ANALYSIS OF AN ENCOURAGED ARRIVAL QUEUING MODEL WITH SERVERS REPEATED VACATIONS AND BREAKDOWNS

Jenifer Princy P *, K Julia Rose Mary

•

[1,2] Department of Mathematics, Nirmala College for Women, Coimbatore, India
* pjeniferprincy2001@gmail.com
juliakuladaisamy@gmailcom

Abstract

The behavior of customers plays a vital role in realizing the nature of a queue. If there is a favor for customers from the side of service facility the arrival rate increases than usual. Also the positive perspective about the service providers also encourages more number of customers to join the system. The arrival rate of the customers follow Poisson distribution. This paper analyses a queuing model with those encouraged customers who urges to join the system. Here the customers are served in batches according to the general bulk service rule along with the phenomenon that the servers undergo repeated vacations until they find minimum number of customers to start the service. In addition this paper interprets the scenario that if there is a breakdown in the service facility, the waiting line of the customers increases which causes a greater impact on the effectiveness of the service providers favoring the customers. On account of this situation the steady state probability solutions and some performance measures are evaluated along with a numerical illustration.

Keywords: Markovian Process, Poisson distribution, repeated vacations, general bulk service rule, breakdowns, encouraged arrival

1. Introduction

In everyday life, we would have come across different circumstances of waiting in a queue. Waiting in checking counter, Ticket booking, for consulting doctor etc. basically in need of a service to be delivered. Queue is built whenever the customers do not receive service instantaneously. Queue size can be reduced by speeding up the service or by increasing the number of servers to meet up the demand. Mathematically we can figure out every queuing situation into a queuing model which becomes the topic of issue. Agner K. Erlang introduced the concept of queuing theory by modeling the system of incoming calls at the Copenhagen telephone exchange company

In past decades queuing models with breakdowns was one of the area of interest for many researchers. White and Christie [19] proposed the concept of queuing systems with server interruption and evaluated the probabilities of performance in means of geometric function approach. Neuts and Lucantoni [11] discussed the M/M/N queuing model along with sever failures. Xiaolan Yang and Attahiru S. Alfa [20] had analyzed a class of multiserver queuing system with server failures due to undesirable servers.

In the similar way research on vacation queuing models has also become a crucial area of study for the past decades. The term vacation points out the behavior of the server who leaves the system for a random period for various reasons. The server takes the vacation whenever there are no customers in the queue. B. T Doshi [4] attempted to provide a methodological overview of the queuing systems with two vacation models. K.J.R Mary et. al [10] investigated a queuing system with second optional service channel under bi-level control policy and server's single vacation. Wang and Ruiling [18] made a strategic analysis on both single and multiple working vacations. S. Sindhu et.al [15] illustrated a theoretical comparison between single working vacation model with interdependent and independent arrival and service process. William J. Gray et.al. [5] studied the vacation queuing model with service breakdowns. Jeyakumar. S and Senthilnathan. B [6] analysed a single service queue with batch service along with multiple working vacations and server breakdowns where the server works with different rates without stopping the service completely during vacation period. M. Seenivasan and S. Chandiraleka [14] dicussed on the queuing model with multiple working vacation queuing model with breakdown using Matrix Geometric Approach. R.K. Srivastava et.al.[17] analyzed a model with bulk arrival where the service is provided in two categories. Lidiya and K. J. R. Mary [9] aimed to interpret the queuing model M/M (a, b)/1 with multiple working vacations and breakdown where the customers are served in batches.

In multiserver queuing models the servers usually leave the system for vacation whenever they are idle. In order to reduce the waiting time of customers arriving during servers vacation it is better if the system contains some additional servers to provide service so that the customers are served instantaneously. Afthab Begum and Nadarajan [1] had been analysed this situation where the system comprises of atleast r number of servers while the s-r servers are in vacation. Srinivas R. Chakravarthy et.al. [3] depicted a queuing model with a backup server in which the backup server helps the system to run continuously irrespective of the breakdown, repairs, vacation.

On completion of a random vacation period if a server does not find the minimum accessible number of customers to start the service he leaves the system again for another random period. This type of server's behavior can be defined as repeated vacation. S. Palaniammal et.al.[12] analysed M/M (a, b)/ (2,1) queuing model with servers repeated vacation in which at least one server should remain in the system always among the two servers. S.Baskar and Palaniamal [2] discussed the M/M (a, b)/ (2,1) queuing model along with servers repeated and delayed vacation.

Service providers usually try to fascinate customers by providing profitable deals which encourage many customers to take service than usual. Those customers can be called as encouraged customers. Som B.S and Seth. S [16] interprets the concept of encouraged arrivals, Impatient customers and retention of impatient customers to design effective business strategies. Khan I.E and Paramasivam. R [7] studied encouraged arrival Markovian Model with breakdown and numerous vacations where the customers are served in batches in which PGF is determined in Laplace transforms. Khan I.E and Paramasivam. R [8] evaluated the performance of the reduction in waiting time of single server Markovian encouraged arrival queuing model using control chart technique. Prakati and K. J. R. Mary [13] made comparative study on single working vacation and multiple working vacation under encouraged arrival.

Moreover if the encouraged customers join the system then the queue length will increase than usual. Perhaps disruption in the service facility may further extends the queue length leading to an extended waiting time. This paper aims to interpret the steady state probability solution, the average queue length, and the probability measures of idle and busy servers in M/M (a, b)/ (2,1) queuing model with servers repeated vacation,encouraged arrival of customers and breakdown in the system which will be useful to manage the service facility in an effective sense.

2. Mathematical Model Description

This model involves two service facility (servers) where the arrival of customers into the system follows Poisson distribution with parameter λ. Also the encouraged arrival rate follows Poisson distribution with parameter $\lambda(1+\eta_e)$. Both the service and the breakdown rate of the servers follow exponential distribution with parameters μ and β. Service is done in batches according to the General Bulk Service Rule introduced by Neuts. The service is provided, if there are minimum 'a' number of customers in the queue. If the number of customers is more than the maximum limit 'b' only the first b customers are allowed for taking the service.

The servers are in the state of idle if there is not even a minimum number of customers in a queue. In this case a server leaves the system for a random period called vacation since only one server will be allowed to go for a vacation at a time. This follows an exponential distribution with parameter θ. On returning from the vacation if the server finds less than 'a' customers in the queue while the other server is busy or idle, the server leaves the system for another random period called repeated vacation. The server will continue the same activity until he finds minimum 'a' number of customers to start his service.

Breakdown of the system in the busy period will keep the customers to be in the service for some extent which also cause an impact in the waiting line of the system. If this situation happens for the customers who were being encouraged for getting service from service facility the queue length of the system increases furthermore and even it prolongs the waiting time duration of the customers in the queue.

On the state space $(j,n)\,, j = 0,1,2; n \geq 0$, the queue is studied as a Markov process where $n \geq 0$ denotes the number of waiting customers in the queue and j denotes the level of the server.

1. State $(0,n)$, where $0 \leq n \leq a-1$ represents that one server is idle and the other server is on vacation.
2. State $(1,n)$, where $n \geq 0$ represents that one server is busy and the other server is on vacation.
3. State $(2,n)$, where $n \geq 0$ represents that both the servers are busy.

Defining $P_{jn}(t) = \text{Prb}\ \{$ At time t, the system is in the state $(j,\ n) j = 0,1,2; n \geq 0\}$ and considering that the steady state probabilities are $P_{0n} = \lim_{t\to\infty} p_{0n}(t)$,$P_{1n} = \lim_{t\to\infty} p_{1n}(t)$ and $P_{2n} = \lim_{t\to\infty} p_{2n}(t)$.

The steady state equations are

$$\lambda p_{00} = \mu p_{10} \tag{1}$$

$$\lambda p_{0n} = \lambda p_{0n-1} + \mu p_{1n} \quad (1 \leq n \leq a-1) \tag{2}$$

$$(\lambda\,(1+\eta_e) + \beta + \mu) p_{10} = \lambda p_{0a-1} + 2\mu p_{20} + \mu \sum_{n=a}^{b} p_{1n} \tag{3}$$

$$(\lambda\,(1+\eta_e) + \beta + \mu) p_{1n} = (\lambda\,(1+\eta_e) + \beta)\, p_{1n-1} + 2\mu p_{2n} + \mu p_{1n+b} \quad (1 \leq n \leq a-1) \tag{4}$$

$$(\lambda\,(1+\eta_e) + \mu + \beta + \theta) p_{1n} = (\lambda\,(1+\eta_e) + \beta)\, p_{1n-1} + \mu p_{1n+b} \quad (n \geq a) \tag{5}$$

$$(\lambda\,(1+\eta_e) + \beta + 2\mu) p_{20} = \theta \sum_{n=a}^{b} p_{1n} + 2\mu \sum_{n=a}^{b} p_{2n} \tag{6}$$

$$(\lambda\,(1+\eta_e) + 2\mu + \beta) p_{2n} = (\lambda\,(1+\eta_e) + \beta)\, p_{2n-1} + \theta p_{1n+b} + 2\mu p_{2n+b} \quad (n \geq 1) \tag{7}$$

3. Steady State solutions

Let E denote the forward shifting operator defined by $E(p_{1n}) = p_{1n+1}$. Equation (5) implies $[\mu E^{b+1} - (\lambda(1+\eta_e) + \mu + \beta + \theta)E + (\lambda(1+\eta_e) + \beta)]p_{1n} = 0. \quad (n \geq 1)$.

The corresponding characteristic equation is

$$\mu Z^{b+1} - (\lambda(1+\eta_e) + \mu + \beta + \theta)Z + (\lambda(1+\eta_e) + \beta) = 0. \tag{8}$$

Then by Rouche's Theorem., it has only one real root inside the circle $|z| = 1$ when $\rho = \dfrac{\lambda(1+\eta_e) + \beta + \theta}{b\mu} < 1$. Let r_0 be the root of the above characteristic equation with $|r_0| < 1$. Therefore the homogeneous difference equation has the solution of the form

$$p_{1\,n} = A_1 r_0^n \quad (n \geq a-1)$$

so we get

$$p_{1\,n} = r_0^{n-a+1} p_{1\,a-1} \quad (n \geq a) \tag{9}$$

Using equation (7) we get

$$(2\mu E^{b+1} - (\lambda(1+\eta_e) + 2\mu + \beta)E + (\lambda(1+\eta_e)+\beta))p_{2\,n} = -\theta p_{1\,n+b+1} \quad (n \geq 1)$$

The corresponding characteristic equation is

$$(2\mu Z^{b+1} - (\lambda(1+\eta_e) + 2\mu + \beta)Z + (\lambda(1+\eta_e)+\beta))p_{2\,n} = 0 \tag{10}$$

If r_1 is the root of the above characteristic equation with $|r_1| < 1$ which exists when $\rho_1 = \left(\frac{\lambda(1+\eta_e)+\beta}{2b\mu}\right) < 1$. This non - homogeneous difference equation (7) has the solution

$$p_{2\,n} = (A_1 r_1^n + K r_0^n) p_{1\,a-1} \quad (n \geq 0) \tag{11}$$

where A_1 is constant and $K = \frac{\left(-\theta r_0^{b-a+2}\right)}{(\lambda(1+\eta_e)+\beta+2\theta)\, r_0 - (\lambda(1+\eta_e)+\beta)}$

Using equation (4) and substituting for $p_{2\,n+1}$ and $p_{1\,n+b+1}$ we have after simplification.

$$p_{1\,n} = [A_2 R^n + G_1(r_0) r_0^n + G_2(r_1) r_1^n]\, p_{1\,a-1} \quad (0 \leq n \leq a-1) \tag{12}$$

where $R = \frac{\lambda(1+\eta_e)+\beta}{\lambda+\lambda\eta_e+\beta+\mu}$, $G_1(r_0) = \frac{\mu k\,(\lambda(1+\eta_e)+\beta)\,(1-r_0)}{\theta(\lambda+\lambda\eta_e+\beta+\mu)r_0 - (\lambda(1+\eta_e)+\beta)}$ and $G_2(r_1) = \frac{2\mu A_1 r_1}{(\lambda+\lambda\eta_e+\beta+\mu)r_1 - (\lambda(1+\eta_e)+\beta)}$

Adding equation (2) over $k = 1$ to n and substituting for $p_{1\,k}$ from equation (12) and simplifying, we get

$$p_{0\,n} = \frac{\mu}{\lambda}\left[A_2 \frac{1-R^{n+1}}{1-R} + G_1(r_0)\frac{1-r_0^{n+1}}{1-r_0} + G_2(r_1)\frac{1-r_1^{n+!}}{1-r_1}\right] p_{1\,a-1} \quad (0 \leq n \leq a-1) \tag{13}$$

Using equation (9) and (11) in (6) then we obtain

$$A_1 = \frac{(1-r_1)}{(1-r_1^a)\,(1-r_0)}\left[\frac{r_0\theta}{2\mu} - k\,(1-r_0^a)\right] \tag{14}$$

Further, In equation (12) substituting $n = a-1$ we get the value of A_1 as

$$A_2 = \frac{1}{R^{a-1}}\left[1 - G_1(r_0) r_0^{a-1} - G_2(r_1) r_1^{a-1}\right] \tag{15}$$

The value of $p_{1\,a-1}$ is obtained by using the normalizing condition

$$\sum_{n=0}^{\infty} p_{2\,n} + \sum_{n=a}^{\infty} p_{1\,n} + \sum_{n=0}^{a-1} (p_{0\,n} + p_{1\,n}) = 1 \tag{16}$$

substituting for $p_{2\,n}$, $p_{1\,n}$ and $p_{0\,n}$ and simplifying we get,

$$p_{1\,a-1}^{-1} = A_1 J(R) + G_1(r_0) J(r_0) + G_2(r_1) J(r_1) + \frac{A_1}{1-r_1} + \frac{K}{1-r_0} + \frac{r_0}{1-r_0} \tag{17}$$

where $J(x) = \frac{1-y^a}{1-y} + \frac{\mu}{\lambda}\left(\frac{a}{1-y} - \frac{y}{1-y}\frac{1-y^a}{1-y}\right)$ and the values of $R, G_1(r_0), G_2(r_1)$ are obtained from equation (12).

4. Analysis of the average measures of performance

The following are the performance measures for the effective mechanism of the queuing model M/M(a, b)/(2, 1) during the time of encouraged arrivals with servers repeated vacations and breakdowns.

1. **Expected Queue Length**

   The expected queue length is given by

$$L_q = \sum_{n=1}^{\infty} np_{2\,n} + \sum_{n=a}^{\infty} np_{1\,n} + \sum_{n=1}^{a-1} n\left(p_{0\,n} + p_{1\,n}\right)$$

   Using equations (9) to (15) and simplifying, we have

$$L_q = \left[A_2 S(R) + G_1(r_0)S(r_0) + G_2(r_1)S(r_1) + \frac{ar_0}{1-r_0} + \frac{r_0^2}{(1-r_0)^2} + \frac{A_1 r_1}{(1-r_1)^2} + \frac{Kr_0}{(1-r_0)^2}\right] p_{1\,a-1}$$

   where $S(y) = \left(\frac{1-y^a - ay^{a-1}(1-y)}{(1-y)^2}\right)\left(y - \frac{y^2\mu}{\lambda(1-y)}\right) + \frac{\mu a(a-1)}{2\lambda(1-y)}$

2. Let $P_{2\,B}$ denote the probability that both the servers are busy then

$$P_{2\,B} = \sum_{n=0}^{\infty} p_{2\,n} = \left(\frac{A_1}{1-r_1} + \frac{K}{1-r_0}\right) p_{1\,a-1}$$

3. Let $P_{1\,B}$ denote the probability that one server is busy and the other server is on vacation then

$$P_{1\,B} = \sum_{n=0}^{\infty} p_{1\,n} = \left(A_2\frac{1-R^a}{1-R} + G_1(r_0)\frac{1-r_0^a}{1-r_0} + G_2(r_1)\frac{r_0}{1-r_0} + \frac{r_0}{1-r_0}\right) p_{1\,a-1}$$

4. Let $P_{0\,B}$ denote the probability that one server is idle and one server is on vacation then

$$P_{0\,B} = \sum_{n=0}^{a-1} p_{1\,n} = \left[A_2 U(R) + G_1(r_0)U(r_0) + G_2(r_1)U(r_1)\right] p_{1\,a-1}$$

   where $U(x) = \frac{\mu}{\lambda}\left(\frac{a}{1-y} + \frac{y(1-y^a)}{(1-y)^2}\right)$ and the values of $R, G_1(r_0), G_2(r_1)$ are obtained from equation (12).

5. Numerical Analysis

In view of an effective performance of the system a sample numerical outcome is analysed in this section by considering sample values.

For a batch of minimum size $a = 10$ and maximum size $b = 25$ service is delivered at a constant rate $\mu = 1$ and the arrival rate λ of the customers tends to be $\lambda = 10$. With these parameters the following table shows the expected queue length for various values of breakdown rate β, encouraged arrival rate η_e, and for the mean vacation time $\frac{1}{\theta}$.

Table 1: *The Expected Queue Length for $a = 5$ and $b = 25$ with respect to η_e & β*

		L_q				
η_e	$\frac{1}{\theta}$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$
0.01		5.863294871	5.9182236	6.028200339	6.136426578	6.249321606
0.03	1.25	5.972415244	6.0282003	6.136426578	6.249321606	6.358418869
0.06		6.136426578	6.1923198	6.302708582	6.416655323	6.52759823
0.09		6.302708582	6.3563036	6.472789527	6.583415019	6.697392467
0.01		7.134515555	7.2066632	7.355389687	7.502775938	7.655940074
0.03	2.5	7.28025688	7.3553897	7.502775938	7.655940074	7.807667263
0.06		7.502775938	7.5766104	7.731087036	7.878229335	8.034363193
0.09		7.731087036	7.8057204	7.957402426	8.112777881	8.265590233
0.01		8.383532999	8.485142	8.676987246	8.87656302	9.074707742
0.03	5	8.580062218	8.6769872	8.87656302	9.074707742	9.275671618
0.06		8.87656302	8.9697	9.171598075	9.381842427	9.578596515
0.09		9.171598075	9.2747364	9.484788121	9.685588806	9.894134297
0.01		9.367526009	9.4843712	9.726184801	9.966874674	10.21860956
0.03	10	9.603887979	9.7261848	9.966874674	10.21860956	10.4684483
0.06		9.966874674	10.084971	10.34215714	10.59752669	10.84996592
0.09		10.34215714	10.468027	10.7152536	10.98782382	11.24189079

It has been interpreted that as there is an increase in encouraged arrival and the breakdown rate the queue length also increases. Figure 1 shows that the queue length increases with increase in encouraged arrival rate η and breakdown rate β for the mean vacation time $\frac{1}{\theta} = 10$ by taking the service rate $\mu = 1$ and also the arrival rate $\lambda = 10$.

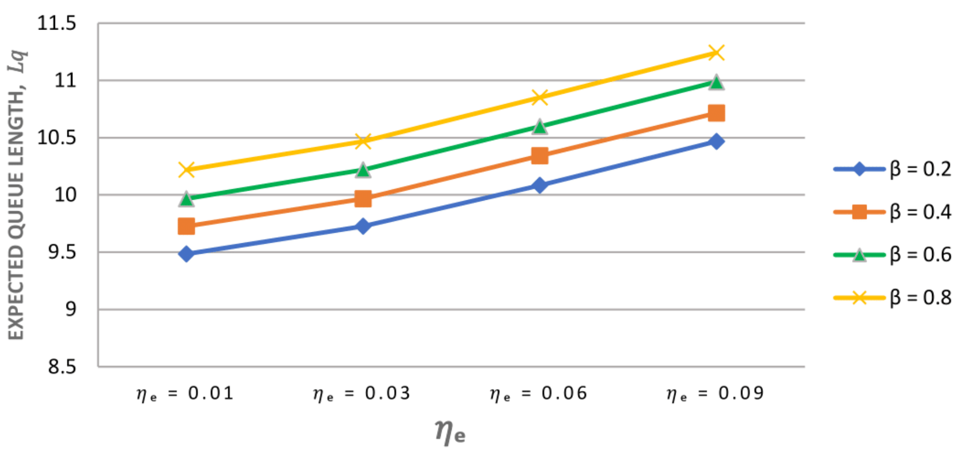

Figure 1: *Expected Queue length L_q for various values of η_e and β*

Figure 2 shows that the queue length increases with increase in encouraged arrival rate η_e and breakdown rate β for the mean vacation time $\frac{1}{\theta} = 2.5$ with the arrival rate $\lambda = 10$.

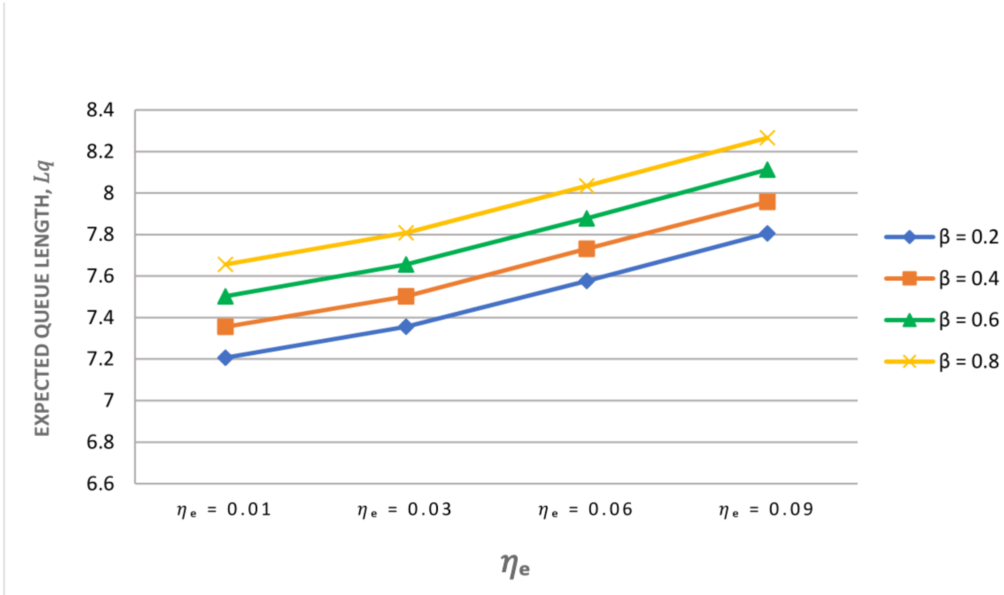

Figure 2: *Expected Queue length L_q for various values of η_e and β*

Thus it has been noted from both Figure 1., and Figure 2., that queue length of the system M/M(a, b)/(2, 1) with repeated vacation, encouraged arrival and breakdown is increased with the mean vacation time, $\frac{1}{\theta} = 10$ when compared to the system with mean vacation time, $\frac{1}{\theta} = 2.5$.

For the batch of maximum size $b = 40$ considering $\eta_e = 0.03$ and $\beta = 0.5$ and opting various values for minimum size a of the batch it has been noted from the following Table 2 that increase in minimum capacity of the batch size increases the queue length which is also depicted graphically in Figure 3.

Table 2: *The Expected Queue Length for $b = 40$ and for various values of a*

λ	$a = 15$	$a = 20$	$a = 25$
10	9.461203	11.03285	13.00848
12	10.81249	12.04423	13.7487
18	13.33766	14.12254	15.40794
20	16.44243	16.88795	17.7878
25	18.84208	19.12746	19.80789
36	26.08044	26.19581	26.50847

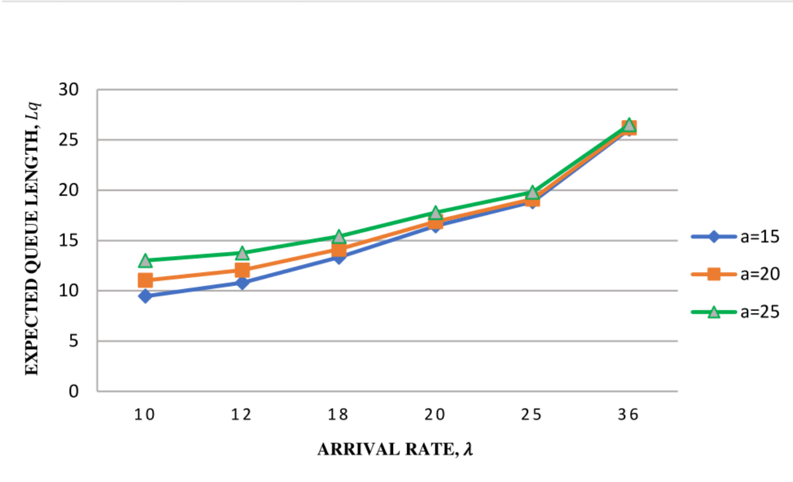

Figure 3: *Expected Queue length L_q for various values of a*

For the batch of minimum size $a = 10$ and opting various values for the maximum size b of the batch and also considering $\eta_e = 0.03$ and $\beta = 0.5$ it has been noted from the following Table 3 that increase in maximum capacity of the batch size decreases the queue length which is also depicted graphically in Figure 4.

Table 3: *The Expected Queue Length for $a = 10$ and for various values of b*

λ	$b = 20$	$b = 30$	$b = 40$
10	9.302594	8.473836	8.266449
12	11.9158	10.3027	9.867244
18	17.06309	13.59169	12.60109
20	23.84891	17.61646	15.73194
25	29.2534	20.72457	18.05442
36	35.34298	24.24377	20.57053

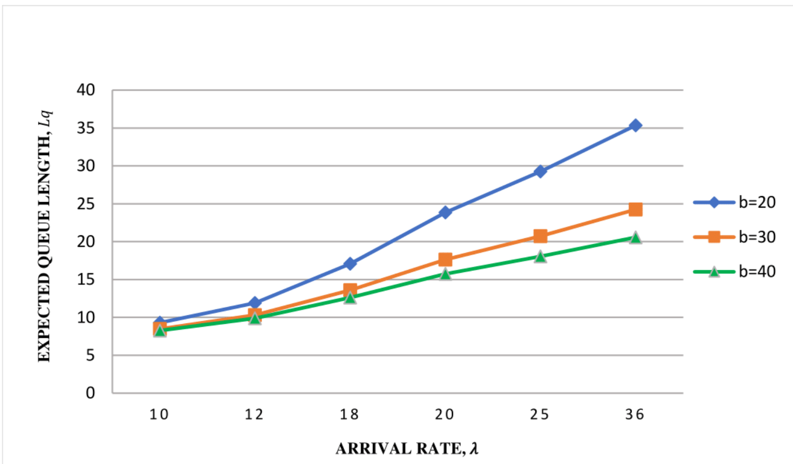

Figure 4: *Expected Queue length L_q for various values of b*

6. Conclusion

This paper analysed the steady state solutions of the M/M(a, b)/(2, 1) queuing model with servers repeated vacations and encouraged arrival of customers together with the breakdown mechanism. The expected queue length, probability measures of the idle and the busy servers are formulated and verified with an numerical illustration which helps to review the system for future effectiveness. Further a comparative study can be made with M/M(a, b)/1 queuing model with servers repeated vacation.

References

[1] Afthab Begum, M. I. and Nadarajan, R. (1997). Multiserver Markovian queueing system with vacation. *Optimization*,41(1), 71-87.

[2] Baskar, S. and Palaniammal, S. (2014). M/M(a,b)/(2,1) queueing system with servers repeated and delayed vacation. *Asian-European Journal of Mathematics*,7(02), 1450032.

[3] Chakravarthy, S. R. and Kulshreshtha, R. (2020). A queueing model with server breakdowns, repairs, vacations, and backup server. *Operations research perspectives*, 7, 100131.

[4] Doshi, B. T. (1986). Queuueing systems with vacations - a survey. *Queueing systems*, 1, 29 - 66.

[5] Gray, W. J. Wang, P. P. and Scott, M. (2000). A vacation queueing model with service breakdowns. *Applied Mathematical Modeling*, 24(5 - 6), 391 - 400.

[6] Jeyakumar, S. and Senthilnathan, B. (2017). Modelling and analysis of a bulk service queueing model with multiple working vacations and server breakdown. *RAIRO - Operations Research - Recherche Operationnelle*, 51(2), 485 - 508.

[7] Khan, I. E. and Paramasivam, R. (2023). Analysis of Batch Encouraged Arrival Markovian Model Due to a Secondary Optional Service, Break-Down and Numerous Vacations. *Mathematical Statistician and Engineering Applications*, 72(1), 1166 - 1177.

[8] Khan, I. E. and Paramasivam, R. (2023). Reduction in waiting time of single server Markovian queuing encouraged arrival model. *Reliability:Theory & Applications*, 18(3(74)), 776 - 784.

[9] Lidiya, P. and Mary, K. J. R (2024). Analysing the Performance of Queuing Model with the Busy period Breakdown. *Baghdad Science Journal*, 22(2).

[10] Mary, K. J. R. and Begum, M. A. and Parveen, M. J. (2011). Bi - level threshold policy of MX/(G 1, G 2)/1 queue with early setup and single vacation. *International Journal of Operational Research*, 10(4), 469 - 493.

[11] Neuts, M. F. and Lucantoni, D. M. (1979). A Markovian queue with N servers subject to breakdowns and repairs. *Management Science*, 25(9), 849 - 861.

[12] Palaniammal, S. and Nadarajan, M. A. B. R. (2005). Servers Repeated Vacation. *Computational Mathematics*, 232.

[13] Prakati, P. (2024). M/M/C Queue with multiple working vacations and single working vacation under encouraged arrival with impatient customers. *Reliability:Theory & Applications*, 19(1(77)) 650-652.

[14] Seenivasan, M. Chandiraleka, S. (2023). Single Server Queueing Model with Multiple Working Vacation and with Breakdown. *Second International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies(ICAECT)*, (pp. 1-5) IEEE.

[15] Sindhu, S. Krishnamoorthy, A. and Kozyrev, D. (2023). On Queues with working vacation and interdependence in arrival and service processes. *Mathematics*, 11(10), 2280.

[16] Som, B. S. & Seth, S. (2018). Queuing System with Encouraged Arrivals, Impatient Customers and Retention of Impatient Customers for Designing Effective Business Strategies. *AISECT University Journal*, 6(13).

[17] Srivastava, R. K. Singh, S. and Singh, A. (2020). Bulk Arrival Markovian Queueing System with Two Types of Services and MUltiple Vacations. *International Journal of Mathematics and Computer Reserach*, 8(8), 2130 - 2136.

[18] Wang, Y. and Tian, R. (2020). Strategic analysis in Markovian queues with a single working vacation and multiple vacations. *International Journal of Innovative Computing, Information and Control*, 16(1), 349 - 366.

[19] White, H. and Christie, L. S. (1958). Queing with preemptive priorities or with breakdown. *Operations research*, 6(1), 79 - 95.

[20] Yang, X. and Alfa, A. S. (2009). A class of multi - server queueing system with server failures. *Computers and industrial Engineering*, 56(1), 33 - 43.