A NEW ATTRIBUTE CONTROL CHART BASED ON EXPONENTIATED EXPONENTIAL DISTRIBUTION UNDER ACCELERATED LIFE TEST WITH HYBRID CENSORING

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Abstract

In this article, we propose a new attribute np control chart for monitoring the median lifetime of the products under accelerated life test with hybrid censoring scheme assuming that the lifetime of the products follows an exponentiated exponential distribution. The optimal parameters for constructing the proposed control chart are determined so that the average run length for the in-control process is as closest to the prescribed average run length as possible. The control chart parameters are estimated for various set of values, and the developed control chart's performance is analysed using the average run length. The proposed control chart is illustrated with numerical examples, and its applicability is demonstrated with simulated data.

Keywords: exponentiated exponential distribution, accelerated life test, hybrid censoring scheme, control chart, average run length

I. Introduction

A control chart is a graphical description of data collected from manufacturing industries. The information may apply to the measurement of quality characteristics or the evaluation of quality characteristics of a sample. It monitors the consistency of the process and raises an alarm if it finds any deviations from the specified tolerance limit. Control charts are effective tool for monitoring the behaviour of a production process in order to determine whether it is stable or not. Control charts visualize the data throughout the order in which it occurred.

In order to monitor the production process in a various circumstances, many new control chart techniques have been developed. A control chart is a type of graph that can be used to analyse how a process varies over time. Every control chart has a minimum of two control limits, known as the lower control limit (LCL) and upper control limit (UCL). It is claimed that the process is in-control if the control statistic falls within the control limits. There are two different kinds of control charts, which are referred to as attribute control charts and variable control charts. The attribute control charts are used to distinguish between conforming and non-conforming items. When the industrial data collected from the measurement process, the variable control charts are implemented.

Lifetime is regarded as a quality characteristic for some products. Life tests are used to monitor the manufacturing process for these products. The product may be classed as conforming or non-conforming based on the results of the life test. This form of product testing takes a considerable amount of time because the duration of the test is lengthy. In this circumstance, the censoring technique is essential and cannot be ignored. Some of the censoring techniques used in life testing include Type-I censoring, Type-II censoring, and hybrid censoring. According to the hybrid censoring, the life test terminates at the earliest of the specified test time t or the time at which the $(UCL + 1)^{th}$ failure is discovered. When the number of failures observed during the life test is between the UCL and LCL at time t, the production process is in control; otherwise, the production process is out of control. Accelerated life testing (ALT) is a test and analytical technique for figuring out how failures will probably happen in the future. As a result of its capacity to "speed up time," ALT is a widely used testing technique. When we cannot wait for failures to occur at their regular rate but need to know how failures are expected to occur in the future, we frequently employ ALT. In an ALT, items are subjected to accelerated conditions including stress, strain, temperature, pressure, etc.; as a result, the products fail faster compared to their performance in a conventional life test. Therefore, ALT under hybrid censoring can minimise total time to failure and inspection cost.

The fraction non-conforming is based on the attribute control chart, such as the *np* control chart, which is constructed by assuming that the quality characteristic follows the normal distribution. The distribution of the quality characteristics may not be normal in reality. Industrial engineers may be misled and the number of non-conforming products may increase if the current control chart is used in this scenario. Numerous authors, including [1-9] and [14-24] have contributed to the literature and worked on attribute control charts for various lifetime distributions. According to a review of the available literature, no research on control charts with a life test for a non-normal distribution, such as an exponentiated exponential distribution with hybrid censoring under accelerated life test, have been attempted.

In reliability and life testing in recent years, exponentiated models have drawn more attention. By exponentiating the corresponding cumulative distribution function with a parameter, the probability distributions are constructed. The exponentiated exponential distribution was first introduced by [13]. It has been revealed that this distribution is more suitable for particular lifetime data than other commonly used lifetime distributions. It has been noted that lifetime data analysis has been quite effective when using the exponentiated exponential distribution. The exponentiated exponential distribution is found to fit data in many cases better than the Weibull, gamma, log-normal, and generalized Rayleigh distributions (see more information on this distribution from [10-12].

This work presents an attribute control chart based on an exponentiated exponential distribution under an accelerated life test with a hybrid censoring scheme. The control chart coefficient was determined, and the performance of the proposed control chart was discussed in relation to the average run length (ARL). When the process scale parameter (median) is changed, the proposed control chart is designed. With the use of simulated data, the proposed control chart's applicability is described. Simulated data is used to illustrate the proposed chart. Exponentiated exponential distribution is introduced in Section 2 along with an explanation of the control chart's design. A simulation study is described in Section 3. The last Section described the overall summary.

II. Design of proposed control chart based on Exponentiated Exponential Distribution under Accelerated Life Test

Let "T" be the product's lifetime, which is distributed to the Exponentiated Exponential Distribution (EED (λ, θ)) with scale parameter "λ" and shape parameter "θ". The probability density function for the variable "T" is given by,

$$f(t|\lambda;\theta) = \left(\frac{\theta}{\lambda}\right) e^{-\left(\frac{t}{\lambda}\right)} \left[1 - e^{-\left(\frac{t}{\lambda}\right)}\right]^{\theta-1} ; t, \lambda, \theta > 0 \quad (1)$$

The underlying distribution's cumulative distribution function is defined by,

$$F(t|\lambda;\theta) = \left[1 - e^{-\left(\frac{t}{\lambda}\right)}\right]^{\theta} ; t, \lambda, \theta > 0 \quad (2)$$

With this distribution, the product's mean as well as median lifetime are given as,

$$\mu = Mean(T) = \lambda[\Psi(\theta + 1) - \Psi(1)]$$

Here, $\Psi(x) = \frac{d}{dx}\Gamma(x)$ and $\Gamma(x) = \int_0^{\infty} x^{u-1} e^{-x} dx$

$$M = Median(T) = -\lambda \ ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]$$

The product's lifetime under ordinary conditions is represented by the symbol t_U, and it follows an exponentiated exponential distribution with specified shape (θ) and scale (λ_U) parameters. As a result, the following equation may be used to determine the product's lifetime under ordinary conditions.

$$F_U(t_U|\lambda_U;\theta) = \left[1 - e^{-\left(\frac{t_U}{\lambda_U}\right)}\right]^{\theta} \quad (3)$$

Similar to this, the lifetime of the product under accelerated conditions is denoted by t_A, and it is assumed that t_A follows an exponentiated exponential distribution with specified shape (θ) and scale (λ_A) parameters. As a result, the product's lifetime under accelerated conditions is stated as follows.

$$F_A(t_A|\lambda_A;\theta) = \left[1 - e^{-\left(\frac{t_A}{\lambda_A}\right)}\right]^{\theta} \quad (4)$$

The product's mean and median lifetime under accelerated conditions as

$$\mu_A = \lambda_A[\Psi(\theta + 1) - \Psi(1)] \ and \ M_A = -\lambda_A \ ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right] \quad (5)$$

By using the Acceleration Factor (AF) definition, we can write

$$\lambda_A = \frac{\lambda_U}{AF}$$

Therefore, Equation (4) becomes

$$F_A(t_A) = \left[1 - e^{-\left(\frac{t_A \times AF}{\lambda_U}\right)}\right]^{\theta}$$

The failure probability of the product, denoted by p_0 while its process is under control, is characterized by the probability that the product will fail before the censoring time t_A.

$$p_0 = \left[1 - e^{-\left(\frac{t_A \times AF}{\lambda_U}\right)}\right]^{\theta} \tag{6}$$

We express the censoring time as a constant multiple of the product's median lifetime, such as $t_A = a^*M_A$, where 'a' is referred to as a test termination ratio. Equation (6) may be rewritten by substituting values for t_A and M_A.

$$p_0 = \left[1 - e^{\left(\frac{\lambda_A}{\lambda_U} \times a \times AF \times ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]\right)}\right]^{\theta} \tag{7}$$

It is assumed that there is no change in scale parameter when the process is in-control. i.e., $\lambda_A = \lambda_U$. Hence, Equation (7) becomes

$$p_0 = \left[1 - e^{\left(a \times AF \times ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]\right)}\right]^{\theta} \tag{8}$$

We can determine the failure probability of the product under accelerated life test using the above equation. The process is considered as out-of-control if there is a change in scale parameter, under this situation the scale parameter shifts from λ_U to λ_A. The ratio between the actual scale parameter and the shifted scale parameter as accelerated life test is denoted by '*f*' and is called as shift constant. That is, $\frac{\lambda_U}{\lambda_A} = f$. Therefore, the failure probability of the product under accelerated life test when the process is out-of-control is denoted by p_1 and determined by

$$p_1 = \left[1 - e^{\left(\frac{a \times AF}{f} \times ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]\right)}\right]^{\theta} \tag{9}$$

Based on the number of products in each subgroup, we propose the following *np* control chart for EED (λ, θ) under accelerated life test with hybrid censoring:

Step 1: Select a set of n products randomly from the production process.

Step 2: Conduct the life test on the selected products considering t_A as the test termination time under the fixed acceleration factor. Observe the number of failed items (D, say)

Step 3: Terminate the life test either after reached at time t_A or D > UCL before reaching time t_A, whichever is earlier.

Step 4: Declare the process as out of control if $D>UCL$ or $D<LCL$. Declare the process as in control if $LCL \leq D \leq UCL$.

The above is considered to as a *np* control chart since it shows the number of failures (D) instead of the proportion nonconforming (p). While the process is in control, the random variable D follows a binomial distribution with parameters n and p_0, where p_0 is the probability that an item fails before time t_A. As a result, the control limits for the control process are as follows:

$$UCL = np_0 + k\sqrt{np_0(1 - p_0)} \tag{10a}$$
$$LCL = max\left[0, np_0 - k\sqrt{np_0(1 - p_0)}\right] \tag{10b}$$

Where, k is the control limit coefficient. The fraction nonconforming in the control process (p_0) can be determined using equation (8).

$$p_0 = \left[1 - e^{\left(a \times AF \times ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]\right)}\right]^{\theta}$$

The control limits for the practical application are provided as, because in practise, probability p_0 is generally unknown.

$$UCL = \bar{D} + k\sqrt{\bar{D}\left(1 - \frac{\bar{D}}{n}\right)}$$
$$LCL = max\left[0, \bar{D} - k\sqrt{\bar{D}\left(1 - \frac{\bar{D}}{n}\right)}\right]$$

Here, $\bar{D}$is the average number of failures over the subgroups.

The following is the probability of declaring as in control for the proposed control chart:

$$p_{in}^0 = P(LCL \le D \le UCL|p_0)$$
$$p_{in}^0 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1 - p_0)^{n-d} \tag{11}$$

The control chart's performance is measured by the ARL. The ARL for the process of in control as follows:

$$ARL_0 = \frac{1}{1 - \left[\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1-p_0)^{n-d}\right]} \tag{12}$$

I. ARL when the scale parameter is shifted:

Assume the process median has been changed from M_0 to M_1. The probability in (9) now becomes

$$p_1 = \left[1 - e^{\left(\frac{a \times AF}{f} \times ln\left[1 - \left(\frac{1}{2}\right)^{\left(\frac{1}{\theta}\right)}\right]\right)}\right]^{\theta}$$

The probability that the process is declared to be in control after shifting to M_1 is now determined by

$$p_{in}^1 = P(LCL \le D \le UCL|p_1)$$
$$p_{in}^1 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1 - p_1)^{n-d} \tag{13}$$

The ARL for the shifted process is given as follows:

$$ARL_1 = \frac{1}{1 - \left[\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1-p_1)^{n-d}\right]} \tag{14}$$

To construct the tables for the proposed control chart, we applied the following algorithm.

Step 1: Specify the values of ARL, say $r_0$and sample size n.

Step 2: Determine the control chart parameters and truncated time constant a values for which the ARL from equation (8) approach r_0.

Step 3: By using the values of the control chart parameters obtained in step 2, determine the ARL_1 in accordance with shift constant f by using equation (14).

For various values of r_0 and n, we determine the control chart parameters and ARL_1, which are shown in Tables 1 & 2. Table 1 & Table 2 shows that the ARLs tend to get smaller when the shift constant f gets less.

Table 1: *ARLs when the process average (median) shifted when the sample size fixed*

	n = 25								
	AF = 1			AF = 1.5			AF = 2		
	r_0 = 300	r_0 = 350	r_0 = 450	r_0 = 300	r_0 = 350	r_0 = 450	r_0 = 300	r_0 = 350	r_0 = 450
UCL	14	16	18	14	16	18	14	16	18
LCL	1	2	3	1	2	3	1	2	3
a	0.6444	0.7689	0.8298	0.4296	0.5126	0.5532	0.3222	0.3844	0.4149
k	2.6331	2.8254	2.9472	2.6331	2.8254	2.9668	2.6331	2.8249	2.9668
Shift (f)	ARL								
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.40	1.04	1.03	1.08	1.04	1.03	1.08	1.04	1.03	1.08
0.50	1.42	1.33	1.69	1.42	1.33	1.69	1.42	1.33	1.69
0.60	2.99	2.64	4.46	2.99	2.64	4.46	2.99	2.64	4.46
0.70	8.97	7.68	17.28	8.97	7.68	17.21	8.97	7.68	17.21
0.80	33.01	28.40	82.95	33.01	28.40	82.95	33.01	28.39	82.95
0.90	128.99	117.81	391.28	128.99	117.81	391.28	128.99	117.81	391.28
1.00	300.01	350.01	450.01	300.01	350.01	450.01	300.01	350.01	450.01

Table 2: *ARLs when the process average (median) shifted when r_0 fixed*

	n = 500								
	AF = 1			AF = 1.5			AF = 2		
	n = 20	n = 25	n = 30	n = 20	n = 25	n = 30	n = 20	n = 25	n = 30
UCL	12	15	19	12	15	19	12	15	19
LCL	0	1	3	0	1	3	0	1	3
a	0.6161	0.6926	0.7439	0.4258	0.46176	0.4959	0.3081	0.3463	0.3890
k	2.9166	2.8550	2.9467	2.7402	2.8547	2.9471	2.9158	2.8550	2.9581
Shift (f)	ARL								
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.40	1.20	1.04	1.02	1.14	1.04	1.02	1.20	1.04	1.01
0.50	2.25	1.43	1.34	1.94	1.43	1.34	2.25	1.43	1.21
0.60	6.28	3.08	2.97	4.92	3.08	2.98	6.27	3.08	2.29
0.70	22.20	9.56	10.51	16.00	9.57	10.51	22.17	9.56	6.86
0.80	87.50	36.94	49.86	59.10	36.97	49.90	87.35	36.93	28.19
0.90	317.92	157.16	261.95	218.69	157.30	262.19	317.42	157.16	137.85
1.00	500.05	500.13	500.34	500.06	500.35	500.16	500.24	500.13	500.08

II. Illustration 1:

Suppose that the lifetimes of the products follow an exponentiated exponential distribution with parameters λ, θ = 2 and AF = 1. Consider the following values for the products: M_0 = 1000 hours, r_0 = 300 and n = 25. Table 1 provided the following control chart parameters: k = 2.6331, a = 0.6444, LCL = 1 and UCL = 14. As a consequence, the control chart was established in the following manner:

Step 1: Select a sample of 25 products from each subgroup and submit them to the life test for 644 hours. During the testing, count the number of failed items (D).

Step 2: Declare that the process is in control if the value of D is between 1 and 14; otherwise, declare that it is out of control.

III. Illustration 2:

Assume that the product lifetimes follow an exponentiated exponential distribution with parameters λ, θ = 2 and AF = 2. Consider the following product values: M_0 is 1000 hours, r_0 is 500, and n is 25. The following control chart parameters were presented in Table 2: k = 2.8550, a = 0.3463, LCL = 1 and UCL = 15. As a result, the control chart was established in the following manner:

From each subgroup, choose a sample of 25 products, and put them through a life test that lasted at least 346 hours while being accelerated. Count the number of failed items during testing (D). If the value of D is between 1 and 15, the process is in control; otherwise, it is out of control.

III. Simulation Study

This section discusses the application of the produced control chart with simulated data. The data was generated and shown in Table 3, using an exponentiated exponential distribution with an average (median) lifetime (M_0) of 1000 hours, the value of acceleration factor (AF) is 1, and the value of the shape parameter (θ) is 2. Assuming that the sample size (n) is 20 and the specified ARL (r_0) is 500. At M_0 = 1000 hours and f = 1, the process is considered to be in control. In-control parameters are employed to construct the first 15 observations of subgroup size 20. Let's now assume that since the median of the exponentiated exponential distribution has shifted, the process has shifted as well. The value of the shift constant f is 0.5. When f is set to 0.5, the shifted median is used to produce the next 15 observations (shown in Table 4).

Take into account that the experiment was conducted at t = 616 hours. Table 3 and Table 4 shows the number of failures, indicated by the letter D, for each subgroup. Figure 1 shows the calculated LCL = 0 and UCL = 12 for simulated data.

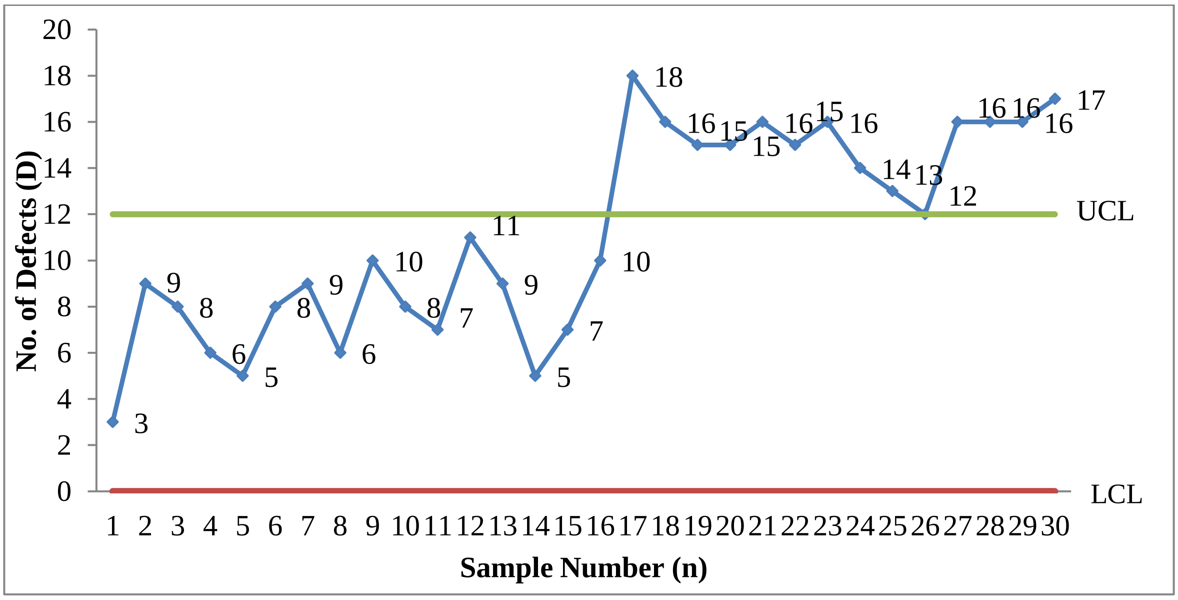

Figure 1: *Control Chart for Simulated Data*

Table 3: *Simulated Data when M_0 = 1000 hours, AF = 1, θ = 2, ARL_0 = 500, n = 20 and f = 1*

S. No		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Sample	**1**	1075	558	1334	520	832	479	545	1683	361	1862	639	756	1169	814	365
	2	1379	881	976	335	324	523	1392	901	918	2901	223	1208	894	751	681
	3	1253	763	1096	487	2703	386	918	824	560	860	370	203	450	685	769
	4	673	273	486	356	3318	1049	499	1264	1242	1090	888	113	2837	822	1560
	5	913	920	377	510	984	1963	2969	990	424	543	1393	4303	2023	2015	1517
	6	1465	2041	1026	704	2708	1940	707	3080	390	1055	1259	107	394	356	873
	7	492	351	264	347	451	220	805	1409	1396	1010	961	1878	426	1017	494
	8	693	389	796	1169	1397	697	665	2177	678	144	614	1023	485	807	2407
	9	1304	913	1562	1483	371	1681	401	225	188	594	501	547	394	132	1439
	10	887	585	255	1079	963	300	2463	257	203	996	2092	2414	1720	446	1265
	11	750	851	359	1852	1296	621	430	901	1458	780	1279	2128	2741	235	2100
	12	681	898	571	1736	1333	796	317	1031	253	1016	1420	354	302	1093	2160
	13	2263	1382	733	1879	394	483	2328	345	223	504	819	413	126	750	1618
	14	420	599	162	955	1614	712	1346	513	404	2911	1403	1682	1689	539	2929
	15	3150	582	756	1415	882	1231	577	3107	2142	228	440	124	777	1828	478
	16	971	675	1097	980	364	2052	667	1841	1615	1094	234	2067	1822	1869	2107
	17	931	545	2520	983	1169	491	548	584	977	204	1917	324	335	691	521
	18	1902	535	665	1069	719	453	676	1232	1004	477	724	187	1049	829	1469
	19	272	990	2264	1353	820	914	117	223	702	463	2186	198	1838	779	471
	20	3368	1858	454	744	1285	897	462	1030	382	2883	3209	470	521	1019	449
D		3	9	8	6	5	8	9	6	10	8	7	11	9	5	7

Table 4: *Simulated Data when M_0 = 1000 hours, AF = 1, θ = 2, ARL_0 = 500, n = 20 and f = 0.5*

	S. No	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Sample	**1**	159	303	792	933	1168	436	298	191	364	320	622	173	361	536	462
	2	709	209	480	447	959	87	3035	244	926	887	130	555	174	511	333
	3	1661	361	1165	183	177	590	141	1460	433	315	291	447	688	633	553
	4	183	310	213	545	106	867	496	285	539	177	904	511	1064	691	893
	5	927	130	426	558	345	144	102	1030	595	1030	292	252	325	1029	478
	6	948	233	223	245	192	202	399	247	300	1718	303	193	163	435	530
	7	170	112	421	132	231	923	740	134	480	338	1811	211	865	94	460
	8	935	205	534	1121	327	1070	550	94	1545	186	185	216	279	343	84
	9	1308	278	423	325	778	126	714	267	575	664	591	237	204	360	747
	10	498	481	510	662	628	397	1214	108	490	942	137	382	416	613	282
	11	216	571	66	568	499	506	509	329	54	1065	503	699	25	437	466
	12	1119	42	255	519	602	423	538	606	783	447	367	1063	263	317	330
	13	1544	861	108	238	674	323	441	621	42	531	1951	442	147	330	685
	14	373	322	125	948	189	750	391	107	952	252	315	366	27	162	611
	15	1727	131	135	193	324	139	315	162	537	897	872	121	1424	352	135
	16	1336	307	368	160	122	573	135	192	893	611	125	78	560	730	292
	17	537	313	784	207	131	344	1086	457	856	171	725	217	251	459	288
	18	76	73	398	162	192	312	332	150	109	274	626	712	219	233	158
	19	306	116	438	827	258	199	477	105	60	453	280	1128	180	178	58
	20	367	620	1294	215	463	226	447	637	202	208	639	443	149	302	272
D		10	18	16	15	15	16	15	16	14	13	12	16	16	16	17

The constructed control chart, as shown in Figure 1, reveals a shift at the 17th (2nd observation following the shift) observation, with a calculated ARL of 2.25. As a consequence, the developed control chart efficiently identifies the shift in the production process.

IV. Conclusion

In this article, a new attribute control chart based on exponentiated exponential distribution under accelerated life test with hybrid censoring scheme is proposed to ensure the median lifetime of the product as the quality criterion. The newly established control chart is highly adaptable and may be used to monitor the lifetime of quality products. The tables are provided for industrial usage and are explicated with the use of simulated data. It is generated using *R* software from an exponentiated exponential distribution. The performance of the proposed control chart is expressed in terms of ARLs for various shift constants (*f*). It should be noted that if the hybrid censoring scheme is used to carry out the life test, executing the sampling inspection will reduce the time and cost of conducting the life test. The developed attribute control chart has the potential to be extended for use with various other statistical distributions as part of a continuing research study.

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