TWO-STAGE GROUP ACCEPTANCE SAMPLING PLAN FOR HALF-NORMAL DISTRIBUTION

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Abstract

This paper proposes a time-truncated life test based on a two-stage group acceptance sampling plan for the percentile lifetime following a half-normal distribution. The optimal parameters for this plan are determined to simultaneously satisfy both producer's and consumer's risks for a given experimentation time and sample size. The efficiency of the proposed sampling plan is evaluated by comparing the average sample number with that of existing sampling plans. Industrial examples are provided to illustrate the application of the proposed sampling plan.

Keywords: Two-stage group sample, producer and consumer risk, normal distribution.

I. Introduction

In today's modern industrial environment, producing high-quality products using modern statistical quality control techniques has become essential. Product quality is a crucial factor contributing to business success, growth, and competitive advantage [1]. SQC techniques are now vital for any manufacturing process, as their application helps improve product quality by reducing process and product variability. SQC plays a significant role in the success of any industry. SQC involves a set of operational activities that an enterprise implements to ensure that its products meet the required quality levels set by consumers. According to [2], product quality can be evaluated based on various dimensions, including durability, serviceability, performance, aesthetics, features, reliability, and conformance to standards. These dimensions collectively determine the overall quality of a product. Therefore, quality has become the most significant factor in consumer satisfaction when selecting among competing products and services.

SQC involves a set of operating activities that an enterprise implements in order to get certified that the quality of its products is at required levels of the consumers. According to [3] one can evaluate the quality of the product in terms of its durability, serviceability, performance, aesthetics, features, reliability, quality and its conformance to standards collectively and they are termed as the dimensions of quality. Hence, we can say that in selecting among competing products and services, the quality has become the most significant factor for consumer's satisfaction.

The quality of any finished product can be judged by inspecting a few items taken randomly from a lot of products and the process of taking such samples is called the sampling. In quality management, the acceptance sampling plans are vital tools in making a decision about the product whether to accept or reject based on the inspection of sampled items and the sampling plans prescribe the experimenter how many items in the sample should be selected from the submitted lot for inspection and how many defectives can be allowed in this sample in order to satisfy both the producer and the consumer. The probability of rejecting a good lot is called the producer's risk and the chance of accepting bad lot is called the consumer's risk. The cost of any life test experiment is directly proportional to the sample size. Therefore, sampling plans that provide smaller sample size for inspection and minimize two risks are considered as efficient sampling plans.

In general, the acceptance sampling plans can be classified into two types namely attribute sampling plans and variables sampling plans. The attribute sampling plans are implemented for quality characteristics which are expressed on a ''go, no go'' basis whereas the variables sampling plans can be applied where the quality characteristics of interest are measured on a numerical scale [4]. Various types of sampling plans such as single sampling plan, double sampling plan, multiple sampling plan, and sequential sampling plan are available in the literature [5]. Group acceptance sampling plan is one of the types of sampling plans which involves a number of testers to be used for testing so that cost and time can be reduced. The inspection of multiple items simultaneously can be made easy to the experimenter for testing. Two stage group acceptance sampling plan is the extension of GASP which involves two groups. The GASP is more advantageous than the conventional sampling plans in terms of minimum inspection so that the considerable testing time and cost can be reduced [6]. The advantage of two stage group sampling plan is that it reduces the average sample number as compared to the GASP.

Several authors have investigated the sampling plans under various life time distributions, which are available in the literature of acceptance sampling [7,8,9,10,11,12] proposed the SSP based on half normal distribution. By exploring the literature on two stage group sampling plans, it can be seen that no work is available based on the half normal distribution. In this paper, we will present the designing of two stage group sampling plan when the life time of an item follows the half normal distribution. The structure of proposed plan is presented and efficiency is compared with the existing sampling plan. The application of the proposed sampling plan is explained with the help of industrial illustrative examples.

2. Half-normal distribution

As far as the variables sampling plans are concerned, the normal distribution is the most preferred statistical distribution. But for life testing problems, normal distribution is not preferred because of its range $[-\infty, \infty]$. However, one of the normal family distributions called the half-normal distribution is the widely used probability distribution for nonnegative data modeling, particularly, in life time testing. [13] investigated the properties of half normal distribution. [14] investigated the maintenance performance of the system under half-normal failure lifetime model as well as a repair-time model. The probability density function of a half normal distribution with 0 mean and its parameter θ with domain $y \in [0, \infty]$ is given by

$$f(x) = \frac{2\theta}{\pi} e^{-\frac{y^2\theta^2}{\pi}}, y > 0, \theta > 0 \tag{1}$$

Its cumulative distribution function is given by

$$F(\mathrm{y}) = \mathrm{erf}\left(\frac{\theta y}{\sqrt{\pi}}\right), \; y > 0, \theta > 0 \tag{2}$$

Here erf is the "Error Function" defined by

$$\text{errf}\ (y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-t^2} dt \tag{3}$$

Consider that life time of product follows a half-normal distribution with σ as a scale parameter. Its cdf is given by

$$F(y) = \text{erf}\left(\frac{\theta\left(\frac{t}{\sigma}\right)}{\sqrt{\pi}}\right),\ t > 0, \theta > 0, \sigma > 0 \tag{4}$$

The 100th percentile of the half normal distribution with $0 < p < 1$, is defined as

$$t_p = \frac{\sigma\sqrt{\pi}}{\theta}\text{erf}^{-1}\ (\text{p}),\ \text{ for } 0 < p < 1 \tag{5}$$

Here erf^{-1} is inverse function of error function. The Maclaurin series of erf^{-1} (.) is given by

$$erf^{-1}(y) = \sqrt{\pi}\left(\frac{1}{2}y + \frac{1}{24}\pi y^3 + \frac{7}{960}\pi^2 y^5 + \frac{127}{80640}\pi^3 y^7 + \cdots\right) \tag{6}$$

According to Pewsey, "If Z is a standard normal random variable, Z~N(0,1), then Y=|Z| follows a standard positive half normal distribution and -Y=-|Z|. follows a standard negative half-normal distribution. The half-normal distribution is a central chi-square distribution with one degree of freedom and a special case of truncated and folded normal distributions". The half-normal distribution is also a limiting case of skewed normal distribution [15]. The applications of half-normal in reliability analysis can be seen in Ayman and Kristen [15]. As half-normal distribution has positively skewed shape, there is a need to model monotone hazard rates. The half-normal distribution is very widespread model to describe the lifetime process of any device under fatigue (see [16].

3. Two Stage Group Acceptance Sampling Plan

Naveed et al. [17] proposed the two-stage group sampling plan. The operating procedure of this plan is explained below.

3.1 Stage one

- Extract the first random sample of size n_1 from a lot submitted for inspection.
- Randomly assign r items to each of g_1 groups or testers so that $n_1 = rg_1$ and set them on test for the duration of t_0 units of time.
- Accept the lot if the total number, Y_1, of failures from g_1 groups is smaller than or equal to c_{1a}.
- Truncate the test and reject the lot as soon as the number of failures Y_1 reaches $c_{1r}(> c_{1a})$ during the test. Otherwise, go to stage two.

3.2 Stage two

- Extract a second random sample of size n_2 from the same lot.
- Randomly assign r items to each of g_2 groups so that $n_2 = rg_2$ and set them on test for the duration of t_0 units of time again.
- Let the total number of failures from the second sample be Y_2.
- Accept the lot if the total number, $Y_1 + Y_2$, of failures from g_1 and g_2 groups is smaller than or equal to $c_{2a}(> c_{1a})$. Otherwise, truncate the test and reject the lot.

The design parameters of above two stage group sampling plans are g_1, g_2, c_{1a}, c_{1r}, and c_{2a}. The acceptance number, c_{2a}, in the second stage is larger than the acceptance number, c_{1a}, in the first stage, since in two stage sampling total number of failures from both stages is used in decision making.

4. Designing of the proposed sampling plan

Based on the operating procedure of two-stage group sampling plan, the lot acceptance probability at the first stage is given as

$$P_a^{(1)} = P\{Y_1 \leqslant c_{1a}\} = \sum_{j=0}^{c_{1a}} \binom{rg_1}{j} p^j (1-p)^{rg_1-j} \quad (7)$$

The lot rejection probability at the stage one is as follows (see Aslam et al. (2012)).

$$P_r^{(1)} = \sum_{j=c_{1r}}^{rg_1} \binom{rg_1}{j} p^j (1-p)^{rg_1-j} = 1 - \sum_{j=0}^{c_{1r}-1} \binom{rg_1}{j} p^j (1-p)^{rg_1-j} \quad (8)$$

To accept the lot based on stage two, the total number, $Y_1 + Y_2$, of failures from both groups g_1 and g_2 must be smaller than or equal to c_{2a}. So, the lot acceptance probability at this stage under the proposed two stage sampling plan is as follows

$$P_a^{(2)} = P\{c_{1a} + 1 \leqslant Y_1 \leqslant c_{1r} - 1, Y_1 + Y_2 \leqslant c_{2a}\} \quad (9)$$

Thus, under the proposed two stage group acceptance sampling plan, the lot acceptance probability is as follows.

$$L(p) = P_a^{(1)} + P_a^{(2)} \quad (10)$$

Stated that in order to implement an acceptance sampling plan to assure the percentile lifetime in a truncated life test, it is convenient to determine the experiment time in terms of the specified percentile lifetime as $t_0 = \delta_{\mathrm{p}} t_{\mathrm{p}}^0$, where δ_{p} is called the termination time ratio. The probability of failure of an item before time t_0 is given as

$$p = F(t_0) = F(\delta_{\mathrm{p}} t_{\mathrm{p}}^0) \quad (11)$$

or

$$p = F(t_0) = \operatorname{erf}\left(\delta_{\mathrm{p}} erf^{-1}(pd)/(t_{\mathrm{p}}/t_{\mathrm{p}}^0)\right) \quad (12)$$

Where t_{p} = true unknown population pth percentile. The erf (.) and erf^{-1} (.) functions have been defined in Eqs. (3) and (6) respectively. The above expression is represented in terms of $t_{\mathrm{p}}/t_{\mathrm{p}}^0$ and is also called failure probability. Let α be the producer's risk and β be the consumer's risk. The producer wishes that the lot acceptance probability should be larger than $1-\alpha$ at various values of percentile ratio $t_{\mathrm{p}}/t_{\mathrm{p}}^0$ and the consumer wishes that it should be smaller than β at $t_{\mathrm{p}}/t_{\mathrm{p}}^0 = 1$. Therefore, the plan parameters of the proposed plan will be determined by minimizing ASN at consumer's risk using the following non-linear optimization problem [16]. Minimize ASN $(p_2) = \mathrm{rg}_1 + \mathrm{rg}_2\left(1 - P_a^{(1)} - P_r^{(1)}\right)$, Subject to $L(p_2) \geqslant 1-\alpha$

$$L(p_1) \leqslant \beta \quad (13)$$

$$g_1 > 1, g_2 > 1, r > 1, c_{1r} > c_{1a} > 0, c_{2a} > c_{1a} > 0 \quad (14)$$

5. Description of tables and industrial examples

Tables for the selection of optimal parameters of the proposed sampling plan are given for various specified requirements such as consumer's risk ($\beta = 0.25, 0.10, 0.05, 0.01$), producer's risk ($\alpha = 0.05$), percentile ratio ($d_2 = 2,4,6,8$), ($r = 5$ or 10), $\delta_{\mathrm{p}} = 0.5$ or 1.0 and $d_1 = 1$. The tables are presented for average life ($p = 0.5$ and $p = 0.25$). The tables can be made for any other specified parameters. The R codes are available with the authors upon request.

The optimal parameters of the proposed sampling plan along with the ASN and the lot acceptance probability are presented in Tables 1-4. Tables 1 and 2 provide the optimal parameters of the two-stage group acceptance sampling plan to ensure median life time of the products and Tables 3 and 4 show the optimal parameters of the two-stage group acceptance sampling plan to ensure lower percentile life. From Tables 1-4, we observe that when the ratio d_2 increases the number of groups for the experiment and the acceptance numbers decrease. Tables 2-4 will be made available on request.

6. Case study

For the implementation of proposed sampling plan, we will consider the data from a leading ball-bearing manufacturing company in Korea. The failure data are well fitted to the half normal distribution. Suppose that the company is interested to test the product using the proposed sampling plan. Let $\alpha = 5\%, \beta = 5\%$, and $p = 0.5$. The number of testers of the product is limited to $r = 5$ From Table 1, we have $c_{1r} = 3$, $c_{1a} = 0, c_{2a} = 2, g_I = 3$ and $g_2 = 2$. A sample of size 15 items are selected and distributed into three groups. The failure times for each of three groups are given as follows

Table 1: *Optimal parameters of the two-stage group sampling plan under half-normal distribution with δq=0.5, q=0.5*

β	d_2	$r = 5$							$r = 10$						
		c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_2)$	c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_2)$
0.25	2	9	6	13	7	5	41.34	0.953	10	7	13	4	2	44.54	0.956
	4	4	2	3	3	1	16.06	0.9671	4	2	5	2	1	21.14	0.9558
	6	3	0	2	2	1	12.19	0.9732	4	2	3	2	1	21.14	0.9717
	8	2	0	3	2	1	10.84	0.9574	3	1	9	2	1	20.53	0.9717
0.1	2	13	0	17	11	6	63.20	0.9554	16	10	17	6	3	72.45	0.9552
	4	4	2	5	4	3	21.72	0.9505	5	1	4	2	1	23.42	0.9534
	6	3	1	3	3	2	16.36	0.9644	4	2	4	2	1	21.14	0.9857
	8	3	1	2	3	1	15.68	0.9753	3	1	7	2	1	20.53	0.9717
0.05	2	14	4	20	12	9	71.26	0.9502	15	10	26	7	6	77.64	0.9529
	4	5	2	6	5	4	27.96	0.9614	5	3	8	3	2	30.92	0.9514
	6	4	1	3	4	2	21.67	0.9598	4	1	3	2	1	21.67	0.9598
	8	3	0	2	3	2	16.90	0.9519	3	1	4	2	1	20.53	0.9713
0.01	2	19	8	30	18	15	97.62	0.9521	19	2	31	9	8	98.13	0.9540
	4	6	1	8	7	5	36.74	0.9634	7	3	7	4	2	41.28	0.9528
	6	4	0	4	5	3	26.08	0.9580	4	0	6	3	2	30.51	0.9547
	8	3	0	3	4	3	21.03	0.9569	4	2	3	3	1	30.19	0.9648

Group-1	Group-2	Group-3
0.6825	1.5650	0.9232
1.8024	0.8981	0.0607
0.0509	0.7322	0.4541
1.2080	2.1866	1.0035
0.4275	0.4223	0.6611

Let $\delta_{\mathrm{p}} = 0.5$ and $t_{\mathrm{p}}^0 = 1.5$, these leads $t_0 = 0.075$. We note one failure from group 1, no failure from group 2 and 1 failure from group 3 before time $t_0 = 0.075$. So, the total number of failures from three groups is 2. As the total number of failures lies between $c_{1a} = 0$ and $c_{Ir} = 3$, a decision about the disposition of lot will be made on the basis of second sample. A second sample of size 10 is selected from the lot and distributed into two groups. The number of failures from two groups is given in the following table

Group-1	Group-2
0.8472	0.0701
0.7845	0.4341
0.5452	0.1104
0.1316	0.7054
0.2624	0.8239

From group 1, we note that no failure occurs before experiment time and from group 2, we note that only one failure occurs before the experiment time. Since, the total number of failures from both samples is larger than $c_{2a} = 2$, the lot of products will be rejected.

7. Comparison

In this section, a comparison is made between the GASP and the proposed two-stage group sampling plan. This comparison is made based on the ASN needed for both sampling plans. It is to be pointed out that the two-stage group sampling plan could have a chance to use the sample from the first stage or combined samples from both stages to make a decision. For example, if the product lifetime has a half-normal distribution, the next step would be to decide whether to use a GASP or to use the proposed two stage group sampling plan which will have a minimum ASN. Here we compare both plans for experiment termination time ($\delta = 0.50$ and 1.0) with $p = 0.5$ median life time quality level.

Tables 5 and 6 provide the ASN for both acceptance sampling plans, where presumed producer's risk was set as $\alpha = 0.05$. It has been witnessed from tables that the ASN for the proposed two-stage group sampling plan is much smaller than the GASP at lower ARL mean ratios with termination time $\delta_p = 0.5$ and 1.0. So, it is concluded that the two-stage group acceptance sampling plan is better than the GASP as it provides lesser ASN at lower percentile ratios for accepting or rejecting a lot in case of median life time quality.

As in Table 5, when $\delta_p = 0.5$ with $p = 0.5$ and consumer's risk is set as 0.05 with ratio 2, ASN for GASP when $r = 5$ is 110 and the ASN of the proposed two stage group acceptance sampling plan reduces to 71.26. Likewise, from Table 6, when $\delta_p = 1.0$ and $p = 0.5$ with same consumer's risk and ratio, the ASN for GASP is 50 but for proposed plan, it reduces to 38.50. Table 6 will be made available on request.

8. Concluding remarks

In this paper, two stage sampling plan has been proposed for the inspection of products whose life time follows a half-normal distribution. Tables have been presented for industrial applications of the proposed sampling plan. The efficiency of the proposed sampling has been compared with the existing single stage group sampling plan. It is concluded that the proposed sampling plan is more efficient in reducing the ASN for the life test experiment. The real time applications of the proposed sampling plan are given using the industrial data. The proposed sampling plan can also be used in testing of software

Table 5: *ASN for GASP and two-stage group sampling plan under half-normal distribution $\delta_p = 0.5$ when $p = 0.5$.*

B	d_2	GASP with $r = 5$	GASP with $r = 10$	Proposed two-stage plan	
				$r = 5$	$r = 10$
0.25	2	65	70	41.34	44.54
	4	20	*	16.06	*
	6	15	*	12.19	*
0.1	2	95	100	63.20	72.45
	4	30	30	21.72	23.42
	6	25	30	16.36	21.14
	8	20		15.68	
0.05	2	110	40	71.26	77.64
	4	40	30	27.96	30.92
	6	30	160	21.67	21.67
	8	30	60	16.90	20.53
0.01	2	155	40	37.62	98.13

4	60	40	26.74	41.28
6	40		21.03	30.51
8	35			

Optimal plan does not exist. reliability, testing/lot sizing of electronic product, automobile industry and mobile manufacturing industry. The efficiency of the proposed sampling plan using a cost model can be considered as future research.

Discloser statement

The authors declare no potential conflict of interest.

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