RT&A, No 4(80)
Volume 19, December, 2024

OPTIMIZING HIDDEN MARKOV MODELS WITH FUZZIFICATION TECHNIQUES

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Abstract

This work explores using fuzzified techniques to enhance the performance of Hidden Markov Models (HMMs) in handling uncertainties and imprecise inputs. We construct and evaluate three types of fuzzy HMMs: the Trapezoidal fuzzy HMM, the Sigmoidal fuzzy HMM, and the Gaussian fuzzy HMM. As part of our process, parameter estimations are calculated and models are chosen based on AIC, BIC, AICc, and HQIC criteria. Each state's mean, variance, and stationary distribution are calculated and examined to evaluate the predictability and stability of the models. We use the Viterbi technique to identify the most likely state sequences for the next five years. According to the results, the Gaussian Fuzzy HMM offers superior predicted accuracy and durability when compared to the other models. This paper emphasizes the advantages of using fuzzy membership functions in HMMs and provides the foundation for future research in different areas, such as agricultural data prediction.

Keywords: Trapezoidal, Sigmoidal, Gaussian, Stationary Distribution, Viterbi Algorithm

I. Introduction

HMMs were introduced by Baum and his colleagues in the late 1960s, and they have been widely employed in domains like speech recognition, the biological sciences, and finance. Hidden states, observable emissions, and state transition probabilities characterize an HMM. Hidden Markov Models are a potent tool for modeling sequential data in the fields of statistical modeling and machine learning. Statistical models represent systems having hidden states called Hidden Markov Models. They are composed of observable symbols, a collection of hidden states, transition probabilities between states, and the likelihood that symbols will be emitted from states. The conventional HMM relies on clear and accurate observations, however noise and fuzziness can affect observations in a lot of real-world situations. Traditional statistical methods are sometimes inadequate in addressing difficulties and uncertainties present in agricultural data, such as the oilseed area from 1992 to 2022. Even though standard HMMs are reliable when modeling sequential data, they may not be sufficient when handling fuzziness of this kind. As a result, it is imperative to create and assess models that can manage these uncertainties more skillfully to guarantee greater forecast accuracy and dependability.

Introduced by Zadeh in 1965, fuzzy logic incorporates degrees of membership instead of binary true or false logic, offering a mathematical framework to handle such inaccurate information. Fuzzy logic is an appropriate method for handling uncertainty since it extends the traditional set theory to incorporate partial membership. The act of utilizing membership functions to convert crisp inputs into fuzzy values is known as "fuzzification." The degrees of membership within the interval [0, 1] are mapped to input values by these membership functions. The resulting fuzzy model's performance and interpretability are greatly influenced by the membership function selection. By adding fuzzy logic to the observation and state representation, FHMMs go beyond conventional HMMs. With this adjustment, the model can now handle observational uncertainty more effectively and generate more reliable predictions.

Ahmed T. Salawuden, et al. [2] introduced a genetic algorithm and hidden Markov model-based FTS forecasting model. Conventional techniques, such as the Baum-Welch algorithm, have trouble capturing the fuzziness present in real data. The model optimizes the HMM parameter estimation problem using GA and an objective function. Qi Deng and Dirk Soffker [11] proposed a driving behavior prediction model based on a novel approach that combines HMM and Fuzzy Logic. The model divides driving scenes into three scenarios: very safe, safe, and risky, with associated HMMs for each. A prefilter reduces collected signals to observed sequences with predefined features. NSGA-II is utilized to maximize model performance, and experimental data from real-world human driving behaviors confirms the efficacy of the fuzzy-based HMM. Ying-Kui Gu, et al. [17] proposed a gearbox vibration experiment, the study gathers vertical vibration signals from three gear fault states: normal, worn, and fractured teeth. A hidden Markov model and fuzzy comprehensive evaluation are used to provide an online diagnosis and performance evaluation model.

This study's main goal is to investigate how fuzzified approaches can be used to improve HMM performance. Our specific purpose is to build and assess three different kinds of fuzzy HMMs: the Gaussian fuzzy HMM, the Sigmoidal fuzzy HMM, and the Trapezoidal fuzzy HMM. We aim to increase the predictability and stability of the models and better handle imprecise inputs by using fuzzy membership functions. The concepts of fuzzy logic, HMMs, and how they combine to form FHMMs form the theoretical basis of this study. Fuzzy logic defines fuzzy sets and membership functions to give a mathematical framework for handling ambiguity and imprecision.

II. Methods

Fuzzy logic is an effective framework for dealing with uncertainty and imprecision in data, making it especially helpful in complicated systems where classic binary logic fails. Unlike classical logic, which uses strict true or false values, fuzzy logic allows for degrees of truth, expressed by values ranging from 0 to 1. This flexibility is done using fuzzy sets and membership functions, such as Sigmoidal, trapezoidal, and Gaussian functions, which determine an element's degree of membership in a set. Fuzzy logic systems use "if-then" rules to process inputs and generate outputs, providing a more sophisticated approach to decision making. The fuzzy inference system (FIS) automates this process by mixing fuzzy rules, and its output is frequently "defuzzified" to convert fuzzy results into crisp, actionable conclusions. Fuzzy logic, which is widely employed in control systems, expert systems, and forecasting, excels in circumstances where data is ambiguous or imprecise, effectively dealing with uncertainty using fuzzy membership functions.

I. Hidden Markov Model

A Statistical model known as a Hidden Markov Model (HMM) can be used to forecast the development of observable events that consist of non-observable internal components. The phenomenon under observation is referred to as a 'state', whereas the witnessed event is called a ‘symbol’. A hidden state invisible process and an observable symbol visible process are the two stochastic processes that construct an HMM. Within a Markov chain formed by the hidden states, the underlying state determines the probability distribution of the observed symbol. A doubly embedded stochastic process is another name for an HMM [5].

Definition: The triplicate *(S, O, A, B, π)* defines HMM [4] where,

- A set of hidden states is denoted by $S = S_1, S_2, S_3, \dots, S_n$.
- A set of m observable symbols at each time interval is represented by $O(t) = O_1, O_2, \dots, O_m$.
- State transition probability is represented by A, which is defined as, $A = a_{ij} = \{ P(X_{t+1} = S_j | X_t = S_i) | 1 \leq i, j \leq n\}$. The probability of changing from state i at time t to state j at time $t+1$ is shown here by the symbol a_{ij}.
- B is the probability of generating a symbol $O(t)$ from state j, and it can be expressed by the formula $B = b_j(t) = \{P(O(t) | X(t) = S_j) | 1 \leq j \leq n\}$.
- The initial state probability is $\pi = \{\pi_i = P(X_1 = S_i) | 1 \leq i \leq n\}$.

Using the forward method, one may determine the likelihood of a sequence of observed symbols, $\alpha_t(i) = P(o_1, o_2, o_3, \dots, o_m, s_t = s_i) = \sum_{j=1}^{n} \alpha_{t-1}(j) a_{ji} b_i(o_t)$, where $\alpha_t(i)$ indicates the chance of noticing the observations and being in states s_i at time t. The whole observation sequence's likelihood is

$$P(O) = \sum_{i=1}^{n} \alpha_T(i) \tag{1}$$

The Viterbi method determines which hidden state sequence is most likely to exist and HMM parameters are developed using the Baum-Welch algorithm to maximize the likelihood of the observed data.

II. Fuzzy Hidden Markov Model

To more effectively handle uncertainties and imprecisions in the data, a Fuzzy Hidden Markov Model (FHMM) combines fuzzy logic concepts with the conventional HMM architecture. An FHMM's primary concept is to use fuzzy sets to represent the states and observations, which enables more flexible and intricate modeling of the underlying stochastic processes. In a FHMM, states are represented using fuzzy sets, where each state is associated with a membership function that specifies the degree to which an observation belongs to that state. This fuzzy representation allows the model to handle imprecise and noisy data more effectively. Observations in an FHMM can also be fuzzy, paralleling the fuzziness of states, which enhances the model's capability to process uncertain data. Membership functions, such as sigmoidal, Gaussian, and trapezoidal, characterize the degree of fuzziness in both observations and states.

Transition probabilities in an FHMM indicate the likelihood of transitioning from one state to another and can be adjusted to reflect the fuzziness in the states. Similarly, emission probabilities represent the chances of observing a specific output from a given state and can incorporate the uncertainty in observations. Additionally, the initial state probabilities, denoted as π_i, can be fuzzified by describing the likelihood of the system being in a specific state at the beginning of the time step. This fuzzified approach to initial probabilities further enhances the model's ability to handle uncertainty from the onset of the observation sequence.

Hidden Markov Models use fuzzy membership functions to address uncertainty and imprecision in state allocations. Traditional HMMs confine observations to discrete states, which might be restrictive since real-world state transitions are frequently lagging or confusing. HMMs use fuzzy membership functions to allow observations to belong to various states, capturing data variations and reflecting smoother transitions. This strategy is especially useful for dealing with noisy or ambiguous data since it enhances prediction accuracy and robustness. Furthermore, fuzzy HMMs provide more versatility by allowing the use of various membership function shapes to depict complex relationships. This makes fuzzy HMMs more adaptive and capable of addressing the complexities encountered in real-world applications such as forecasting and pattern recognition.

III. Trapezoidal Fuzzy Hidden Markov Model

Trapezoidal membership functions are used by a Trapezoidal Fuzzy Hidden Markov Model (TrFHMM) to reflect the fuzziness of states and observations. Trapezoidal membership functions, with their flat top that can capture more information, offer a more flexible technique to model uncertainty than triangular functions. The trapezoidal membership function Ψ is defined as:

$$\Psi(x; \acute{a}, b, \c{c}, d) = \begin{cases} 0, & if\ x \le \acute{a} \\ \frac{x-\acute{a}}{b-\acute{a}}, & if\ a < x \le b \\ 1, & if\ b < x \le \c{c} \\ \frac{d-x}{d-\c{c}}, & if\ \c{c} < x \le d \\ 0, & if\ x \ge d \end{cases} \tag{2}$$

where b and $\c{c}$ indicate the trapezoid's top and $\acute{a}$ and d, respectively, denote the lower and upper bounds [8].

IV. Sigmoidal Fuzzy Hidden Markov Model

To describe the fuzziness of states and observations, sigmoidal membership functions are used in a Sigmoidal Fuzzy Hidden Markov Model (SFHMM). Sigmoidal membership functions are especially helpful as a more realistic and gradual approach to dealing with uncertainty and imprecision. This is because they smoothly go from 0 to 1. The Sigmoidal membership function Ψ can be defined as:

$$\Psi(x; \acute{a}, \c{c}) = \frac{1}{1+e^{-\acute{a}(x-\c{c})}} \tag{3}$$

where $\c{c}$ represents the sigmoid curve's center, or midway, and $\acute{a}$ determines the function's slope [16].

V. Gaussian Fuzzy Hidden Markov Model

The Gaussian Fuzzy Hidden Markov Model (GFHMM) models the fuzziness of states and observations by using Gaussian membership functions. To effectively depict uncertainty in data, Gaussian functions provide effortless and gradual transitions. The Gaussian membership function Ψ is defined as:

$$\Psi(x; \c{c}, \sigma) = exp\left(-\frac{(x-\c{c})^2}{2\sigma^2}\right) \tag{4}$$

where $\c{c}$ is the center (mean) of the Gaussian function, and σ is the standard deviation [15].

VI. Fuzzification of Observations

Considering a set of $\ddot{O} = \{ \ddot{O}_1, \ddot{O}_2, \ldots, \ddot{O}_T \}$, the fuzzified observation $\tilde{\ddot{O}}_t$ at time t for a fuzzy state $\check{S}_i$ is

$$\tilde{\ddot{O}}_t(\check{S}_i) = \Psi_{\check{S}_i}(\ddot{O}_t) \tag{5}$$

where $\Psi_{\check{S}_i}(\ddot{O}_t)$ is the membership function value of the observation $\ddot{O}_t$ concerning state $\check{S}_i$.

The likelihood of a state $\check{S}_i$ to state $\check{S}_j$ transition accumulates the same as in the conventional HMM;

$$\hat{A}_{ij} = P(\check{S}_{t+1} = \check{S}_j \mid \check{S}_t = \check{S}_i) \tag{6}$$

Given a state $\check{S}_j$, the emission probability of detecting $\ddot{O}_t$ is fuzzily defined as,

$$\mathcal{B}_j(\ddot{O}_t) = \Psi_{\check{S}_i}(\ddot{O}_t) \tag{7}$$

The forward probability $\acute{\alpha}_t(i)$ is defined as;

$$\acute{\alpha}_t(i) = P(\ddot{O}_1, \ddot{O}_2, \ldots, \ddot{O}_t, \check{S}_t = \check{S}_i \mid \lambda) \tag{8}$$

with the initial condition,

$$\acute{\alpha}_1(i) = \pi_i \, \mathcal{B}_i(\ddot{O}_1) \tag{9}$$

and the recursive formula,

$$\acute{\alpha}_{t+1}(j) = \left(\sum_{i=1}^{N} \acute{\alpha}_t(i) \, \hat{A}_{ij}\right) \mathcal{B}_j(\ddot{O}_{t+1}) \tag{10}$$

The backward probability $Б_t(i)$ is defined as;

$$Б_t(i) = P(\ddot{O}_{t+1}, \ddot{O}_{t+2}, \ldots, \ddot{O}_t \mid \check{S}_t = \check{S}_i \mid \lambda) \tag{11}$$

with the initial condition,

$$Б_T(i) = 1 \tag{12}$$

and the formula for recursion,

$$Б_t(i) = \sum_{j=1}^{N} \hat{A}_{ij} \, \mathcal{B}_j(\ddot{O}_{t+1}) Б_{t+1}(j) \tag{13}$$

One well-known method for estimating HMM parameters is the Baum-Welch algorithm. It is a particular kind of Expectation-Maximization (EM) method created with HMMs. The algorithm iteratively refines the model parameters to optimize the likelihood of the observed data. The probabilities that follow are defined for parameter re-estimation [14],

- $\xi_t(i, j) = P(\check{S}_t = \check{S}_i, \check{S}_{t+1} = \check{S}_j \mid \ddot{O}, \lambda)$
- $\ddot{Y}_t(i) = P(\check{S}_t = \check{S}_i \mid \ddot{O}, \lambda)$

The following are the re-estimation formulas [13]:

$$\pi_i = \ddot{Y}_1(i) \tag{14}$$

$$\hat{A}_{ij} = \frac{\sum_{ţ=1}^{T-1} \xi_{ţ}(i,j)}{\sum_{ţ=1}^{T-1} \ddot{Y}_{ţ}(i)} \tag{15}$$

$$Ɓ_j(k) = \frac{\sum_{ţ=1,\ddot{O}_ţ=k}^{T} \ddot{Y}_ţ(j)}{\sum_{ţ=1}^{T} \ddot{Y}_ţ(j)} \tag{16}$$

The Viterbi method is a dynamic programming technique that determines the most likely sequence of hidden states, or Viterbi path, that leads to a sequence of observable events. Based on the observed sequence of events and the model parameters, the algorithm improves in forecasting, helping to predict future states.

Initialization

$$V_1(i) = \pi_i \ \mathcal{B}_i(\ddot{\bar{O}}_1) \tag{17}$$

Recursion

$$V_ţ(j) = \max_i \left(V_{ţ-1}(i)\ \hat{A}_{ij} \right) \mathcal{B}_j(\ddot{\bar{O}}_ţ)$$

$$\text{Path}(ţ)(j) = \arg\max_i \left(V_{ţ-1}(i)\ \hat{A}_{ij} \right) \tag{18}$$

Termination

$$\text{Optimal path (T)} = \arg\max_i V_T(i) \tag{19}$$

Backtracking

$$\text{Optimal path } (ţ) = \text{path}(ţ+1) \text{ optimal path}(ţ+1), \text{ for } ţ = T-1, T-2, \ldots, 1. \tag{20}$$

For every future time step T+k,

$$s_{T+k} = \arg\max_j \hat{A}_{s_{T+k-1},j} \tag{21}$$

The essential elements of the Viterbi algorithm and the procedures required to predict future states can be found in these equations [14].

VII. Data Source

The "Economics & Statistics Division within the Ministry of Agriculture & Farmers Welfare" website, may be accessed at **http://desagri.gov.in.**

VIII. Algorithm

The following are the steps for constructing and assessing Fuzzified HMM:

1. Import historical oilseed area data from 1992 to 2022.
2. Define the fuzzy membership functions: trapezoidal, sigmoidal, and Gaussian.
3. Assign fuzzy states (Low, Medium, and High) to observations using each fuzzification method.
4. Create and assign the sequence according to the highest membership value for each observation.
5. Set up the transition matrix, emission matrix, and initial probabilities for each F-HMM.

6. Using the Baum-Welch algorithm, estimate the transition and emission probability, as well as the initial state distribution.
7. Calculate the AIC, BIC, AICc, and HQIC for each FHMM.
8. Calculate the stationary distribution of each model.
9. Calculate the mean and variance of data for each state.
10. Use the Viterbi method to estimate the most likely state sequence during the next five years.
11. Compare models using AIC, BIC, AICc, HQIC, and prediction metrics to determine the best-performing model.

III. Results and Discussion

The findings from our examination of the three different kinds of FHMM are shown in this section. The main findings are fuzzy values, sequence prediction, parameter estimations, state parameters, and model selection criteria, which are compiled in the tables. Using fuzzy membership functions equations (2), (3), and (4), the fuzzification process transforms crisp values into fuzzy values.

Table 1: Fuzzy Values for oilseed area data.

Trapezoidal Fuzzy Value	Sigmoidal Fuzzy Value	Gaussian Fuzzy Value
0.870	0.224	0.528
1	0.668	0.980
1	0.643	0.995
1	0.692	0.953
1	0.613	0.999
1	0.450	0.782
1	0.802	0.546
1	0.435	0.749
1	0.316	0.928
1	0.345	0.950
0.745	0.624	0.968
1	0.158	0.708
0.52	0.882	0.891
0.86	0.913	0.990
1	0.733	0.878
1	0.766	0.788
0.56	0.886	0.907
1	0.613	0.999
0.78	0.848	0.737
1	0.692	0.953
1	0.727	0.891
1	0.927	0.998
1	0.524	0.923
1	0.643	0.995
1	0.663	0.983
0.745	0.271	0.454
0.79	0.329	0.481
0.86	0.837	0.691

1	0.965	0.708
1	0.975	0.504

The state sequences reported by each model are quite consistent, notably in differentiating times of low (1), medium (2), and high (3) oilseed areas. However, variances in state transitions highlight the complex nature represented by various fuzzy membership functions.

Table 2: FHMM-identified state sequence

Models	Sequences														
TrFHMM	1	2	2	2	2	2	2	2	1	1	1	1	3	3	2
	2	3	2	2	2	2	3	2	2	2	1	2	2	3	3
GFHMM	1	2	2	2	2	2	3	2	1	1	1	1	3	3	2
	2	3	2	3	2	2	3	2	2	2	1	2	3	3	3
SFHMM	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2
	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3

Table 3: Parameter Estimation for FHMMs

Models	Parameter	State-1	State-2	State-2
TrFHMM	TPM	[0.50, 0.33, 0.16]	[0.11, 0.722, 0.16]	[0, 0.5, 0.33]
	EPM	[0.66, 0.16, 0.16]	[0.38, 0.33, 0.27]	[0, 0.16, 0.83]
	π	[0.2, 0.6, 0.2]		
SFHMM	TPM	[0.75, 0.25, 0]	[0.04, 0.92, 0.04]	[0, 0, 1]
	EPM	[0.50, 0.25,0.25]	[0.37, 0.29, 0.33]	[0, 0, 1]
	π	[0.133, 0.8, 0.06]		
GFHMM	TPM	[0.5, 0.33, 0.16]	[0.13, 0.53, 0.33]	[0, 0.62, 0.37]
	EPM	[0.66, 0.16, 0.16]	[0.46, 0.40, 0.13]	[0, 0.11, 0.88]
	π	[0.2, 0.5, 0.3]		

TrFHMM is appropriate for systems in which State-2 dominates and transitions between states are more evenly dispersed. Its balanced transition dynamics and emission probabilities make it adaptable to diverse conditions. GFHMM is extremely stable, particularly in State-2, and has an absorbing State-3, making it ideal for scenarios in which the system is intended to converge to a final state. The deterministic emission probabilities make precise predictions. The sigmoidal model strikes a balance between stability and flexibility, with frequent transitions between states and high emission probabilities for specific data. This model can handle systems with more complex state transitions and moderate stability.

Table 3: Performance of model parameters.

Models	Parameter	State-1	State-2	State-3
TrFHMM	δ	0.134	0.608	0.250
	μ	23.2216	26.165	28.165
	σ	1.1376	0.6439	0.6841
SFHMM	δ	0.138	0.532	0.330
	μ	22.64	26.2208	29

	σ	0.89065	1.0295	0.24041
GFHMM	δ	0.1452	0.5331	0.3214
	μ	23.2216	26.0657	27.805
	σ	1.1376	0.4312	0.768

Systems with moderate variability and a dominant State-2 are most suited for TrFHMM. Strongly preferred State-2 scenarios with very consistent observations in State-3 are well suited for SFHMM. For evenly distributed state preferences, GFHMM is the best option since it provides a balanced stationary distribution and minimal variability in State-2. The optimal model selection for a given prediction task is determined by the desired balance between state stability, variability, and distribution.

Table 4: Measures of Model Performance.

Models	Log-likelihood	AIC	BIC	AICC	HQIC
TrFHMM	-3.44	34.89	54.507	62.891	41.166
GFHMM	-3.367	**34.734**	**54.351**	**62.734**	**41.009**
SFHMM	-3.376	34.752	54.368	62.752	41.027

GFHMM performs better than TrFHMM and SFHMM models according to the evaluation measures. With the highest log-likelihood and the lowest values of AIC, BIC, AICC, and HQIC, the SFHMM offers the best fit and strikes the ideal balance between goodness of fit and model complexity. As a result, the GFHMM is the best model to use with the given data. These measurements also show that the TrFHMM and SFHMM are less efficient, despite their good performance.

The graph illustrates the various degrees of membership for each fuzzy state by representing it over various intervals. Evaluating the model's performance and capacity to capture the variations in the data requires an awareness of how the membership values are spread among the fuzzy states, which is made easier with the aid of this graphic.

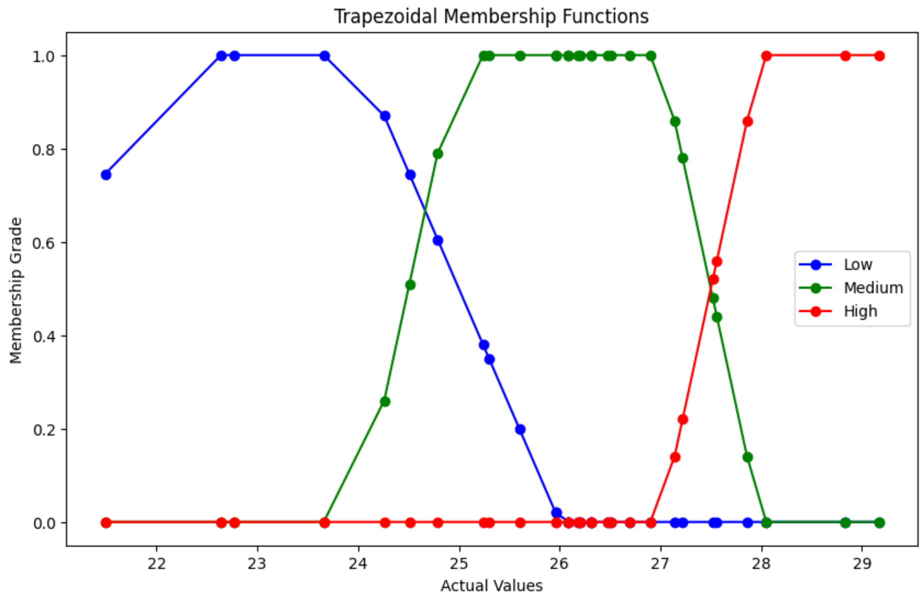

Figure 1: Shows the fuzzy states of trapezoidal membership functions.

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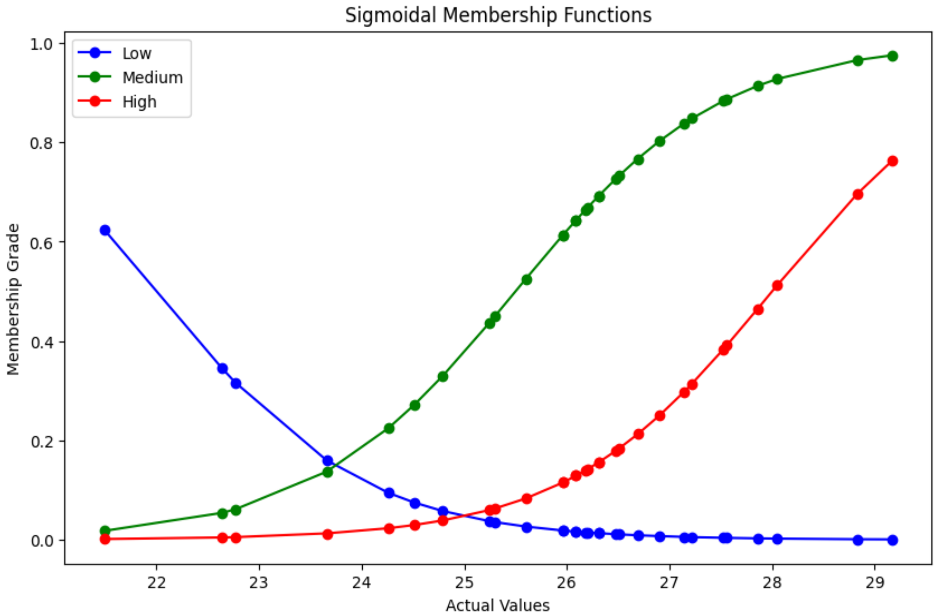

Figure 2: Demonstrates the fuzzy states of sigmoidal membership functions.

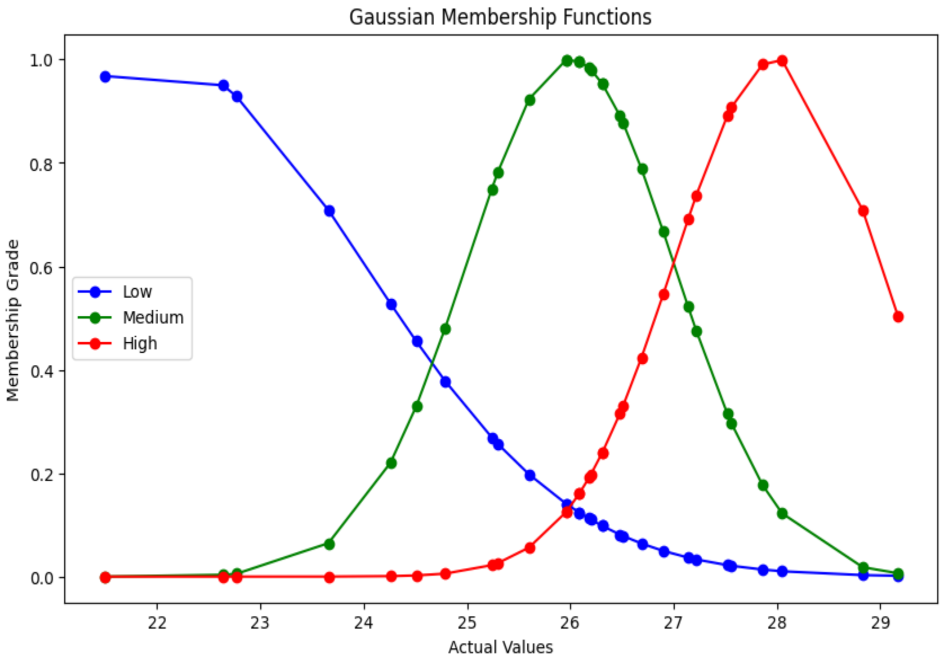

Figure 3: Fuzzy states are shown using Gaussian membership functions.

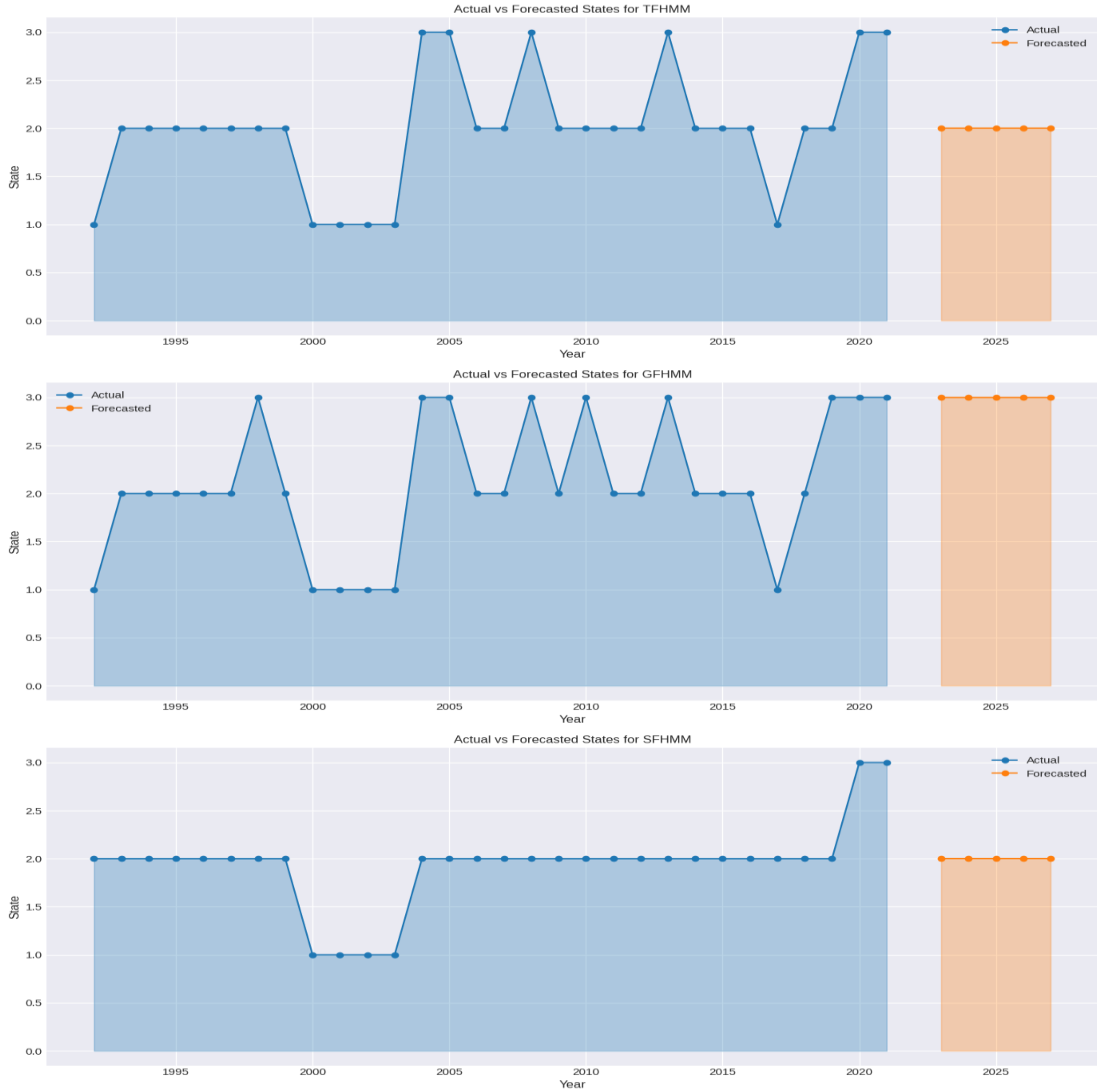

Figure 4: Shows the actual and forecasted states.

IV. Conclusion

The present study developed and evaluated three types of Fuzzy Hidden Markov Models. The study includes evaluating the parameters of each FHMM and comparing their performance against historical data from 1992 to 2022. Among the models, the Gaussian FHMM had the highest prediction accuracy and stability, as evidenced by favorable outcomes in model selection criteria. The estimates for the next five years indicated different results across the models: both the Sigmoidal and Trapezoidal FHMMs predicted that the oilseed area would remain in the "medium" state. In contrast, the Gaussian FHMM predicted that the oilseed area would reach a "high" state. Despite these variations, the Gaussian FHMM remains a reliable instrument for forecasting agricultural trends due to its superior performance. This study provides a significant framework for interpreting and forecasting agricultural commodity data, which will help the industry plan and make better decisions.

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