RT&A, No 4(80)
Volume 19, December, 2024

M/M(A,B)/1 MULTIPLE WORKING VACATIONS QUEUING SYSTEM WITH HETEROGENOUS ENCOURAGED ARRIVAL

PRAKATI.P*, JULIA ROSE MARY.K

•

Department of Mathematics, Nirmala College for Women, India
*prakatidhanam@gmail.com
juliakulandaisamy@gmail.com

Abstract

The concept of Queuing system is most commonly used in our everyday life. It is essential to characterize the practical queuing characteristics in order to improve the performance of the queuing model. This study investigates M/M(a,b)/1/MWV queuing model with heterogeneous encouraged arrival occurring in the regular busy period. The considered model follows General bulk service rule and if the system is not in use, or when it is vacant, the server goes on vacation, thus there occurs multiple working vacations which are exponentially distributed. In this study, a model of multiple working vacation queues in which with heterogeneous encouraged arrivals following Poisson process is examined. With the mentioned conditions, the explicit formulations for the steady state probabilities and the performance measures of the proposed model is derived. Also,some particular cases have been developed and compared with existing models. Finally, the numerical impact of various parameters on performance attributes are also analysed.

Keywords: Heterogeneous Encouraged Arrival, General Bulk Service Rule, Multiple Working Vacations (MWV), Poisson Distribution, Mean Queue Length

1. Introduction

Several fields including telecommunications, traffic signals, the medical sector, inventory and control etc., widely use queuing theory. In a vacations queuing system, there may be a chance of unavailability of a server from a primary service center for a period of time. A vacation is a period of time spent away from the main service location and it can be caused by a variety of circumstances. Neuts'[7] general bulk service rule (GBSR), states that the server processes the clients in batches. Many Researchers have contributed their findings in Queuing theory with server vacations.

Initially, a general type of bulk queues is discussed by Neuts, who also examines the length of the queue and its busiest times. Further,Y. Levy and U. Yechiali[4] discussed about the vacations in queuing model. A class of semi-vacations policies were first presented by L.D Servi and S.G Finn[9], in which the servers operate at a reduced pace rather than suspending all primary service altogether while on vacations and such vacations is termed as Working Vacations.

De-An Wu and Hideaki Takagi[1] examined M/G/1 queue with multiple working vacations and evaluated its performance measures. Liu et al.,[5] stated the queue size probabilities of M/M/1 Multiple Working Vacation Queuing model. Further, K. Julia Rose Mary and M.I. Afthab Begum[3] analysed a single server with bulk service queue with general arrival pattern following multiple working vacations period. Later, K. Santhi and S. Pazhani Bala Murugan[8] examined on the concept of heterogeneous service on M/G/1 queue with two-stage service under single working vacation. Numerous Researchers analysed the concept of multiple working vacations. Further, O.C. Ibe, O.A. Isijola [2] also discussed on M/M/1 multiple vacations queuing systems with differentiated vacations.

While B.K. Som, S. Seth [10] evaluated M/M/1/N Queuing system with Encouraged Arrivals. Recently, S. Malik, R. Gupta [6] analysed the Finite Capacity Queuing System with Multiple Vacations and Encouraged customers. This paper discusses the concept of heterogeneous arrival specifically encouraged arrival in the busy period state of M/M(a,b)/1/MWV and obtained with performance measures.Also, Prakati and Julia Rose Mary[**?**] discussed the concept of encouraged arrival with both single and multiple working vacations performing with single server. Further, the numerical analysis of the considered model is evaluated.

2. Methods

An M/M(a,b)/1 queuing model of multiple working vacations with heterogeneous encouraged arrival is considered with the following assumptions:

The three different states of arrival is assumed to be heterogeneous and are uniquely denoted. The heterogeneous arrival process in this model is considered to follow Poisson distribution with parameter λ_{wi} in the idle state , λ_{wv} in the vacation state, whereas the heterogeneous encouraged arrival process also follows Poisson distribution with parameter $\lambda_{wb}(1+\delta)$. According to Neuts (1967) general bulk service rule (GBSR), the server processes the clients in batches. This rule states that the server will only begin providing service if at least "a" customers is present. After completing a service, if the server discovers "a" (or) more clients but not more than "b" clients in the system, he serves them all at once; if he discovers more than b, he serves the first b-customers in turn while the others wait.

As a result, there are minimum of "a" units and maximum of "b" units in each batch for service. The assumption is that the service time of batches of size s$(a \leq s \leq b)$ is an independent random variable with identical distribution and a parameter with an exponential distribution $''\mu_w''$. When a service is finished and there are less than 'a' clients in the queue, the server starts vacations, which is an exponentially distributed random variable with parameter η' .If the system length is still less than "a" after finishing one vacations, the server takes another vacations, and so on, until the server detects at least "a" customers in the queue (i.e., multiple vacations is used).

During vacations, if the queue size reaches at least "a" customers, then the server begins providing service at a service rate $''\mu_{wv}''$ which is different from the regular service rate. The size of the batch in service is "k" with $a \leq k \leq b$ and the service rates are independent of the size of the batch in service, thus when the vacations is over the server will shift the service rate from μ_{wv} to μ_w, when the server is operating. In this model, there is an increase in the arrival rate, i.e., encouraged arrival occurs in the regular busy period and then the server still continues to serve following GBSR. The above queuing model is denoted as M/M(a,b)/1/MWV with heterogeneous encouraged arrival.

3. Steady State Equations

Let $N_K(t)$ = number of customers waiting at the time, "t"
and M(t) = 0,1 or 2 denotes that the server is idle during the vacations or working during vacations or in the regular busy period respectively.

Let $IR_n(t) = Pr\{N_{k(t)} = n, M(t) = 0\} \quad ; \quad 0 \leq n \leq a-1$
$VQ_n(t) = Pr\{N_{k(t)} = n, M(t) = 1\}; n \geq 0$
$BP_n(t) = Pr\{N_{k(t)} = n, M(t) = 2\}; n \geq 0$
when M(t) = 0, the size of the queue and the system are same,
when M(t) = 1 or 2, then the size of the system is the sum of total no. of customers waiting in the queue or the size of the service batch containing $a \leq k \leq b$ customers.

The Steady State Probabilities satisfying the Chapman Kolmogrov equations are assumed as follows:

$$VQ_n = t \xrightarrow{lt} \infty VQ_n(t);$$

$$IR_n = t \xrightarrow{lt} \infty IR_n(t);$$

$$BP_n = t \xrightarrow{lt} \infty BP_n(t)$$

For the specified model, the steady state equations are expressed below:

$$\lambda_{wi} IR_0 = \mu_w BP_0 + \mu_{wv} VQ_0 \tag{1}$$

$$\lambda_{wi} IR_n = \lambda_{wi} IR_{n-1} + \mu_w BP_n + \mu_{wv} VQ_n \quad ; \quad 1 \leq n \leq a-1 \tag{2}$$

$$(\lambda_{wv} + \dot{\eta} + \mu_{wv}) VQ_0 = \lambda_{wi} IR_{a-1} + \mu_{wv} \sum_{n=a}^{b} VQ_n \tag{3}$$

$$(\lambda_{wv} + \dot{\eta} + \mu_{wv}) VQ_n = \lambda_{wv} VQ_{n-1} + \mu_{wv} VQ_{n+b} \quad ; \quad n \geq 1 \tag{4}$$

$$(\lambda_{wb}(1+\delta) + \mu_w) BP_0 = \mu_w \sum_{n=a}^{b} BP_n + \dot{\eta} VQ_0 \tag{5}$$

$$(\lambda_{wb}(1+\delta) + \mu_w) BP_n = (\lambda_{wb}(1+\delta)) BP_{n-1} + \mu_w BP_{n+b} + \dot{\eta} VQ_n) \quad ; \quad n \geq 1 \tag{6}$$

In the above mentioned steady state equations, Eq (1) and Eq (2) represent idle state, working vacations are defined in Eq (3) and Eq (4). Furthermore, encouraged arrival occurs during the regular busy period i.e., in Eq (5) & Eq (6)

3.1. Steady State Solution

The concept of forward shifting operator (E) is introduced on BP_n and VQ_n to solve the above defined steady state equations,

$$E(BP_n) = BP_{n+1}; E(VQ_n) = VQ_{n+1}; for(n \geq 0)$$

The homogeneous difference equation is obtained from Eq (4)

$$(\mu_{wv} E(b+1) - (\lambda_{wi} + \dot{\eta} + \mu_{wv}) E + \lambda_{wi}) VQ_n = 0; n \geq 0 \tag{7}$$

The characteristic equation of the difference equation is expressed as follows
$h(w) = (\mu_{wv} w(b+1) - (\lambda_{wi} + \dot{\eta} + \mu_{wv}) w + \lambda_{wi}) = 0.$

By assuming $f(w) = (\lambda_{wi} + \dot{\eta} + \mu_{wv}) w$ and $g(w) = \mu_{wv} w(b+1) + \lambda_{wi}$, it is known that if $|g(w)| \leq |f(w)|$ on $|w| = 1$, then by Rouche's theorem, h(w) has only one root z_v inside the contour $|w|$. As the root is real, solution of the homogeneous difference equation is obtained as

$$VQ_n = z_v^n VQ_0; n \geq 0 \tag{8}$$

Similarly, Eq(6) can be written as

$$[\mu_w E(b+1) - (\lambda_{wb}(1+\delta) + \mu_w) E + \lambda_{wb}(1+\delta)] BP_n = -\dot{\eta} z_v^{(n+1)} VQ_0; n \geq 0 \tag{9}$$

Again by Rouche's Theorem, Eq (9) is obtained as

$[\mu_w w^{(b+1)} - (\lambda_{wb}(1+\delta) + \mu_w)w + \lambda_{wb}(1+\delta)] = 0$ has a unique root 'z' with $|z| < 1$ provided $\frac{\lambda_{wb}(1+\delta)}{b\mu} < 1$

The non-homogeneous difference equation is solved and the solution obtained is given by

$$BP_n = (Xz^n + Yz_v^n)VQ_0 \tag{10}$$

where

$$Y = \frac{\dot{\eta} z_v}{(\lambda_{wb}(1+\delta))(z_v - 1) + \mu_w z_v(1 - {z_v}^b)}; if z_v \neq z \tag{11}$$

by adding Eqs (1) & (2) over 0 to n, and substituting VQ_n & BP_n , IR_n is obtained as follows

$$IR_n = [\frac{\mu_w}{\lambda_{wi}}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-{z_v}^{n+1})}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_{wi}}\frac{(1-{z_v}^{n+1})}{(1-z_v)}]VQ_0$$

Hence, the steady state queue size probabilities are expressed in terms of the unknowns X and VQ_0.

Now to calculate X , consider Eq (5) and on substituting the value of BP_n, it is found that

$$X(\lambda_{wb}(1+\delta) + \mu_w) - \mu_w \frac{(z^a - z^{b+1})}{(1-z)} = \dot{\eta} - Y((\lambda_{wb}(1+\delta) + \mu_w) - \mu_w \frac{({z_v}^a - {z_v}^{b+1})}{(1-z_v)}) \tag{12}$$

which can be simplified as

$$\frac{X\mu_w(1-z^a)}{(1-z)} = \left(\left(\frac{\dot{\eta}}{(1-z_v)}\right) - \frac{Y\mu_w(1-{z_v}^a)}{(1-z_v)}\right) \tag{13}$$

Moreover, Eq (3) is also verified and the steady state queue size probabilities are expressed in terms of VQ_0 and obtained as

$$VQ_n = {z_v}^n VQ_0 \quad ; \quad n \geq 0 \tag{14}$$

$$BP_n = (Xz^n + Y{z_v}^n)VQ_0 \quad ; \quad n \geq 0 \tag{15}$$

$$IR_n = [\frac{\mu_w}{\lambda_{wi}}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-{z_v}^{n+1})}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_{wi}}\frac{(1-{z_v}^{n+1})}{(1-z_v)}]VQ_0; \quad 0 \leq n \leq a-1 \tag{16}$$

where

$$X = \frac{(1-z)}{\mu_w(1-z^a)}\left(\left(\frac{\dot{\eta}}{(1-z_v)}\right) - \frac{Y\mu_w(1-{z_v}^a)}{(1-z_v)}\right) \tag{17}$$

The value for VQ_0 is evaluated by using the normalizing condition
$\sum_{n=0}^{\infty}(VQ_n + BP_n) + \sum_{n=0}^{a-1} IR_n = 1$

$$\sum_{n=0}^{\infty}({z_v}^n)VQ_0 + (Xz^n + Yz_v^n)VQ_0 + \sum_{n=0}^{a-1}[\frac{\mu_w}{\lambda_{wi}}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-{z_v}^{n+1}}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_{wi}}\frac{(1-{z_v}^{n+1})}{(1-z_v)}]VQ_0 = 1$$

Thus $(VQ_0^{-1}) = F(z_v, \mu_{wv}) + XF(z, \mu_w) + YF(z_v, \mu_w)$;

where $F(r,t) = \frac{1}{1-r}(1 + \frac{t}{\lambda_{wi}}(a - \frac{r(1-r^a)}{(1-r)}))$

4. Performance Measures

One of the objectives of this paper is to deduce the expected queue length of the considered model. The expected queue length is also known as Mean Queue length and denoted as L_q

4.1. Mean Queue Length

The expected queue length L_q is calculated as

$$L_q = \sum_{n=1}^{\infty} n(VQ_n + BP_n) + \sum_{n=0}^{a-1} nIR_n \tag{18}$$

by substituting the values of VQ_n, BP_n and IR_n , L_q is simplified as

$$L_q = XH(z, \mu_w) + YH(z_v, \mu_w) + H(z_v, \mu_{wv})$$

where

$$H(r,t) = \frac{r}{(1-r)^2} + \frac{t}{\lambda_w(1-r)}\left(\frac{a(a-1)}{2} + \frac{ar^{a+1}(1-r) - r^2(1-r^a}{(1-r)^2}\right)$$

as X and Y are given by Eq (15) & Eq (16)

Also,the other characteristics of the queuing model includes if P_v, P_{busy} and P_{idle} respectively which denotes the probability that the server is in working vacations state, in regular and is idle in vacations state, then

$$P_v = \sum_{n=0}^{\infty}(VQ_n) = \frac{VQ_0}{(1-z_v)} \tag{19}$$

$$P_{busy} =_{\sum_{n=0}^{\infty}} (BP_n) = \left(\frac{X}{(1-z)} + \frac{Y}{(1-z_v)}\right) VQ_0 \tag{20}$$

$P_{busy} = \left(\frac{X}{(1-z)} + \frac{Y}{(1-z_v)}\right) VQ_0$,where X & Y are specified in Eq (17) & Eq (11)

$$P_{idle} = \sum_{n=0}^{a-1} IR_n \tag{21}$$

where IR_n is given by Eq (16). Thus, the above characteristics helps in deciding the efficiency of the queuing model.

5. Particular Cases

5.1. Case 1: M/M/1 Multiple Working Vacations Model

The M/M/1 Multiple working vacations queuing model's steady state queue size probabilities are calculated.

By letting in equations Eq (14) to Eq (16) at a= b = 1 and $\lambda_{wb}(1+\delta) = \lambda_w$, it is observed that

$$\begin{aligned} VQ_n &= z_v^n VQ_0 \quad ;n \geq 0 \\ BP_n &= \frac{y}{z_v}\left(z_v^{n+1} - z_v^{n+1}\right) \quad ;n \geq 0 \\ IR_n &= \frac{VQ^0}{z_v} \text{where 'z'} = \frac{\lambda_{wi}}{\mu} = \rho, \quad ;n \geq 0 \end{aligned}$$

where

$$X = -\frac{Y\rho}{z_v} \quad \text{and} \quad Y = \frac{\dot{\eta} z_v}{\mu_w(1-z_v) + (z_v - \rho)}$$

which results in the queue size probabilities of $M/M/1$ Multiple Working Vacations Queuing model (Liu et al.2007)

5.2. Case 2: M/M/(a,b)/1 Multiple Working Vacations Model

Letting $\lambda_{wb}(1+\delta) = \lambda_w$,it is obtained as $M/M(a,b)/1$ under working vacations.

$$\begin{aligned} VQ_n &= z_v^n VQ_0 \quad ; n \geq 0 \\ BP_n &= (Xz^n + Yz_v^n)VQ_0 \quad ; n \geq 0 \\ IR_n &= \left[\frac{\mu_w}{\lambda_w i}(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-z_v^{n+1})}{(1-z_v)} + \frac{\mu_{wv}}{\lambda_w i}\frac{(1-z_v^{n+1})}{(1-z_v)}\right] VQ_0; 0 \leq n \leq a-1 \end{aligned}$$

By substituting the values of VQ_n, BP_n and IR_n in (L_q) , it is observed that (L_q)= X H(z,μ_w) + YH(z_v,μ_w) + H(z_v, μ_{wv})

As $H(s,t) = \frac{s}{(1-s)^2} + \frac{t}{\lambda_w i(1-s)}\left(\frac{a(a-1)}{2} + \frac{as^{a+1}(1-s)-s^2(1-s^a)}{(1-s)^2}\right)$;

where $X = \frac{(1-z)}{\mu_w(1-z^a)}\left(\left(\frac{\dot\eta}{(1-z_v)}\right) - \frac{Y\mu_w(1-{z_v}^a)}{(1-z_v)}\right)$

$Y = \frac{\dot\eta z_v}{\lambda_{wi}(z_v-1)+\mu_w z_v\left(1-{z_v}^b\right)}$ if $z_v \neq z$

which coincides with the queue size probabilities of M/M(a,b)/1 Multiple Working Vacations (K.Julia Rose Mary,2011)

6. Sensitivity Analysis

The numerical values of the mean queue length are derived for $M/M(a,b)/1$ MWV queuing system with heterogeneous encouraged arrival during busy period. The obtained numerical values are examined to study the impact of the vacation parameter $\dot\eta$ and also various other parameters like encouraged arrival rate $\lambda_w b(1+\delta)$, regular service rate (μ_w), service rate during vacation (μ_{wv}).

Based on the above parameters, the mean queue length (L_q) i.e., the expected system size of batch arrival has been calculated. Further, a comparative study is made between L_q of $M/M(a,b)/1$ MWV Encouraged Arrival and L_q of $M/M(a,b)/1$ MWV under Heterogeneous Encouraged arrival.

Thus the impact of the mentioned parameters on the system size probabilities and on the expected system size during the idle state, working vacation state and the regular busy period are also analysed by considering z_v and z as the roots of the following characteristic equations

$$(\mu_{wv}w^{(b+1)} - (\lambda_{wv} + \dot\eta + \mu_{wv})w + \lambda_{wv}) = 0 \quad \&$$

$$(\mu_w w^{(b+1)} - (\lambda_{wb}(1+\delta) + \mu_w)w + \lambda_{wb}(1+\delta) = 0$$

respectively.By considering homogeneous arrival rate during idle and vacation state as $\lambda_w = 4.05$ and during encouraged arrival $\lambda_w(1+\delta) = 5.051$, also, by assuming for heterogeneous arrival queuing system arrival rate as $\lambda_{wi} = 3.9; \lambda_{wv} = 4.0\&$ during encouraged arrival $\lambda_{wb}(1+\delta) = 5.051$ and further by assuming $\mu_w = 0.9$ as constant and by varying $\dot\eta$ & μ_{wv}, the mean queue length (L_q) is calculated and tabulated below.

Table 1: *L_q of $M/M(a,b)/1$ MWV Queuing system with Heterogeneous EA and $M/M(a,b)/1$ MWV Queuing system under Homogeneous EA*

μ_{wv}	$\dot{\eta}$	L_q (Heterogeneous EA)	L_q (Homogeneous EA)
0.05	0.02	211.4142	211.5021
	0.04	159.4418	159.5500
	0.06	89.8687	89.9613
	0.08	64.5690	64.6545
	0.1	48.6767	48.7549
0.1	0.02	127.0639	127.1303
	0.04	105.3322	105.4188
	0.06	74.3123	74.3973
	0.08	52.9088	52.9864
	0.1	46.01627	46.0943
0.15	0.02	100.1150	100.1765
	0.04	83.3706	83.44902
	0.06	64.1551	64.2356
	0.08	45.1962	45.2689
	0.1	41.5132	41.5887
0.2	0.02	75.1551	75.2117
	0.04	72.1117	72.1873
	0.06	52.6958	52.7703
	0.08	41.8486	41.9209
	0.1	39.3368	39.41266

From the above tabulation, it is evident that the queue length of the heterogeneous encouraged arrival is comparatively less than that of the homogeneous encouraged arrival, as the arrival rate differs in each state of the heterogeneous encouraged arrival model. The obtained numerical values are represented graphically.

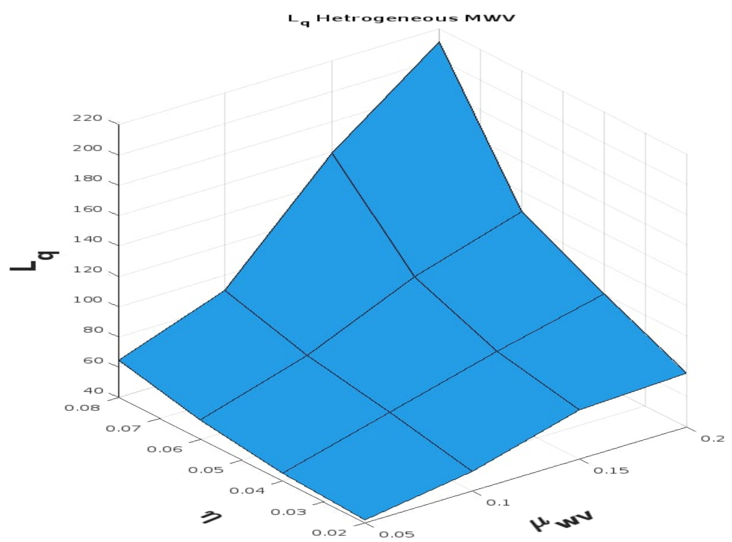

Figure 1: *$M/M(a,b)/1$ MWVwith Heterogeneous Encouraged Arrival*

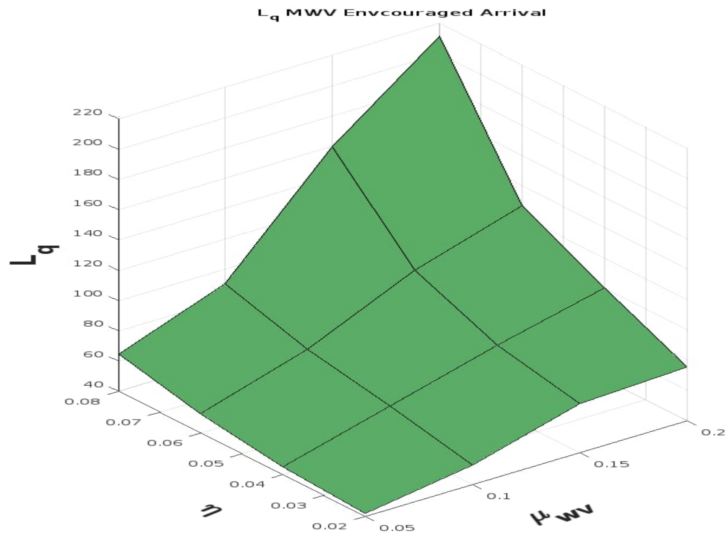

Figure 2: *$M/M(a,b)/1$ MWV with HomogeneouEncouraged Arrival*

The above graphical representations demonstrate that the mean queue length reaches its peak at the minimum values of $\dot{\eta}$&μ_{wv} and the mean queue length reduces at a great margin level when $\dot{\eta}$&μ_{wv} are maximum. For example if μ_{wv} = 0.05 , and $\dot{\eta}$= 0.02 , the mean queue length of M/M(a,b)/1 MWV with heterogeneous encouraged arrival is 211.4142 while the mean queue length of M/M(a,b)/1 MWV queuing model with encouraged arrival is 211.5021. Thus with the varied arrival rates there arises a gradual variation in the queue length.

As discussed earlier, the encouraged arrival is an impact factor in deciding the queue length. Thus on varying the encouraged arrival rate during the regular busy period as $\lambda_{wb}(1+\delta)$ = 5.051 ; $\lambda_{wb}(1+\delta)$ = 5.053 ; $\lambda_{wb}(1+\delta)$ = 5.055 and by having λ_{wi}= 3.9, λ_{wv}= 4.0 &μ_w = 0.9 as constant and on varying μ_{wv} from 0.05 to 0.2 and $\dot{\eta}$ from 0.02 to 0.1 , the mean queue length is computed and tabulated below

Table 2: *L_q of $M/M(a,b)/1$ MWV Heterogeneous Queuing system with varied EA*

μ_{wv}	$\dot{\eta}$	$L_q(\lambda_{wb}(1+\delta)=5.051)$	$L_q(\lambda_{wb}(1+\delta)=5.053)$	$L_q(\lambda_{wb}(1+\delta)=5.053)$
0.1	0.02	127.0639	127.06636	127.0687
	0.04	105.3322	105.3360	105.3399
	0.06	74.3123	74.3164	74.3205
	0.08	52.9088	52.91281	2.968
	0.1	46.01627	46.02055	46.0248
0.15	0.02	100.1150	100.1170	100.11900
	0.04	83.3706	83.37881	83.37708
	0.06	64.1551	64.1588	64.1624
	0.08	45.1962	45.1998	45.2034
	0.1	41.5132	41.5172	41.5212
0.2	0.02	75.1551	75.1566	75.1582
	0.04	72.1117	72.1146	72.1174
	0.06	52.6958	52.6990	52.7022
	0.08	41.8486	41.8520	41.85550
	0.1	39.3368	39.3407	39.3446

Thus from the above tabulations, it is clear that as the encouraged arrival rate increases gradually from $\lambda_{wb}(1+\delta)$= 5.051 to $\lambda_{wb}(1+\delta)$= 5.055 the queue length increases gradually from 211.4142705 to 211.4180277. Similarly, when the encouraged arrival rate increases from $\lambda_{wb}(1+\delta)$= 5.051 to $\lambda_{wb}(1+\delta)$= 5.055 the queue length increases from **211.4142705 to 211.4217896** . Thus it is clear that as the encouraged arrival rate increases,the queue length increase. Hence the encouraged arrival rate significantly decides the queue length of the considered model. The above tabulations is represented graphically below

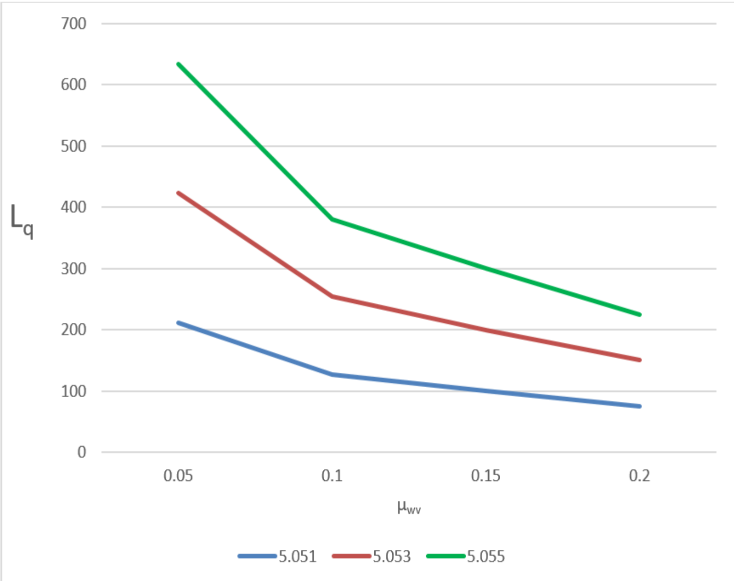

Figure 3: *$M/M(a,b)/1$ MWV Heterogeneous Queuig system with varied Encouraged Arrival*

Thus for the vacation service rate μ_{wv} varying from 0.05 to 0.2, with the increased encouraged arrival rate, the corresponding mean queue length is denoted in the above graph.

Now, the other performance measures of the discussed model like P_{idle}. $P_{vacation}$, P_{busy} are computed by using Eq (19), Eq (20) & Eq (21) and tabulated below:

Table 3: *Other Characteristics of $M/M(a,b)/1$ MWV Heterogeneous Queuing system under encouraged arrival*

μ_{wv}	$\dot{\eta}$	P_{idle}	$P_{vacation}$	P_{busy}
0.1	0.02	0.01438	0.7468	0.2387
	0.04	0.02275	0.6294	0.3477
	0.06	0.0319	0.5935	0.3745
	0.08	0.04156	0.5786	0.37973
	0.1	0.04845	0.54449	0.4070
0.15	0.02	0.0170	0.7791	0.2038
	0.04	0.0261	0.6685	0.3053
	0.06	0.0352	0.6168	0.3478
	0.08	0.04602	0.6010	0.3529
	0.1	0.05232	0.5584	0.3892
0.2	0.02	0.02104	0.8105	0.1683
	0.04	0.02933	0.6895	0.2811
	0.06	0.0400	0.6451	0.3147
	0.08	0.0496	0.61013	0.3402
	0.1	0.05560	0.5644	0.3799

From the above table, as the working vacation service rate (μ_{wv}) increases and with the vacation parameter varying from 0.02 to 0.1, the probability values during the idle state and busy period increases and the probability during the vacation period decreases.

7. Conclusion

M/M(a,b)/1 MWV queue under heterogeneous encouraged arrival of customers is studied with steady state and the steady state solutions are derived. Additionally, the mean queue length and various other performance measures were also calculated and numerically examined. Moreover, other queuing models were deduced as particular cases.

From the numerical examples computed for the mean queue length of the considered model and compared with M/M(a,b)/1/MWV with encouraged arrival, evidently concluded that many factors like vacation parameter, service rate plays a role in deciding the queue length, the key factor is the encouraged arrival rate that makes a notable impact in deciding the queue length of the considered batch service heterogeneous queuing model with single server.

Conflict of Interest: The authors declare no conflict of interest

8. Acknowledgement

The corresponding author would like to acknowledge the UGC Scholarship team for approving and providing (SJSGC) Scholarship Grants, which is more likely helpful and beneficial to complete this study.

References

[1] Wu, D. A. and Takagi, H. (2006). M/G/1 queue with multiple working vacations. *Performance Evaluation*, 63:654–681.

[2] Ibe, O. C. and Isijola, O. A. (2014). M/M/1 multiple vacation queueing systems with differentiated vacations.*Modelling and Simulation in Engineering*, 2014:158247.

[3] Mary, J. R. K. and Afthab Begum, MI (2009). Closed form Analytical solution of the General Bulk service queuing model M/M (a,b)/1 under working vacation. *International conference on Mathematical and Computational models, PSG College of Tech*, 1:92–100.

[4] Levy, Y. and Yechiali, U. (1976). An M/M/s queue with servers vacations. *INFOR: Information Systems and Operational Research*, 14:153–163.

[5] Liu, W. Y., Xu, X. L. and Tian, N. S. (2007). Stochastic decompositions in the M/M/1 queue with working vacations. *Operations Research Letters*, 35:595–600.

[6] Malik, S., and Gupta, R. (2022). Analysis of Finite Capacity Queuing System with Multiple Vacations and Encouraged Customers. *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 9:17–22.

[7] Neuts, M. F. (1967). A general class of bulk queues with Poisson input. *The Annals of Mathematical Statistics*, 38:759–770.

[8] Santhi, K. and Pazhani Bala Murugan, S. (2013). An M/G/1 queue with two-stage heterogeneous service and single working vacation. *International Mathematical Forum*, 8:1323–1336.

[9] Servi, L. D. and Finn, S. G. (2002). M/M/1 queues with working vacations (M/M/1/WV). *Performance Evaluation*, 50:41–52.

[10] B.K. Som, S. Seth.(2017). An M/M/1/N Queuing system with Encouraged Arrivals. *Global Journal of Pure and Applied Mathematics*, 7:3443–3453.

[11] Prakati,P. and Julia Rose Mary.K (2024). M/M/C Queue With Multiple Working Vacations and Single Working Vacation Under Encouraged Arrival With Impatient Customers. *Reliability:Theory and Applications*, 19:650–662.