A SHORTAGES MULTI WAREHOUSE HAVING IMPERFACT ITEMS AND DIFFERENT DISCOUNT POLICY

Krishan Pal[1], Ajay Singh Yadav[1*], Seema Agarwal[1]
•
[1]Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad, India, 201204
kp8910@srmist.edu.in, ajaysiny@srmist.edu.in, seemas@srmist.edu.in
*Corresponding author

Abstract

A multi-warehouse shortage model has been developed where demand is assumed to be deterministic. In reality, machines run for long periods during production and random failures may occur as the system transitions from a controlled to an uncontrolled state. During this time the production system produces defective products. Demand is assumed to be deterministic. Retailers offer a quantity discount per unit on the selling price of an item and in return receive a quantity- based discount on the purchase price of the item. A retailer has limited storage capacity and therefore requires additional space with unlimited storage capacity. This additional space is called a rented warehouse and its storage cost is higher than accompany- owned warehouse. The objective of this model is to study a multiple inventory model of defective items under quantity- based discounts, where defective items can be sorted and sold in a single batch with decision variables set to the optimal order quantity and optimal inventory and shipment quantity to increase overall profits to maximize the value for the retailer. A solution procedure for determining the optimal solution is presented and a numerical example is given to illustrate this study. A sensitivity analysis is also performed to examine the effect of changing parameter values on the optimal solution.

Keywords: Warehouse, Multi-warehouse; Demand, Imperfect items; Shortages; Quantity Discount

I. Introduction and Literature Survey

The essential part of business is Inventory management, as it constitutes a major part of capital expenditure. Hence, inventory management plays a vital role in running a business flexibly. There are several features that affect a business such as customer demand, deterioration of goods, manufacturing defects, stock shortages, quantity based discounts offered by suppliers etc. Managing these features can directly affect a retailer's overall profits. Inventory consists of raw materials, partially finished goods and finished goods. This type of inventory involves ordering costs, carrying costs, and stock out costs. Purchasing costs may also include carrying costs, but most models assume these costs are constant unless volume discounts are involved. The quantity and timing of orders are the most important decisions in inventory management. These decisions help increase a company's overall profits. It is common for suppliers to offer discounts for larger batch sizes. There are several reasons why suppliers offer discounts to retailers. These reasons include increasing cash flow, reducing product inventory, increasing market share, or simply retaining the retailer. For retailers, inventory costs increase while bulk-ordering costs can be reduced.

These costs can be controlled through mathematical models, allowing retailers to make more profits. It is assumed that products are perfectly manufactured during the manufacturing stage, but in reality, this is not possible due to machine breakdowns after long hours of operation. Therefore, retailers need to inspect defective products and separate defective from good products. In addition, retailers have limited space to store inventory. In this situation, retailers need a separate space to store their inventory, the so-called rental warehouses. Rental warehouses offer unlimited storage space and better preservation options. That is why the storage costs in rental warehouses are higher than in their own warehouses. When demand increases, bottlenecks can occur. Product imperfections can also cause stock outs.

This paper describes a company that sells goods in a deterministic demand environment across multiple warehouses. It assumes discounts on every unit, defects, and shortages. The goal of this study is to optimize quantities and reorders to maximize profits across the inventory. The model becomes more realistic when considering volume discounts, a feature that is important for small retailers. Here, the procedure to obtain the optimal solution is used and a numerical example is provided to verify the results. A sensitivity analysis is also performed to study the effect of the parameters on the optimal solution. This article start with assumptions and notations and end with the conclusion of the article The model is based on a multiple warehouse environment with markdowns and shortages of incomplete items. S. Papachristos, K. Skouri [1] which ensures that the rate of backlogged demand increases as the waiting time to the following replenishment point decreases. Seto et al. [2] described a two-camp model with deterioration and demand growth over time. Jaggi et al. [3] developed a two-camp model with deterioration and inflation, where demand is based on the selling price. Jaggi et al. [4] proposed a two-camp model for deteriorating items with credit financing policy. Furthermore, the model assumes incomplete items. Assuming inventory-dependent demand, Jaggi et al. [5] developed a model with defects and damaged goods in two warehouse environments. This model was based on maximizing retailer profits with deterministic demand. Mandal & Giri [6] proposed a two-inventory model with incomplete items, where demand is based on inventory. An EOQ model was developed by Shah & Naik [7] with price-sensitive demand and discount policy. Volume discounts were offered in this model. Shaikh et al. [8] use a two-camp model under inflationary conditions. . Dhaka et al. [9] developed an inventory model with defects. They assumed inventory-dependent demand due to credit lending policy. By accepting incomplete articles, another two-camp model is proposed by Mashud et al. [10] assuming discountability and non-instantaneous deterioration rates. There were also some bottlenecks in the credit lending policy. Mandeep et al. [11] developed a two-warehouse model that considered the impact of human error on inventory levels and stockouts. Priyanka & Pareek [12] developed a two-inventory model by incorporating non-instantaneous deterioration and stochastic demand. The model also assumes credit financing Gilotra et al. [13] developed another model.

Additionally, we added carbon emissions and human error to the model. The impact of defective items is accounted for in an inventory model developed by Aastha et al. [14]. They assumed shortages in two warehouse environments. Another inventory model was developed by Aastha et al. [15] which is based on volume discounts under credit financing policies. In this model, the authors assumed that non-instantaneous deterioration in demand depends on advertising and selling prices. Mishra et al. [16] studied green technologies and developed a supply chain inventory model in the context of shortage and surplus payment systems. For fresh food products, Jayaswal et al. also developed an EOQ model. [17]. They developed it with a two-stage credit and lending policy under the learning effect and illustrated it with a numerical example. Shah et al. [18] formulated an EPQ model for perishables with reliability and inflation under a two-stage credit policy.

The total cost function was also minimized. Alamri et al. [19] developed an EOQ model that includes learning effect and carbon emissions, taking into account several aspects of naturally spoiled products. The Authors also discussed the implications for various related fields. Kumar et al. [20] Slack inventory ordering policy with pre-pay and post-pay strategies. Mittal et. al. [21] gave a model of the retailer's ordering policy for deteriorating imperfect quality items when demand and price are time-dependent under inflationary conditions and permissible delay in payments Masae et. al. [22] developed a model of order picker routing in warehouse. Ibrahim Hassan [23] gave a model on the role of warehouse layout and operations in warehouse efficiency: a review on it. Here, a mathematical model is develop with the warehouse environment. This include the analysis as well as the sensitivity analysis of the parameters. The given tables discuss the account of the article.

II. Assumptions and Notation

Assumptions

1. A known, and constant demand rate is considered.
2. Replenishment rate is instantaneous.
3. Inventory to be sold in single batch only.
4. Proportion of defective items p follows a uniform distribution$[\alpha, \beta]$where $[0 \leq \alpha \leq \beta \leq 1]$ (Wee et al., 2007)
5. A fully backlogged shortage are allowed.
6. The purchase price is offered under the following, all-units quantity discount scheme: (Papachristos & Skouri, 2003)

$$u_i(\mu) = \begin{cases} u_1 \ for \ m_1 < q \leq m_2 \\ u_2 \ for \ m_2 < q \leq m_3 \\ \vdots \\ u_n \ for \ m_n < q \end{cases}$$

Notation: The notation is used in the present study is describe in Table 1.

Table 1: *Notation and description of the inventory life cycle mode*

Notation	Description
y_1	Lot size units/cycle
W	limited space of OW units/cycle
$y_1 - W$	unlimited space in RW units/cycle
D	demand rate unit / year
j, A	variable cost/ unit and ordering cost / order
p	proportion of imperfect items in y_1
f(p)	p.d.f of p

s	selling price of items with good quality
v	selling price of defective items,$v < j$
b	backordering cost per unit
h_r	holding cost for the items in RW
h_o	holding cost for the items in OW
m_i	ith price breaking point, i=1, 2, 3, …, n
$u_i = u_i(\mu)$	per unit material cost
x	rate of screening
d	cost for screening
t_r	screening rate in RW
t_w	screening rate in OW
t_1	time for RW when all the units used
t_2	time for OW when all the units used
t_3	time period of shortages
B	backorder quantity in units
T	length of a cycle
y_{10pt}	optimal lot size
$TR(B,\mu)$	sale revenue

III. Model Description

I. Problem Definition

Initially, at time $t = 0$, y_1 units come in the system. As the system has rent-warehouse too, W units are stored in own-warehouse and rest of the units are stored in rent-warehouse. But due to high holding costs in rent-warehouse, first units are consumed from rent-warehouse and then from own-warehouse. Generally, it is assumed that all the units come perfectly in the system but practically this is not possible due to improper transport, low quality of raw material. In this case, retailer must apply the screening process with the rate of x units per unit time. This paper assumes the p percent of imperfect items found in y_1 units, where pa random variable is whose p.d.f. is, $f(p)$, and its mean is $E(p) = p$. Thus, py_1 defective items and $(1-p)y_1$ non-defective items found in y_1 units. These imperfect items sold at v per unit salvage value, where $v < j$. After this screening process, the demand is fulfilled from rent warehouse by time t_1 and then upcoming demand is fulfilled from own warehouse. After t_3, shortages occur when both the warehouse is exhausted. (Figure. 1)

II. Mathematical Model

The time horizon of total inventory is given by,

$$T = (1-p)\frac{y_1}{D} \quad (1)$$

Sale revenue $(TR(p, y_1))$ is the sum of sales of imperfect and the perfect items.

$$TR(p, y_1) = sy_1(1-p) + vpy_1 \quad (2)$$

And Total cost $(TC(B, y_1))$ = ordering cost + purchase cost + holding cost + backordering cost + screening cost

$$TC(B, y_1) = u_i y_1 + A + jy_1 + dy_1 + h_r\left[\left(\frac{(t_1(y_1 - W)(1-p)}{2}\right)t_r p(y_1 - W)\right]$$
$$+h_r\left[Wt_w + W(t_1 - t_w)(1-p) + \frac{(w(1-p) - B(t_2 - t_1)}{2}\right] + \frac{Bbt_3}{2} \quad (3)$$

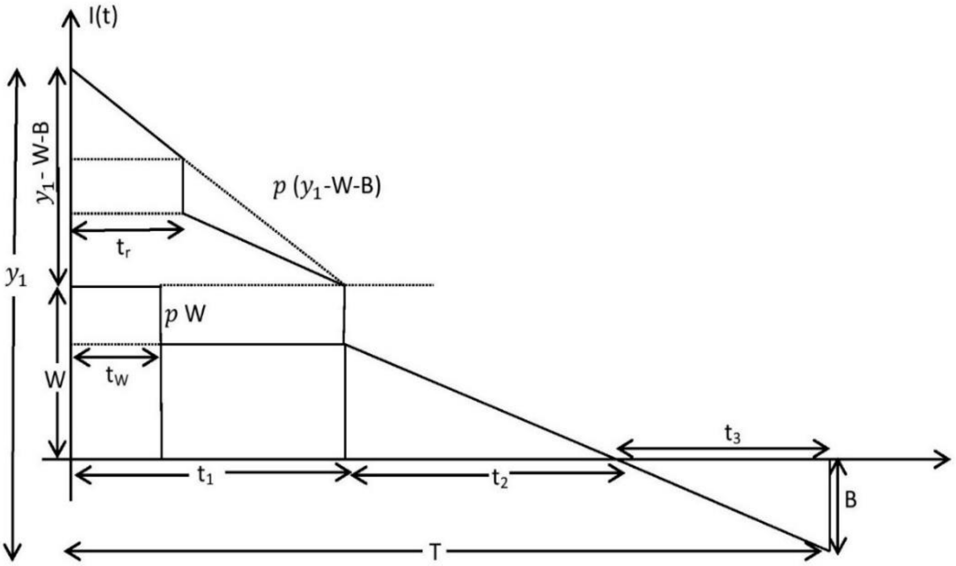

Figure 1: *Multi warehouse Modelling of Inventory*

Since $t_1 = \frac{(y_1-W)(1-p)}{D}$,$t_r = \frac{y_1-W}{x}$, $t_W = \frac{W}{x}$, $t_2 = \frac{W(1-p)-B}{D}$, $t_3 = \frac{B}{D}$

Then Eq (3) is

$$TC(B,y_1) = u_i y_1 + A + jy_1 + dy_1 + h_r\left(\frac{1}{2D}(y_1-W)^2(1-p)^2 + \frac{p(y_1-W)^2}{x}\right) + h_o\left(\frac{W(y_1-W)(1-p)^2}{D} + \frac{W^2p}{x} + \frac{1}{2D}\left((W(1-p)-B)^2 - W(y_1-W)(1-p)^2 + B(1-p)(y_1-W)\right)\right) + \frac{B^2b}{2D} \quad (4)$$

The total profit per unit of time ($TPU(B,y_1)$) is calculated with the help of total revenue per unit of time and total cost per unit of time. Hence,

$$\text{TPU}(p,y_1) = \frac{TR(p,y_1) - TC(B,y_1)}{T} \quad (5)$$

$$= \text{D}\left(s - v + h_r\frac{(y_1-w)^2}{xy_1} + h_o\frac{W^2}{xy_1}\right) + \left(\frac{D}{1-p}\right)\left(v - u_i - \frac{A}{y_1} - j - d - h_r\frac{(y_1-w)^2}{xy_1} - h_o\frac{W^2}{xy_1}\right)$$

$$-h_r(y_1-W)^2\left(\frac{1-p}{2y_1}\right) - \text{h}_o\left(\frac{W(y_1-W)(1-p)}{2y_1} + \frac{W^2(1-p)}{2y_1} - \left(\frac{WB}{y_1}\right) + \frac{B(y_1-W)}{2y_1}\right) - \left(\frac{1}{1-p}\right)\left(\frac{\text{h}_oB^2}{2y_1} - \frac{B^2b}{y_1}\right) \quad (6)$$

Since p is a random variable that follows a uniform distribution with known p.d.f. f (p) then the expected value of total profit ($ETPU(B,y_1)$) is

$$ETPU(B,y_1) = D\left(s - v + \text{h}_r\frac{(y_1-W)^2}{xy_1} + \text{h}_o\frac{W^2}{xy_1}\right) + E\left(\frac{1}{1-p}\right)D\left(v - u_i - \frac{\text{A}}{y_1} - j - d - \text{h}_r\frac{(y_1-W)^2}{xy_1} - \text{h}_o\frac{W^2}{xy_1}\right) - \text{h}_r(y_1-W)^2\left(\frac{1-E(p)}{2y_1}\right)$$

$$-\text{h}_o\left(\frac{W(y_1-W)(1-E(p))}{2y_1} + \frac{W^2(1-E(p))}{2y_1} - \left(\frac{WB}{y_1}\right) + \frac{B(y_1-W)}{2y_1}\right) - E\left(\frac{1}{1-p}\right)\left(\frac{\text{h}_oB^2}{2y_1} - \frac{B^2b}{y_1}\right) \quad (7)$$

Optimal lot size is $y_1^* = \sqrt{W^2 + \frac{u_i + A + \frac{B^2 b}{D} + h_o\left(\frac{pW^2}{x} + \frac{B^2}{2D} - \frac{3WB(1-p)}{2D}\right)}{h_r\left(\frac{(1-p^2)}{2D} + \frac{p}{x}\right)}}$ (8)

III. Solution Procedure

The objective of this model is to optimize order quantity (y_1^*) for both the warehouses and optimal shortages B^*, from these we get expected total profit $ETPU(B, y_1)$, therefore the necessary condition for expected profit to be optimal are $\frac{\partial ETPU(B,y_1)}{\partial y_1}$ and $\frac{\partial ETPU(B,y_1)}{\partial B}$which can be evaluated by the eq. (7) and equating the result to zero

$$\frac{\partial ETPU(B, y_1)}{\partial B} = -h_o\left(\frac{-W}{y_1} + \frac{1}{2}\frac{y_1 - W}{y_1}\right) - E\left(\frac{1}{1-p}\right)\left(\frac{h_o B}{y_1} - \frac{2Bb}{Dy_1}\right) = 0 \quad (9)$$

$$\frac{\partial ETPU(B, y_1)}{\partial y_1} = D\left(\frac{2h_r(y_1 - W)}{xy_1} - \frac{h_r(y_1 - W)^2}{xy_1^2} - \frac{h_W W^2}{xy_1^2}\right)$$
$$+E\left(\frac{1}{1-p}\right)D\left(\frac{A}{y_1^2} - \frac{2h_r(y_1-W)}{xy_1} + \frac{h_r(y_1-W)^2}{xy_1^2} + \frac{h_o W^2}{xy_1^2}\right) - \frac{h_r(y_1-W)(1-E(p))}{y_1} + \frac{h_r(y_1-W)^2(1-E(p))}{2y_1^2}$$
$$-h_o\left(\frac{1}{2}\frac{W(1-E(p))}{y_1} - \frac{1}{2}\frac{W(y_1-W)(1-E(p))}{y_1^2} - \frac{1}{2}\frac{W^2(1-E(p))}{y_1^2} + \frac{WB}{y_1^2} + \frac{B}{2y_1} - \frac{B(y_1-W)}{2y_1^2}\right)$$
$$-E\left(\frac{1}{1-p}\right)\left(-\frac{h_o B^2}{2y_1^2} + \frac{B^2 b}{y_1^2}\right)$$
$$= 0 \quad (10)$$

Eqs. (9) and (10) can be solved simultaneously for B* and y_1* using Maple. Here, the sufficient condition for expected total profit to be concave. First taking the second derivative.

$$\frac{\partial^2 ETPU(B,y_1)}{\partial B^2} = -E\left(\frac{1}{1-p}\right)\left(\frac{h_0}{y_1} - \frac{2b}{y_1}\right) \quad (11)$$

$$\frac{\partial^2 ETPU(B, y_1)}{\partial y_1^2} = D\left(\frac{2h_r}{xy_1} - \frac{4h_r(y_1 - W)}{xy_1^2} + \frac{2h_r(y_1 - W)^2}{xy_1^3} + \frac{2h_o W^2}{xy_1^3}\right)$$
$$+E\left(\frac{1}{1-p}\right)D\left(-\frac{2A}{y_1^3} - \frac{2h_r}{xy_1} + \frac{4h_r(y_1 - W)}{xy_1^2} - \frac{2h_r(y_1 - W)^2}{xy_1^3} - \frac{2h_o W^2}{xy_1^3}\right)$$
$$-\frac{h_r(1-E(p))}{y_1} + \frac{2h_r(y_1 - W)(1-E(p))}{y_1^2} - \frac{h_r(y_1 - W)^2(1-E(p))}{y_1^3}$$
$$-h_o\left(\frac{-W(1-E(p))}{y_1^2} + \frac{W(y_1-W)(1-E(p))}{y_1^3} + \frac{W^2(1-E(p))}{y_1^3} - \frac{2WB}{y_1^3} - \frac{B}{y_1^2} + \frac{B(y_1-W)}{y_1^3}\right) - E\left(\frac{1}{1-p}\right)\left(\frac{h_o B^2}{y_1^3} - \frac{2B^2 b}{y_1^3}\right) \quad (12)$$

$$\frac{\partial^2 ETPU(B, y_1)}{\partial y_1 \partial B} = -h_o\left(\frac{W}{y_1^2} + \frac{1}{2y_1} - \frac{1}{2}\frac{y_1 - W}{y_1^2}\right) - E\left(\frac{1}{1-p}\right)\left(\frac{-h_o B}{y_1^2} + \frac{2b\,B}{y_1^2}\right) \quad (13)$$

$$\left(\frac{\partial^2 ETPU(B, y_1)}{\partial B^2}\right) \times \left(\frac{\partial^2 ETPU(B, y_1)}{\partial y_1^2}\right) - \left(\frac{\partial^2 ETPU(B, y_1)}{\partial y_1 \partial B}\right)^2 \geq 0 \quad (14)$$

And $\left(\frac{\partial^2 ETPU(B,y_1)}{\partial B^2}\right) \leq 0, \left(\frac{\partial^2 ETPU(B,y_1)}{\partial y_1^2}\right) \leq 0$ (15)

By necessary and sufficient condition, the above equations show that the function *ETPU (B,y_1)* is strictly concave with a negative-definite Hessian matrix with optimal *(B*, y_1*)* values. With the optimal B* and y_1* values known, the net profit can be derived from Eq. (7).

IV. Results & Discussion

I. Numerical Example

In order to assess the mention inventory system, a few parameters are required, and these parameters are acquired from (Aastha. et al., 2020)

$D = 50{,}000\ unit\ /\ year;\ W\ =\ 800\ units/cycle;\ \mathrm{A}\ = \$\ 100\ /cycle;$
$j\ =\ \$\ 20\ /unit;\ h_r\ =\ \$\ 7\ /unit/year;\ h_o\ =\ \$\ 5/unit/year;\ x\ =\ 1\ unit/min;\ d\ =\ 0.5/unit;\ u_i$
$= 5/unit;\ s\ =\ \$\ 50/unit;\ v\ =\ \$\ 20\ /unit; b = 10/units$

In this example, the inventory model works on a 8 $hours/day$ for whole year, Hence, the screening rate per year,$x = 1 * 60 * 8 * 365 = 175200\ units/year$.

The proportion of imperfect random variable, p, follows uniformly distribution with p.d.f. as

$$f(p) = \begin{cases} 25, & 0 \leq \rho \leq 0.04 \\ 0, & otherwise \end{cases}$$

From this $E(p) = 0.02$ and$E\left(\frac{1}{1-p}\right) = 1.02055$.

The optimal solution is obtained by solving equations (15) and (16).

Table 2: *Optimum values of the model*

${y_1}^*$	B^*	$ETPU(B^*, {y_1}^*)\$$	${t_2}^*$(Yr.)	${t_3}^*$(Yr.)	T^*(Yr.)
1,373.892	167.651	12130.342	0.012	0.001	0.027

II. Sensitivity Analysis

A sensitivity analysis is conducted for the aforementioned case. The effects of fraction defective items ([p]), ordering cost, selling price of defective items and discount on the expected total profit ($ETPU(B^*, {y_1}^*)$) are shown in Figure 1, Figure 2, Figure 3 and Figure 4, respectively.

Table 3: *Sensitivity analysis with different parameters of the inventory system*

Parameter	Value	T (years)	t_1 (years)	t_2 (years)	y_1 (units/cycle)	B (units/cycle)	TP ($)
y_1	0.02	0.027	0.0112	0.012	1,373.80	167.7	12130.4
	0.03	0.026	0.0113	0.0122	1,383.60	164.5	12101.3
	0.04	0.0267	0.0114	0.0121	1,393.50	161.1	12071.4
	0.05	0.0267	0.0115	0.0120	1,403.40	157.8	12040.6
A	100	0.027	0.0112	0.012	1,373.80	167.7	12130.5
	125	0.026	0.0113	0.0122	1,383.60	164.5	12121.6
	150	0.0267	0.0114	0.0121	1,393.50	161.1	12113.5
	175	0.0267	0.0115	0.0120	1,403.40	157.70	12105.6
v	20	0.027	0.0112	0.012	1,373.89	167.7	12130.5
	25	0.026	0.0113	0.0122	1,383.67	164.4	12181.5
	30	0.0267	0.0114	0.0121	1,393.52	161.1	12232.6
	35	0.0267	0.0115	0.0120	1,403.46	157.70	12283.4
u	6	0.027	0.0112	0.012	1,373.89	167.7	11624.5
	5	0.026	0.0113	0.0122	1,383.67	164.4	12130.2
	4	0.0267	0.0114	0.0121	1,393.52	161.1	12640.6
	3	0.0267	0.0115	0.0120	1,403.46	157.70	13150.8

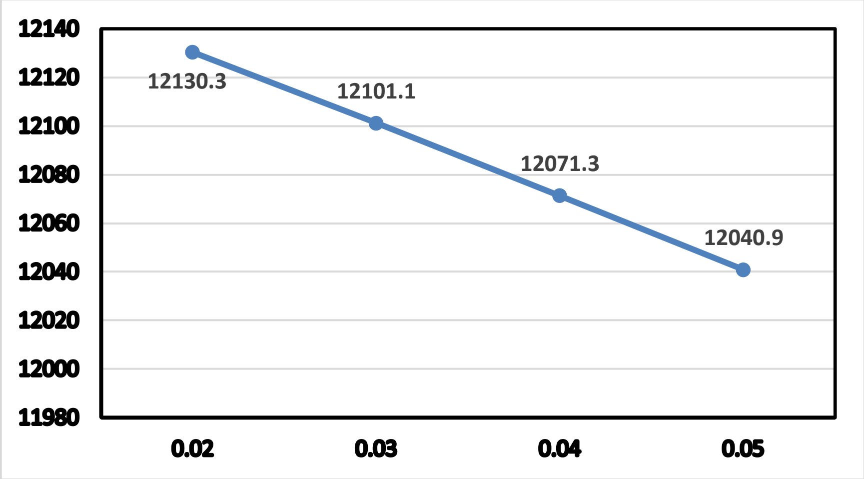

Figure 2: *Plotting of total profit with defective items*

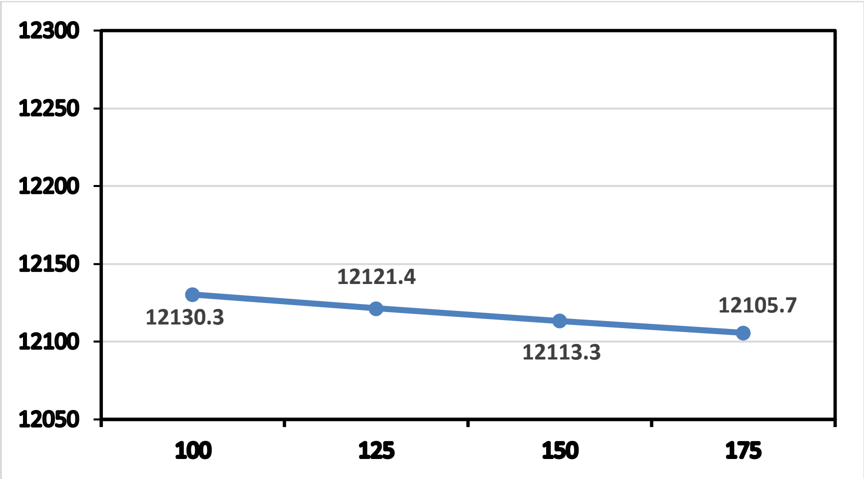

Figure 3: *Plotting of total profit with the ordering cost*

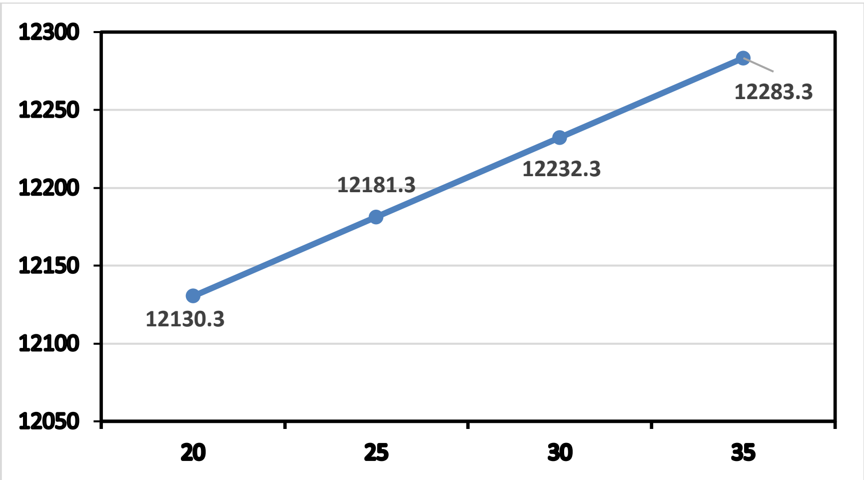

Figure 4: *Plotting of total profit with its selling price of defective items*

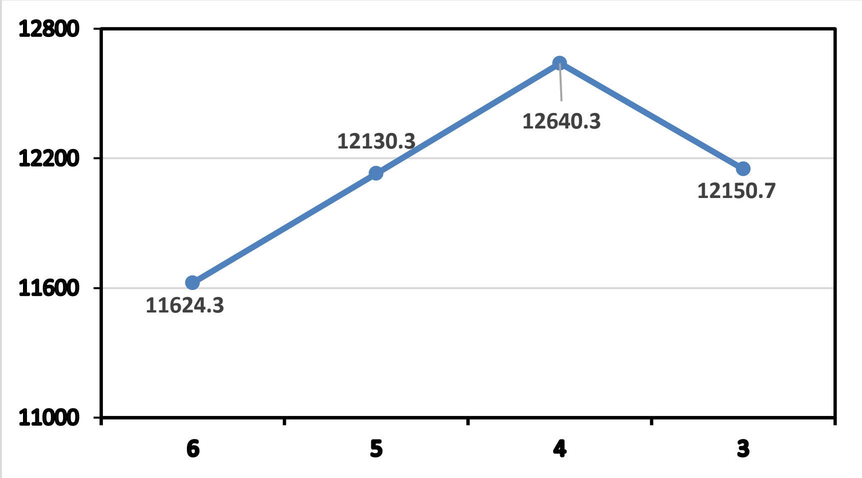

Figure 5: *Plotting of total profit with the offered discounts*

The graph above (see Figure 2) shows that as the percentage of defective goods increases, the expected total profit decreases significantly. (See Figure 3) As the acquisition cost of goods increases, the expected total profit also increases. In this scenario, buyers need to be careful when ordering and sellers need to make their systems more efficient to enhance the quality of goods produced. (See Figure 4) As the selling price of defective goods increases, the expected total profit increases significantly. (See Figure 5) As the discount offered decreases, the total expected profit increases. So, in this scenario, if the buyer receives a defective product, the buyer should pay attention to the sale price of the defective product as well as the discount offered.

V Conclusion

In this article, an EOQ model for defective items with stock outs in a multiple warehouse environment and determine the total expected profit if a discount is offered. It is presented as if the buyer has stock of defective items and can make a profit even with the defective items. To make more profit, a volume discount can also be offered. In this model, businesspersons need to look at the cost of purchasing the goods and the discount offered as these directly affect the overall expected profit. This paper is very useful for retail and manufacturing industries. This is also useful in the business world. The model can be extended in several ways. It can be extended to specify demand types through credit policies under CO_2 emission limit.

References

[1] S. Papachristos*, K. Skouri (2003). An inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging *Int. J. Production Economics 83 (2003) 247–256.*

[2] Seto, B. K., Sarkar, B., & Goswami, A. (2012). A two-warehouse inventory model with increasing demand and time varying deterioration. *Scientia Iranica, 19(6), 1969–1977*

[3] Jaggi, C., Pareek, S., Khanna, A., & Sharma, R. (2015). Two-warehouse inventory model for deteriorating items with price-sensitive demand and partially backlogged shortages under inflationary conditions. *International Journal of Industrial Engineering Computations, 6(1), 59–80.*

[4] Jaggi, C. K., Cárdenas-Barrón, L. E., Tiwari, S., & Shafi, A. (2017). Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments. *Scientia Iranica, 24(1), 390–412.* https://doi.org/10.24200/sci.2017.4042

[5] Mandal, P., & Giri, B. C. (2017). A two-warehouse integrated inventory model with imperfect pro duction process under stock-dependent demand and quantity discount offer.

International Journal of System Science Operations & Logistics

[6] Shah, N. H., & Naik, M. K. (2018). EOQ model for deteriorating item under full advance payment availing of discount when demand is price-sensitive. *International Journal of Supply Chain and Operations Resilience, 3(2), 163–197.*

[7] Chung, K.-J., Liao, J.-J., Ting, P.-S., Lin, S.-D., & Srivastava, H. M. (2018). A unified presentation of inventory models under quantity discounts, trade credits and cash discounts in the supply chain management. *Revista de La Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Mathematics, 112(2), 509–538. https://doi.org/10.1007/s13398-017-*

[8] Dey, B. K., Sarkar, B., Sarkar, M., & Pareek, S. (2019). An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling price dependent demand, and investment. *RAIRO - Operations Research, 53(1), 39–57. https://doi.org/10.1051/ro/2018009*

[9] Dhaka, V., Pareek, S., & Mittal, M. (2019). Stock-Dependent Inventory Model for Imperfect Items Under Permissible Delay in Payments. *Optimization and Inventory Management (pp. 181–194). Springer. https://doi.org/10.1007/978-981-13-9698-4_10*

[10] Agarwal, G. L., & Mandeep, M. (2019). Inventory Classification Using Multi-Level Association Rule Mining. *International Journal of Decision Support System Technology, 11, 1–12. https://doi.org/10.4018/IJDSST.2019040101*

[11] Mashud, A. H. M., Wee, H.-M., Sarkar, B., & Li, Y.-H. C. (2020). A sustainable inventory system with the advanced payment policy and trade-credit strategy for a two-warehouse inventory system. *Kybernetes.*

[12] Mittal, M., Pareek, S. & Aastha, (2020). Effect of Human Errors on an Inventory Model Under Two Warehouse Environments. *Recent Advances in Computer Science and Communications, 13, 1–10.*

[13] Priyanka, & Pareek, S. (2020). Two Storage Inventory Model for Non-Instantaneous Deteriorating Item with Stochastic Demand Under Credit Financing Policy. *2020 8th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), 978–983. https://doi.org/10.1109/ICRITO48877.2020.9197872*

[14] Gilotra, M., Pareek, S., Mandeep, M., & Dhaka, V. (2020). Effect of Carbon Emission and Human Errors on a Two-Echelon Supply Chain under Permissible Delay in Payments. *International Journal of Mathematical, Engineering and Management Sciences, 5,* 225–236. https://doi.org/10.33889/IJMEMS.2020.5.2.018

[15] Aastha, Pareek, S., & Mittal, M. (2020). Non Instantaneous Deteriorating Inventory Model under Credit Financing When Demand Depends on Promotion and Selling Price. *2020 8th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO), 973–977. https://doi.org/10.1109/ICRITO48877.2020.9197990*

[16] Dey, B. K., Pareek, S., Tayyab, M., & Sarkar, B. (2021). Autonomation policy to control work-in-process inventory in a smart production system. *International Journal of Production Research, 59(4), 1258–1280. https://doi.org/10.1080/00207543.2020.1722325*

[17] Jayaswal, M. K., Mittal, M., and Sangal, I. (2021). Effect of credit financing on the learning model of perishable items in the preserving environment. In Decision Making in Inventory Management, *pages 49–60. Springer*

[18] Mishra, U., Mashud, A. H. M., Tseng, M.-L., and Wu, J.-Z. (2021). Optimizing a sustainable supply chain inventory model for controllable deterioration and emission rates in greenhouse farm.*Mathematics,9(5):495*

[19] Shah, N. H., Rabari, K., and Patel, E. (2022). Economic production model with reliability and inflation for deteriorating items under credit financing when demand depends on stock displayed. Revista Investigacion *Operacional., 43(2):203–213.*

[20] Alamri, O. A., Jayaswal, M. K., Khan, F. A., and Mittal, M. (2022). An eoq model with carbon emissions and inflation for deteriorating imperfect quality items under learning effect. Sustainability, 14(3):1365.

[21] Kumar, B. A., Paikray, S. K., and Padhy, B. (2022). Retailer's optimal ordering policy for deteriorating inventory having positive lead time under pre-payment interim and post- payment strategy. *International Journal of Applied and Computational Mathematics, 8(4):1–33.*

[22] Ruidas, S.; Seikh, M.R.; Nayak, P.K. (2022). A production inventory model with interval-valued carbon emission parameters under price-sensitive demand. *Computer. Ind. Eng., 154, 107154*

[23] Mittal, M., Khanna, A., & Jaggi, C. K. (2023). Retailer's ordering policy for deteriorating imperfect quality items when demand and price are time-dependent under inflationary conditions and permissible delay in payments. *International Journal of Procurement Management, 10(4), 461–494.*

[24] Masae, M., Glock, C.H., Grosse, E.H. (2023). Order picker routing in warehouses: A systematic literature review. *International Journal of Production Economics, 224: 107564. https://doi.org/10.1016/j.ijpe.2019.107564*

[25] Ibrahim Hassan (2024). The role of warehouse Layout and Operations in Warehouse Efficiency: A literature Review. *Journal Europeen des Systemes Automatises, 56(1), 61-68.*