for deteriorating items with an exponential declining demand over a fix time horizon. A production inventory model for deteriorating items with exponential declining demand was discussed by Kumar and Sharma [9]. Time horizon was fixed and the production rate at any instant was taken as the linear combination of on hand inventory and demand. A single-vender and multiple-buyers production-inventory policy for a deteriorating item was formulated by Yang and Wee [17]. Production and demand rates were taken to be constant. A mathematical model incorporating the costs of both the vender and the buyers was developed. Goyal and Giri [6] considered the production-inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. Shortages of a cycle were allowed to be partially backlogged. An order-level inventory model for a deteriorating item with Weibull distribution deterioration, time-quadratic demand and shortages was suggested by Ghosh and Chaudhuri [5]. An economic production quantity model for deteriorating items was discussed by Teng and Chang [16]. Demand rate was taken as dependent on the display stock level and the selling price per unit. An order level inventory system for deteriorating items has been discussed by Manna and Chaudhuri [10]. The demand rate was taken as ramp type function of time and the finite production rate was proportional to the demand rate at any instant. The deterioration rate was time proportional and the unit production cost was inversely proportional to the demand rate. A note on the inventory models for deteriorating items with ramp type demand was developed by Deng et al. [2]. They have proposed an extended inventory model with ramp type demand rate and its optimal feasible solution. In their study, Yadav et al. [18] explore an inventory optimization model for decaying items that reduces carbon emissions through investments in environmentally friendly technologies. Inventory management aims to achieve a balance between environmental and financial objectives.

Yadav, Yadav, and Bansal [19] recommend utilizing an interval number strategy that combines investments in preservation technology with analytical optimization techniques to optimize the two-warehouse inventory management of degrading commodities. Their methodology enhances inventory efficiency and makes better decisions in the face of uncertainty. Mahata and Debnath [20] study a profit-maximizing inventory model that accounts for storage deterioration for a single item. Their study focuses on profit maximization but also considers how preservation techniques impact inventory management. Mahata and Debnath [21] look at the effects of RD investment and screening on an interval number unknown Economic Production Rate (EPR) inventory model. Their objective is to optimize inventory control while accounting for manufacturing defects and research expenses. Yadav, Yadav, and Bansal's [22] study optimizes a model for degrading products that considers storage degradation in a two-warehouse system with the goal of improving inventory management. Their approach builds on prior research by combining complex deterioration dynamics with multi-warehouse logistics to enhance inventory efficiency and management.

In this paper a production model is developed for deteriorating items with two-storage facility and inflation. We assume that the deterioration rates of the items stored are different in the two warehouses due to the difference in the environment conditions or preserving conditions. We assume that demand rate, production rate and deterioration rates all are functions of time. Planning horizon is taken finite and the parameters of demand function are assumed to remain constant during the finite planning horizon, rather than being reset at the beginning of each production cycle.

2. Identified Gaps and Our Contributions

The existing literature on inventory management often addresses time-dependent demand, production rates, and deterioration rates in isolation instead of combining them into a single model. Moreover, most models ignore the real significance of time restrictions in industries with discrete product life cycles by assuming infinite planning horizons. Furthermore, previous research has a propensity to ignore the complex interactions that occur between different types of storage in dynamic situations and oversimplify dual storage systems. Furthermore, insufficient research has been conducted to investigate the interplay of these time-dependent variables in various storage systems, leading to less-than-optimal decisions. To close these gaps, our work creates a comprehensive production inventory model that takes into account two distinct storage strategies and time-dependent output, deterioration, and demand across a limited planning horizon. We also propose feasible methods to handle the related complex inventory problems, closing the gap between theoretical models and practical application.

3. Assumptions and Notations

3.1. Assumptions

The following assumptions are used in this study:

1. Lead time is zero and the initial inventory level is zero.
2. Shortages are not allowed.
3. Deterioration is considered only after the inventory is stored in the warehouse.
4. The OW has a fixed capacity of w units and the RW has unlimited capacity.
5. The inventory cost (including carrying cost and deterioration cost) in RW are higher than those in OW.
6. The difference between transportation costs from OW to customers and that from RW to customers is negligible.

7. Production rate is time-dependent, given by $p = a + bf(t)$.
8. Deterioration rates in OW and RW are considered as a linear function of t, as follows $\alpha = a_1 + b_1 t$ and $\beta = a_2 + b_2 t$.

3.2. Notations

The notation used in this model are shown as follows

Table 1: *Notations*

Notation	Units	Description
H	–	total planning horizon
$p = a + bf(t)$	–	production rate
$f(t)$	–	demand rate at time t with 0<f(t)<p.
W	unit	fixed capacity level of OW.
$\alpha = a_1 + b_1 t$	–	deterioration rate of inventory items in OW with $a_1, b_1 > 0$
$\beta = a_2 + b_2 t$	–	deterioration rate of inventory items in RW with $a_2, b_2 > 0$
r	–	inflation rate.
n	constant	number of production cycles during the entire horizon H.
L_1	–	a category of production cycle that only OW is used within the cycle.
L_2	–	a category of production cycle that both OW and RW are used.
m	–	the index of production cycle whose type switches from L_2 to L_1 or from L_1 to L_2.
t_{i0}	Time	the time at the beginning of the ith production cycle belonging to L_2.
t_{i1}	Time	the time at which the inventory level in OW first reaches W units within the ith production cycle.
t_{i2}	Time	the time at the end of production of the ith production cycle.
t_{i3}	Time	the time at which all inventory units in RW are depleted within the ith production cycle.
$I_{i1}(t)$	–	inventory level in OW at time t with $t \in [t_{i0}, t_{i1}]$.
$I_{i2}(t)$	–	inventory level in RW at time t with $t \in [t_{i1}, t_{i2}]$.
$I_{i3}(t)$	–	inventory level in RW at time t with $t \in [t_{i2}, t_{i3}]$.
$I_{i4}(t)$	–	inventory level in OW at time t with $t \in [t_{i3}, t_{i+1,0}]$.
$I_{i5}(t)$	–	inventory level in OW at time t with $t \in [t_{i1}, t_{i3}]$.
t_{j0}	Time	the time at the beginning of the jth production cycle belonging to L_1.
t_{j2}	Time	the time at the end of production of the jth production cycle.
U_j	–	the maximum inventory level during the jth production cycle.
D_i	–	the quantity of deteriorated items during the ith production cycle.
D_j	–	the quantity of deteriorated items during the jth production cycle.
C_1	Cost	setup cost per production run.
C_2	Cost	cost of a deteriorated unit.
C_{OW}	Cost	carrying cost per inventory unit held in OW per unit time.
C_{RW}	Cost	carrying cost per inventory unit held in RW per unit time..
TC	Cost	total system cost during H.

4. The Mathematical model

The behavior of inventory level with time-dependent demand and production rate per deteriorating items can be represented as shown in figure1. Fig.1 (a) portrays the inventory level during a production cycle in which both OW and RW are used. Within any arbitrary production cycle i for L_2-system the cycle starts from t_{i0}, at which production, demand and deterioration occur simultaneously. The amount of stock is zero at $t_{i+1,0}$. At t_{i1} the OW is filled to its capacity and then excess of the items are stored in the RW. During $[t_{i2}, t_{i3}]$ the inventory level in the RW gradually decreases due to demand and deterioration, and it becomes zero at t_{i3}. In the OW, the inventory decreased during $[t_{i1}, t_{i3}]$ due to deterioration only, and during $[t_{i3}, t_{i+1,0}]$ the decrease in inventory is both due to demand and deterioration. At $t_{i+1,0}$ all inventory in OW will be fully exhausted.

Fig.1 (b) depicts the inventory level during a production cycle in which only OW is used. Within any arbitrary cycle j for L_1-system, the model can be considered according to two time intervals $[t_{j0}, t_{j1}]$ and $[t_{j1}, t_{j+1,0}]$. During $[t_{j0}, t_{j1}]$ the level of inventory in OW gradually increases but always remains less than W and during $[t_{j1}, t_{j+1,0}]$ the stocks in OW gradually decreases due to demand and deterioration and will be exhausted at $t_{j+1,0}$.

For L_2-system, the differential equations describing the inventory level within any production cycle i are given as follows:

$$\frac{dI_{i1}(t)}{dt} + (a_1 + b_1 t) I_{i1}(t) = a + (b-1) f(t); \quad t_{i0} \le t \le t_{i1} \tag{1}$$

$$\frac{dI_{i2}(t)}{dt} + (a_2 + b_2 t) I_{i2}(t) = a + (b-1) f(t); \quad t_{i1} \leq t \leq t_{i2} \tag{2}$$

$$\frac{dI_{i3}(t)}{dt} + (a_2 + b_2 t) I_{i2}(t) = -f(t); \quad t_{i2} \leq t \leq t_{i3} \tag{3}$$

$$\frac{dI_{i4}(t)}{dt} + (a_1 + b_1 t) I_{i4}(t) = -f(t); \quad t_{i3} \leq t \leq t_{i+1,0} \tag{4}$$

$$\frac{dI_{i5}(t)}{dt} + (a_1 + b_1 t) I_{i5}(t) = 0; \quad t_{i1} \leq t \leq t_{i3} \tag{5}$$

With the boundary conditions

$$I_{i1}(t_{i0}) = 0,\ I_{i2}(t_{i1}) = 0,\ I_{i3}(t_{i3}) = 0,\ I_{i4}(t_{i+1,0}) = 0 \ \ and \ \ I_{i5}(t_{i1}) = W.$$

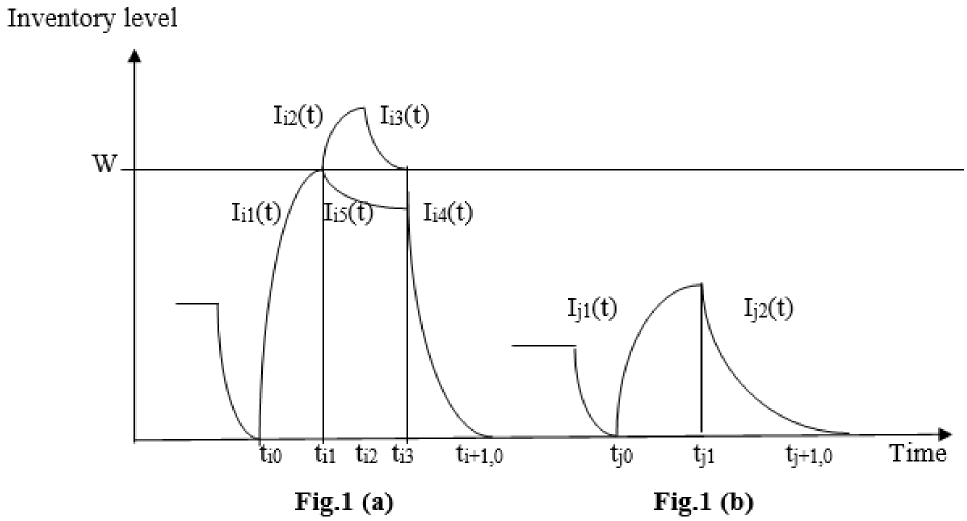

Figure 1: *Inventory level in a production system for deteriorating items with time-dependent demand.*

The above equations can be solved, respectively, as follows:
Therefore solution of equation (1) is

$$I_{i1}(t) = \int_{t_{i0}}^{t} e^{-a_1(t-u) - \frac{b_1}{2}(t^2 - u^2)} [a + (b-1) f(u)] du \tag{6}$$

Similarly, solutions of others equations are

$$I_{i2}(t) = \int_{t_{i1}}^{t} e^{-a_2(t-u) - \frac{b_2}{2}(t^2 - u^2)} [a + (b-1) f(u)] du \tag{7}$$

$$I_{i3}(t) = \int_{t}^{t_{i3}} e^{-a_2(t-u) - \frac{b_2}{2}(t^2 - u^2)} f(u) du \tag{8}$$

$$I_{i4}(t) = \int_{t}^{t_{i+1,0}} e^{-a_1(t-u) - \frac{b_1}{2}(t^2 - u^2)} f(u) du \tag{9}$$

$$I_{i5}(t) = W \cdot e^{a_1(t_{i1} - t) - \frac{b_1}{2}(t_{i1}^2 - t^2)} \tag{10}$$

Now, the inventory level in RW can be calculated as

$$I_{RW,i}(t) = \int_{t_{i1}}^{t_{i2}} e^{-rt} I_{i2}(t) dt + \int_{t_{i2}}^{t_{i3}} e^{-rt} I_{i3}(t) dt \tag{11}$$

$$I_{RW,i}(t)=\left\{\int_{t_{i1}}^{t_{i2}} e^{a_2 t+\frac{b_2}{2}t^2}[a+(b-1)f(t)]\left[(t_{i2}-t)-\frac{(a_2+r)}{2}(t_{i2}^2-t^2)-\frac{b_2}{6}(t_{i2}^3-t^3)\right]dt\right.$$
$$\left.\int_{t_{i2}}^{t_{i3}} e^{a_2 t+\frac{b_2}{2}t^2} f(t)\left[(t_{i2}-t)-\frac{(a_2+r)}{2}(t_{i2}^2-t^2)-\frac{b_2}{6}(t_{i2}^3-t^3)\right]dt\right\}$$

$$=\left(t_{i2}-\frac{(a_2+r)}{2}t_{i2}^2-\frac{b_2}{6}t_{i2}^3\right)\left\{\int_{t_{i2}}^{t_{i3}} e^{a_2 t+\frac{b_2}{2}t^2}[a+(b-1)f(t)]dt-\int_{t_{i2}}^{t_{i3}} e^{a_2 t+\frac{b_2}{2}t^2} f(t)dt\right\}$$
$$+\int_{t_{i2}}^{t_{i3}} e^{a_2 t+\frac{b_2}{2}t^2}[t-\frac{(a_2+r)}{2}t^2-\frac{b_2}{6}t^3]f(t)dt$$
$$-\int_{t_{i1}}^{t_{i2}} e^{a_2 t+\frac{b_2}{2}t^2}[t-\frac{(a_2+r)}{2}t^2-\frac{b_2}{6}t^3][a+(b-1)f(t)]dt$$

$$=-\int_{t_{i1}}^{t_{i2}}\left(1+a_2 t+\frac{b_2}{2}t^2\right)\left[t-\frac{(a_2+r)}{2}t^2-\frac{b_2}{6}t^3\right]\big(a+(b-1)f(t)\big)dt$$
$$+\int_{t_{i2}}^{t_{i3}}\left(1+a_2 t+\frac{b_2}{2}t^2\right)\left[t-\frac{(a_2+r)}{2}t^2-\frac{b_2}{6}t^3\right]f(t)dt$$

By using the relation $I_{i2}(t_{i2})=I_{i3}(t_{i2})$, i.e.,

$$\int_{t_{i1}}^{t_{i2}} e^{-a_2(t_{i2}-t)-\frac{b_2}{2}(t_{i2}^2-t^2)}\Big(a+(b-1)f(t)\Big)dt=\int_{t_{i2}}^{t_{i3}} e^{-a_2(t_{i2}-t)-\frac{b_2}{2}(t_{i2}^2-t^2)}f(t)dt$$

$$=\int_{t_{i1}}^{t_{i3}}\left[t+\frac{(a_2-r)}{2}t^2+\frac{b_2}{3}t^3+\right]f(t)dt-b\int_{t_{i1}}^{t_{i2}}\left[t+\frac{(a_2-r)}{2}t^2+\frac{b_2}{3}t^3+\right]f(t)dt$$
$$-a\left[\frac{1}{2}(t_{i2}^2-t_{i1}^2)+\frac{(a_2-r)}{6}(t_{i2}^3-t_{i1}^3)+\frac{b_2}{12}(t_{i2}^4-t_{i1}^4)\right]\quad(12)$$

Similarly, in OW, the inventory level can be derived as

$$I_{OW,i}=\int_{t_{i0}}^{t_{i1}} e^{-rt}I_{i1}(t)dt+\int_{t_{i3}}^{t_{i+10}} e^{-rt}I_{i4}(t)dt+\int_{t_{i1}}^{t_{i3}} e^{-rt}I_{i5}(t)dt\quad(13)$$

Now

$$I_{OW,i}=\int_{t_{i0}}^{t_{i1}} e^{a_1 t+\frac{b_1}{2}t^2}\Big[a+(b-1)f(t)\Big]\left[(t_{i1}-t)-\frac{(a_1+r)}{2}(t_{i1}^2-t^2)-\frac{b_1}{6}(t_{i1}^3-t^3)\right]dt$$
$$-\int_{t_{i3}}^{t_{i+1,0}} e^{a_1 t+\frac{b_1}{2}t^2}f(t\left[(t_{i3}-t)-\frac{(a_1+r)}{2}(t_{i3}^2-t^2)-\frac{b_1}{6}(t_{i3}^3-t^3)\right]dt$$
$$+we^{a_1 t_{i1}+\frac{b_1}{2}t_{i1}^2}\left[(t_{i3}-t_{i1})-\frac{(a_1+r)}{2}(t_{i3}^2-t_{i1}^2)-\frac{b_1}{6}(t_{i3}^3-t_{i1}^3)\right]$$

$$=\left[\left(t_{i1}-\frac{(a_1+r)}{2}t_{i1}^2-\frac{b_1}{6}t_{i1}^3\right)\right]\left\{\int_{t_{i0}}^{t_{i1}} e^{a_1 t+\frac{b_1}{2}t^2}\Big[a+(b-1)f(t)\Big]dt-we^{a_1 t_{i1}+\frac{b_1}{2}t_{i1}^2}\right\}-\int_{t_{i0}}^{t_{i1}} e^{a_1 t+\frac{b_1}{2}t^2}$$
$$\left[t-\frac{(a_1+r)}{2}t^2-\frac{b_1}{6}t^3\right]\Big[a+(b-1)f(t)\Big]dt-\left[\left(t_{i3}-\frac{(a_1+r)}{2}t_{i3}^2-\frac{b_1}{6}t_{i3}^3\right)\right]\left\{\int_{t_{i3}}^{t_{i+1,0}} e^{a_1 t+\frac{b_1}{2}t^2}f(t)dt\right.$$
$$\left.-we^{a_1 t_{i1}+\frac{b_1}{2}t_{i1}^2}\right\}+\int_{t_{i3}}^{t_{i+1,0}} e^{a_1 t+\frac{b_1}{6}t^2}\left[t-\frac{(a_1+r)}{2}t^2-\frac{b_1}{6}t^3\right]f(t)dt$$

On using the conditions $I_{i4}(t_{i3}) = I_{i5}(t_{i3})$

$$\Rightarrow \int_{t_{i3}}^{t_{i+1,0}} e^{-a_1}(t_{i3} - t) + \frac{b_1}{6}(t_{i3}^2 - t^2)f(t)dt = we^{a_1(t_{i1}-t_{i3})+\frac{b_1}{2}(t_{i1}^2-t_{i3}^2)}$$

$$\int_{t_{i3}}^{t_{i+1,0}} e^{a_1} + \frac{b_1}{6}t^2 f(t)dt = we^{a_1+\frac{b_1}{2}t_{i1}^2}$$

and

$$I_{i1}(t_{i1}) = W$$

$$\Longrightarrow \int_{t_{i0}}^{t_{i1}} e^{-a_1(t_{i1}-t^2)+\frac{b_1}{2}(t_{i1}^2-t^2)}\Big[a + (b-1)f(t)\Big]dt = W$$

$$\Longrightarrow \int_{t_{i0}}^{t_{i1}} e^{a_1 t+\frac{b_1}{2}(t_{i1}^2-t^2)}\Big[a + (b-1)f(t)\Big]dt = We^{a_1 t_{i1}+\frac{b_1}{2}t_{i1}^2}$$

$$I_{OW,i} = \int_{t_{i3}}^{t_{i+1,0}} e^{a_1 t+\frac{b_1}{2}t^2}\left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right]f(t)dt + \int_{t_{i0}}^{t_{i1}} e^{a_1 t+\frac{b_1}{2}t^2}\left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right]\Big[a + (b-1)f(t)\Big]f(t)dt$$

$$I_{OW,i} = \int_{t_{i3}}^{t_{i+1,0}} \left\{1 + a_1 t + \frac{b_1}{2}t^2\right\}\left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right]f(t)dt - \int_{t_{i0}}^{t_{i1}} \left\{1 + a_1 t + \frac{b_1}{2}t^2\right\}\left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right]\Big[a + (b-1)f(t)\Big]f(t)dt$$

$$I_{OW,i} = \int_{t_{i3}}^{t_{i+1,0}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{6}t^3\right]f(t)dt - \int_{t_{i0}}^{t_{i1}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{6}t^3\right]\Big[a + (b-1)f(t)\Big]f(t)dt$$

$$I_{OW,i} = \int_{t_{i3}}^{t_{i+1,0}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{6}t^3\right]f(t)dt - (b-1)\int_{t_{i0}}^{t_{i1}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{6}t^3\right]f(t)dt - a\left[\frac{1}{2}(t_{i1}^2 - t_{i0}^2) + \frac{(a_1-r)}{2}(t_{i1}^3 - t_{i0}^3) + \frac{b_1}{12}(t_{i1}^4 - t_{i0}^4)\right] \quad (14)$$

Additionally, the inventory units deteriorated during the production cycle i,

$$D_i = \int_{t_{i0}}^{t_{i1}} (a_1 + b_1 t)e^{-rt} I_{i1}(t)dt + \int_{t_{i3}}^{t_{i+1,0}} (a_1 + b_1 t)e^{-rt} I_{i4}(t)dt + \int_{t_{i1}}^{t_{i3}} (a_1 + b_1 t)e^{-rt} I_{i5}(t)dt + \int_{t_{i1}}^{t_{i2}} (a_2 + b_2 t)e^{-rt} I_{i2}(t)dt + \int_{t_{i2}}^{t_{i3}} (a_2 + b_2 t)e^{-rt} I_{i3}(t)dt$$

$$D_i = a_1 I_{OW,i} + a_2 I_{RW,i} + b_2\left[\int_{t_{i1}}^{t_{i2}} te^{-rt} I_{i2}(t)dt + \int_{t_{i2}}^{t_{i3}} te^{-rt} I_{i3}(t)dt\right] + b_1\left[\int_{t_{i0}}^{t_{i1}} te^{-rt} I_{i1}(t)dt + \int_{t_{i3}}^{t_{i+1,0}} te^{-rt} I_{i4}(t)dt + \int_{t_{i1}}^{t_{i3}} te^{-rt} I_{i5}(t)dt\right]$$

$$D_i = a_2\left[\int_{t_{i1}}^{t_{i3}}\left(t-\frac{r}{2}t^2\right)f(t)dt - b\int_{t_{i1}}^{t_{i2}}\left(t-\frac{r}{2}t^2\right)f(t)dt - a\left\{\frac{1}{2}(t_{i2}^2-t_{i1}^2)-\frac{r}{6}(t_{i2}^3-t_{i1}^3)\right\}\right]$$
$$a_1\left[\int_{t_{i3}}^{t_{i+1,0}}\left(t-\frac{r}{2}t^2\right)f(t)dt-(b-1)\int_{t_{i0}}^{t_{i1}}\left(t-\frac{r}{2}t^2\right)f(t)dt - a\left\{\frac{1}{2}(t_{i1}^2-t_{i0}^2)-\frac{r}{6}(t_{i2}^3-t_{i1}^3)\right\}\right]$$

$$D_i = \int_{t_{i1}}^{t_{i3}}\left\{a_2t+\frac{(b_2-a_2r)}{2}t^2-\frac{b_2r}{3}t^3\right\}f(t)dt - b\int_{t_{i1}}^{t_{i2}}\left\{a_2t+\frac{(b_2-a_2r)}{2}t^2-\frac{b_2r}{3}t^3\right\}f(t)dt$$
$$+\int_{t_{i3}}^{t_{i+1,0}}\left\{a_1t+\frac{(b_1-a_1r)}{2}t^2-\frac{b_1r}{3}t^3\right\}f(t)dt-(b-1)\int_{t_{i0}}^{t_{i1}}\left\{a_1t+\frac{(b_1-a_1r)}{2}t^2-\frac{b_1r}{3}t^3\right\}f(t)dt$$
$$+a\left\{\frac{a_2}{2}(t_{i2}^2-t_{i1}^2)+\frac{(b_2-r)}{6}(t_{i2}^3-t_{i1}^3)-\frac{b_2r}{12}(t_{i2}^4-t_{i1}^4)\right\}$$
$$-a\left\{\frac{a_1}{2}(t_{i1}^2-t_{i0}^2)+\frac{(b_1-r)}{6}(t_{i1}^3-t_{i0}^3)-\frac{b_1r}{12}(t_{i1}^4-t_{i0}^4)\right\} \quad (15)$$

Since W is known and $p > f$, a unique solution of t_{i1} can be obtained by using the condition $I_{i1}(t_{i1}) = W$, for a given t_{i0}

$$\int_{t_{i0}}^{t_{i1}} e^{-a_1(t_{i1}-t)-\frac{b_1}{2}(t_{i1}^2-t^2)}\Big[a+(b-1)f(t)\Big]dt = W \quad (16)$$

Furthermore, from $I_{i4}(t_{i3}) = I_{i5}(t_{i3})$ a relationship between $t_{i3}, t_{i+1,0}$ and t_{i1} can be derived as

$$\int_{t_{i3}}^{t_{i+1,0}} e^{a_1t+\frac{b_1}{6}t^2}f(t)dt = we^{a_1t_{i1}+\frac{b_1}{2}t_{i1}^2} \quad (17)$$

For $f(t) > 0$, a unique solution of t_{i1} can be obtained from (6) for given $t_{i+1,0}$ and t_{i1}. .Additionally, from the condition $I_{i2}(t_{i2}) = I_{i3}(t_{i2})$, we have

$$\int_{t_{i1}}^{t_{i2}} e^{a_2t+\frac{b_2}{2}t^2}[a+bf(t)]dt = \int_{t_{i1}}^{t_{i3}} e^{a_2t+\frac{b_2}{2}t^2}f(t)dt \quad (18)$$

Then, for given t_{i1} and t_{i3}, corresponding value of t_{i2} can be determined from (18).Again, for those cycles using the L_1 -system, the above results can be simplified to express the inventory level in the one-warehouse model. For any arbitrary production cycle j in L_1 -system, I_{j1} is similar to I_{i1} behavior and I_{j2} is similar to I_{i4}. Therefore, by using the analogous results, we have

$$I_{j1}(t) = \int_{t_{j0}}^{t} e^{-a_1(t-u)-\frac{b_2}{2}(t^2-u^2)}[a+(b-1)f(u)]du, t_{j0} \le t \le t_{j1}$$

and

$$I_{j2}(t) = \int_{t}^{t_{j+1,0}} e^{-a_1(t-u)-\frac{b_2}{2}(t^2-u^2)}[a+(b-1)f(u)]du, t_{j1} \le t \le t_{j+1,0}$$

Then the inventory levels for cycle j can be calculated as

$$I_{RW,j} = 0 \quad (19)$$
$$I_{OW,j} = \int_{t_{j0}}^{t_{j1}} e^{-rt}I_{j1}(t)dt + \int_{t_{j1}}^{t_{j+1,0}} e^{-rt}I_{j2}(t)dt$$

and

$$I_{OW,j} = \left[t_{j1}-\frac{(a_1+r)}{2}t_{j1}^2-\frac{b_1}{6}t_{j1}^3\right]\left\{\int_{t_{j0}}^{t_{j1}} e^{a_1t+\frac{b_1}{2}t^2}\Big[a+(b-1)f(t)\Big]dt - \int_{t_{j1}}^{t_{j+1,0}} e^{a_1t+\frac{b_1}{2}t^2}f(t)dt\right\}$$
$$-\int_{t_{j0}}^{t_{j1}} e^{a_1t+\frac{b_1}{2}t^2}\left[t-\frac{(a_1+r)}{2}t^2-\frac{b_1}{6}t^3\right]\Big[a+(b-1)f(t)\Big]f(t)dt$$
$$+\int_{t_{j1}}^{t_{j+1,0}} e^{a_1t+\frac{b_1}{2}t^2}\left[t-\frac{(a_1+r)}{2}t^2-\frac{b_1}{6}t^3\right]f(t)dt$$

By using the condition $I_{j1}(t_{j1}) = I_{j2}(t_{j1})$

$$\int_{t_{j0}}^{t_{j1}} e^{-a_1(t_{j1}-t)-\frac{b_1}{2}(t_{j1}^2-t^2)} \Big[a + (b-1)\Big] f(t)dt = \int_{t_{j1}}^{t_{j+1,0}} e^{-a_1(t_{j1}-t)-\frac{b_1}{2}(t_{j1}^2-t^2)} f(t)dt$$

$$\implies \int_{t_{j0}}^{t_{j1}} e^{a_1 t+\frac{b_1}{2}t^2} \Big[a + (b-1)\Big] f(t)dt = \int_{t_{j1}}^{t_{j+1,0}} e^{a_1 t+\frac{b_1}{2}t^2} f(t)dt$$

$$I_{OW,j} = \int_{t_{j0}}^{t_{j+1,0}} e^{a_1 t+\frac{b_1}{2}t^2} \left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right] f(t)dt$$
$$+ \int_{t_{j0}}^{t_{j1}} e^{a_1 t+\frac{b_1}{2}t^2} \left[t - \frac{(a_1+r)}{2}t^2 - \frac{b_1}{6}t^3\right] \Big[a + bf(t)\Big] f(t)dt$$

$$I_{OW,j} = \int_{t_{j0}}^{t_{j+1,0}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{3}t^3\right] f(t)dt - \int_{t_{j0}}^{t_{j1}} \left[t + \frac{(a_1-r)}{2}t^2 + \frac{b_1}{6}t^3\right] f(t)dt$$
$$- a\left[\frac{1}{2}(t_{j1}^2 - t_{j0}^2) + \frac{(a_1-r)}{6}(t_{j1}^3 - t_{j0}^3) + \frac{b_1}{12}(t_{j1}^4 - t_{j0}^4)\right] \quad (20)$$

Now, the amount of deteriorated items during the production cycle j is

$$D_j = \int_{t_{j0}}^{t_{j1}} (a_1 + b_1 t)e^{-rt} I_{j1}(t)dt + \int_{t_{j1}}^{t_{j+1,0}} (a_1 + b_1 t)e^{-rt} I_{j2}(t)dt$$

$$D_j = a_1.I_{OW,j} + b_1\left[\int_{t_{j0}}^{t_{j1}} te^{-rt} I_{j1}(t)dt + \int_{t_{j1}}^{t_{j+1,0}} te^{-rt} I_{j2}(t)dt\right]$$

$$D_j = a_1\left[\int_{t_{j0}}^{t_{j+1,0}} \left(t - \frac{r}{2}t^2\right) f(t)dt - b\int_{t_{j0}}^{t_{j1}} \left(t - \frac{r}{2}t^2\right) f(t)dt\right] - aa_1\left[\frac{1}{2}(t_{j1}^2 - t_{j0}^2) + \frac{r}{6}(t_{j1}^3 - t_{j0}^3)\right]$$
$$+ b_1\left[\int_{t_{j0}}^{t_{j+1,0}} \left(t - \frac{r}{2}t^2\right) f(t)dt - b\int_{t_{j0}}^{t_{j1}} \left(t - \frac{r}{2}t^2\right) f(t)dt\right] - ab_1\left[\frac{1}{6}(t_{j1}^3 - t_{j0}^3) + \frac{r}{12}(t_{j1}^4 - t_{j0}^4)\right]$$

$$D_j = (a_1 + b_1)\left[\int_{t_{j0}}^{t_{j+1,0}} \left(t - \frac{r}{2}t^2\right) f(t)dt - b\int_{t_{j0}}^{t_{j1}} \left(t - \frac{r}{2}t^2\right) f(t)dt\right]$$
$$- a\left[\frac{a_1}{2}(t_{j1}^2 - t_{j0}^2) - \frac{a_1 r}{6}(t_{j1}^3 - t_{j0}^3) + \frac{b_1}{6}(t_{j1}^3 - t_{j0}^3) - \frac{b_1 r}{12}(t_{j1}^4 - t_{j0}^4)\right] \quad (21)$$

The condition $I_{j1}(t_{j1}) = I_{j2}(t_{j1})$ gives a relation among t_{j0}, t_{j1} and $t_{j+1,0}$ as follows:

$$\int_{t_{j0}}^{t_{j1}} e^{a_1 t+\frac{b_1}{2}t^2} \Big[a + (b-1)f(t)\Big] dt = \int_{t_{j0}}^{t_{j+1,0}} e^{a_1 t+\frac{b_1}{2}t^2} f(t)dt$$

Clearly, within a cycle j, at t_{j1}, maximum inventory level occurs and then

$$U_j = I_{j1}(t_{j1})$$

The total system cost, which consists of carrying cost, setup cost and deterioration cost incurred in each production cycle within the planning horizon H can be given as

$$TC = nC_1 + C_{RW}\sum_i I_{RW,i} + C_{OW}\sum_i I_{OW,i} + C_{OW}\sum_j I_{OW,i} + C_2\sum_i D_i + C_2\sum_j D_j$$

$$
\begin{aligned}
TC &= nC_1 + C_{RW}\sum_i \Bigg[\int_{t_{i1}}^{t_{i3}}\left[t+\frac{(a_2-r)}{2}t^2+\frac{b_2}{3}t^3+\right]f(t)dt - b\int_{t_{i1}}^{t_{i2}}\left[t+\frac{(a_2-r)}{2}t^2+\frac{b_2}{3}t^3+\right]f(t)dt \\
&- a\left[\frac{1}{2}(t_{i2}^2-t_{i1}^2)+\frac{(a_2-r)}{6}(t_{i2}^3-t_{i1}^3)+\frac{b_2}{12}(t_{i2}^4-t_{i1}^4)\right]\Bigg] + C_{OW}\sum_i\Bigg[\int_{t_{i3}}^{t_{i+1,0}}\left[t+\frac{(a_1-r)}{2}t^2+\frac{b_1}{6}t^3\right]f(t)dt \\
&-(b-1)\int_{t_{i0}}^{t_{i1}}\left[t+\frac{(a_1-r)}{2}t^2+\frac{b_1}{6}t^3\right]f(t)dt - a\left[\frac{1}{2}(t_{i1}^2-t_{i0}^2)+\frac{(a_1-r)}{2}(t_{i1}^3-t_{i0}^3)+\frac{b_1}{12}(t_{i1}^4-t_{i0}^4)\right]\Bigg] \\
&+ C_{OW}\sum_j\Bigg[\int_{t_{j3}}^{t_{j+1,0}}\left[t+\frac{(a_1-r)}{2}t^2+\frac{b_1}{6}t^3\right]f(t)dt-(b-1)\int_{t_{j0}}^{t_{j1}}\left[t+\frac{(a_1-r)}{2}t^2+\frac{b_1}{6}t^3\right]f(t)dt \\
&- a\left[\frac{1}{2}(t_{j1}^2-t_{j0}^2)+\frac{(a_1-r)}{2}(t_{j1}^3-t_{j0}^3)+\frac{b_1}{12}(t_{j1}^4-t_{j0}^4)\right]\Bigg] + C_2\sum_i\Bigg[\int_{t_{i1}}^{t_{i3}}\left\{a_2t+\frac{(b_2-a_2r)}{2}t^2-\frac{b_2r}{3}t^3\right\}f(t)dt \\
&- b\int_{t_{i1}}^{t_{i2}}\left\{a_2t+\frac{(b_2-a_2r)}{2}t^2-\frac{b_2r}{3}t^3\right\}f(t)dt + a\left\{\frac{a_2}{2}(t_{i2}^2-t_{i1}^2)+\frac{(b_2-r)}{6}(t_{i2}^3-t_{i1}^3)-\frac{b_2r}{12}(t_{i2}^4-t_{i1}^4)\right\} \\
&- a\left\{\frac{a_1}{2}(t_{i1}^2-t_{i0}^2)+\frac{(b_1-r)}{6}(t_{i1}^3-t_{i0}^3)-\frac{b_1r}{12}(t_{i1}^4-t_{i0}^4)\right\}\Bigg] + C_2\sum_j\Bigg[(a_1+b_1)\Bigg[\int_{t_{j0}}^{t_{j+1,0}}\left(t-\frac{r}{2}t^2\right)f(t)dt \\
&- b\int_{t_{j0}}^{t_{j1}}\left(t-\frac{r}{2}t^2\right)f(t)dt\Bigg] - a\left[\frac{a_1}{2}(t_{j1}^2-t_{j0}^2)-\frac{a_1r}{6}(t_{j1}^3-t_{j0}^3)+\frac{b_1}{6}(t_{j1}^3-t_{j0}^3)-\frac{b_1r}{12}(t_{j1}^4-t_{j0}^4)\right]\Bigg]
\end{aligned} \tag{22}
$$

Note that when the demand is increasing with time the type of production cycle may switch from L_1 to L_2. On the contrary, when the demand is decreasing with time, the switch may have the opposite direction.Therefore, the utilization of warehouse is affected by not only the capacity of OW but also the characteristics of demand.

5. Solution Process

Due to the complexity of (22) which is immensely increased by the adoption of a general demand function and production rate, it is extremely difficult to establish the property of convexity analytically, for the optimal solution. However the optimum values of t_{i0}, t_{i1}, t_{i2} and t_{i3} which minimize the cost function TC are the solutions of the equations

$$\frac{\partial TC}{\partial t_{i0}} = 0,\ \frac{\partial TC}{\partial t_{i1}} = 0,\ \frac{\partial TC}{\partial t_{i2}} = 0,\ \ and\ \ \frac{\partial TC}{\partial t_{i3}} = 0$$

Provided these values of $T_{ik},\ \ (k = 0,1,2,3)$ satisfy the conditions $D_k > 0,\ \ (k = 0,1,2,3)$ where D_k is the Hessian determinant of order k given by

$$D_k = \begin{vmatrix} C_{11} & C_{12} & \cdots & C_{1k} \\ C_{21} & C_{22} & \cdots & C_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ C_{k1} & C_{k2} & \cdots & C_{kk} \end{vmatrix}$$

Where $C_{kl} = \frac{\partial^2 TC}{\partial t_k \partial t_i},\ \ (k, l = 0,1,2,3)$.
Also optimum values of t_{j0} and t_{j1} for TC are the solutions of the equations

$$\frac{\partial TC}{\partial t_{j0}} = 0,\ \ and\ \ \frac{\partial TC}{\partial t_{j1}} = 0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 TC}{\partial t_{j0}^2} > 0,\ \frac{\partial^2 TC}{\partial t_{j1}^2} > 0,\ \ and\ \ \frac{\partial TC}{\partial t_{j0}}\frac{\partial TC}{\partial t_{j1}} - \left(\frac{\partial^2 TC}{\partial t_{j0}\partial t_{j1}}\right)^2 > 0.$$

6. Numerical Example

The application of our model is illustrated by the following example:

$$\begin{array}{ccc} H=10, & p=1800, & \alpha = 0.02, \\ \beta = 0.01, & C_1 = 10,000\ Rs, & C_2 = 750\ Rs\ per\ unit, \\ C_{OW} = 200\ Rs, & C_{RW} = 300\ Rs, & W=500\ \text{units}, \\ f(t)=a\ e^{bt}, & a=200, & b=0.1, \end{array}$$

Solution of above example is illustrated in the table as follows: From above table it is ob-

n	t_{i0}	TC
1	$t_{10} = 0.00$	152871.89
2	$t_{10} = 0.00,\ t_{20} = 6.4312$	87012.47
3	$t_{10} = 0.00,\ t_{20} = 4.3719,\ t_{30} = 7.0432$	64243.77
4	$t_{10} = 0.00,\ t_{20} = 3.4379,\ t_{30} = 5.7524,\ t_{40} = 7.2845$	43174.42
5	$t_{10} = 0.00,\ t_{20} = 2.3765,\ t_{30} = 4.9432,\ t_{40} = 6.4369,\ t_{50} = 8.2784$	32114.74
6	$t_{10} = 0.00,\ t_{20} = 1.9827,\ t_{30} = 4.4326,\ t_{40} = 5.5274,\ t_{50} = 7.3239,\ t_{60} = 8.5768$	27332.57
7	$t_{10} = 0.00,\ t_{20} = 1.7292,\ t_{30} = 3.4662,\ t_{40} = 4.8829,\ t_{50} = 6.0021,\ t_{60} = 7.5341,\ t_{70} = 8.7598$	26998.69
8	$t_{10} = 0.00,\ t_{20} = 1.5492,\ t_{30} = 2.9793,\ t_{40} = 4.2324,\ t_{50} = 5.1789,\ t_{60} = 6.7249,\ t_{70} = 7.9842,\ t_{80} = 8.9598$	27124.71
9	$t_{10} = 0.00,\ t_{20} = 1.3247,\ t_{30} = 2.6744,\ t_{40} = 3.9896,\ t_{50} = 4.7249,\ t_{60} = 5.9215,\ t_{70} = 7.4889,\ t_{80} = 8.3647,\ t_{90} = 9.2343$	27843.29
10	$t_{10} = 0.00,\ t_{20} = 1.0421,\ t_{30} = 2.1257,\ t_{40} = 3.9653,\ t_{50} = 4.1257,\ t_{60} = 5.3237,\ t_{70} = 7.0187,\ t_{80} = 7.9839,\ t_{90} = 8.7698,\ t_{10,0} = 9.1430$	29834.23

served that the total system cost is convex of n. The minimum of TC is achieved when seven inventory cycles ($n^* = 7$) are involved and both OW and RW are used in each cycle. However if we exceed the number of cycles more than seven then we see that total cost increases, which is verified by the table.

7. Graphical Representations

The graphical depictions in this part provide a comprehensive grasp of the dynamics within the production inventory model. Figure 2 and Figure 3 explains the link between deterioration rates and time, while Figure 4 illustrates how demand impacts inventory management usnig different type of functions of time together with the adjustments needed to effectively balance production, demand, and degradation.

1. In an inventory model, the rate of deterioration often increases as the time period is extended. This is due to the fact that goods that are exposed to the environment for extended periods of time degrade or decay more quickly. Therefore, longer time horizons usually result in larger quantities of depreciating inventory, which might impact inventory costs overall and management strategies.

2. If the demand is time-dependent and increases over time, then extending the time horizon is probably going to result in more demand. In a time-dependent demand model, the demand rate often varies with time, often increasing over time. A longer planning horizon therefore suggests that the model will account for this increasing demand, which could affect order quantities, inventory levels, and overall plans for inventory management.

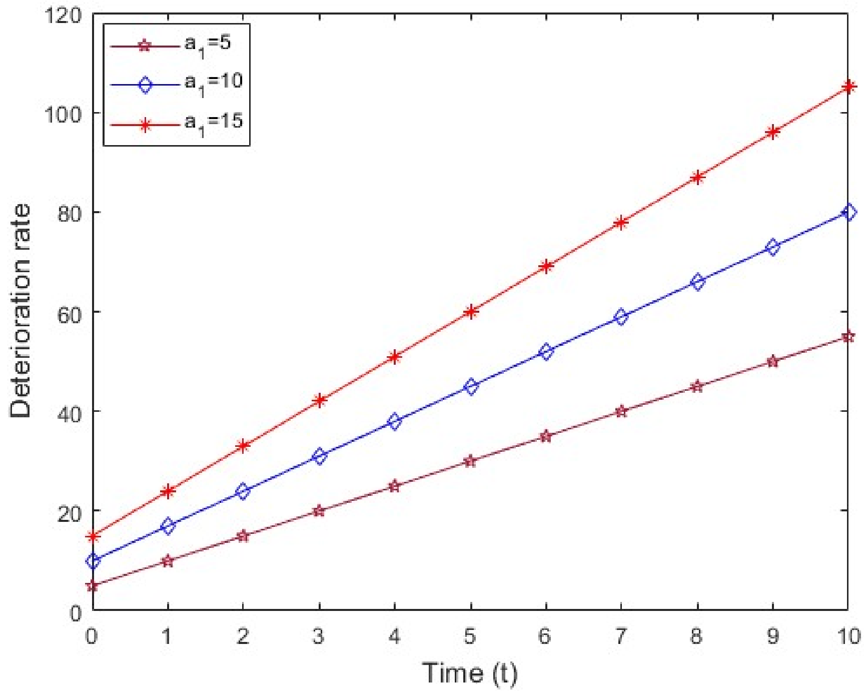

Figure 2: *Relation between deterioration rate and time, where b_1 is constant.*

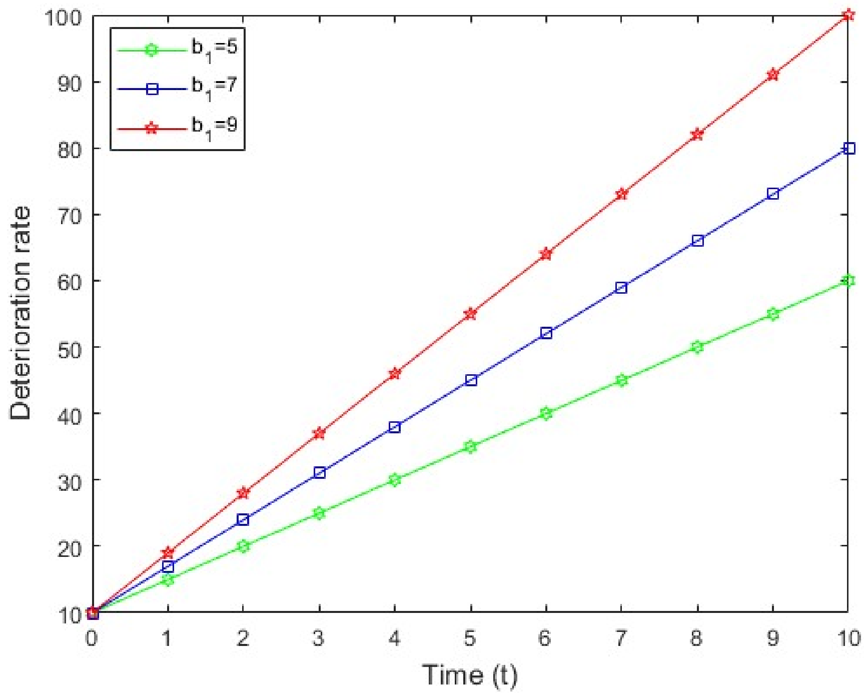

Figure 3: *Relation between deterioration rate and time, where a_1 is constant.*

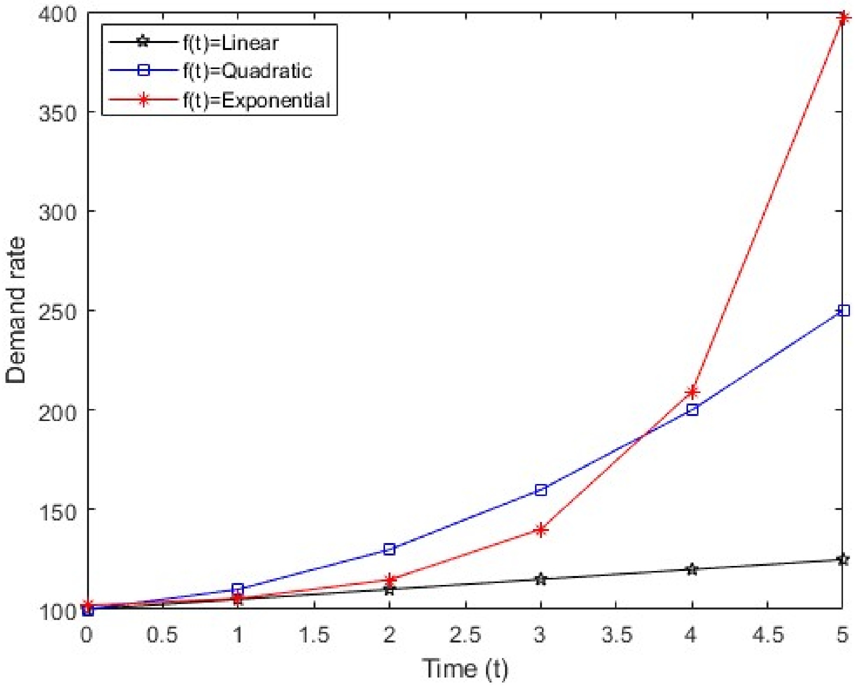

Figure 4: *Relation between demand rate and time using different functions of t.*

8. Conclusions

Most products experience a period of rapid demand increase during the introduction phase of product life cycle, level off in demand after reaching their maturity period, and will enter a period of sales decline due to new competing products or changes in consumer preference. The two-warehouse inventory control is an intriguing yet practicable issue of decision science when time-dependent demand is involved. As most of the work on the inventory model with time-dependent market demand has to determine the planning horizon before a meaningful inventory policy can be implemented, we rectify the two-warehouse inventory model with time-dependent demand, production and deterioration. This problem is different from that with constant demand case where keeping a consistent inventory level in the rented warehouse is the best solution. Due to the complexity of modeling, previous studies either adopted a heuristic approach of equal production cycle times or made ample assumptions to achieve the solution.

This general model can be applied to the inventory problem of either time increasing or time decreasing market demand.Numerical analyses indicate that both the total system cost and the number of production runs are significantly sensitive to the variation of length of planning horizon, setup cost, and demand parameters.

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