IMPACT OF PREVENTIVE MAINTENANCE AND FAILURE RATE ON A COMPLEXLY CONFIGURED SYSTEM: A SENSITIVE ANALYSIS

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Abstract

The availability of uninterrupted performance time has become essential for any industry seeking to maximize profits while incurring minimal maintenance costs. However, the system's components become weary as a result of the constant burden, resulting in decreased system efficiency and automatic full failure in the end. Complete failure is not always manageable; it might result in a significant loss of profit or productivity. In this regard, preventative maintenance is critical to ensuring that the industry runs smoothly, even with lower efficiency. Preventive maintenance is required in any sector to satisfy the demands of maximum profit and low cost for good output. This study examines the reliability of a complexly organized system of three units, A, B and C in order to determine its sensitivity to the effects of deteriorated rate and preventative maintenance rate over time. The three units are further made up of subunits which are in series or parallel configuration. The mathematical design work is based on the Markov process and the Laplace transformation. Different system parameters such as mean time to system failure, Available performance time, reliability, and profit, are analysed with respect to time and various rates. Further, A sensitivity analysis is used to explore how the rate of deterioration and preventative maintenance affects the system over time. Various malfunction and repair rates effect the system parameters in increasing or decreasing manner and sensitive analysis evaluated the impact of one unit on another or whole system. Here is a numerical example generated with the help of an appropriate model; the results are visually represented which concluded that with the passage of time reliability and other system parameters of system decreased under the influence of different rates. Utilizing the service cost, Profit is analysed which help to estimate the overall gain by the presented system. Also, by sensitive analysis it is concluded that out of three units A, B and C, Unit C has more effect as compared to B and C which is shown graphically. The purposed study can elaborate the profit after examined the reliability indices which become a key point for different industries like as diary plant ,fertilizer plant etc to have good outcomes with less maintenance cost.

Keywords: Preventive maintenance, Laplace Transformation, Relaibility and Sensitive Analysis .

I. Introduction

The availability and dependability of manufacturing machines determine the capacity to create any commodity. To have the most, quality output that meets demand is required. The highest level of system dependability may be reached if all components perform well. However, it is not possible to operate at 100% efficiency for a lengthy period of time since certain system components wear down due to friction or other factors, even if they fail completely. rare systems' full failure cannot be fixed, resulting in a loss of money and, in rare cases, life. As a result, increasingly reliable systems require daily gains in loss minimization, which may be accomplished by performing the appropriate preventive or corrective maintenance methods. Improving dependability necessitates excessive preventative maintenance. Preventive maintenance conducted on a regular basis increases availability and profits. Creating an appropriate mathematical programming model has proven useful in system work. Numerous studies have been conducted in this topic.

Singla et al. [1], determined the plant's production, which comprises of four polytube units. The Chapman-Kolmogorov differential equations are solved using the method of extra variables and total performance time, thus reliability has been calculated using the likelihood of malfunctions and the number of repairs in a certain period of time. Naithani et al. [2], Using semi-Markov processes and the effects of cold standby, the dependability of induced draft fans in thermal plants was investigated, and it was determined that how the reliability and availability of the whole industry changes over time, as well as the impact of induces fans on them. The issue of assessing a banking server while considering a warm standby system was addressed Kumar and Goel, [3]. The reliable server has been derived by overcoming the effect of failure concept with the help of maintenance strategy. The lucrative idea for industry takes into account the appropriate choice of failure and repair rate. Employing computational models, the Markov method, the complementary variable technique, and the Laplace transform of the power plant's gas turbine system, Ram and Nagiya,[4], explored a number of reliability metrics, including mean time before failure, accessibility, reliability, and anticipated revenue of the system of the gas power plant and inspected out that which portion of the entire structure accomplished the reliability and MTTF more and also which component required greater scrutiny in order to have a growing profit using the sensitive analysis. According to Mohajan [5], the dependability of a measuring device is measured by how much one can trust the data produced by the instrument or how effectively any measuring apparatus corrects for the odd inaccuracy. The validity of a measuring instrument is determined by two factors: what it measures and how well it measures. Issue characteristics and problem-solving strategies continue to emphasize the reliability of a series-parallel system. Many research has also been undertaken in an attempt to improve these systems. Kumar et al. [6], have focused on the operational efficiency of a wiping unit used in the paper sector that employs Regenerative point graphical technique (RPGT) to analyse reliability characteristics, as well as the influence of failure and repair rates on reliability parameters. Yang and Tsao [7], explored a matrix-analytic technique to examine the dependability and availability of backup systems with operational escapes. They used a sensitivity analysis with the Laplace transformation to evaluate the MTTF and reliability function. The results show that increasing the number of spare components and maintenance frequency can increase system dependability. Tyagi et al. [8], conducted sensitivity analysis and reliability modeling on a flood alerting system (FAS) based on the Internet of Things (IoT). The authors employed a Markov method to calculate the probability of state change, which were then corrected using Laplace transformation. Kumar et al. [9], investigated the dependability of tripod turnstile machines operating in parallel configuration for extremely secure considerations utilizing the Laplace solver and the Markov idea. The sensitivity analysis for reliability has been investigated to determine the influence of one machine on another, and hence on the entire system. Modibbo et al. [10], offered two separate strategies to maximize system dependability under degradation in order to have lower costs for component maintenance while increasing profit. This work introduced a hybrid idea combining estimate and optimization theory. Reliability, availability, and maintainability of a threshing machine are three critical metrics described by Anchal et al. [11], and are highly useful in agriculture to get high reliability results. The influence of working units over time on one another and total productivity has been examined in order to make a profit in the agricultural industry. Particle swarm optimization, a nature-inspired algorithm, is being addressed to optimize the cost of rubber cultivation, and negotiations about availability and other reliability parameters have been graphically depicted to understand the effect of failure and repair rate on overall system performance, Singla et al.,[12]. Shakuntla and Pooja, [13] devised the mathematical analysis of the Regenerative Point Graphical Technique (RPGT) to examine the reliability metrics and total profit benefit of the proposed system. The variation of failure and repair rates on reliability metrics has been provided, and the value of each rate is computed to determine which rate is best for increased output and profit. Shubham et al.'s analysis [14], concentrated on executing competence evaluation, validation, and optimization activities for the steam production system of a coal-fired thermal power plant to understand the influence of one component's performance rate on the other, and therefore on the entire system. Saini et al. [15] discussed the availability of a steam turbine plant, the exponential failure time behavior, and the arbitrary repair time behavior. The Particle swarm optimization with Genetic Algorithm approach was used to solve the differential equation and get various system metrics at several places during the system's malfunction and recovery, and the system's overall profit and efficiency were calculated. According to Khan et al.,[16], this study bridged the gap that was existing by employing the BLLP-Bi-level programming plan to handle the optimization problem concerning the liability of a system undergoing chosen maintenance.

A numerical demonstration of the Khun-Trucker approach with linear constraints is shown. This study examines how changes to reliability indices affected overall dependability and performance. Singla et al. [17], investigated a deep learning technique in order to maximize reliability characteristics, increase industrial revenues, and manufacture a 2:3 good system. Deep learning algorithms are compared to one another based on their availability. Singla et al. [19], study a failing system using a genetic algorithm to maximize the mean time to system failure and availability acquired by RPGT, as well as to determine the reliability metrics driven by degradation rate and preventative maintenance rate. Ahmadini et al. [20], conducted a study on dependability metrics under the influence of preventative maintenance using a heuristic method and an artificial bee colony algorithm to determine the impact of degraded rate and system failure. The analysis of poly-tube manufacturing plant has been done to discuss the availability regarding each unit of plant with the optimizing tool PSO by Singla et. al. [21].

Different research studies have counted the variation in various reliability attributes related to failure and repair parameters, but the Laplace transformation methodology, which accounts for time variation, enables the industry to operate for an extended duration. Additionally, the sensitivity analysis has revealed the following: The current project's purpose is to increase the dependability of a presentation system while keeping it operational throughout time and at variable rates. The purpose of this research is to focus on how the sensitivity of system units affects the reliability, accessibility, and profitability of complex unit arrangements under preventive maintenance which was studied in less content in previous year. The paper is organized as follows: Section 2 describes the model's characteristics, including the state overview, assumptions, notations, and model frame. Section 3 discusses the methodology behind mathematical simulations. The MTTF, available performance time, reliability, and profit analysis are among the primary topics covered in Section 4. Section 5 addresses the system's sensitivity to parameters and reliability. Section 6 concludes with the outcome discussion. Section 7 has the conclusion.

II. Model description

I. System description

For the sake of this study, the complete system is represented by a complicated configuration made up of three major units (A, B, and C) plus subunits with mixed configurations stacked in series as seen in many plants like fertilizer plant, yarn mill, Soap industry, soft drink plant etc. Figure 1 depicts how the system is organized with its units and subunits. Unit A can run at low efficiency and be restored to full efficiency with a single preventive maintenance treatment; however, it will shut down after a second decline. There is no such situation for Unit C, which means it may undergo a catastrophic failure, whereas the Unit B component functions in parallel, causing it to collapse when both fail. The Markov process is used to generate the Champman-Kolmogorov differential equations. These equations can be solved using the concept of the Laplace transformation. There is a constructed mathematical model whose functioning is determined by the rates of failure, degradation, preventative maintenance, and corrective maintenance. Following a sensitivity analysis, the different metrics are calculated using the Laplace concept.

II. Assumptions

- At first, the system can execute its mission perfectly and efficiently, as if it were a new system.
- When a system degrades to a certain point, it loses part of its efficiency or becomes less efficient at accomplishing the task.
- After repair, the system operates as new, with each subcomponent taking the same amount of time to correct.
- Subunits A_1 and A_2 are assumed to have the same deterioration and failure rates. Components C_1, C_2, and C_3 have identical failure rates, as do components B_1 and B_2.
- It is assumed that failure and deterioration rates follow exponential distributions.

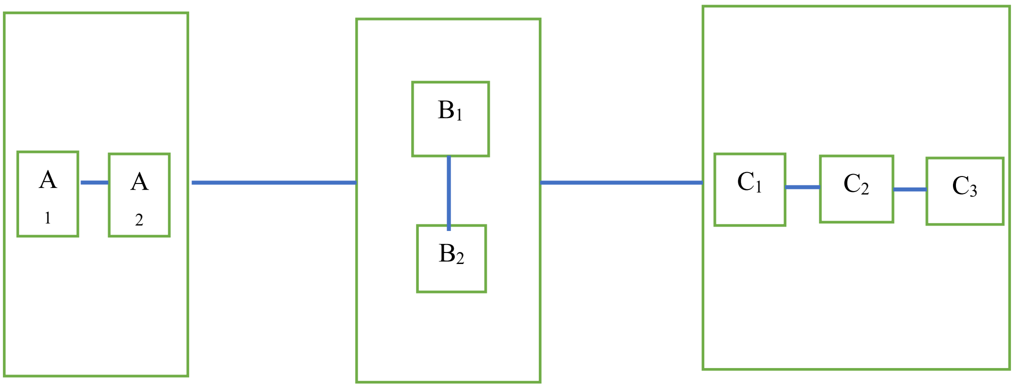

Figure 1: *System configuration*

III. Notations:

Table 1: *Various phrases pertaining to the work being presented.*

⬭ ⬠ □	Represent full functional state, reduced state and down state respectivly
MTTF	Mean time to system failure
Av(t)	Performance time of system over t
R(t)	The reliability of system over t
U_i	Represent transitions state where , $0 \le i \le 11$.
A	Represent 100% functional state of system
$\bar{A}$	The unit is reduced to some low efficiency due to some external or internal cause and need preventive maintenance.
A	The system is demised completely.
λ_j	Failure rate from excellent to malfunctioned state for Units B and C or from deteriorated to destroyed state For Unit A, where j = A, B, C
α_A	It is the degraded rate going from good condition to reduced condition for the unit A i.e. from A to $\bar{A}$.

μ_A	It represents the preventive maintenance rate to reversed the condition from reduced state to good state, only one time applicable for unit A.
M	Repair rate for taking back state from failed to good working state by applying corrective maintenance
$P_i(t)$	Represent the probability of various changing states where $0 \leq i \leq 11$.
P(t)	Represent the overall probability vector and its associated differential vector.
S	Laplace transformation variable
k_1	Revenue cost
k_2	Service cost

The potential state transition diagram for the model that is being presented can be seen as in Figure 2.

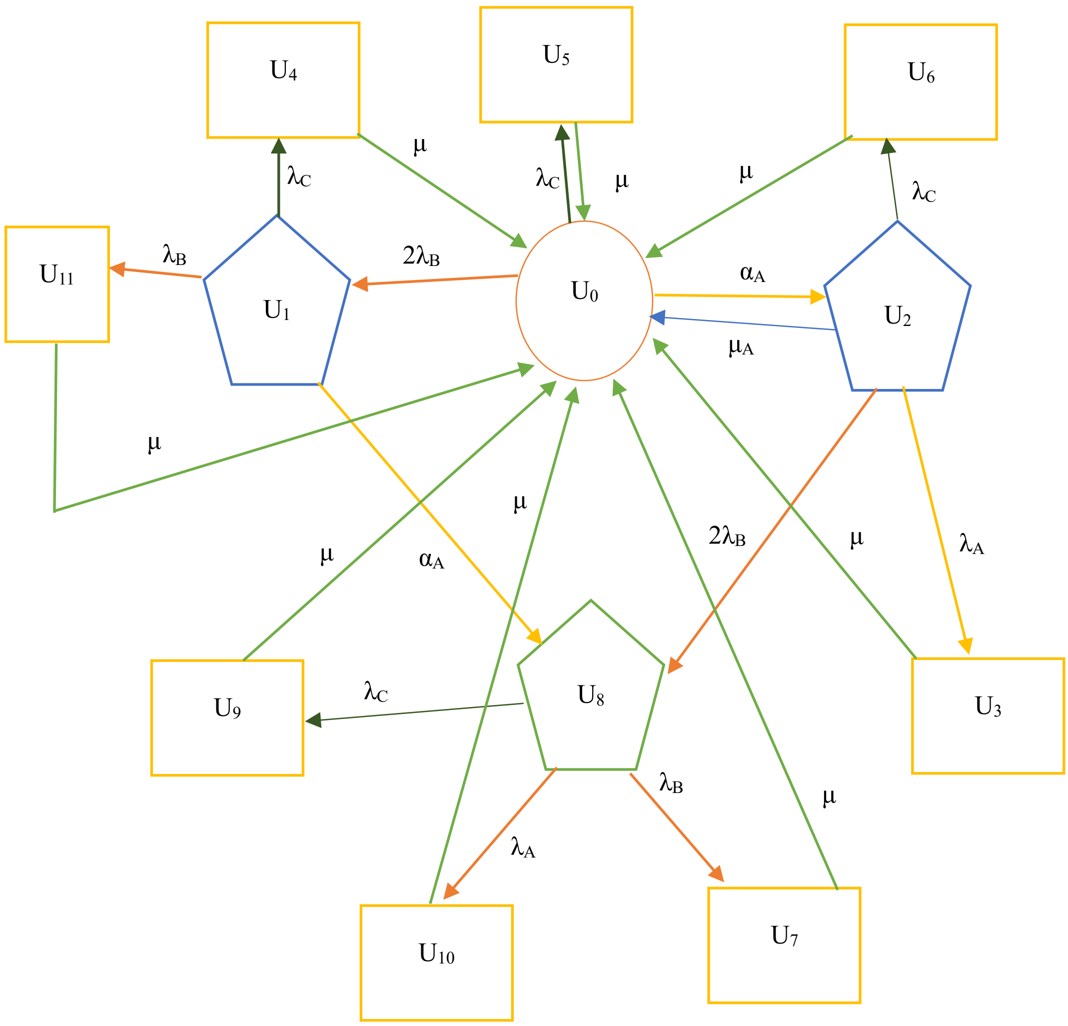

Figure 2: *The state transition diagram for different possibilities of working of three unit A,B,C.*

IV. State Description:

Table 2: *Various descriptions of states associated with the transition diagram.*

U_0	(A,B,C)	This is the full functional state with 100% efficiency.

U_1	(A,B(b),C)	One of the component of B is failed but 100% efficient state due to parallel arrangement of parts of unit B
U_2	($\bar{A}$,B,C)	Reduced state due to deteioration of unit A and efficiency of whole system reduced to some level and undergo preventive maintenance
U_3	(a,B,C)	Represting a breakdowned state due to complete breakdown of Unit A
U_4	(A,B(b),c)	It is a down state as Unit C breakdowned.
U_5	(A,B,c)	Breakdowned state as Unit C failed.
U_6	($\bar{A}$,B,c)	Down state as unit c malfunctioned fully while A is in reduced state and B is functional
U_7	($\bar{A}$,b,C)	Breakdown state as both component of unit B stop working while A is in reduced state and C is functional
U_8	($\bar{A}$,B(b),C)	whole condition refer to reduced state.
U_9	($\bar{A}$,B(b),c)	It is a down state as Unit C stopped while A is in reduced state and B is functional
U_{10}	(a,B(b),C)	Breakdowned state due to Unit A while other two are functional.
U_{11}	(A,b,C)	Breakdowned state due to Unit B while other two are functional

III. Mathematical Modelling of the Presented Model

The computational foundation for the mechanism being discussed incorporates the concept of the Markov process. The creation of first-order Chapman-Kolmogorov differential equations, which correspond to the numerous stable states depicted in the transition diagram, assists in the determination of reliability parameters. Pi(t) is the chance that the system will be in state Ui at time t ≥0. Furthermore, let P(t) be the probability vector at time t in hours given a starting condition.

$$P_i(0) = \begin{cases} 1 & if\ i = 0 \\ 0 & if\ i \neq 0 \end{cases} \tag{1}$$

The differentioal equations associated to Figure 2 are:

$$P'_0(t) = -(\alpha_A + \lambda_C + 2\,\lambda_B)P_0(t) + \mu_A P_2(t) + \mu\,(\textstyle\sum_{i=3}^{11} P_i(t) - P_8(t)) \tag{2}$$
$$P'_1(t) = -(\,\alpha_A + \lambda_C + \lambda_B\,)\,P_1(t) + 2\,\lambda_B\,P_0(t) \tag{3}$$
$$P'_2(t) = -(\,\mu_A + 2\lambda_B + \lambda_A + \lambda_C)P_2(t) + \alpha_A\,P_0(t) \tag{4}$$
$$P'_3(t) = -\mu P_3(t) + \lambda_A\,P_2(t) \tag{5}$$
$$P'_4(t) = -\mu P_4(t) + \lambda_C\,P_1(t) \tag{6}$$
$$P'_5(t) = -\mu P_5(t) + \lambda_C\,P_0(t) \tag{7}$$
$$P'_6(t) = -\mu P_6(t) + \lambda_C\,P_2(t) \tag{8}$$
$$P'_7(t) = -\mu P_7(t) + \lambda_B\,P_8(t) \tag{9}$$
$$P'_8(t) = -(\lambda_A + \lambda_C + \lambda_B)P_8(t) + 2\lambda_B\,P_2(t) + \alpha_A P_1(t) \tag{10}$$
$$P'_9(t) = -\mu P_9(t) + \lambda_C\,P_8(t) \tag{11}$$
$$P'_{10}(t) = -\mu P_{10}(t) + \lambda_A\,P_8(t) \tag{12}$$
$$P'_{11}(t) = -\mu P_{11}(t) + \lambda_B\,P_1(t) \tag{13}$$

Equations (2) to (13) can be transformed using the Laplace transform, and the initial condition, i.e., equation (1), That is what we understand.

$$(s + \alpha_A + \lambda_C + 2\,\lambda_B)\bar{P}_0(s) = 1 + \mu_A\bar{P}_2(s) + \mu\,(\textstyle\sum_{i=3}^{11} \bar{P}_i(s) - P_8(t)) \tag{14}$$
$$(s + \alpha_A + \lambda_C + \lambda_B\,)\,\bar{P}_1(s) = 2\,\lambda_B\,\bar{P}_0(s) \tag{15}$$
$$(s + \mu_A + 2\lambda_B + \lambda_A + \lambda_C)\bar{P}_2(s) = \alpha_A\,\bar{P}_0(s) \tag{16}$$
$$(s + \mu)\bar{P}_3(s) = \lambda_A\,\bar{P}_2(s) \tag{17}$$
$$(s + \mu)\bar{P}_4(s) = \lambda_C\,\bar{P}_1(s) \tag{18}$$
$$(s + \mu)\bar{P}_5(s) = \lambda_C\,\bar{P}_0(s) \tag{19}$$
$$(s + \mu)\bar{P}_6(s) = \lambda_C\,\bar{P}_2(s) \tag{20}$$

$(s+\mu)\bar{P}_7(s) = \lambda_B \bar{P}_8(s)$ (21)
$(s+\lambda_A+\lambda_C+\lambda_B)\bar{P}_8(s) = 2\lambda_B \bar{P}_2(s) + \alpha_A \bar{P}_1(s)$ (22)
$(s+\mu)\bar{P}_9(s) = \lambda_C \bar{P}_8(s)$ (23)
$(s+\mu)\bar{P}_{10}(s) = \lambda_A \bar{P}_8(s)$ (24)
$(s+\mu)\bar{P}_{11}(s) = \lambda_B \bar{P}_1(s)$ (25)

Now solving equation from (14) –(25) , We get the trasition state probabilities:

$$\bar{P}_0(s)=\frac{1}{\left[(s+C_1)-\left\{\frac{\mu_A\alpha_A}{(s+C_3)}+\frac{\mu}{(s+\mu)}\left(\frac{(\lambda_A+\lambda_C)\,\alpha_A}{(s+C_3)}+\frac{2(\lambda_B+\lambda_C)\lambda_B}{(s+C_2)}+\lambda_C+\frac{2\lambda_B\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\right)\right\}\right]}$$
(26)

$$\bar{P}_1(s) = \left(\frac{2\lambda_B}{s+C_2}\right)\bar{P}_0(s) \quad (27)$$
$$\bar{P}_2(s) = \left(\frac{\alpha_A}{s+C_3}\right)\bar{P}_0(s) \quad (28)$$
$$\bar{P}_3(s) = \left(\frac{\lambda_A\alpha_A}{s+C_3}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (29)$$
$$\bar{P}_4(s) = \left(\frac{2\lambda_B\,\lambda_C}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (30)$$
$$\bar{P}_5(s) = \left(\frac{\lambda_C}{s+\mu}\right)\bar{P}_0(s) \quad (31)$$
$$\bar{P}_6(s) = \left(\frac{\alpha_A\lambda_C}{s+C_3}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (32)$$
$$\bar{P}_7(s) = \frac{2\lambda_B\,\lambda_B\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (33)$$
$$\bar{P}_8(s) = \left(\frac{2\lambda_B\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\right)\bar{P}_0(s) \quad (34)$$
$$\bar{P}_9(s) = \frac{2\lambda_B\,\lambda_C\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (35)$$
$$\bar{P}_{10}(s) = \frac{2\lambda_B\,\lambda_A\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (36)$$
$$\bar{P}_{11}(s) = \left(\frac{2\lambda_B\,\lambda_B}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\bar{P}_0(s) \quad (37)$$

Subsequently, the Laplace transformation of the system's up-state probability looked like this:

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_8(s)$$
$$\bar{P}_{up}(s) = \left(1+\left(\frac{2\lambda_B}{s+C_2}\right)+\left(\frac{\alpha_A}{s+C_3}\right)+\frac{2\lambda_B\ \alpha_A}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\right)\bar{P}_0(s) \quad (38)$$

Furthermore, the Laplace transformation is used to change the system's down-state probability as follows:

$$\bar{P}_{down}(s) = \sum_{i=3}^{11}\bar{P}_i(s) - \bar{P}_8(s)$$
$$\bar{P}_{down}(s) = \begin{bmatrix}\left(\frac{\lambda_A\alpha_A}{s+C_3}\right)\left(\frac{1}{s+\mu}\right)+\left(\frac{2\lambda_B\,\lambda_C}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)+\left(\frac{\lambda_C}{s+\mu}\right)+\left(\frac{\alpha_A\lambda_C}{s+C_3}\right)\left(\frac{1}{s+\mu}\right)+\\ \frac{2\lambda_B\ \alpha_A C_4}{(s+C_4)}\left(\frac{1}{s+C_3}+\frac{1}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)+\left(\frac{2\lambda_B\,\lambda_B}{s+C_2}\right)\left(\frac{1}{s+\mu}\right)\end{bmatrix} \quad (39)$$

IV. Mathematical Measure of the presented Model

I. Availability Analysis (Av)

A system's availability may be summarized as the frequency of problems and the speed with which they are resolved. By using the different rate values as $\alpha_A = 0.01 = \mu_A$, $\lambda_C = 0.03$, $\lambda_B = 0.02$, μ =0.04 and λ_A =0.01 in (38) and applying inverse Laplace transformation. The system's available performance time is expressed as follow:

$$\text{Av} = 0.1520e^{-0.1353t} + 0.0370e^{-0.0523t} + 0.4988 + 0.3122e^{-0.0712t}\cos(0.0463t) + 0.3184e^{-0.0712t}\sin(0.0463t) \quad (40)$$

Equation (40) allows us to change the time from 0 to 30 hours, resulting in numerical availability metrics, as shown in Table 3 and Figure 3.

Table 3: *Variation in Availability w.r.t time*

Time	Availability
0	1.0000
1	0.9708
2	0.9432
3	0.9170
4	0.8922
5	0.8686
6	0.8461
7	0.8248
8	0.8045
9	0.7852
10	0.7669
11	0.7494
12	0.7329
13	0.7172
14	0.7023
15	0.6881
16	0.6748
17	0.6622
18	0.6502
19	0.6390
20	0.6284
21	0.6184
22	0.6090
23	0.6003
24	0.5920
25	0.5843
26	0.5771
27	0.5703
28	0.5641
29	0.5582
30	0.5528

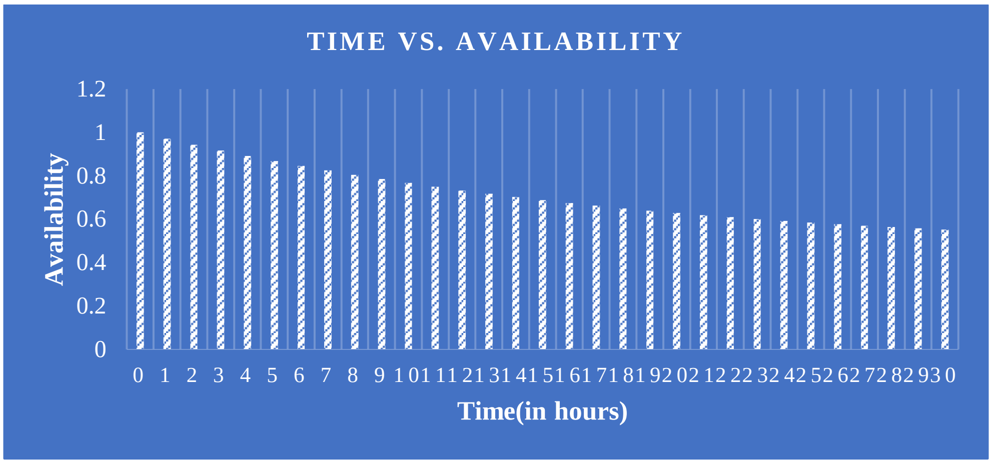

Figure 3: *The variation in Availability with variation in time.*

II. Mean time to system failure (MTTF) Analysis

MTTF calculates the average time projected before an initial malfunction, i.e. the length of system operation before the first failure. Using μ=0 and the limit approaching zero in (38), we may obtain MTTF as

$$\text{MTTF}=\lim_{s\to 0}\overline{P_{up}}(s)$$

$$\text{MTTF}=\left(\frac{1+\frac{2\lambda_B}{\alpha_A+\lambda_C+\lambda_B}+\frac{\alpha_A}{\mu_A+2\lambda_B+\lambda_A+\lambda_C}+\frac{2\lambda_B\ \alpha_A}{(\lambda_B+\lambda_A+\lambda_C)(\alpha_A+\lambda_C+\lambda_B)}+\frac{2\lambda_B\ \alpha_A}{(\lambda_B+\lambda_A+\lambda_C)(\mu_A+2\lambda_B+\lambda_A+\lambda_C)}}{(\alpha_A+\lambda_C+2\lambda_B)-\frac{\mu_A\alpha_A}{\mu_A+2\lambda_B+\lambda_A+\lambda_C}}\right) \tag{41}$$

Putting the value of $\alpha_A = 0.01$ and $\mu_A = 0.01$ and then varying the both one by one, respectively, from going 0.1 to 0.9 in (41), we get the values represented in Table 4 and Figure 4.

Table 4 : *Variation in MTTF with variation in degraded rate and preventive maintenance rate.*

Variable in α_A and μ_A	MTTF	
	α_A	μ_A
0.1	21.7817	25.1244
0.2	21.5087	25.2179
0.3	21.4242	25.2636
0.4	21.3577	25.2907
0.5	21.2619	25.3086
0.6	21.1956	25.3214
0.7	21.1471	25.3309
0.8	21.1101	25.3383
0.9	21.0809	25.3442

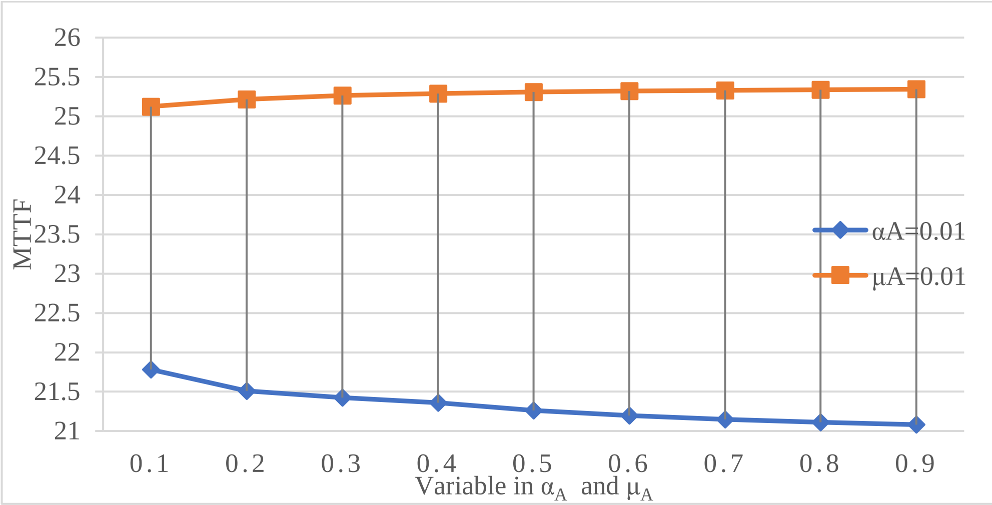

Figure 4: *Variation in MTTF with degraded and preventive maintenance rate*

III. Reliability Analysis

Reliability refers to working time without any failure in a given period of time. By using the different rate values as $\alpha_A = 0.01 = \mu_A$, $\lambda_C = 0.03, \lambda_B = 0.02$, λ_A =0.01 and recovery rate μ=0 in (38) and by taking inverse Laplace transformation, Reliability may be described as

$$R(t) = 0.1025e^{-0.0962t} - 0.7025\, e^{-0.0738t} + 1.6\, e^{-0.06t} \tag{42}$$

Varying time from 0 to 30 hours, the following data is obtained, depicted by Table 5 and represented graphically in Figure 5.

Table 5: *Variation in Reliability vs. time*

Time	Reliability
0	1
1	0.9474
2	0.8975
3	0.8503
4	0.8054
5	0.7629
6	0.7227
7	0.6845
8	0.6483
9	0.6140
10	0.5814
11	0.5506
12	0.5214
13	0.4937
14	0.4674
15	0.4425
16	0.4189
17	0.3966

18	0.3754
19	0.3553
20	0.3363
21	0.3183
22	0.3012
23	0.2851
24	0.2698
25	0.2552
26	0.2415
27	0.2285
28	0.2162
29	0.2045
30	0.1934

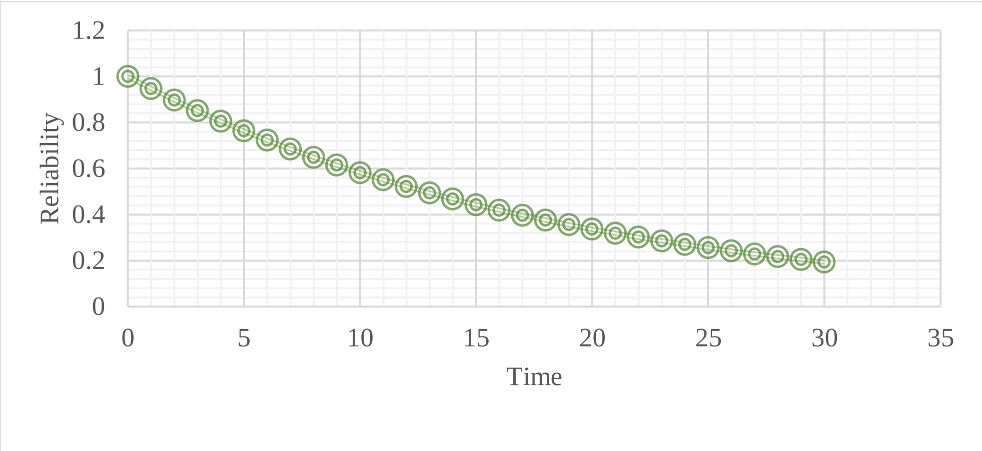

Figure 5: *Time vs. Reliability*.

IV. Profit Analysis

If the facility for service is assumed to have been readily available at all times, the expected profit is as follows, accounting for maintenance expenditures for the range [0, t].

Profit = $k_1\int_0^t P_{up}(t)dt - tk_2$ (43)

Using (38) and Having k_1 =1 and k_2 = 0.1 ,0.2 ,0.3 ,0.4 ,0.5 respectively we get data which is depicted by the Table 6 and Graph 6.

Table 6: *Profit analysis with respect to time*

Time (t)	k_2 = 0.1	k_2 =0.2	k_2 =0.3	k_2 =0.4	k_2 =0.5
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8853	0.7853	0.6853	0.5853	0.4853
2	1.7422	1.5422	1.3422	1.1422	0.9422
3	2.5722	2.2722	1.9722	1.6722	1.3722
4	3.3767	2.9767	2.5767	2.1767	1.7767
5	4.1570	3.6570	3.1570	2.6570	2.1570
6	4.9143	4.3143	3.7143	3.1143	2.5143

7	5.6497	4.9497	4.2497	3.5497	2.8497
8	6.3642	5.5642	4.7642	3.9642	3.1642
9	7.0590	6.1590	5.2590	4.3590	3.4590
10	7.7350	6.7350	5.7350	4.7350	3.7350

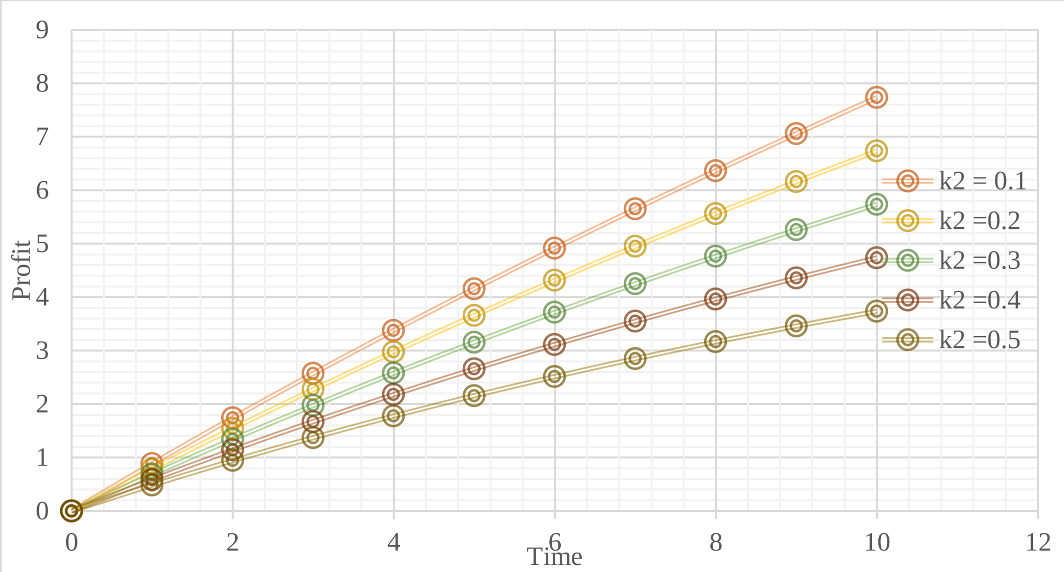

Figure 6: *Time vs. profit*

V. Sensitive Analysis

I. Sensitivity of Availability

The inverse Laplace equation(38) is differentiated to assess the sensitivity of availability, and then time is varies from 0 to 30 hours with respect to, λ_A,λ_B and λ_C one by one, as demonstrated by tabular form using Table 7 and graphically by Figure 7.

Table 7: *Sensitivity Analysis of System Availability*

Time(t)	**Reliability's** $\boldsymbol{\lambda_A}$	**Sensitivity** $\boldsymbol{\lambda_B}$	$\boldsymbol{\lambda_C}$
0	0	0	0
1	-0.0979	-0.9856	-1.4892
2	-0.1903	-1.4804	-1.6677
3	-0.2690	-1.9190	-1.8259
4	-0.3360	-2.3066	-1.9654
5	-0.3926	-2.6480	-2.0877
6	-0.4405	-2.9473	-2.1940
7	-0.4807	-3.2086	-2.2858
8	-0.5143	-3.4355	-2.3643
9	-0.5424	-3.6313	-2.4305
10	-0.5656	-3.7990	-2.4856

11	-0.5848	-3.9414	-2.5306
12	-0.6005	-4.0612	-2.5665
13	-0.6132	-4.1604	-2.5940
14	-0.6235	-4.2412	-2.5665
15	-0.6317	-4.3060	-2.5940
16	-0.6383	-4.3560	-2.6140
17	-0.6433	-4.3931	-2.6274
18	-0.6473	-4.4186	-2.6346
19	-0.6502	-4.4339	-2.6337
20	-0.6525	-4.4403	-2.6266
21	-0.6541	-4.4387	-2.6017
22	-0.6552	-4.4303	-2.5848
23	-0.6560	-4.4160	-2.5655
24	-0.6565	-4.3965	-2.5442
25	-0.6568	-4.3725	-2.5211
26	-0.6569	-4.3449	-2.4966
27	-0.6570	-4.3140	-2.4709
28	-0.6571	-4.2806	-2.4442
29	-0.6571	-4.2451	-2.4169
30	-0.6571	-4.2079	-2.3892

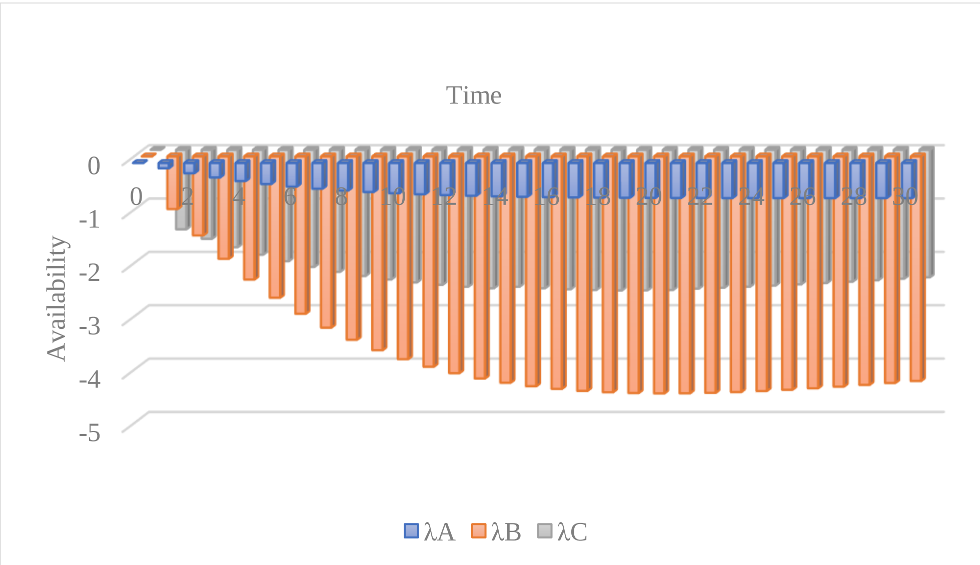

Figure 7: *Time vs. Availability 's sensitivity*

II. Sensitivity of MTTF

By differentiating equation (41) with respect to three failure rates, λ_A ,λ_B and λ_C one by one, repectively, then varing the value form 0.1 to 0.9 .

Table 8: *Sensitivity of MTTF.*

Variable in λ_A, λ_B and λ_C	MTTF

	λ_A	λ_B	λ_C
0.1	-4.7269	-70.4246	-80.1074
0.2	-1.9436	-24.8599	-23.1077
0.3	-1.0526	-12.5387	-10.6718
0.4	-0.6587	-7.5156	-6.0982
0.5	-0.4506	-4.9840	-3.9340
0.6	-0.3276	-3.5294	-2.7444
0.7	-0.2488	-2.6153	-2.0220
0.8	-0.1953	-2.0020	-1.5509
0.9	-0.1574	-1.5691	-1.2269

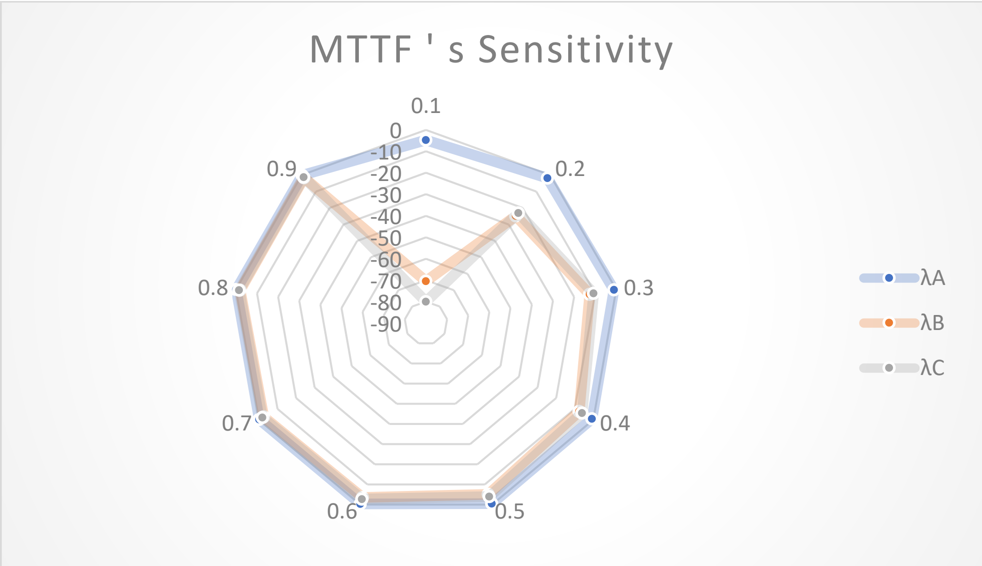

Figure 8: *MTTF's Sensitivity*

III. Sensitivity of Reliability

By taking inverse Laplace of (38) with value of μ=0, and differentiating equation with respect to three failure rates, λ_A ,λ_B and λ_C one by one, repectively, then by changing time from 0 to 30 hours. we get the Figure 9 and Table 9.

Table 9: *Reliability Sensitivity*

	Reliability's	**Sensitivity**	
Time(t)	λ_A	λ_B	λ_C
0	0	0	0
1	-0.0046	-0.0557	-0.9696
2	-0.0169	-0.2071	-1.8775
3	-0.0351	-0.4328	-2.7226
4	-0.0574	-0.7149	-3.5048
5	-0.0825	-1.0379	-4.2245
6	-0.1094	-1.3889	-4.8829
7	-0.1371	-1.7569	-5.4812

8	-0.1649	-2.1329	-6.0215
9	-0.1922	-2.5092	-6.5057
10	-0.2185	-2.8797	-6.9361
11	-0.2435	-3.2395	-7.3152
12	-0.2669	-3.5847	-7.6455
13	-0.2886	-3.9120	-7.9296
14	-0.3084	-4.2192	-8.1702
15	-0.3263	-4.5046	-8.3699
16	-0.3421	-4.7671	-8.5314
17	-0.3560	-5.0060	-8.6572
18	-0.3679	-5.2211	-8.7498
19	-0.3779	-5.4122	-8.8117
20	-0.3861	-5.5798	-8.8454
21	-0.3925	-5.7244	-8.8529
22	-0.3973	-5.8466	-8.8367
23	-0.4005	-5.9473	-8.7986
24	-0.4023	-6.0273	-8.7409
25	-0.4027	-6.0877	-8.6652
26	-0.4019	-6.1296	-8.5735
27	-0.3999	-6.1540	-8.4673
28	-0.3969	-6.1621	-8.3484
29	-0.3930	-6.1549	-8.2181
30	-0.3882	-6.1336	-8.0779

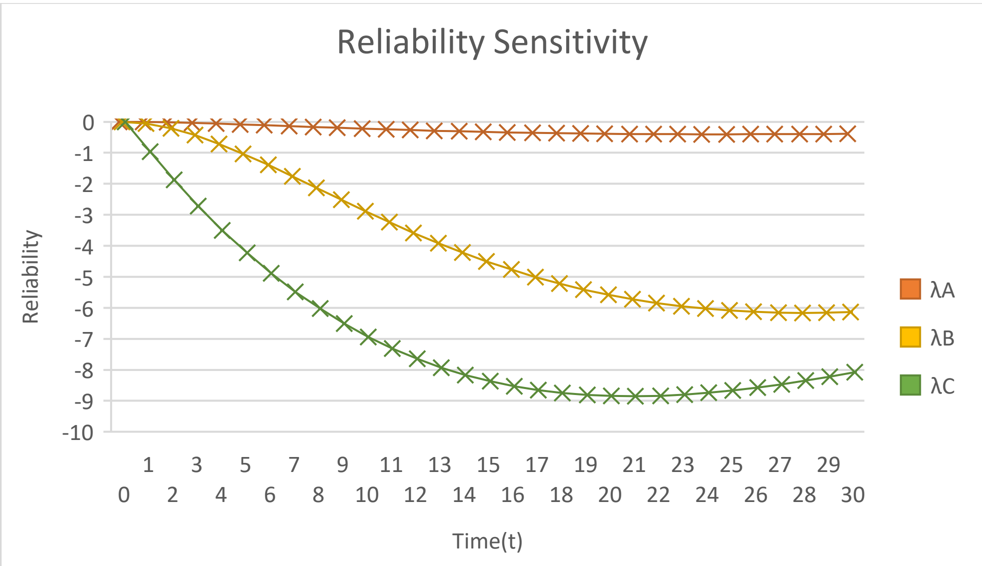

Figure 9: *Reliability's Sensitivity*

VI. Result and Discussion

The authors of the current study investigated the sensitivity analysis and reliability metrics of a multi-configuration complex system that incorporates failure rates, degradation, and preventative maintenance. The choice of parameters used in work is appropriate to have a good reliable system, Shivani et al. [18]. The framework's essential outlines are given below once the approach has been implemented.

Using Table 3 and Figure 3, the authors demonstrate how a system's available performance time evolves over time. After setting factors such as failure rate, repair rate, degradation rate, and preventive maintenance rate, availability drops with time as the chance of failure increases. After a long period, it becomes consistent and, to a lesser extent, continuous, affecting many portions of the system.

The MTTF is determined while accounting for system variances. This demonstrates that the MTTF of the system rises with the use of preventative maintenance (as depicted graphically in figure 4) and falls with regard to the deteriorated rate.

Figure 5 depicts the system's behavior, illustrating how dependability degrades over time and finally approaches zero. Table 5 displays the system's influence on reliability over time with fixed settings.

Figure 6 depicts an estimated profit analysis based on service cost and time. Critical inspection of the graphs reveals that the system creates a bigger profit over time; nevertheless, this profit decreases when service expenses grow.

In terms of failure rate, Table 7 and Figure 7 focus on the system's sensitivity analysis. The visual portrayal shows that the availability value decreases with time until t = 10 units, at which point it stabilizes. According to the analysis, the value declines until t = 23 units, at which point it climbs somewhat. When considering time, the availability value decreases until t = 21 units, then increases from t = 22 units to 30 units.

A critical examination of Figure 8 reveals that at a failure rate of 0.1 to 0.2 units, the sensitivity of the MTTF increases with some difference in value across all failure rates. As the failure rate climbed, so did the value of MTTF.

Figure 9 depicts how the system's dependability is affected by each of the three failure rates. As time passes, the value of dependability decreases until t = 28, at which point it increases, as opposed to initially declining until t = 25. This demonstrates that variation in the failure rate of unit A has the greatest impact on system dependability, followed by unit B.

VII. Conclusion

A complicated three-unit system with numerous configurations is investigated, as well as the sensitivity of dependability measurements. In this work, the reliability indices are obtained using the Laplace transformation after the system's equation has been created using the Champman-Kolmogorov differential approach and the Markov process notion. Many academic papers have been produced regarding series-parallel systems, but none of them have considered a complex system that employs preventative maintenance. These findings lead us to the following conclusions: as time passes, the framework's dependability and available performance time decline, and as failure rates grow, MTTF reduces as well. This emphasizes the importance of preventive maintenance and its significance in the choice of strategy to be used in the maintenance field to reduce the cause of degradation, as well as in identifying the unit with the highest failure rate that should be repaired as soon as possible to minimize loss.

A sensitivity investigation of the three units indicated that unit C is more sensitive than the other two in starting level, thus a maintenance concept is applied to it. However, when compared to unit A, it is clear that unit B is more susceptible to failure rates. As a result, the system becomes incredibly lucrative as the failure rate is reduced through preventative and corrective maintenance. By taking care of units B and C, the researchers may improve productivity while requiring less maintenance, considerably increasing profit. This article offers research findings that highlight the importance of parameter and unit selection for engineers and designers in developing more lucrative and low-maintenance systems.

To design cost-effective systems in the future, writers might develop mathematical models that optimize dependability while decreasing cost and sensitivity. Another element pushing designers to reduce service costs is the study's use of preventative maintenance. A meta-heuristic technique may be used to maximize reliability and other elements of the system by creating a model containing the reliability function.

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