MARKOV CHAIN MODEL FOR COMPARISON OF PRICE MOVEMENT OF FRUITS IN SALEM DISTRICT, TAMILNADU

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Abstract

Statistical forecasting requires mathematical models and techniques to predict future outcomes based on historical data. Markov chains are statistical models that can be utilized to analyze the movement of prices in agriculture price, financial market price, business process, fuel prices and etc., They are particularly relevant in the context of price movements because they provide a framework for understanding and predicting the future state of a system based on its current state. In a Markov chain process, there are a set of states and we progress from one state to another based on a fixed probability. In these decades many articles are showed that modeling a market as a random walk was applicable and that a market may be viewed as having the Markov property. The objective of this paper is to construct the Markov chain model for daily fruit price movement in Salem District, Tamil Nadu. Two models are highlighted, where the price movement is considered as being in a state of gain, loss and no change and large gain, or small gain or loss, or large loss and no change. Ten different types of fruits are considered which are cultivated Salem areas and above two models are used to analyze the price movement of each fruit. These models were used to obtain transitional probabilities, steady state probabilities and mean recurrence times. Our results indicate that the pattern of price movement of Banana is similar to price movements of other fruits, in both models. The investor is encouraged to invest in the fruit market at any time in away which leads to a greater chance of getting more gain than loss.

Keywords: Markov chain, Transition probability, Steady state distribution, Ergodic chain, Mean recurrence time, Price movement.

I. Introduction

As fruit is one of the most important consumer goods for residents, the fluctuation of the prices has a direct impact on people's daily life. Especially in recent years, the continuous increase in fruit prices has drawn much attention of the government as well as the people, Salem, District has all a long been one of the Districts in the state with a credit able performance in agricultural production with the farmers relatively more responsive and receptive to changing technologies and market forces. Fruits are produced on aye around basis and a large number of farmers are involved in the production process. Fruits are being cultivated throughout the year with the help of lift irrigation from the dug wells. Uzhavarsandai marketing system has played an important role in deciding the fruit prices in Salem markets. It ensures that the farmers will get a better price for their produce and to enable the consumers to get fresh fruits at a less or price than the retail market price.

Fruits are seasonal crops, and their supply in town markets is influenced by natural factors. Due to their perishable nature, they must be quickly dispatched to markets. Fruit is the pulpy or dry ripened ovary part of a flowering plant, enclosed by the seed or seeds. Bananas, Custard Apple, Gooseberry, Guava, Mango, Mosambi, Grapes, Sapota, Papaya and Watermelon are the examples of fruits. The cultivation of fruits and its treatment are done by fruit farming. Many animals are attracted by fruits that are pulpy and contain sugar amounts and these animals then disperse the seeds of fruits to new locations. Non-fleshy fruits use different mechanisms for seed dispersal. There are some plants in which fruits can develop without fertilization. This process is called parthenocarpy, and those fruits are seedless. The wall thickens and becomes differentiated into three, more or less distinct, layers during the development of the ovary.

1.1 Importance of Fruits

An apple a day keeps the doctor away! You must have heard this classic saying and understood the importance of fruits in keeping ourselves healthy and keeping the doctors away. Fruits are wholesome food. They are rich in vitamins and other nutrients. It is almost impossible to even think of any doctor who does not recommend fruits for good health and diet. Fruits do not just have a taste, but they also have a health benefits, such as:

- Lower blood pressure
- Reduce the risk of heart disease and stroke
- Prevent some types of cancer
- Lower risk of eye and digestive problem

II. Basics of Markov chain

A random movement (or walk) is said to exhibit the Markov property if the position of the movement at time (n+1) depends only upon the position of the movement at time n. Let Y_n denote the position of the random movement at time n, then equation (1):

$$P\ (Y_{n+1} = j \mid Y_n = i) = p_{ij} \quad (1)$$

is independent of $Y_{n-1}, Y_{n-1}, \ldots, Y_0$ so that the state of Y at time (n+1) depends only upon the state Y of at time n . Here each p_{ij} for j = 1, 2,…, N is a probability row vector describing every possible transition from state i to any other existing in N possible states in the process. Then equation (2):

$$\sum_{j=1}^{N} p_{ij} = 1 \quad (2)$$

Generally, a random movement exists in N possible states in the system. Then, in chains, P (Y_{n+1} = j) will depend on the whole sequence of random variables starting with the initial value Y_0 (Jones and Smith, 2001); it leads to equation (3):

$$P\ (Y_{n+1}=j \mid Y_n= i, Y_{n-1}=i_{n-1}, \ldots, Y_0=i_0) = P(Y_{n+1}=j \mid Y_n=i) \quad (3)$$

Intuitively, one interprets equation (3) to means that, given the “present” of the process, the "future" is independent of its "past" found by Parzen E [8]. The random process of moving from one state of the system to another with the associated probabilities of each transition is known as the chain. It is said that every step taken in a chain possessing the Markov property depends only on immediately preceding step.

This expresses the fact that, if the system is in one of the states at one observed value, it will certainly be in one of the states at the next observed value. With these transition probabilities, a N×N matrix, P = (p_{ij}), called the first step transition probability matrix of the Markov chain.

$$P = (p_{ij}) = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{bmatrix}$$

Each row of is the probability distribution relating to a transition from state i to state j.

The probability vectors $p^{(n)}$ for n = 0, 1, 2 … are said to be the state vectors of a Markov chain, where ${p_i}^{(n)} = P(Y_n = i)$is the probability that the system is in the i^{th}state at the n^{th}step. In particular, the state vector $p^{(0)}$ is called the initial probability or initial state vector of the Markov chain. If P is the transition matrix and $p^{(n)}$ is the state vector at the n^{th}step, one can write,

$$p^{(n+1)} = p^{(n)}P$$

where $p^{(n+1)}$ is the state vector at the$(n + 1)^{th}$ step . From this it follows that

$$p^{(n)} = p^{(0)}P^n \quad (4)$$

i.e., the initial state vector $p^{(0)}$ and the transition matrix P determine the state vector $p^{(n)}$ at the $n^{(th)}$time point. The $n^{(th)}$step transition probabilities are called conditional Probabilities and are denoted by ${p_{ij}}^{(n)} = P(Y_{(n+k)} = j|Y_k = i)$, where i , j € {1,2,…,N} with $0 \leq P_{ij}^{(n)} \leq 1$ for n = 0,1,2,…and

$$\sum_{j=1}^{N} p_{ij}^{(n)} = 1$$

$p^{(n)} = p^{(0)}P^n$ in matrix notation for the finite state homogeneous Markov chain can be written as;

$$\left({p_1}^{(n)}, \ldots, {p_N}^{(n)}\right) = \left({p_1}^{(0)}, \ldots, {p_N}^{(0)}\right) \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{bmatrix}^n$$

Therefore, the future state vector $P^{(n)}$can be evaluated if the initial state vector and the transition matrix are known. Classifying the states of a finite Markov chain according to certain basic properties, such a classification can be based on the definitions as follows: State j is said to be accessible from state i, if j can be reached from i in a finite number of steps. If two states i and j are accessible to each other, then they are said to communicate. The period of a state i is defined as the greatest common divisor of all integers n≥1, for which $P_{ij}^{(n)}$>0. When the period is one, the state is referred to as a periodic. If all states of a chain is communicate and are a periodic, then the chain is said to be ergodic said to Bhat. U. N [2].

A chain is to have a stationary (or steady state) distribution, if there exists a vector π such that given a transition probability matrix P: π = πP
If a finite Markov chain is Ergodic then

$$\lim_{n\to\infty} p^n = \pi = \begin{bmatrix} \pi_1 & \pi_2 & \dots & \pi_N \\ \pi_1 & \pi_2 & \dots & \pi_N \\ \dots & \dots & \dots & \dots \\ \pi_1 & \pi_2 & \dots & \pi_N \end{bmatrix}$$

Where $\pi = (\pi_1, \pi_2, \dots, \pi_N)$ with $0 < \pi_j < 1$ and $\sum_{j=1}^{N} \pi_j = 1$.

This stationary probability vector π can be viewed as the unique distribution of a random variable in the long-run. Consequently expected (or mean) recurrent times μ_j are given by

$$\mu_j = \frac{1}{\pi_j} \tag{5}$$

2.1 Review of Literature

Xiaoxia Zhu and Xiuquan Xu [14] have established the prediction Markov chain model for fruit price fluctuation in China. Many scholars have conducted deep research in the rise of fruit price. Ravi shankar et. al., [11] had a study on effect of price of other seasonal fruits on Mango price in Uttar Pradesh, India. The study examines shifting consumer demand and production patterns towards fruits and vegetables, noting that horticultural commodity production now exceeds food grain production. It specifically analyzes how the prices of other seasonal fruits impact mango prices in major markets of Uttar Pradesh.

Eunice Wacu Mutahi et.al., [4] study was intended to investigate determinants of price fluctuations on fruits and vegetables. Aparna Bairagi and Sarat Kakaty [1] made Markov chain modeling for prediction on future market prices of potatoes with special reference to Nagaon district ,Assam, India. They predicted the arrival market price movement of potatoes in near future. Also found the long term price behavior by using transition probability matrix. Prasenjit et.al., [9] made a case study in price and arrivals of vegetable in different market of south west Bengal, India. They Analysed seasonal and other time series data on whole sale prices and marketing efficie ncy. Tamilselvi et. al., [13] made a study on behaviour of market arrival and price of vegetables in Koyambedu market in Chennai. Also price patterns and correlation between monthly arrivals and price were observed. Khem Chand et.al., [7] made a case study on Fruit marketing. Its efficiency and supply chain constraints in India. This research highlights significant challenges in the marketing of kinnow and aonla and offers practical recommendations to enhance the efficiency and profitability of their supply chains.

Hongyu Yang et.al., [5] had a study on the dynamic impacts of weather changes on vegetable price fluctuations in Shandong Province, China. In this article, the Granger causality test was utilized to demonstrate changes in weather factors and vegetable price fluctuations. Jhade Sunil et.al., [6] had a study on export performance and comparative analysis of Onion with reference to India. In this paper they shown that, inconsistent export, policies, rising domestic onion prices and export bans in India necessitate evaluating export competitiveness and the direction of onion exports. They also noted that the most consistent importer of Indian Onion based on first order Markov chain method. Sachin kumar Verma et.al., [12] had a study on Food Grain Trade Prospect of India .In this study underscores the importance of area expansion over productivity growth in enhancing food grain production in India. Especially the correlation analysis shows a positive relationship between area and output. Also the linear programming analysis identifies stable and unstable export markets.

Divya et.al. [3] done a research on comprehensive analysis of production and import dependency of pulses in India. In this article provides valuable insights for policy makers and industry stakeholders serve as guiding principle in making strategic decisions bolster India's role in the global agricultural sector for contributing economic growth and food security. Priyanka et.al., [10] made a agribusiness performance appraisal on export competitiveness and direction of trade of Banana from India. In this paper, the Markov chain analysis was applied to the export data to the export data and the transitional probability matrix provided a great deal of information on where to sell Indian Banana to get the highest benefits.

III. Results Data Source and Its Properties

The daily price data of Fruit at Salem Uzhavar Santhai 808 days (October 2021 –December 2023) have been collected from the websitehttps://vegetablemarketprice.com/market/salem.

Table 1: *Basic Statistics*

Fruits	*Basic Statistics*						
	Max	*Min*	*Mean*	*Median*	*Mode*	*Variance*	*SD*
Banana	*48*	*22*	*38.62*	*40*	*44*	*37.16*	*6.11*
Custardapple	*65*	*40*	*57.70*	*59*	*60*	*31.54*	*5.62*
Gooseberry	*135*	*67*	*111.61*	*115*	*105*	*222.29*	*14.89*
Guava	*56*	*35*	*45.93*	*45*	*44*	*20.65*	*4.55*
Mango	*160*	*47*	*103.54*	*100*	*145*	*1211.36*	*34.77*
Mosambi	*80*	*41*	*62.62*	*61*	*62*	*79.01*	*8.88*
Grapes	*118*	*26*	*37.00*	*31*	*29*	*489.65*	*22.11*
Sapota	*50*	*21*	*41.19*	*44*	*46*	*47.48*	*6.89*
Papaya	*49*	*18*	*26.24*	*25*	*21*	*37.41*	*6.11*
Watermelon	*28*	*11*	*19.69*	*19*	*22*	*15.10*	*3.88*

IV. Methodology

4.1 Markov Chain

An elementary form of dependence between values of X_n in successive transitions was introduced by Russian mathematician A. A.Markov and it is known as Markov dependence. Markov dependence is a form of dependence which states that X_{n+1} depends only on $X_n$when it is known and is independent of $X_{n-1}, X_{n-2}, \ldots, X_0$. This implies that the future of the process depends only on the present, irrespective of the past. This property is known as Markov property. In probabilities terms, the Markov property can be stated as follows:

$$P\,[X_{n+1} = i_{n+1} | X_n = i_n, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0] = P\,[X_{n+1} = i_{n+1} | X_n = i_n] \quad (6)$$

For all states $i_0, i_1, \ldots, i_{n+1}$ and for all n, this is called Markov dependence of the first order.

4.2 Transition Probability Matrix

The state transition probability matrix of a Markov chain gives the probabilities of transitioning from one state to another in a single time unit. It will be useful to extend this concept to longer time intervals.

The square matrix P consisting of the elements $p_{ij}^{(1)}$ for all possible states i and j is called one – step transition probability matrix of the chain. Therefore

$$P = [p_{ij}^{(1)}] = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

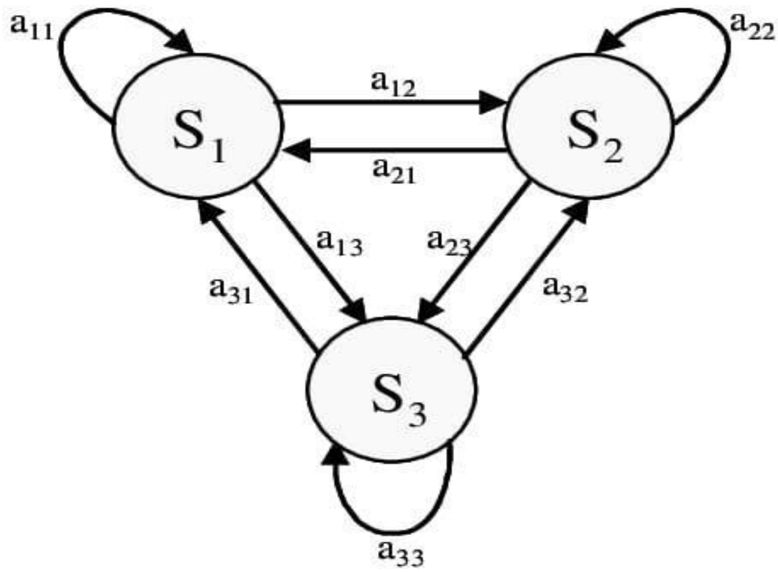

Figure 1: *Three State Transition Probability Matrix*

Similarly, the square matrix $P^{(m)}$ consisting of the elements $p_{ij}^{(m)}$ for all possible values of the states i and j is called them m – step TPM of the chain. Hence, $p^{(m)} = [p_{ij}^{(m)}]$

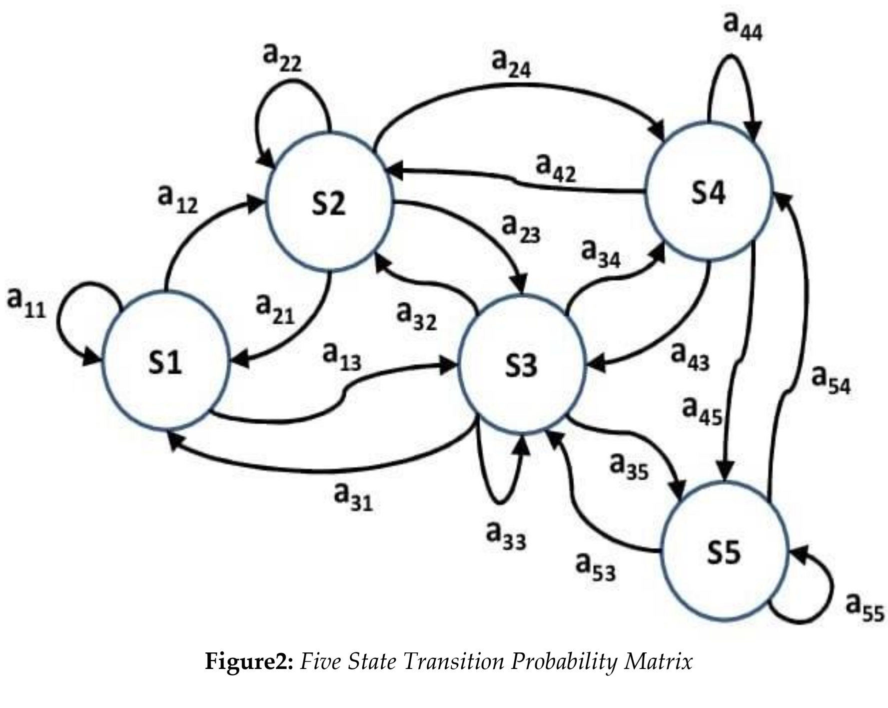

Figure2: *Five State Transition Probability Matrix*

4.3 Chapman – Kolmogorov Equation

We have, considered unit-step or one-step transition probabilities the probability of X_n given X_{n-1}. i.e. the probability of the outcome at the nth step or trial given the outcome at the previous step; P_{jk} gives the probability of unit - step transition from the state j at a trial to the state k at the next following trial. The m-step transition probability is denoted by $P_r\{X_{m+n}$=k | X_n= j}=$P_{jk}^{(m)}$; $P_{jk}^{(m)}$gives the probability that from the state j at n^th trial, the state k is reached at (m+n)^th trial in m steps, i.e. the probability of transition from the state j to the state k in exactly m steps. The number 'n' does not occur in the RHS of the relation and the chain is homogeneous. The one-step transition probabilities $P_{jk}^{(1)}$are denoted byP_{jk}. The state k can be reached from the state j in two steps through some intermediate state r. Consider a fixed value of r, we have

$$P\ (X_{n+2}= k.\ X_{n+1}= r \mid X_n=j) = P_{rk}^{(1)} P_{jr}^{(1)} = P_{jr} P_{rk} \quad (7)$$

Since these intermediate states r can assume values r=1, 2…, we have

$$P_{jk}^{(2)} = \Pr\{X_{n+2}=k \mid X_n=j\} = \sum_r Pr\{X_{n+2}=k,\ X_{n+1}= r \mid X_n=j\} = \sum_r P_{jr}\, P_{rk} \quad (8)$$

(summing over for all the intermediate states)
Similarly, we get

$$P_{jk}^{(m+1)} = \sum_r P_{jr} P_{rk}^{(m)} \quad (9)$$

In general,

$$P_{jr}^{(m+n)} = \sum_r P_{rk}^{(n)} P_{jr}^{(m)} = \sum_r P_{jr}^{(n)} P_{rk}^{(m)} \quad (10)$$

This equation is a special case of Chapman - Kolmogorov equation, which is satisfied by the transition probabilities of a Markov chain, $P_{jr}^{(m+n)} \geq P_{jr}^{(m)} P_{rk}^{(n)}$.

Let P = P_{jk} denote the transition matrix of the unit - step transitions and $P^{(m)}$= ($P_{jr}^{(m)}$) denoted the transition matrix of the m - step transitions. For m=2, we have the matrix $P^{(2)}$whose elements are given. It follows that the elements of $P^{(2)}$are the elements of the matrix obtained by multiplying the matrix P by itself, i.e, $P^{(2)}$= P . P = P^2 . Similarly, $P^{(m+1)}$ = P. P^m and $P^{(m+n)}$= P^m . P^n

It should be noted that there exist non – Markovian chain whose transition probabilities satisfy Chapman – Kolmogorov equation.

4.4 Construction of Models

The daily market prices of 10 different types of fruits namely Banana , Custard Apple , Gooseberry, Guava, Mango, Mosambi, Papaya, Sapota, Grapes, Watermelon at https://vegetablemarketprice.com/fruits/tamilnadu/todaywere used in this modeling study. The period February 1st 2022 to November 10th 2023 was chosen in the estimation of the model, but the chain took into account the behavior of the market price for consecutive days, each classified as increase or decrease or remains the same. The data was set up into two types of models and studied separately. One of the main assumptions in Markov chain is stationarity. Let $X_n$denote the price of fruits at the market during the n^{th} day. Then the random variable Z_n is defined as:

$$Z_n = X_n - X_{n-1} \quad (11)$$

Here fruit price changes are taken as the first order of the chain since the nature of the perishable and particular fruit cannot permit selling after a day. Therefore, the first order change is only appropriate for study related to fruit markets. So that, for this study, two models of Markov chain analysis will be considered as follows:

4.5 Construction of Model-I

Each day was classified as having price value higher than or lower than or within "a "rupees from the previous day for this experiment considering the movement from a category of large gain, or small gain/loss, or large loss, let the classification of three states, namely:

- State 1(large gain): Today's price value is more than "a" rupees than the price value of the previous day;
- State 2 (small gain /loss): Today's price value change is within or equal to "a" rupees v is-a-v is the price value of the previous day;
- State 3(large loss): Today's price value is lower than "a" rupees than the price value of the previous day.

The state of this system may be able to for matrinary random variable denoted by

$$Y_n = \begin{cases} 0, & if\ Z_n > a, \\ 1, & if\ |Z_n| \leq a, \\ 2, & if\ Z_n < -a. \end{cases}$$

The random variable $\{Y_n\}$ defined by above equation is known as a Markov chain with state space {0, 1, 2}. Here "a" is treated as threshold value of absolute price changes. There is no unique way of determining the threshold in this nature of studies. Generally, researchers used common parameters like measures of central tendency to find the threshold point. In this study, the main focus is only on the changes of the price pattern not on the price values. Here threshold value was fixed by determining the absolute median of the market daily changes of each fruits separately.

4.6 Construction of Model-II

Each day was classified as having price value higher or lower or within "a" than the previous day for this experiment considering the movement from a category of highest gain (or) lowest gain (or) no gain/no loss (or) lowest loss (or) highest loss to the next day, thus letting classification of five states namely:

- State 1 (Highest gain): Today's price value change is greater than "a" rupees than the price value of the previous day.
- State 2 (Lowest gain): Today's price value change is lower than "a" rupees than the price value of the previous day.
- State 3 (No gain / No loss): Today's price value change is equal to "0" rupees than the price value of the previous day.
- State 4 (Lowest loss): Today's price value change is greater than"-a" rupees than the price value of the previous day.
- Step 5(Highest loss): Today's price value change is lower than "-a" rupees than the price value of the previous day.

$$
Y_n = \begin{cases} 0, & if\ Z_n < a \\ 1, & if\ Z_n > a \\ 2, & if\ Z_n = a \\ 3, & if\ Z_n < -a \\ 4, & if\ Z_n > -a \end{cases}
$$

The random variable $\{Y_n\}$ defined by above equation is known as a Markov chain with state space {0, 1, 2, 3, 4}. Here "a" is treated as threshold value of absolute price changes. There is no unique way of determining the threshold in this nature of studies. Generally, researchers used common parameters like measures of central tendency to find the threshold point. In this study, the main focus is only on the changes of the price pattern not on the price values. Here threshold value was fixed by determining the absolute median of the market daily changes of each fruits separately.

V. Result and Discussion

Probability values for each vector movement of the Model-I is described as follows:

Table 2: *π and μ Values for Model–I*

Model-I	
Banana State 0, 1, 2: $\begin{bmatrix} 0 & 0.9756 & 0.0243 \\ 0.1659 & 0.7261 & 0.1078 \\ 0.0370 & 0.9629 & 0 \end{bmatrix}$ $\pi = [0.1325\ \ 0.7796\ \ 0.0875]$ $\mu = [7.5471\ \ 1.2827\ \ 11.4285]$	*Custard Apple* State 0, 1, 2: $\begin{bmatrix} 0 & 0.9878 & 0.0121 \\ 0.4347 & 0.4239 & 0.1413 \\ 0.0217 & 0.9782 & 0 \end{bmatrix}$ $\pi = [0.2760\ \ 0.6305\ \ 0.0926]$ $\mu = [3.6231\ \ 1.5860\ \ 10.7991]$
Gooseberry State 0, 1, 2: $\begin{bmatrix} 0 & 0.9726 & 0.0273 \\ 0.1540 & 0.7004 & 0.1455 \\ 0 & 0.9726 & 0.0273 \end{bmatrix}$ $\pi = [0.1177\ \ 0.7644\ \ 0.1177]$ $\mu = [8.4961\ \ 1.3082\ \ 8.4961]$	*Guava* State 0, 1, 2: $\begin{bmatrix} 0.0123 & 0.9876 & 0 \\ 0.1767 & 0.6241 & 0.1991 \\ 0.0111 & 0.9777 & 0.0111 \end{bmatrix}$ $\pi = [0.1309\ \ 0.7228\ \ 0.1456]$ $\mu = [7.6394\ \ 1.3835\ \ 6.8681]$
Mango State 0, 1, 2: $\begin{bmatrix} 0.0111 & 0.9666 & 0.0222 \\ 0.2027 & 0.5990 & 0.1981 \\ 0 & 1 & 0 \end{bmatrix}$ $\pi = [0.1474\ \ 0.7102\ \ 0.1422]$ $\mu = [6.7842\ \ 1.4080\ \ 7.0323]$	*Mosambi* State 0, 1, 2: $\begin{bmatrix} 0 & 1 & 0 \\ 0.1834 & 0.6241 & 0.1923 \\ 0.0229 & 0.9655 & 0.0114 \end{bmatrix}$ $\pi = [0.1358\ \ 0.7229\ \ 0.1408]$ $\mu = [7.3637\ \ 1.3833\ \ 7.1022]$
Grapes State 0, 1, 2: $\begin{bmatrix} 0 & 0.9770 & 0.0229 \\ 0.1914 & 0.6193 & 0.1891 \\ 0.0229 & 0.9655 & 0.0114 \end{bmatrix}$ $\pi = [0.1406\ \ 0.7181\ \ 0.1409]$ $\mu = [7.1123\ \ 1.3925\ \ 7.0972]$	*Sapota* State 0, 1, 2: $\begin{bmatrix} 0 & 0.975 & 0.025 \\ 0.1741 & 0.6294 & 0.1964 \\ 0.0222 & 0.9777 & 0 \end{bmatrix}$ $\pi = [0.1293\ \ 0.7245\ \ 0.1455]$ $\mu = [7.7339\ \ 1.3802\ \ 6.8728]$

Papaya	*Watermelon*
$\begin{matrix} State & 0 & 1 & 2 \\ 0 & 0 & 0.9888 & 0.0111 \\ 1 & 0.1982 & 0.6280 & 0.1737 \\ 2 & 0.0126 & 0.9873 & 0 \end{matrix}$	$\begin{matrix} State & 0 & 1 & 2 \\ 0 & 0.0298 & 0.9552 & 0.0149 \\ 1 & 0.1371 & 0.7046 & 0.1582 \\ 2 & 0 & 0.9870 & 0.0129 \end{matrix}$
π = [0.1455 0.7263 0.1279]	π = [0.3840 0.5289 0.0859]
μ = [6.8728 1.3768 7.8186]	μ = [2.6041 1.8971 11.6414]

First vector (0, 0.9756, 0.0243), indicates that if a given day has increased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 0%,97.5%,2.43% respectively. Second vector (0.1659, 0.7261, and 0.1078) indicates that if a given day remains within the threshold limits of price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 16.5%, 72.6%, 10.7% respectively. The last vector (0.0370, 0.9629, and 0) indicates that if a given day has decreased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 3.7%, 96.2%, 0% respectively.

Based on the probability matrix P_1, all states are communicated and a periodic, so that it is an ergodic chain. Hence, π = [0.1325 0.7796 0.0875] is interpreted as banana's price movement among categories is being a Markov chain with transition probability matrix P_1, after many days, consist of the following proportions in each category: 13.2% in the state-0, 77.9% in the state-1, and 8.7% in the state-2. Further, the mean recurrence time of the corresponding states 0, 1and 2 are [7.5471 1.28271 1.4285] days respectively.

In a similar manner, remaining fruits: Custard Apple, Gooseberry, Guava, Mango, Mosambi, Grapes, Sapota, Papaya, Watermelon results are estimated and which are presented in Table, consist of the transition probability matrix, long-run invariant distribution and mean recurrence time for each model separately.

For the second model, the transition matrix was found to be:

$$P_2 = \begin{matrix} State & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0.9821 & 0.0178 & 0 \\ 1 & 0.0285 & 0 & 0.9142 & 0.0285 & 0.0285 \\ 2 & 0.1172 & 0.0774 & 0.6460 & 0.0796 & 0.0796 \\ 3 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0.0540 & 0 & 0.9459 & 0 & 0 \end{matrix}$$

$$\pi = [0.0905 \quad 0.0566 \quad 0.7315 \quad 0.0614 \quad 0.0598]$$

$$\mu = [11.0497 \quad 17.6678 \quad 1.3605 \quad 16.2866 \quad 16.7224]$$

Probability values, for each vector movement of the Model 2is described as follows:

First vector (0, 0, 0.9821, 0.0178, 0), indicates that if a given day has increased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 0%, 0%, 98.2% ,1.7% and 0% respectively. Second vector (0.0285, 0, 0.9142, 0.0285, 0.0285), indicates that if a given day remains within the threshold limits of price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 2.8%, 0%, 91.4%,2.8% and 2.8% respectively. The third vector (0.1172, 0.0774, 0.6460, 0.0796 and 0.0796), indicates that if a given day has decreased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 11.7%, 7.7%, 64.6%, 7.9% and 7.9% respectively.

The fourth vector (0, 0, 1, 0, 0), indicates that if a given day has decreased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 0%,0%,1%, 0% and 0%respectively. The last vector (0.0540, 0, 0.9459, 0, 0), indicates that if a given day has decreased price value, then the next day's price will increase or remain within the threshold limits or decrease in the following percentages 5.4 0%, 94.5%, 0% and 0%respectively.

Based on the probability matrix P_2, all states are communicated and a periodic, so that it is an ergodic chain. Hence, π = [0.0905 0.0566 0.7315 0.0614 0.598] is interpreted as banana's price movement among categories is being a Markov chain with transition probability matrix P_2, after many days, consist of the following proportions in each category:9.0%in the state-0,5.6% in the state1, 73.1% in the state-2, 6.1% in the state-3, 5.9% in the state-4. Further, the mean recurrence time of the corresponding states 0, 1, 2, 3 and 4 are [11.0497 17.667 81.3605 16.2866 16.7224] days respectively.

In a similar manner, remaining fruits: Custard Apple, Gooseberry, Guava, Mango, Mosambi, Papaya, Sapota, Grapes, Watermelon results are estimated and which are presented in Table3, consist of the transition probability matrix, long-run invariant distribution and mean recurrence time for each model separately.

Table 3: *π and μ Values for Model–II*

Model-II	
Banana State: 0 1 2 3 4 $\begin{bmatrix} 0 & 0 & 0.9821 & 0.0178 & 0 \\ 0.0285 & 0 & 0.9142 & 0.0285 & 0.0285 \\ 0.1172 & 0.0774 & 0.6460 & 0.0796 & 0.0796 \\ 0 & 0 & 1 & 0 & 0 \\ 0.0540 & 0 & 0.9459 & 0 & 0 \end{bmatrix}$ *π = [0.1325 0.7796 0.0875]* *μ = [7.5471 1.2827 11.4285]*	*Custard Apple* State: 0 1 2 3 4 $\begin{bmatrix} 0 & 0 & 0.9870 & 0 & 0.0129 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1689 & 0.0405 & 0.6148 & 0.0630 & 0.1126 \\ 0 & 0 & 1 & 0 & 0 \\ 0.0392 & 0 & 0.9607 & 0 & 0 \end{bmatrix}$ *π = [0.1245 0.0291 0.7186 0.0452 0.0824]* *μ = [8.0321 34.3642 1.3915 22.1238 12.1359]*
Gooseberry State: 0 1 2 3 4 $\begin{bmatrix} 0 & 0 & 0.9726 & 0 & 0.0273 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1540 & 0.0042 & 0.7004 & 0.0021 & 0.1392 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ *π = [0.1181 0.0032 0.7670 0.0016 0.1099]* *μ = [8.4674 312.5 1.3037 625 9.0991]*	*Guava* State: 0 1 2 3 4 $\begin{bmatrix} 0.0138 & 0 & 0.9861 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1543 & 0.0581 & 0.6241 & 0.0715 & 0.0917 \\ 0 & 0 & 1 & 0 & 0 \\ 0.0487 & 0 & 0.9512 & 0 & 0 \end{bmatrix}$ *π = [0.1136 0.0421 0.725 0.0051 0.0664]* *μ = [8.8028 23.7529 1.3793 196.0784 15.0602]*
Mango State: 0 1 2 3 4 $\begin{bmatrix} 0.0120 & 0 & 0.9879 & 0 & 0 \\ 0 & 0 & 0.8888 & 0.0555 & 0.0555 \\ 0.1867 & 0.0410 & 0.5990 & 0.0273 & 0.1457 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ *π = [0.1342 0.0291 0.7105 0.0209 0.1050]* *μ = [7.4515 34.3642 1.4074 47.8468 9.5238]*	*Mosambi* State: 0 1 2 3 4 $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1454 & 0.0648 & 0.6241 & 0.0626 & 0.1029 \\ 0 & 0 & 0.9642 & 0 & 0.0357 \\ 0.0425 & 0 & 0.9574 & 0 & 0 \end{bmatrix}$ *π = [0.1049 0.0454 0.7007 0.0438 0.0736]* *μ = [9.5328 22.0264 1.4271 22.8310 13.5869]*
Grapes	*Sapota*

State	0	1	2	3	4
0	0	0.0188	0.9622	0	0.0188
1	0	0	0.9777	0.0222	0
2	0.1171	0.0990	0.6193	0.0923	0.0720
3	0.0232	0	0.9534	0.0232	0
4	0	0	1	0	0

π = [0.0857 0.0727 0.7189 0.0695 0.0530]

μ = [11.6686 13.7551 1.3910 14.3884 18.8679]

State	0	1	2	3	4
0	0.0151	0	0.9696	0	0.0151
1	0	0	1	0	0
2	0.1406	0.0647	0.6294	0.0915	0.0736
3	0.0243	0	0.9756	0	0
4	0.0294	0	0.9705	0	0

π = [0.1066 0.0469 0.7251 0.0663 0.0549]

μ = [9.3808 21.3219 1.3791 15.0829 18.2149]

Papaya

State	0	1	2	3	4
0	0	0	1	0	0
1	0	0	0.9705	0	0.0294
2	0.1380	0.0757	0.6280	0.0556	0.1024
3	0	0	1	0	0
4	0.0212	0	0.9787	0	0

π = [0.1018 0.0550 0.7266 0.0403 0.0760]

μ = [9.8231 18.1818 1.3762 24.8138 13.1578]

Watermelon

State	0	1	2	3	4
0	0	0	0.9729	0	0.0270
1	0.0256	0.0256	0.9487	0	0
2	0.0759	0.0801	0.7046	0.1181	0.0210
3	0	0	0.9824	0.0175	0
4	0	0	1	0	0

π = [0.0596 0.0630 0.7672 0.0922 0.0177]

μ = [16.7785 15.8730 1.3034 10.8459 56.4971]

Notice that for model II, in the entire transition matrix each row vector contains a more likelihood of probabilities in the first column; indicate that irrespective of what state a day occupies, there is a high probability that the next day will be a day of gain. This is encouraging for investors since the price movement will not be down in a sustained manner, but more likely high.

Conclusion

This paper presents a Markov chain modeling of daily fruit price movement in Salem District, a pioneering study in this context. The method used is valuable for analyzing and predicting price movements in the market. The study's results indicate that the daily price movement pattern for each fruit follows a similar Markov chain model, showing a higher likelihood of gains than consecutive days of loss, leading to slow and steady growth.

However, the functionality of the Salem market is influenced by various factors such as daily demand and investor psychology, making it challenging for any single method, including the Markov chain, to accurately predict daily price changes. While the analysis results are promising, the estimates of mean recurrent times may have limited validity. The study demonstrates that a regional market approach yields better results and suggests that a similar framework could be applied to other markets in the region. Although the current model is used to predict price movement patterns, it can be further developed to forecast the magnitude of price movements.

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