A STOCHASTIC RELIABILITY MODELING APPROACH FOR MULTIPLE SYSTEM SUBSCALES

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Abstract

The article discusses the approach of stochastic simulation of the reliability of technical systems. Stochastic simulation works with variables that are expected to change with a certain probability. A stochastic model creates a projection of a model that is based on a set of random outputs. These are recorded, then the projection is repeated with a new set of random variables. Repetition takes place many times, which can be thousands or more repetitions. At the end of the process, the distribution of these outputs shows not only the most probable values and estimates, but also their limits, which are reasonable to expect. The presented paper presents the possibilities of simulation using the Matlab software package and illustrates the simulation experiment on a specific case of monitored reliability variables.

Keywords: stochastic simulation, Matlab software, reliability, failure

1. Principles of reliability modeling

The efficiency of reliability management and the use of suitable simulation methods is closely related to the perfect knowledge of all described phenomena and processes in the monitored area. Analytical activity is also such a source of research and learning. Such a method of analysis requires the merging of knowledge from various scientific fields and disciplines in the so-called system approach. The system approach describes phenomena comprehensively and focuses on their internal and external connections. The inclusion and active use of applied and theoretical system disciplines, which have their own suitable research methods, but their own research subject may be missing, is assumed [1]. The research method used is simulation modeling. From the research possibilities of the given methods, it follows that the primary position is given to computer simulation and modeling of systems, which allows us to:

- describe and express processes whose analytical solution we do not know,
- to simplify the solution of complex mathematical problems,
- shorten time-consuming experiments,
- carry out experiments of new projects,
- quickly and efficiently analyze changes and assess their consequences,
- examine a large number of failure states and assess their variants [2].

The theoretical basis for the management of such a specific area is knowledge from various scientific disciplines, which enter into the research process and thus enable the research of the given area [3]. The development of these disciplines and the application of analysis to all areas of science was conditioned in the past by the use of mathematics and the use of computer technology for the needs of modeling and simulation. Mathematical analysis of various scientific fields made it possible to apply various research tools such as modeling and simulation of systems [4,5]

Of course, it is important to include external environmental influences in the structure and quality of material inputs in these monitored processes, in the form of random deviations in quantity, etc., about which we obtain objective statistical values and characteristics. With the complexity, we cannot specify the prediction, e.g. where, when, on which machine and technique a malfunction, downtime, what material will be needed. It follows from the conclusions that these are complex, dynamic and stochastic systems [6].

The indicated properties describe the facts that we must take into account:

- some phenomena and states we do not know or cannot express stochastically, they can only be predicted with difficulty,
- connections and ties between elements and the environment are complex, they most often occur between several components. This can lead to the fact that phenomena and the probabilities of their occurrence are always conditional,
- complicated investigation of stimuli and relationships in the system,
- limited possibilities for carrying out the experiment,
- definition, or the description of the system is possible only with an extensive mathematical model, sometimes difficult to achieve, due to the bias of the input data,
- when the level of investigation is reduced, a part is torn out of the whole, distorted outputs occur [7].

In the process of investigating objective processes and phenomena, models are a suitable aid, representing the state and structure and behavior of the system in a way similar to reality, i.e. it is very convenient to consume a model and a suitable simulation.

The concept of systems simulation is based on the following facts:

1) Systems simulation is a specific form of the research process, which is used as a tool or method of researching existing or proposed new systems, we understand it as a means of creating knowledge, supporting the decision-making process, a means of describing and establishing hypotheses or forecasts.

2) The subject of simulation is systems focused on objects of knowledge and their movement, so the subject of monitoring should be dynamic systems. Simulation of systems is possible for deterministic systems as well as for stochastic systems.

3) The basic principle for system simulation is the derivation of assumptions about the simulated system using experiments. Experiments have a physical or mathematical nature, most often in the form of a simulation model. Systems must respond to a stimulus of the same input quantity in such a way that the output quantities of both systems are equal.

4) We can understand the simulation model as a system that defines the current idea of the simulated system and the behavior and reporting of the processes in it. All known knowledge, design and technical conditions, rules of operation of the investigated system must be displayed in the simulation model in such a way as to preserve the stochastic nature of events and the real arrangement of system changes over time. In the case of computer simulation with a program and its implementation on a computer.

Simulation assumes the creation of a model using a computer program that provides information about the analyzed system. The simulation approach to system analysis is different from the analytical approach, the analysis method is purely theoretical. Therefore, this reliability analysis approach provides more flexibility and convenience. Model activities consist of events that are activated at certain points in time and thus affect the overall state of the system. The use of simulation modeling is also very advantageous when predicting system reliability. System reliability is determined or influenced by random phenomena and agents [8].

It has not only a probabilistic approach, but we express the reliability indicators:

- Probability densities
- Distribution functions
- Additional functions
- Intensities of phenomena.

The mathematical apparatus used is probability theory and mathematical statistics.

II. Methods of modeling the probabilistic behavior of random variables

We can describe and express the law of probability distribution:

1. expressing a complete description of the probability distribution of the relevant random variable by the type of probability distribution and its parameters:
   - Distribution function - CDF, F(t)
   - By complementary function - R(t)=1 - F(t)
   - Probability density - PDF, f(t)

2. numerical characteristics of the relevant random variable:
   - Location characteristics (e.g. Mean value)
   - Characteristics of variability (e.g. variance and standard deviation)
   - Quantiles (eg Median, Mode, p-quantile)

Simulation models use the following terminological basis for creating models:

- An attribute is an expression of a certain characteristic of components or elements, e.g. working capacity, type, operation cost, device number and process speed. Each element that enters the model is defined by a set number of attributes.
- A moment is a specified, prescribed value of simulation time in which at least one attribute value of a model element changes. Most often, the realization of the event occurs.
- An interval is a specified period of time between two moments.
- An event is a change in the state of an element that occurs at the moment of starting the necessary activity to change the state or the occurrence of equipment damage, the start of service or inclusion in a queue.
- Activity is the state of an element between two subsequent events describing specific changes in the state of an element such as performing an operation, waiting in a queue, transport to a service location.
- A process is a sequence of states of elements expressed by activities in a certain time span, such as what happens to the part from its removal to assembly [9].

Simulation modeling is very advantageous for analysis, modeling and prediction, especially for the possibilities of monitoring the dynamics of the plot with graphic outputs that give a clearer idea of random processes. The output quantities can be quickly compared and based on the conclusions, changes to the parameters of the system model can be implemented so that the target behavior of the system is adequate. The results of other experiments showed that the level of fault-freeness of individual groups is at a comparable, high level.

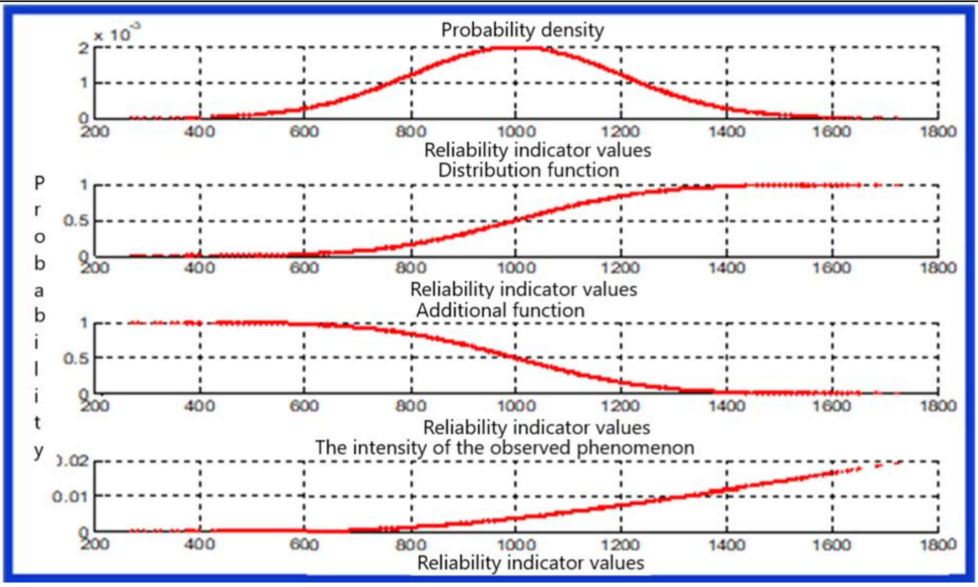

Figure 1: *Numerical characteristics of the relevant random variable*

III. The variable time step principle

We solve simulation models intended for analysis and reliability management by shifting the simulation time, basically in two ways. Constant time step - this defines a simple way of organizing the time structure. The simulation time is increased by a unit constant predetermined step from the initial time to the final time. This method is applicable for mathematical modeling, where we model events by calculating in the time of the time step shift.

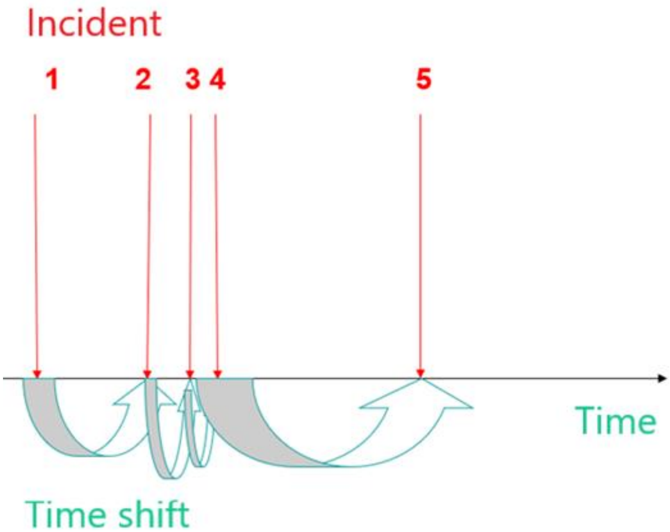

Figure 2: *The variable time step principle*

It assumes that the failure rate of the system is dependent on time, we have data on all object events in the form of a type of probability distribution and distribution parameters. System failure rate analysis is performed with the aim of obtaining the failure distribution of the entire system based on the probability distributions of its part failures.

Each object is characterized by the probability of failure - failure in the form of a statistical expression of the distribution of a random variable in the form of a function. Stochastic analysis can be carried out by mathematical modeling. MATLAB makes it possible to determine the probability of failure in the entire range of the distribution function F(t), the probability density f(t) of each element of the system, given the knowledge of the distribution of the randomly variable time between TTF failures and the parameters of the distribution [10].

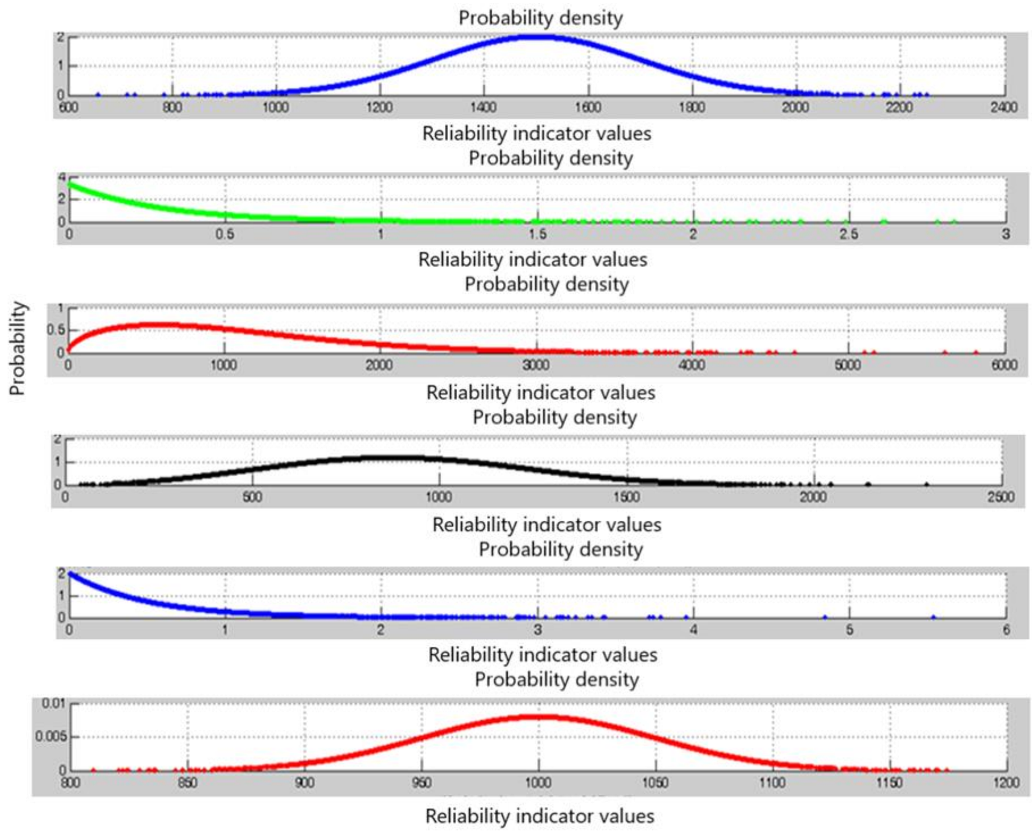

Figure 3: *Plots of simulated element probability densities*

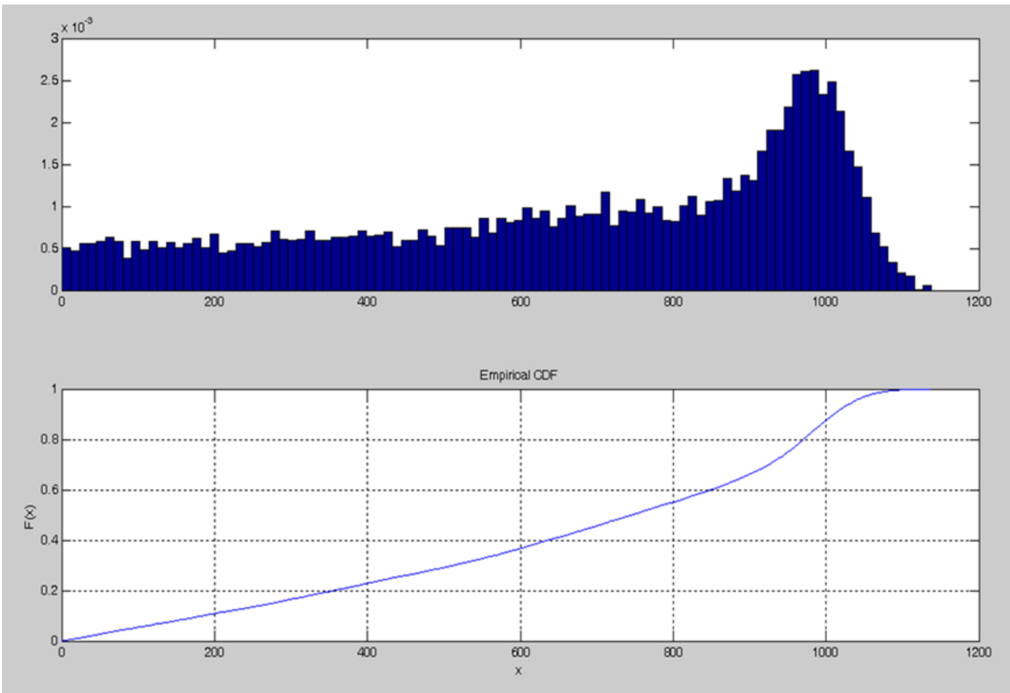

Figure 4: *Histogram of simulated system failure events*

According to the theory, the malfunction rate of the system will reach at any time of operationuntil probability of malfunction 1 is reached, a higher probability than the probability of the most malfunctioning element of the system. The probability of system failures will exceed the quantile value of 0.99.

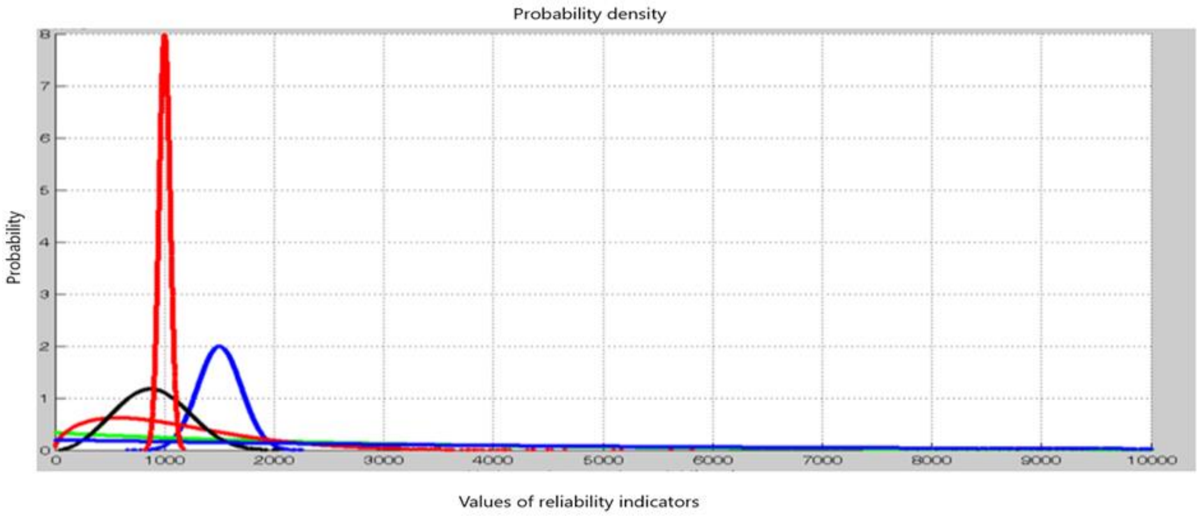

Figure 5: *Results of simulated event times between element failures*

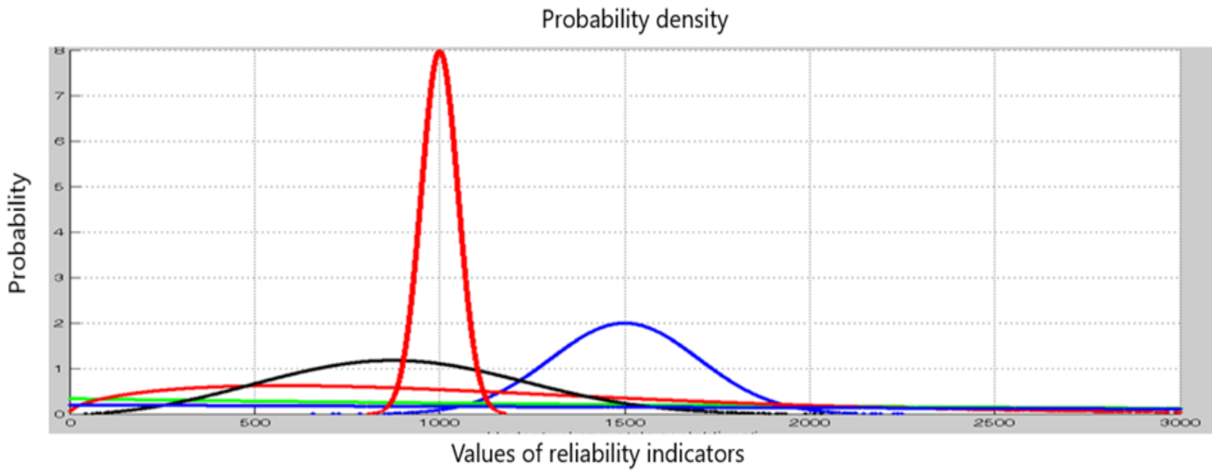

Figure 6: *Results of simulated events of operating units values*

Note the different ranges of values of operating units (times between failures of elements) on the x axis. Elements have a different assumption of occurrence of failures in terms of probability and time.

The generated times to failure of selected elements are shown in the form of probability densities and distribution functions. Static and stochastic quantitative analysis ensures the calculation (estimation) of quantitative numerical values of selected reliability indicators. The numerical value of the indicator is obtained by experimenting with the model using computer technology, considering the elementary phenomena that the model structurally connects to the behavior and analyzed states of the system [11]. The model and all input quantities are stochastic in nature, the result of the analysis is also stochastic, burdened with a certain degree of uncertainty, which can be reduced, but not completely eliminated.

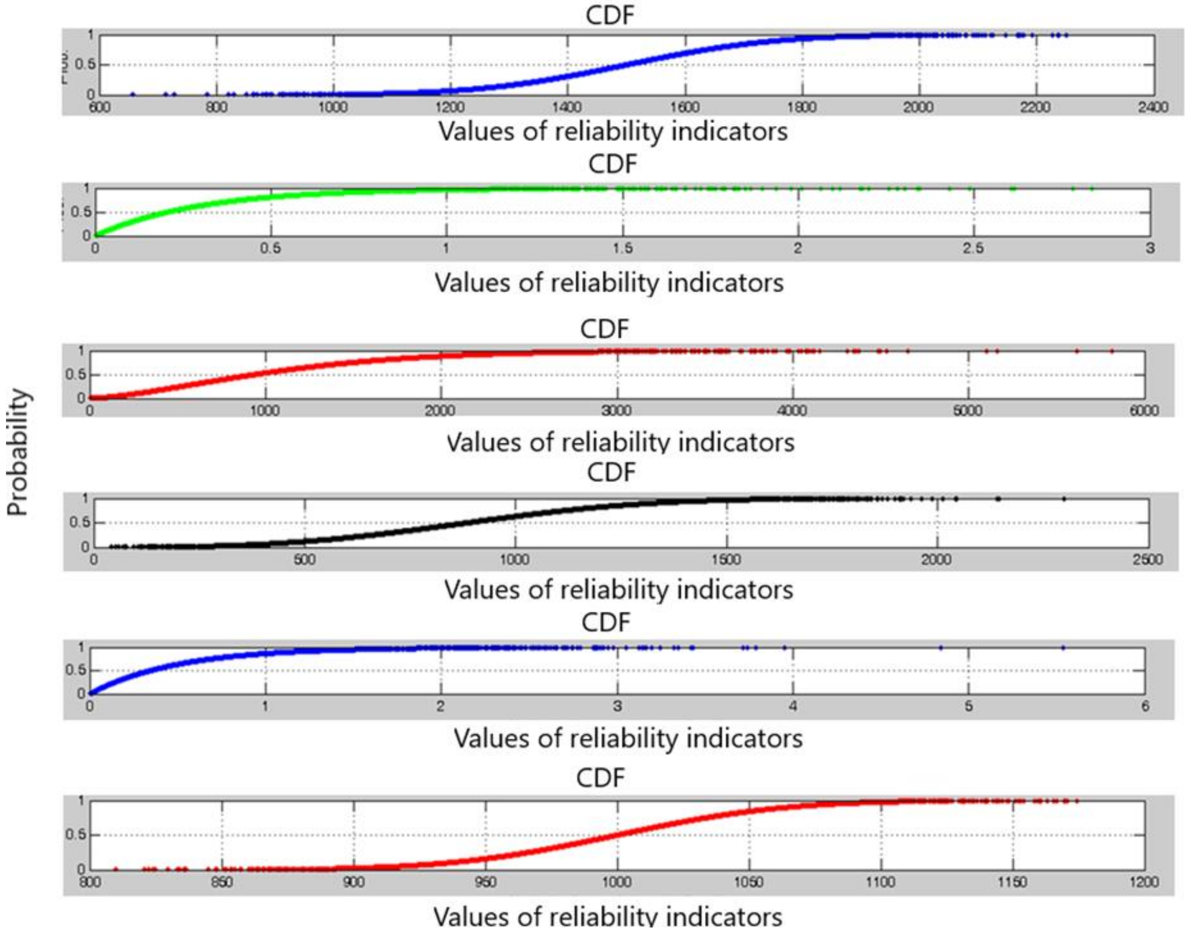

Figure 7: *Results of simulated events assuming the occurrence of faults result 1*

After the unification of the axes, we see that elements 2 and 7 have a range of assumptions for the occurrence of faults up to 30,000 - 55,000 operating units. Elements 1, 3, 4, 5, 6, 8 contribute to the failure rate of the system only in the intervals between failures 1 approx. 2400 km, 3.4- approx. 6000 km, 5.6- approx. 2500 km, 8- approx. 1200 km.

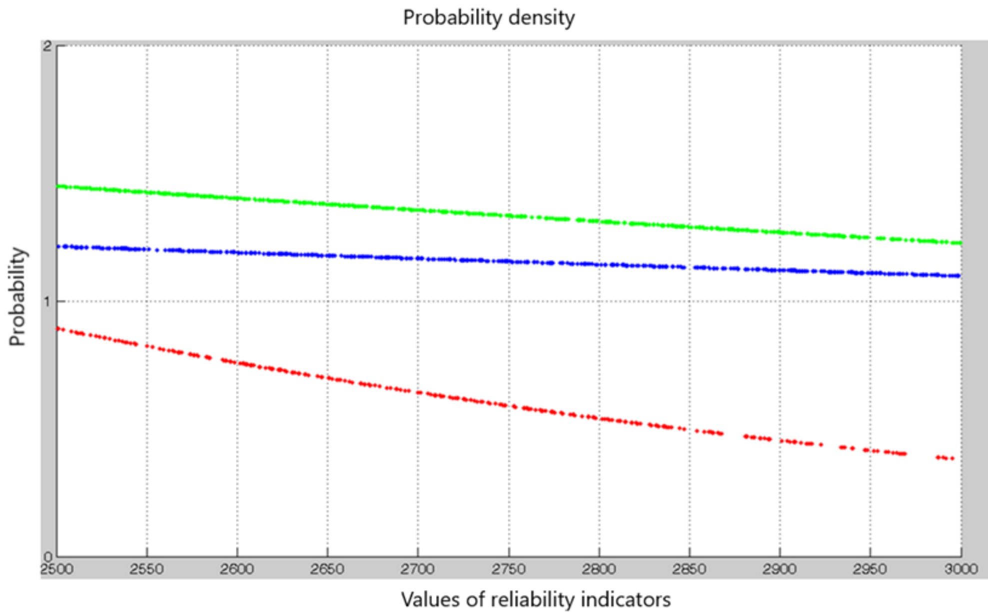

Figure 8: *Analysis of CDF results*

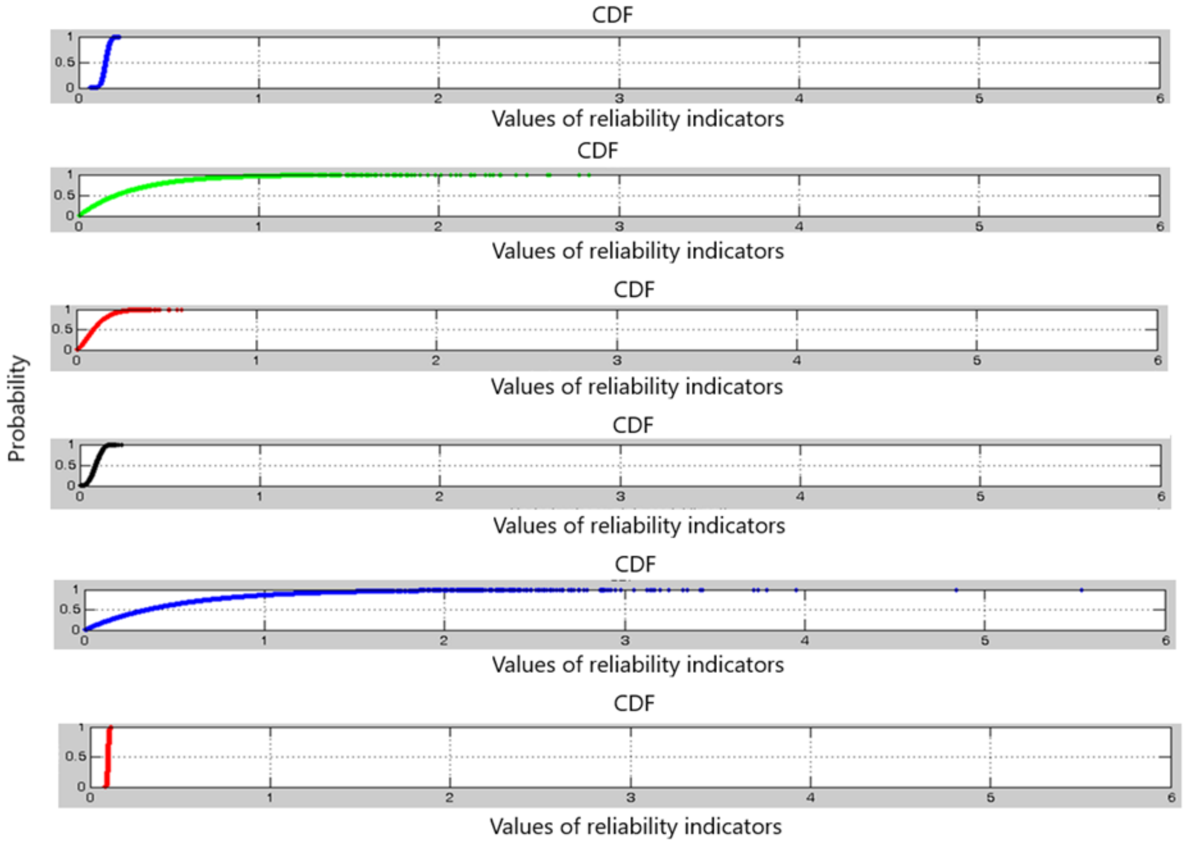

Figure 9: *Results of simulated events assuming the occurrence of faults result 2*

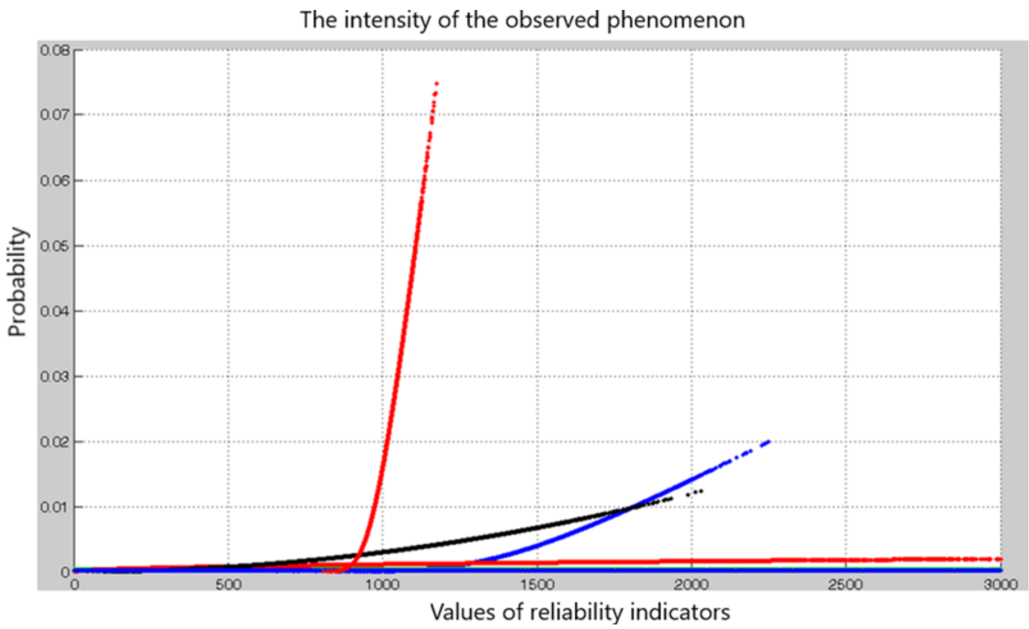

Figure 10: *Analysis of the results of element failure intensities*

IV. Results

The presented simulation is based on the principle of calculation with the possibility of using information about the values of the mean values of failure rate of individual elements from real operation. However, it does not offer the idea of predicting the course of failures of elements and systems over time. However, stochastic simulation with the use of mathematical modeling sufficiently removes these shortcomings, and that is why the authors decided to use just such a modeling approach [12]. In the event that there are different types of distribution of the probability of failure of the elements, the graphical expression of the statistical functions is a significant and important tool to determine the probability of failure for a specific time/time of operation. We can determine the interval in which specific elements can participate in the failure rate and thus make decisions about the method of evaluating the system by changing the reliability properties of the elements [13]. The stochastic mathematical modeling approach is more suitable compared to options with deterministic calculation using mean values, which runs into several shortcomings [14,15]. The stochastic approach offers us an idea of the failure rate of specific elements and systems at a specific time.

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