OPTIMAL AND ECONOMIC DESIGN OF CHAIN SAMPLING PLAN FOR ASSURING MEDIAN LIFE UNDER NEW COMPOUNDED BELL WEIBULL LIFE TIME MODEL

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Abstract

The methodology to design, one of the cumulative results plans called chain sampling plan, is proposed in this paper which ensures the median lifetime of the products under the complementary bell Weibull model. For costly and destructive testing, usually single sampling plan with zero acceptance number is used. But chain sampling plan is an alternative to zero acceptance number single sampling plans. A comparative analysis of proposed plan's OC curve outperforms in discrimination between the lots of varying quality, when compared to the single sampling plan. The advantages of the proposed plan by comparing the performance of the OC curve with other lifetime distributions are also discussed. Tables are constructed to select the optimal parameters for the various combinations of lifetime distributions. The implementation of the proposed plan in industrial scenarios is also explained by using a real time data. Finally, an economic design of the proposed sampling plan is discussed by considering some cost models to minimize the total cost.

Keywords: Consumer's risk, Complementary Bell Weibull distribution, Chain sampling plan, Economic design, Median life, Producer's risk, Truncated life test.

I. Introduction

In today's global marketplace, quality is no longer merely an option or a goal for companies. Instead, it has become an essential requirement for businesses to thrive and succeed. Statistical quality control or SQC uses the technique of statistical approaches like reliable product quality, monitoring, maintenance, and assurance in a variety of sectors. Due to the requirement of testing and destroying every unit in a lot, a 100% inspection plan with destructive testing is not feasible option for both the buyer and the producer when the lot is destroyed, there will be a perpetual shortage for the buyer and also the producers may not achieve the sufficient return on investment. Acceptance sampling plan (ASP) plays a major role in SQC by aiding in the process of decision-making by assessing the quality of a batch and determining whether it should be accepted or rejected. This tool is highly significant in ensuring that only lots with satisfactory quality are accepted. Making decisions about the lot allocation based on samples can increase the chances of errors occurring. Probability of Type I error, also mentioned to as the producer's risk (α), and happens when there is possibility of rejecting a good lot. On the other hand, Probability of Type II error is referred to as the consumer's risk (β) occurs when there is a chance of accepting a defective lot.

ASP's are very important in ensuring the quality and conformity of submitted lot as well as adherence with its acceptance criteria, the utmost number of flawed items may be present in the sample, accepting the entire lot priory. The performance of an ASP's effectiveness lies in the investigation of its operating characteristic (OC) curve. This graphical representation allows the ability of ASP to differentiate between acceptable and unacceptable lots. For costly and destructive testing, an SSP with zero acceptance number is more preferable. But SSP with zero-acceptance number, exhibits limited discriminatory capability. Consequently, the OC curve of these plans lacks effectiveness and does not display any inflection point.

Dodge [5] introduced the idea of chain sampling plan, which is denoted as ChSP-1; to address the drawback of SSP with acceptance number is zero. ChSP-1 applies the accumulated outcomes of multiple samples to determine the acceptance or rejection of a submitted lot. A ChSP-1 aims to enhance the shape of the OC curve compared to SSP with a zero acceptance number, also minimizing the necessary sample size. It is also known as a conditional sampling plan as it leverages information from previous lots to determine how the current lot should be disposed off. The ChSP-1 plan is designed to minimize the risk of rejecting an entire lot, due to the presence of one nonconforming item was detected in the sample. Currently, literature of ASP contains several studies on ChSP-1 across diverse conditions. The procedure outlined by Soundararajan [15] helps practitioners in constructing the ChSP-1 based on specific requirements. Govindaraju and Subramani [6] proposed an alternative method for selecting the ChSP-1 plan that minimizes both producer's and consumer's risk. For further information regarding the concept of ChSP-1, one may refer to Soundararajan and Govindaraju ([16], [17]), Raju [13], Govindaraju and Balamurali [7], Raju and Raghottam [14], Jeyadurga and Balamurali [10] and so on.

The process of developing a product is incomplete without conducting a life testing experiments, because it focuses how to determine the reliability of products over an extended period. Overall, the implementation of a reliability test plan that includes time truncated life (TTL) testing can greatly enhance the product quality assurance efforts and help to ensure that customers receive reliable and high-quality products. Acceptance sampling involves the use TTL, to ascertain, that either the average or percentile lifetime of products meets certain criteria. Several sampling plans have been investigated by using the TTL test, (see for example Aslam and Jun [1], Jeyadurga *et al.* [11], Vijayaraghavan and Saranya [19], Jayalakshmi and Veerakumari [12], Vijayaraghavan and Pavithra [20]). In this paper, we attempt to design of ChSP-1 plan with the intention of assuring complementary Bell Weibull distributed median lifetime of products. The literature survey reveals that there is no prior research has been explored on ChSP-1 for complementary Bell Weibull distribution or compared its performance with other lifetime models. Implementation of the proposed plan is explained with an industrial example of fatigue fracture of Kevlar 373/epoxy composite material, and also the advantages of economic aspect of proposed plan with reduced cost of inspection and total cost for assuring complementary Bell Weibull distribution with different median lifetime models are also discussed.

II. Determination of Failure Probability with Complementary Bell Weibull Model

Combining continuous lifetime distributions with discrete distribution is a powerful approach to create flexible and versatile models that capture the complexity of lifetime data and leading to more accurate predictions and better analysis. Tahir and Cordeiro [18] discussed a comprehensive review of compounding techniques for univariate distributions. Their study focused on generalization classes and explored various complementary compound models. Castellares *et al.* [4] have proposed a single-parameter discrete bell distribution as an alternative to the widely used Poisson distribution. Castellares *et al.* [4] had found that the Poisson model cannot be directly combined into the Bell model, although the bell distribution converges to the Poisson distribution when the parameter value decreases. In other words, the Poisson distribution can be viewed as a special case of the Bell distribution that emerges when the Bell parameter approaches to zero. Recently, Algarni [2] proposed a new compounded model known as complementary Bell Weibull distribution (CBell- WD) that aims to ensure the median lifetime based on group acceptance sampling plan. The cumulative distribution function (CDF) of the CBell- WD is given by

$$F(t)=\frac{exp\left[e^{\theta\left\{1-exp\left[-\left(t/\delta\right)^{\eta}\right]\right\}}-1\right]-1}{exp\,[e^{\theta}-1]-1}\quad 0<t<\infty;\ \delta,\eta>0 \tag{1}$$

where θ denoted as strictly positive Bell model parameter also η and δ stands for shape and scale parameters respectively. The median of CBell- W distribution is discussed by Algarni [2] and it is represented as

$$m=\delta\left[-ln\left(1-\left\{\theta^{-1}ln\left[1+ln\left\{1+0.5\left[exp\left(e^{\theta}-1\right)-1\right]\right\}\right]\right\}\right)\right]^{1/\eta} \tag{2}$$

and now let us take

$$\xi=\left[-ln\left(1-\left\{\theta^{-1}ln\left[1+ln\left\{1+0.5\left[exp\left(e^{\theta}-1\right)-1\right]\right\}\right]\right\}\right)\right]^{1/\eta}$$

The failure probability of an item prior to the time $t_0$is represented by $p=F(t_0)$ and can be amended as

$$p=\frac{exp\left[e^{\theta\left\{1-exp\left[-\left(t_0/\delta\right)^{\eta}\right]\right\}}-1\right]-1}{exp\,[e^{\theta}-1]-1} \tag{3}$$

In the context of developing sampling plans for lifetime assurance, it is common to express the test duration (t_0) as a fixed multiple of the specified median life (m_0). This mathematical relationship can be represented by the equation as $t_0 = am_0$ where *'a'* symbolizes the experiment termination ratio. The probability that an item fails prior to a given experiment time 't_0' as determined by the CBell - W distribution, is calculated as follows:

$$p=\frac{exp\left[e^{\theta\left\{1-exp\left[-\left(\frac{a\xi}{m/m_0}\right)^{\eta}\right]\right\}}-1\right]-1}{exp[e^{\theta}-1]-1} \tag{4}$$

I. Survey of Few Other Lifetime Distributions

In this paper, we have chosen a few lifetime distributions to compare the ChSP-1 for assuring the median lifetime of the products such as extended odd Weibull exponential distribution (EOW-ED), exponentiated Weibull distribution (EWD), exponentiated Frechet distribution (EFD) and Burr XII distribution (BXIID). Our analysis revolves around, analysing the performance of the ChSP-1 using CBell-WD with other lifetime distributions. Additionally, we will utilize these models to compare the OC curve performance of the proposed sampling plan. Table 1 provides the description of these lifetime distributions and their failure probabilities based on median lifetime.

Table 1: *Failure probability based on median life of some selected life time models*

S. No	Models	Median lifetime ($q = 0.5$)	Failure probability
1.	EOW-ED	$m = -\frac{1}{\lambda} log\left[1 - \left(\frac{-1+(1-q)^{-\eta}}{\eta + \left[-1+(1-q)^{-\eta}\right]}\right)^{1/\delta}\right]$ where $\omega = log\left[1 - \left(\frac{-1+(1-q)^{-\eta}}{\eta + \left[-1+(1-q)^{-\eta}\right]}\right)^{1/\delta}\right]$	$p = 1 - \left(1 + \eta\left[e^{-\omega\, a\, (1/r_2)} - 1\right]^{\delta}\right)^{-\frac{1}{\eta}}$ where η and δ are the two shape parameters.
2.	BXIID	$m = \sigma * \omega$ where $\omega = \lambda * \left[(1/(1-q))^{1/\delta} - 1\right]^{1/\eta}$	$p = 1 - \left[1 + \left(\frac{\omega * a}{r_2}\right)^{\delta}\right]^{-\eta}$ where η and δ are the two shape parameters.
3.	EWD	$m = \lambda\left[-ln\left(1 - q^{1/\eta}\right)\right]^{1/\delta}$	$p = \left[1 - exp\left((r_2)^{\delta}\left(a\ ln\left(1 - q^{1/\eta}\right)\right)^{\delta}\right)\right]^{\eta}$ where η and δ are the two shape parameters.
4.	EFD	$m = \sigma * \omega$ where $\omega = \left(-ln\left(1 - (1-q)^{1/\eta}\right)^{-1/\delta}\right)$	$p = 1 - \left[1 - exp\left\{-\left(\frac{a * \omega}{r2}\right)^{-\eta}\right\}^{\delta}\right]$ where η and δ are the two shape parameters.

III. Conditions for Application and Operating Procedure of ChSP-1

The following are the conditions of application and the operating procedure of the proposed ChSP-1 plan.

1. The product to be inspected consists of a sequence of consecutive lots, either material or individual units, produced through a continuous and steady flow.
2. Chain sampling plans rely on a level of trust between the consumer and the producer. The producer will not intentionally supply inferior lots because the plan uses a combination of small sample size and information from previous samples to decide on the current lot's acceptance.
3. The production process should operate in a stable and continuous manner to ensure consistency and reliability. The product originates from a trusted source in which the consumer has high level of confidence.

The operating procedure of ChSP-1 based on TTL test is described in Figure 1 as shown below.

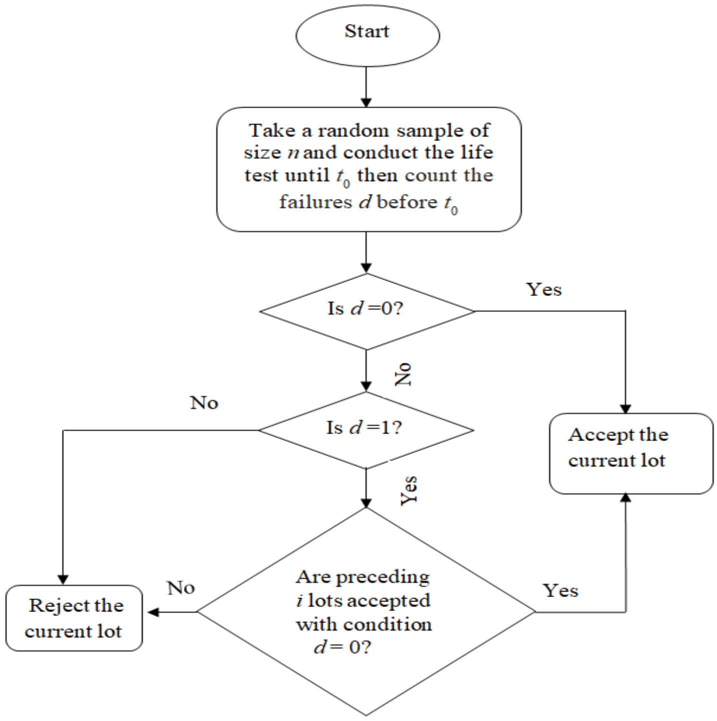

Figure 1: *Operating procedure of ChSP-1*

The ChSP-1 has two parameters namely n and i where n is the sample size, and i denoted as the number of preceding lots that are taken into account when making a decision about the current lot. When the limit of i approaches infinity $(i \to \infty)$, the operating characteristic (OC) function of a ChSP -1 simplifies to the OC function of a SSP with acceptance number zero.

The OC function of ChSP-1, as referenced in Dodge [5], can be expressed using the following equation.

$$L_A(p) = P(d=0) + [P(d=1) \times (P(d=0))^i] \quad (5)$$

where the probability $P(d = 0)$ indicates the possibility that no items in a given sample are non-conforming out of n items. Conversely, $P(d = 1)$ is used to represent the probability of a current sample containing a single non-conforming item also i denoted as the number of preceding samples.

Under the binomial distribution,

$$L_A(p) = (1-p)^n + np(1+p)^{n+ni-1} \quad (6)$$

IV. Designing Methodology

This section presents the methodology for designing the ChSP-1, which aims to ensure the product has a median lifetime that adheres to the CBell - WD. The primary objective of using the two-points on the OC curve approach is to fulfil both the producer's risk and the consumer's risk simultaneously. The optimal plan parameters are determined by passing through the two designated points on the OC curve such as (*PQL*, 1-*α*) and (*CQL*, *β*). By solving the following optimization problem, the optimal parameters of the proposed plan are determined.

$$\begin{aligned} &\text{Minimize } ASN(p) = n \\ &\text{Subject to } L_A(p_1) \geq 1-\alpha \\ &\qquad L_A(p_2) \leq \beta \\ &\qquad n > 1,\ i \geq 1 \end{aligned} \tag{7}$$

The failure probabilities of both the producer's and consumer's risk are represented as p_1 and p_2 respectively. The probability acceptance of the lot at producer quality Level (PQL) and consumer quality level (CQL) under ChSP-1 is as follows:

$$L_A(p_1) = (1-p_1)^n + np_1(1+p_1)^{n+ni-1} \tag{8}$$

$$L_A(p_2) = (1-p_2)^n + np_2(1+p_2)^{n+ni-1} \tag{9}$$

The quality level can be expressed as the median proportionate between product's actual median lifetime and its specified median lifetime. This ratio is commonly denoted as m/m_0.The failure probability, denoted as p_1, can be determined by considering the median ratios m/m_0= 4, 6, 8 and 10. These median ratios are noted as PQL. Let p_2 be the failure probability and its median ratio m/m_0= 1 is considered as CQL.

V. Algorithm

The following algorithm can be used to compute the optimal parameters of the proposed ChSP-1 plan, with the objective of assuring a median lifetime of the submitted products for inspection.

Step 1 : Specify α, β, a, η, θ and median ratios.

Step 2 : For the specified input parameters α,β, a, η, θ and median ratios to calculate the failure probability (p_1) using an equation (4) denoted as PQL similarly to get the failure probability (p_1) when the median ratio is equal to 1 labelled it as CQL.

Step 3 : Set n =1 and i =1

Step 4 : Using an equation (6), replace n and i to calculate the lot acceptance probability at p_1 denoted as $L_A(p_1)$. Likewise, substitute n and i to calculate lot acceptance probability at p_2 denoted as $L_A(p_2)$.

Step 5 : Determine the largest sample size n, say n_A, and satisfying the condition $L_A(p_1) \geq 1-\alpha$ for all $n \leq n_A$.

Step 6 : Determine the largest sample size n, say n_B, and satisfying the condition $L_A(p_1) \leq \beta$ for all $n_B \leq n$.

Step 7 : If the condition, $n_B \leq n_A$ exists, then the corresponding set of plan parameter (n_B and i) is obtained. If not, Steps 4 to 5 must be repeated with various combination of n and i until an optimum plan parameter is successfully acquired.

By using the above algorithm, Tables 1 and 3 are constructed. These tables display the optimal plan parameters of ChSP-1, which have been determined by considering different combinations of the parameters (η, θ) = (2, 1.25), (2, 1.50) and (1, 1.20) for the CBell - WD. We have also considered the producer's risk α = 0.05 and consumer's risk β = 0.25, 0.10, 0.05 and 0.01 and the values of median ratios m/m_0= 4, 6, 8, 10 at the termination ratios a = (0.5 and 1.0). The tables are presented for the calculated performance measures, including the OC functions, at specific failure probabilities of p_1 and p_2. From the tables, we analysed that if any value ofβ,a, m/m_0 and η increases then the value of sample size leads to decrease. Therefore, consumers will be safeguarded from accepting the lots of poor quality and producer's risks are also decreased while reaching the decision for accepting the current lot.

Table 1: *Optimal parameters of the proposed ChSP-1 under the CBell - W model with η = 2 and θ = 1.25*

		a = 0.5				a = 1.0			
β	r_2	n	i	$L_A(p_1)$	$L_A(p_2)$	n	i	$L_A(p_1)$	$L_A(p_2)$
0.25	4	19	3	0.9876	0.2401	3	1	0.9976	0.1718
	6	↑	↑	0.9974	0.2401	↑	↑	0.9995	0.1718
	8	↑	↑	0.9991	0.2401	↑	↑	0.9998	0.1718
	10	↑	↑	0.9996	0.2401	↑	↑	0.9999	0.1718
0.10	4	31	2	0.9772	0.0965	4	1	0.9956	0.0781
	6	↑	↑	0.9951	0.0965	↑	↑	0.9992	0.0781
	8	↑	↑	0.9984	0.0965	↑	↑	0.9997	0.0781
	10	↑	↑	0.9993	0.0965	↑	↑	0.9999	0.0781
0.05	4	40	2	0.9639	0.0479	5	1	0.9932	0.0361
	6	↑	↑	0.9921	0.0479	↑	↑	0.9987	0.0361
	8	↑	↑	0.9974	0.0479	↑	↑	0.9996	0.0361
	10	↑	↑	0.9989	0.0479	↑	↑	0.9998	0.0361
0.01	4	89	1	0.9633	0.0091	7	1	0.9869	0.0082
	6	61	2	0.9825	0.0096	↑	↑	0.9975	0.0082
	8	↑	↑	0.9941	0.0096	↑	↑	0.9992	0.0082
	10	↑	↑	0.9975	0.0096	↑	↑	0.9996	0.0082

Table 2: *Optimal parameters of the proposed ChSP - 1 under the CBell - W model with η = 2 and θ = 1.50*

		a = 0.5				a = 1.0			
β	r_2	n	i	$L_A(p_1)$	$L_A(p_2)$	n	i	$L_A(p_1)$	$L_A(p_2)$
0.25	4	30	3	0.9916	0.2449	3	1	0.9993	0.1718
	6	↑	↑	0.9983	0.2449	↑	↑	0.9998	0.1718
	8	↑	↑	0.9994	0.2449	↑	↑	0.9999	0.1718
	10	↑	↑	0.9997	0.2449	↑	↑	0.9999	0.1718
0.10	4	49	2	0.9845	0.0994	4	1	0.9987	0.0781
	6	↑	↑	0.9968	0.0994	↑	↑	0.9997	0.0781
	8	↑	↑	0.9989	0.0994	↑	↑	0.9999	0.0781
	10	↑	↑	0.9995	0.0994	↑	↑	0.9997	0.0781
0.05	4	63	4	0.9601	0.0499	5	1	0.9980	0.0361
	6	↑	↑	0.9911	0.0499	↑	↑	0.9996	0.0361
	8	↑	↑	0.9970	0.0499	↑	↑	0.9998	0.0361
	10	↑	↑	0.9987	0.0499	↑	↑	0.9999	0.0361
0.01	4	106	1	0.9598	0.0067	7	1	0.9961	0.0082
	6	97	2	0.9882	0.0099	↑	↑	0.9993	0.0082
	8	↑	↑	0.9961	0.0099	↑	↑	0.9997	0.0082
	10	↑	↑	0.9983	0.0099	↑	↑	0.9999	0.0082

(↑) use the plan above

Table 3: *Optimal parameters of the proposed ChSP-1 under the CBell - W model with η = 1 and θ = 1.20*

		a = 0.5				a = 1.0			
β	r_2	n	i	$L_A(p_1)$	$L_A(p_2)$	n	i	$L_A(p_1)$	$L_A(p_2)$
0.25	4	*	*	*	*	*	*	*	*
	6	7	2	0.9556	0.2297	3	1	0.9755	0.1718
	8	7	2	0.9747	0.2297	3	1	0.9870	0.1718
	10	7	2	0.9837	0.2297	3	1	0.9920	0.1718
0.10	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	4	1	0.9576	0.0781
	8	12	1	0.9571	0.0853	4	1	0.9772	0.0781
	10	11	2	0.9627	0.0898	4	1	0.9859	0.0781
0.05	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	*	*	*	*
	8	22	1	0.9528	0.0498	4	1	0.9653	0.0361
	10	15	1	0.9582	0.0412	4	1	0.9783	0.0361
0.01	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	*	*	*	*
	8	*	*	*	*	*	*	*	*
	10	*	*	*	*	7	1	0.9592	0.0082

(*) refers plan doesn't exist

VI. Comparison Using OC Curve

The first subsection, initiates the analysis by comparing the performance of proposed plan with SSP based on CBell-WD in terms of OC function. The subsequent subsection explores the OC curve performance of proposed plan based on CBell-WD with multiple median lifetime distributions.

I. Performance Analysis on OC Curve of Proposed ChSP-1 with SSP

In this section, we will briefly analyze the proposed plan's performance in comparison to SSP with CBell-WD. This comparison will be done using the performance of OC curve. Then the OC curve of proposed plan parameter is drawn for n = 3, i= 1 and the SSP with Ac = 0 is drawn for the same sample size with η = 2 and θ = 1.50 at the level of α = 0.05 and β = 0.25 which are gratified with m/m_0 = 2 and a =1.0 as shown in Figure 2. Also the comparison of acceptance probabilities $L_A(p_1)$ and $L_A(p_2)$ and the optimal plan parameters of ChSP-1 and SSP are calculated in Table 4. From Figure 2, it is clear that the OC curve of the proposed plan surpasses the SSP plan for low failure probabilities. The OC curve of a zero acceptance number (Ac = 0) SSP lacks a point of inflection. This means the probability of accepting a lot drops sharply even for the slightest increase in the proportion of nonconforming items (p). Thus, the proposed plan offers higher probability of acceptance compared to SSP. The proposed plan gives the protection for producers, where their products are exhibiting the best quality simultaneously; this plan also ensures the consumer protection in cases where the product's quality falls on poor quality. Therefore, based on the discussion, it can be concluded that the proposed plan will distinguish the various lots in terms of their quality, in comparison to the discriminative capacity of SSP with Ac = 0.

Table 4: *Comparison of acceptance probabilities proposed ChSP-1 with SSP under the CBell - W model with $\eta = 2$ and $\theta = 1.50$*

β	r_2	$a = 1.0$					
		SSP			ChSP-1		
		(n, c)	$L_A(p_1)$	$L_A(p_2)$	(n, i)	$L_A(p_1)$	$L_A(p_2)$
0.25	4	(3,0)	0.9771	0.125	(3,1)	0.9993	0.1718
	6	(3,0)	0.9905	0.125	(3,1)	0.9998	0.1718
	8	(3,0)	0.9948	0.125	(3,1)	0.9999	0.1718
	10	(3,0)	0.9967	0.125	(3,1)	0.9999	0.1718
0.10	4	(4,0)	0.9697	0.0625	(4,1)	0.9987	0.0781
	6	(4,0)	0.9874	0.0625	(4,1)	0.9997	0.0781
	8	(4,0)	0.9931	0.0625	(4,1)	0.9999	0.0781
	10	(4,0)	0.9956	0.0625	(4,1)	0.9999	0.0781
0.05	4	(5,0)	0.9622	0.0313	(5,1)	0.9980	0.0361
	6	(5,0)	0.9843	0.0313	(5,1)	0.9996	0.0361
	8	(5,0)	0.9914	0.0313	(5,1)	0.9998	0.0361
	10	(5,0)	0.9945	0.0313	(5,1)	0.9999	0.0361
0.01	4	*	*	*	(7,1)	0.9961	0.0082
	6	(7,0)	0.9781	0.0078	(7,1)	0.9993	0.0082
	8	(7,0)	0.9879	0.0078	(7,1)	0.9997	0.0082
	10	(7,0)	0.9924	0.0078	(7,1)	0.9999	0.0082

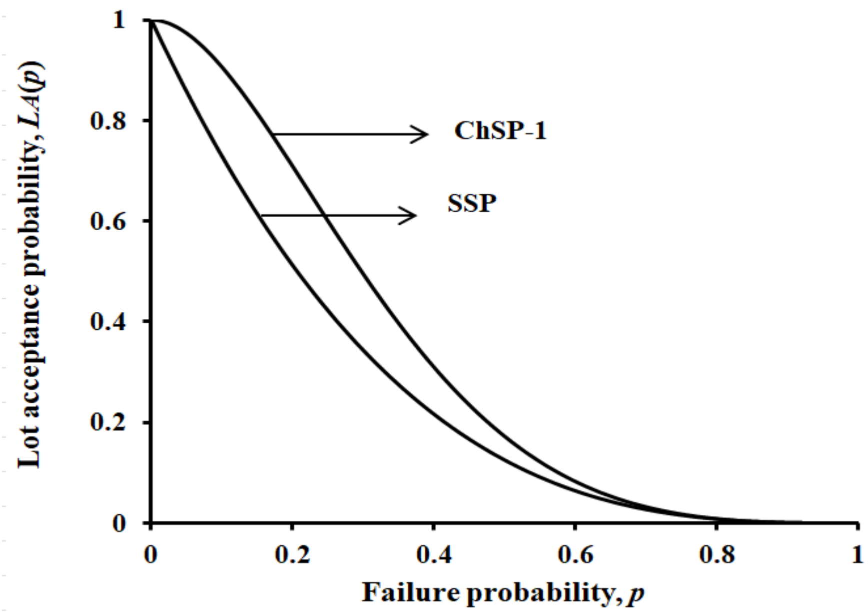

Figure 2*: OC curves of the proposed ChSP-1 and SSP with Ac = 0*

II. OC curve Comparison of Proposed Plan Based on Distinct Lifetime Distributions

In this analysis, we compare the OC curve performance of ChSP- 1 using CBell- WDwith other few other lifetime distributions such as EOW-ED, EWD, EFD, BXIID and CBell- WD. To draw the OC curve of ChSP- 1, the optimal parameters are chosen under each distribution when α = 0.05, β = 0.25, a = 0.5, r_2 = 6 and considering the model's shape parameter as η = 1, θ = 2 for EOW-ED (n = 9, i = 3), and fixing η = 1, θ = 3 for EWD (n = 17, i= 2), fixing η = 1, θ = 3 for EFD (n = 11, i= 3), fixing η = 1, θ = 4 for BXIID (n = 23, i= 4) and fixing the parameter η = 1, θ = 1.2 for CBell- WD (n = 7, i=2). Figure 3 show that the OC curve of the proposed plan under the CBell- WD is dominating the same under all the other lifetime distributions. In simpler terms, when using the ChSP-1 with CBell-WD, is accurately distinguishes between the good batches (when the chances of accepting high probability acceptance, $L_A(p)$) and reject the bad batches (when the failure probability, p is high).Therefore it is clear that the discriminating power of the proposed ChSP- 1 based on the CBell- WD is better than the other lifetime models.

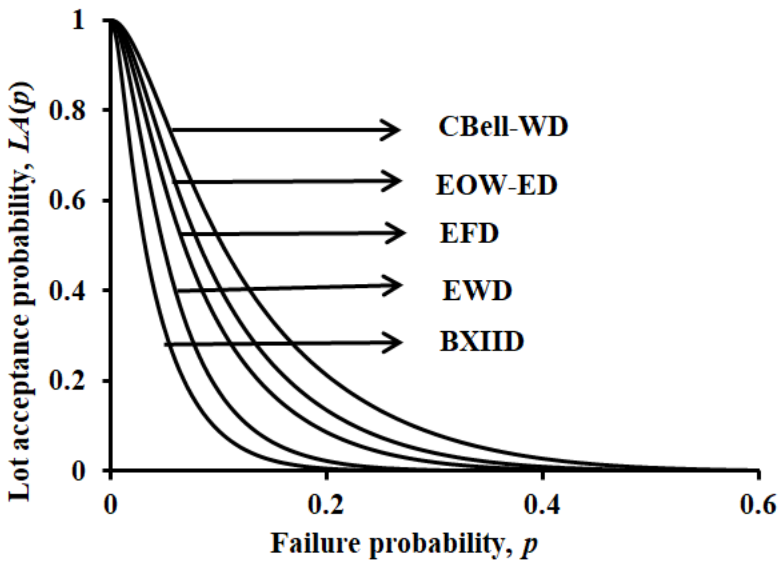

Figure 3: *OC curves of proposed ChSP-1 based on different life time models*

VII. Industrial Application

In this section, we address the execution of the proposed ChSP- 1 plan based on CBell-WD, utilizing the lifetime of Kevlar 373/epoxy fatigue fracture is the well fitted dataset analysed from the study of Algarni [2]. Kevlar 373, is a species of aramid fiber known for its strength, it is intricately combined with epoxy resin, which serves as a binding agent to coalesce the fibers within the composite material. The epoxy resin, which acts as a binder for the fibers, is susceptible to cracking due to repeated stress. Setting a constant pressure at 90% of the stress level is a unique feature. This is likely near to the material's endurance limit, where the stress level of failure is minimal even after testing a high number of cycles. By Understanding the material's behaviour under sustained 90% stress level can be informative for design and facilitate the prediction of long-term performance characteristics. The Kevlar 373/epoxy fatigue fracture dataset specifically measures the lifetime, in terms of number of cycles that a sample can endure before failing, with the constant fatigue stress level set at 90% of the material's limit.To illustrate the implementation of the proposed ChSP- 1 for assuring the median lifetime of Kevlar 373/epoxy fatigue fracture are used in industrial machinery, specifically their lifetime are measured in number of cycles. The dataset contains 76 observations which are given by Gómez-Déniz*et al.* [8].

According to Algarni [2] for the above data set, the maximum likelihood estimator of the model parameters under CBell-WD are found to be η = 1.6316, θ = 0.6734(i.e., $\hat{\eta}$ =1.6, $\hat{\theta}$ =0.7). The manufacturer aims to investigate how the proposed ChSP- 1 can be used to reliably assure the median lifetime for Kevlar 373/epoxyfatigue fracture (measured in cycles) under 90% stress level. Suppose that the median number of cycles in the fatigue fracture of Kevlar 373/epoxy is specified (m_0) as 1.257 cycles but the quality inspector decide to run the experiment before complete the testing for only 0.6285 cycles (t_0 = 0.628 cycles). Therefore, the test termination ratio of a = 0.5.The producer's risk is specified as 5% and the consumer risk's as 25%, when the true median number of cycles (m) in Kevlar 373/epoxy is 7.542 cycles at least thrice the specified number of cycles as 1.257 cycles therefore, the median ratio becomes m/m_0 = 6. It is estimated that the model parameters of CBell - WD using the fatigue fracture of Kevlar 373/epoxy as $\hat{\eta}$ = 1.6, $\hat{\theta}$ = 0.7. Based on the above values, the optimal plan parameters n = 7 and i= 2 are depicted from Table 5. Suppose the simulated breaking strength of Kevlar 373/epoxyfatigue fracture (measured in cycles) of 7 materials are as given below.

0.900	1.5728	2.920	3.399	5.230	7.443	9.061

The implementation of the proposed ChSP- 1 using Kevlar 373/epoxyfatigue fracture (measured in cycles) under 90% stress level is explained below.

Step 1: A random sample of 7 items is selected from the current lot of Kevlar 373/epoxyand count the number of defective items before complete the life testing 0.628 cycles.

Step 2: Accept the current lot if no sample item is found to be defective and rejects the current lot if more than one sample item is defective before 0.628 cycles.

Step 3: If the defective item is equal to one then accept the current lot reaching before 0.628 cycles under the condition that preceding 2 lots have been accepted before 0.628 cycles.

VIII. Economic Design of ChSP-1 Using CBell-WD

The implementation of a cost-effective ChSP-1 has been proactively undertaken, driven by industry recognition of its potential to significantly reduce overall product inspection costs. By implementing the proposed plan, producers can avoid disrupting inspection costs and reworking defective products, thereby ensuring high-quality products reach customers. This approach not only maintains customer satisfaction but also enhances the producers' reputation in the market by offering quality products at minimal cost. Ailor *et al.* [3] determined the economic designing of sampling plan based on the combination of attributes and variables. Hsu and Hsu [9] proposed an economic design for a two-stage supply chain, which utilizes a single acceptance sampling plan to optimize efficiency and cost. Despite the existing literature, no prior work has ventured into an economic model for a chain sampling plan based on different lifetime distribution. Our work bridges this gap by introducing an economic design based on the CBell -WD which outperforms other lifetime distribution in terms of Average total inspection (ATI) and Total Cost (TC). In order to calculate the *TC*, we need to consider few economic measures, such as *ATI*, detection of failure items (D_d) and non- detection of failure items (D_n). These measures are defined as follows. (see Hsu and Hsu [9]).

Table 5: *Optimal parameters of the proposed ChSP-1 under the CBell - W model with* $\hat{\eta}$ *= 0.7 and* $\hat{\theta}$ *= 1.6*

β	r_2	a = 0.5				a = 1.0			
		n	i	$L_A(p_1)$	$L_A(p_2)$	n	i	$L_A(p_1)$	$L_A(p_2)$
0.25	4	*	*	*	*	*	*	*	*
	6	7	2	0.9577	0.2192	3	1	0.9774	0.1718
	8	7	2	0.9748	0.2192	3	1	0.9881	0.1718
	10	7	2	0.9830	0.2192	3	1	0.9926	0.1718
0.10	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	4	1	0.9609	0.0781
	8	12	1	0.9574	0.0787	4	1	0.9791	0.0781
	10	11	2	0.9611	0.0838	4	1	0.9869	0.0781
0.05	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	8	0	0.9510	0.0351
	8	*	*	*	*	5	1	0.9680	0.0361
	10	14	1	0.9611	0.0478	5	1	0.9798	0.0361
0.01	4	*	*	*	*	*	*	*	*
	6	*	*	*	*	*	*	*	*
	8	*	*	*	*	*	*	*	*
	10	*	*	*	*	7	1	0.9620	0.0082

(*) refers plan doesn't exist

$$ATI(p) = n + \{1\text{-}L_A(p)\}(N\text{-}n) \tag{10}$$

$$D_d = np + (1 - L_A(p)) \times (N - n) \times p \tag{11}$$

$$D_n = p \times L_A(p) \times (N - n) \tag{12}$$

where N is the lot size and $\{1{-}L_A(p)\}$ is the probability of rejection of the lot with failure probability p and $L_A(p)$ has been given in equation (5). The probability that the lot will be accepted ($L_A(p)$) under the economic ChSP-1 at the specified failure probability p in equations (10), (11), and (12) which is calculated for the median ratios of PQL and CQL, respectively. To calculate the TC, we consider three cost measures such as C_o, C_i and C_f as used by Hsu and Hsu [9] and are defined as C_o - cost of an outgoing failure item, C_i - life testing cost per item and C_f - replacement cost. In order to determine the economic ChSP-1 which minimizes the TC, we use the following optimization model;

$$\begin{aligned} &Minimize\ TC = C_i \cdot ATI + C_f \cdot D_d + C_o \cdot D_n \\ &Subject\ to\ L_A(p_1) \geq 1 - \alpha \\ &\qquad L_A(p_2) \leq \beta \\ &\qquad n > 1,\ i \geq 1 \end{aligned} \tag{13}$$

where $L_A(p_1)$ and $L_A(p_2)$ are the probabilities of lot acceptance at PQL and CQL and that can be obtained from an equation (5) by substituting p_1 and p_2 respectively instead of p. The plan parameters namely (n, i) and the economic performance measure includes $ATI(p)$, $L_A(p)$, D_d, D_n, and TC have been estimated from the input values of model parameters using median life assurance which are given as (a, η, θ) = (0.5, 3, 1.25) and (0.5, 2, 1) by fixing the lot size N =1000 and cost parameters (C_i, C_f and C_o) = (1.0, 2.0 and 10). The values of failure probability (p) are defined to the ratios 2.5, 3.5, 4.5, 5.5 which are corresponds to the average ratios of PQL (r_2 = 4, 6, 8 and 10) and CQL (r_1=1) which are reported in Table 6 and 7. From these tables we observe that if the consumers risk β decreases, then the values of *ATI(p)*, D_d and *TC* get increased while the value of D_n decreases. Similarly, when the median ratio (r_2) increases, then the values of *ATI (p)* and overall TC get decreased.

Table 6: *Optimal parameters of economic ChSP-1 under the CBell- WD for assuring median life when η = 3 and θ = 1.25, a = 0.5*

β	r_2	$L_A(p)$	n	i	D_d	D_n	$ATI(p)$	TC
0.25	4	0.9860	47	2	0.10	1.60	60.32	76.55
	6	0.9969	44	4	0.02	0.58	46.94	52.88
	8	0.9929	44	4	0.01	0.27	44.67	47.47
	10	0.9997	44	4	0.01	0.15	44.20	45.73
0.10	4	0.9768	79	1	0.17	1.53	100.27	115.96
	6	0.9951	74	2	0.04	0.56	78.45	84.24
	8	0.9985	73	3	0.02	0.26	74.38	77.10
	10	0.9995	73	3	0.01	0.14	73.42	74.91
0.05	4	0.9651	99	1	0.22	1.48	130.41	145.70
	6	0.9922	95	2	0.06	0.55	102.02	107.70
	8	0.9981	95	2	0.02	0.26	96.63	99.31
	10	0.9994	95	2	0.01	0.14	95.50	96.96
0.01	4	0.9301	147	1	0.35	1.35	206.62	220.86
	6	0.9889	147	1	0.10	0.52	156.42	161.82
	8	0.9974	147	1	0.04	0.24	149.20	151.75
	10	0.9982	145	3	0.23	0.13	146.50	147.90

Table 7: *Optimal parameters of economic ChSP-1 under the CBell- WD for assuring median life when η = 2 and θ = 1, a = 0.5*

β	r_2	$L_A(p)$	n	i	D_d	D_n	$ATI(p)$	TC
0.25	4	0.9441	16	1	0.98	12.82	70.99	201.22
	6	0.9840	16	1	0.22	6.72	31.71	99.37
	8	0.9939	16	1	0.09	4.08	21.97	62.98
	10	0.9972	16	1	0.05	2.73	18.72	46.16
0.10	4	*	*	*	*	*	*	*
	6	0.9661	24	1	0.39	6.54	57.05	123.31
	8	0.9867	24	1	0.15	4.02	36.89	77.40
	10	0.9938	24	1	0.08	2.70	29.96	57.15
0.05	4	*	*	*	*	*	*	*
	6	0.9494	30	1	0.54	6.40	79.00	144.03
	8	0.9799	30	1	0.20	3.96	49.48	89.57
	10	0.9906	30	1	0.99	2.67	3.10	66.09
0.01	4	*	*	*	*	*	*	*
	6	*	*	*	*	*	*	*
	8	0.9578	45	1	0.35	3.81	85.22	124.12
	10	0.9798	45	1	0.17	2.60	64.26	90.68

(*) refers plan doesn't exist

IX. Comparative Study of ChSP-1 Performance In Terms of ATI and TC Curves

This section offers a comprehensive analysis of the *ATI* and *TC* curve of ChSP-1 plan using the CBell-WD under median lifetime assurance. The following section consists of two subsections. In Section 7.1, we present the *ATI* and *TC* curves and analyse the performance of the proposed plan using CBell-WD based on median lifetime with SSP. In Section 7.2, we proceed to compare the *ATI*and *TC* curve of the proposed plan assuring the product's median lifetime based on CBell-WD and the same based on some other distinct lifetime distributions.

I. *ATI* and *TC* Comparison of the Proposed ChSP-1 Plan with SSP

This section conducts a comparison analysis of the proposed plan by using the cost metrics such as *ATI*and *TC*. Table 8 compares the measures of *ATI* and *TC* curve based on the economic perspective of SSP and ChSP-1 with the input parameters are η = 3 and θ = 1.250 and the termination ratio a = 0.5 for assuring the median lifetime of the product under CEBT-ED. As shown in Table 8, the proposed ChSP-1 plan consistently outperforms SSP in terms of *ATI* and *TC* with the (%) decrement between these two plans across all combinations of β and r_2 indicating the better decision-making capabilities while reducing the quality inspection costs. Consider the scenarios, where the cost parameters (C_i= 1.0, C_o = 10.0 and C_f= 2.0), N = 1000, α = 0.05, β = 0.05, r_2 = 6. In this case, the *ATI* of SSP is 146.56 and ChSP-1 is 102.02, the percentage reduction of these two plan is 30.40% and finally the TC of SSP is 152.00 and ChSP-1 is 107.70, resulting in a 29.14% reduction. Therefore, Table 8 illustrates that the proposed ChSP-1 plan consistently offers better performance than SSP, in terms of *ATI*and *TC* is minimum, across various input scenarios, confirming our earlier expectations. Additionally, in order to draw the TC curve by following the above specified input parameters. Figure 4 portray the economic ChSP-1 parameter with n = 95, i= 2 and SSP parameters with n = 95 and c = 0. From this figure, it is apparent that a lot inspection has been performed excellent when the TC curves of economic ChSP-1 is more effective than SSP.

Table 8: *Comparison of ATI(p) and TC of economic SSP and ChSP-1 under CBell-WD for median life assurance when η = 3 and θ = 1.250, a = 0.5*

β	r_2	*ATI(p)*		*Reduction* in (%)	*TC*		*Reduction* in (%)
		SSP	*ChSP -1*		*SSP*	*ChSP -1*	
0.25	4	94.83	60.32	36.40%	110.60	76.55	30.78%
	6	69.83	46.94	32.78%	75.60	52.88	30.05%
	8	56.11	44.67	20.38%	58.88	47.47	19.38%
	10	50.64	44.20	12.71%	52.16	45.73	12.32%
0.10	4	139.70	100.27	28.22%	154.85	115.96	25.11%
	6	113.85	78.45	31.10%	119.47	84.24	29.49%
	8	92.41	74.38	19.51%	95.10	77.10	18.93%
	10	83.67	73.42	12.25%	85.15	74.91	12.02%
0.05	4	173.40	130.41	24.80%	188.09	145.70	22.53%
	6	146.56	102.02	30.40%	152.00	107.70	29.14%
	8	119.58	96.63	19.19%	122.20	99.31	18.73%
	10	108.53	95.50	12.00%	109.98	96.96	11.84%
0.01	4	249.96	206.62	17.33%	263.60	220.86	16.21%
	6	216.07	156.42	27.61%	221.17	161.82	26.83%
	8	180.19	149.20	17.20%	182.67	151.75	16.92%
	10	164.43	146.50	10.90%	165.81	147.90	10.80%

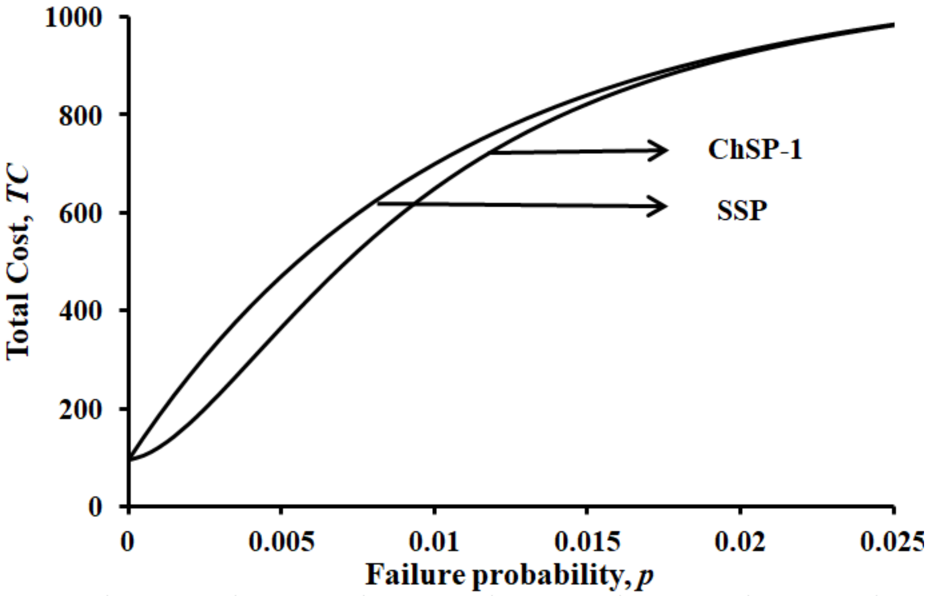

Figure 4: *TC curves of proposed ChSP-1 and SSP*

II. Comparison of *ATI* and *TC* of Proposed ChSP-1 Plan Based on Distinct Lifetime Distributions

This section conducts a comparison analysis of the proposed plan by using the cost metrics such as *ATI* and *TC*. Here, we focus on, how the ChSP-1 plan performs, in terms of *ATI*and *TC* by using the different distributions such as EOW-ED, EWD, EFD, BXIID and CBell- WD respectively. To draw the *ATI*and *TC* curve of ChSP-1, the plan parameters are chosen under each distribution when α = 0.05, β = 0.25, a = 0.5, r_2 = 6 and following the same shape parameter as η = 2, θ = 4 for EOW-ED (n = 35, i = 4), and fixing η = 2, θ = 1.9 for EWD (n = 18, i= 3), fixing η = 2, θ = 1 for EFD (n = 22, i= 3), fixing η = 2, θ = 4 for BXIID (n = 28, i= 3) and fixing η = 2, θ = 1 for CBell- WD (n = 16, i=1) respectively. Figure 5 and 6 shows that economic *ATI*and *TC*curve of the proposed plan under the CBell- WD outperforms other lifetime distributions. Additionally, Table 9 compares the measures of *TC* curve based on the economic perspective of different lifetime models by fixing the same shape parameters as η = 2 and the termination ratio a = 0.5. Consider the scenarios, where N = 1000, α = 0.05, β = 0.25, r_2 = 6. In this case, the *TC*of EFD is 810.73, the *TC*of EOW-ED is 221.11, the *TC*of BXIID is 186.91, the *TC*of EWD is 170.91, and the *TC* of CBell- WD attains 99.37. Table 9 reveals that the *TC*of ChSP-1 plan is minimum based on CBell- WD when compared with other lifetime models.

X. Advantages of ChSP-1 Based on CBell-WD

The following are the main advantages of the proposed ChSP-1 plan based on CBell-WD.

❖ The CBell-W distribution's pdf and cdf can be articulated as a linear combination of Weibull distribution also containing the unique features of Bell numbers. Therefore, CBell-WD is a derivative of the Bell-WD. This distribution has well-defined theoretical properties, making it easier to work and analyse in practical applications.

❖ ChSP-1 based on CBell-WD can be particularly useful for industrial applications like Kevlar 373/epoxy fatigue fracture, where early detection of potential failures may be crucial. ChSP-1 allows early termination of the life testing process (in this case we fix, t_0 = 0.628 cycles for materials) if the sample meets the acceptance criteria.For example, if the previous two lots were good, accepting a current lot with no defective item might be a reasonable decision.

❖ This plan will significantly reduce testing time and the associated costs compared to other sampling plans where all items are tested until failure.

❖ The shape parameter of CBell-WD provides a stable and flexible distribution when compared to other lifetime models also the CBell-WD observed high probability of acceptance when the failure probability is decreased.

❖ By implementing this economic approach, inspection costs can be significantly reduced when compare to other lifetime models and leading to more effective and efficient inspection processes.

Table 9: *Comparing TC of economic ChSP-1 under different lifetime models for median life assurance when a = 0.5*

β	r_2	*TC*				
		EOW-ED	EWD	*EFD*	*BXIID*	*CBell-WD*
		$\eta = 2,\ \delta=3$	$\eta = 2,\ \delta=1$	$\eta = 2,\ \delta=0.5$	$\eta = 2,\ \delta=1$	$\eta = 2,\ \theta=1$
0.25	4	443.59	343.65	*	372.92	201.22
	6	221.11	170.91	810.73	186.91	99.37
	8	132.28	103.37	534.81	112.89	62.98
	10	87.88	70.87	363.77	76.96	46.16
0.10	4	488.84	309.45	*	428.84	*
	6	242.02	196.49	843.76	216.44	123.31
	8	143.26	118.22	562.83	129.15	77.40
	10	95.15	80.62	382.76	87.43	57.15
0.05	4	528.45	430.28	*	463.52	*
	6	261.46	215.88	*	233.64	144.03
	8	155.51	132.25	583.65	140.69	89.57
	10	102.99	89.60	397.30	94.70	66.09
0.01	4	*	*	*	*	*
	6	305.59	263.80	*	281.60	194.62
	8	179.99	160.67	639.67	170.30	124.12
	10	121.41	110.24	438.47	117.14	90.68

(*) refers plan doesn't exist

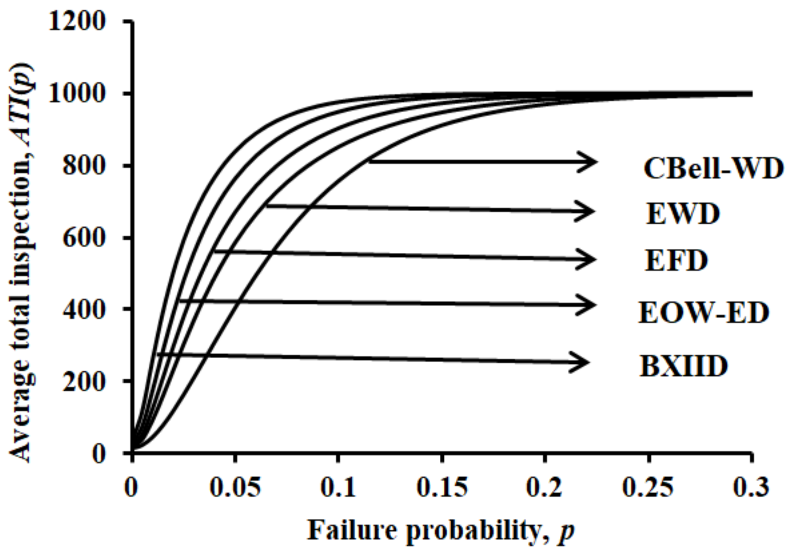

Figure 5: *ATI curves of proposed economic ChSP-1 under different lifetime models*

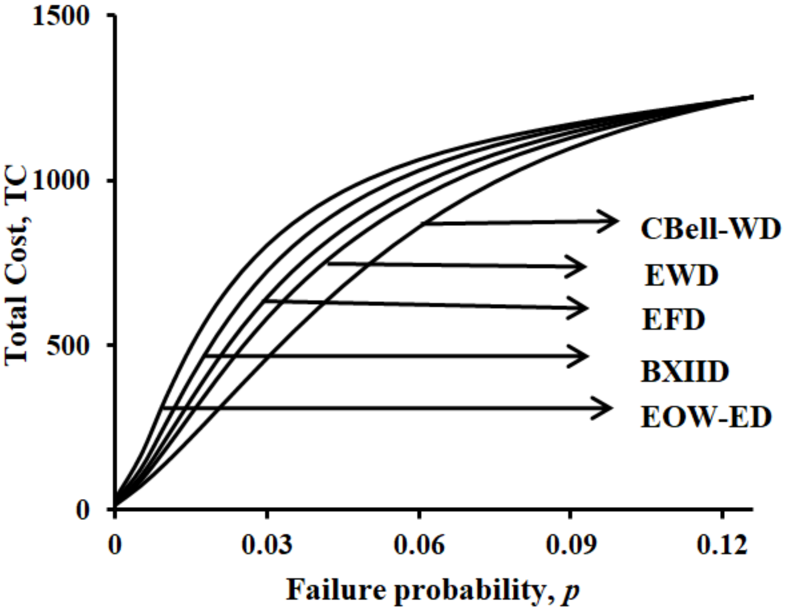

Figure 6. *TC curves of proposed economic ChSP-1 under different lifetime models*

XI. Conclusions

In this article, the proposed ChSP -1 presented for TTL test under median lifetime of the CBell -W model. By considering performance of two specific points on the OC curve, this approach allowed to determine the optimal parameters. The main comparison of the OC curves for both the proposed plan and SSP with Ac = 0 indicated that the proposed plan have higher level of discriminatory power when compared to the SSP. Also, the proposed plan under CBell-WD performed as the better model when compared to ChSP-1 based on other lifetime models. The proposed ChSP-1 plan under CBell-WD is applied in industry for fatigue fracture testing of Kevlar 373/epoxy composites at a 90% stress level, which is striking an optimal balance between test efficiency and result accuracy, thereby ensuring both reliable and efficient testing outcomes. The idea of quality control from an economic perspective may not be new idea of research; still it is relevant and worth considering, as it can lead to cost savings and efficiency gains in production processes by finding the right balance between inspection efforts and production outputs. An economic design of ChSP-1 is more effective than SSP when reducing *ATI* and TC, achieving the best performance at a lower inspection cost per unit. It has been also shown that the proposed sampling plan using CBell-WD is more economical than the existing other lifetime models. Therefore, the proposed plan will not only safeguard the producer needs but also guarantee the consumer satisfaction through the good product quality.

Acknowledgement

The first author wishes to convey her gratitude to the Kalasalingam Academy of Research and Education for granting her a fellowship opportunity.

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