NEW GENERALIZATION OF INVERTED EXPONENTIAL DISTRIBUTION: PROPERTIES AND ITS APPLICATIONS

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Abstract

In this paper, we introduce a new extension of the inverted exponential distribution called as "SMP Inverted Exponential" (SMPIE) distribution through the SMP technique. Various statistical properties of this new distribution have been illustrated, including survival function, hazard function, quantile function, moments, moment generating function, entropy, and order statistics. Method of maximum likelihood estimation is used to evaluate the parameters of the proposed distribution. A simulation study is carried out for illustration of the performance of estimates. Two real-life data sets are incorporated to illustrate the utility and flexibility of the proposed distribution as compared to other existing probability distributions.

Keywords: SMP transformation, inverted exponential, order statistics, Maximum likelihood estimation.

1. Introduction

In Probability theory and statistics, the exponential distribution is a continuous probability distribution that describes the time between events in the Poisson process i.e. a process in which events occur independently and continuously at a constant failure rate. The exponential distribution possesses the constant and bathtub hazard rates. But in real-life problems, there may be a situation where the data shows the inverted bathtub hazard rate (initially increases and then decreases, i.e., uni-modal). So, the exponential distribution becomes unfit for modeling real-life situations. For such data types, another extension of the exponential distribution has been put forward in the statistical literature. That is known as inverted exponential distribution (IED) which possesses the inverted bathtub hazard rate. The inverted exponential distribution was introduced by [5] and has been used by [7] as a lifetime model. It is widely used in biology, medicine, and engineering. Several authors have proposed distributions using inverted exponential (IE) distribution in Statistical literature. The generalized inverted exponential distribution was proposed by [1]. The Bayes estimators of the parameter and reliability function of inverted exponential distribution were obtained by [15]. Exponentiated generalized inverted exponential distribution was proposed by [11]. The exponentiated inverted exponential distribution was proposed by [4]. The alpha power inverted exponential distribution was introduced by [16]. A new Weibull inverted IE distribution was obtained by [3]. The Weibull inverse exponential[Loglogistic] distribution was derived by [8]. New Sine Inverted Exponential distribution was proposed by [2]. The probability density function (PDF) of the IE distribution is given as follows:

$$f(x) = \frac{\lambda}{x^2} e^{\frac{-\lambda}{x}}; \quad x > 0, \lambda > 0, \tag{1}$$

and corresponding cumulative distribution function (CDF) as follows:

$$F(x) = e^{\frac{-\lambda}{x}}; \quad x > 0, \lambda > 0. \tag{2}$$

Where λ is a scale parameter and is greater than 0.
In this paper, we proposed a new extension of inverted exponential (IE) by using the SMP technique. The proposed model is named as SMP inverted exponential distribution (SMPIE). The primary motivation for considering SMP Inverted exponential (SMPIE) distribution is that the proposed model is very efficient and flexible for introducing a new parameter to generalize the existing distributions. The additional parameter can give various desirable properties and is more flexible in the form of hazard and density functions and demonstrates a superior fit compared to other competing models. The rest of this paper is organized as follows: In section 2, we defined the SMP Transformation. In section 3, the PDF and CDF of the proposed model i.e. SMPIE are defined. Section 4 includes the Reliability measures of the model. in section 5, we derive some mathematical properties. Order statistics are investigated in section 6. Maximum likelihood estimation of the model parameters is addressed in section 7. The simulation study and the applicability of the model is discussed in section 8 and 9 respectively. Finally, a concluding remark are addressed in Section 10.

2. SMP Transformation

The SMP transformation was recently proposed by [13] whose CDF and PDF are given by the following equations respectively.

$$G_{SMP}(x) = \begin{cases} \frac{e^{\log(\alpha)\overline{F}(x)} - \alpha}{1-\alpha}, & \alpha \neq 1, \alpha > 0 \\ F(x), & \alpha = 1 \end{cases} \tag{3}$$

where $\overline{F}(x) = 1 - F(x)$
$G_{SMP}(x)$ is a valid CDF, if F(x) is a valid CDF. This is because it satisfies the following properties:

(i) $G_{SMP}(-\infty) = 0$; $G(\infty) = 1$

(ii) $G_{SMP}(x)$ is monotonic increasing function of x

(iii) $G_{SMP}(x)$ is right continous

(iv) $0 \leq G_{SMP}(x) \leq 1$

For $x \in \mathbb{R}$, the PDF of SMP transformation is given as follows:

$$g_{SMP}(x) = \begin{cases} \frac{e^{\log(\alpha)\overline{F}(x)} \log(\alpha) f(x)}{\alpha - 1}, & \alpha \neq 1, \alpha > 0 \\ f(x) \quad , & \alpha = 1 \end{cases} \tag{4}$$

where F(x) and f(x) are the CDF and PDF of the baseline distribution respectively.

3. Smpie Distribution

A random variable X is said to follow SMPIE distribution with scale parameter $\lambda > 0$ and shape parameter $\alpha > 0$, if its CDF is given as

$$G_{SMPIE}(x; \alpha, \lambda) = \begin{cases} \frac{e^{(\log \alpha)(1 - e^{\frac{-\lambda}{x}})} - \alpha}{1-\alpha}, & \alpha \neq 1, \alpha > 0 \\ e^{\frac{-\lambda}{x}} \quad , & \alpha = 1 \end{cases} \tag{5}$$

Figure (1) displays the CDF plot of the SMPIE distribution for different parameter values of α and λ.

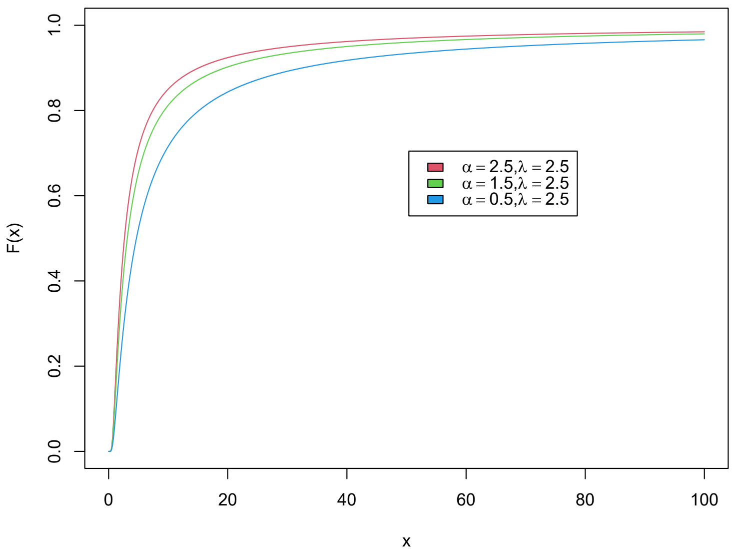

Figure 1: *CDF plot for SMPIE distribution*

The corresponding PDF of SMPIE distribution is given as

$$g_{SMPIE}(x;\alpha,\lambda) = \begin{cases} \frac{\log(\alpha)}{\alpha-1}\frac{\lambda}{x^2}e^{(\log\alpha)(1-e^{\frac{-\lambda}{x}})}e^{\frac{-\lambda}{x}}, & \alpha \neq 1, \alpha > 0 \\ \frac{\lambda}{x^2}e^{\frac{-\lambda}{x}}, & \alpha = 1 \end{cases} \tag{6}$$

Figure (2) displays the pdf plot of the SMPIE distribution for different parameter values.

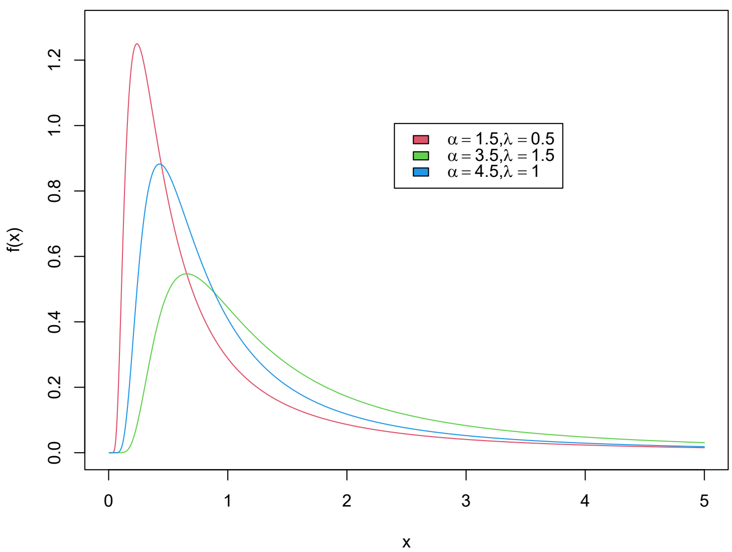

Figure 2: *PDF plot for SMPIE distribution*

4. Relaiability analysis

In this section, we obtain the reliability (survival function), hazard rate (failure rate), reverse hazard function and mills ratio expressions for SMPIE.

4.1. Reliability function

The reliability function is the probability that an item does not fail before time say x and for SMPIE distribution, it is given as

$$R(x;\alpha,\lambda) = 1 - G_{SMPIE}(x;\alpha,\lambda) = \frac{1 - e^{(\log\alpha)(1-e^{\frac{-\lambda}{x}})}}{1-\alpha}, \quad \alpha \neq 1 \tag{7}$$

4.2. Hazard Rate

The hazard rate or failure rate accesses the likelihood of component's failure failure based on how long it has already been in use. Consequently, it has various applications in the analysis of lifetime distributions. The expression for the hazard rate of SMPIE is obtained as

$$h(x;\alpha,\lambda) = \frac{g_{SMPIE}(x;\alpha,\lambda)}{R(x;\alpha,\lambda)} = \frac{\lambda}{x^2}\log\alpha \frac{e^{(\log\alpha)(1-e^{\frac{-\lambda}{x}})}e^{\frac{-\lambda}{x}}}{e^{(\log\alpha)(1-e^{\frac{-\lambda}{x}})} - 1} \tag{8}$$

Figure (3) depicts graphs of the hazard rate of the SMPIE distribution for different parameter combinations. It shows that the hazard function exhibits a unimodal increasing, decreasing and constant shapes for different parameter combination. This implies that the SMPIE distribution can be used to describe real life phenomenon with unimodal failure rates.

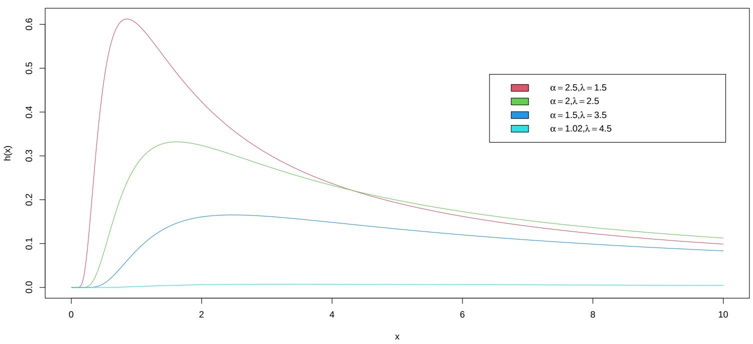

Figure 3: *Hazard plot for SMPIE distribution*

4.3. Reverse Hazard Rate

The concept of reverse hazard rate of a random variable is defined as the ratio between the life probability density to its distribution function and is obtained as

$$h_r(x;\alpha,\lambda) = \frac{g_{SMPIE}(x;\alpha,\lambda)}{G_{SMPIE}(x;\alpha,\lambda)} = \frac{\lambda}{x^2}\log\alpha \frac{e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})}e^{-\frac{\lambda}{x}}}{\alpha - e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})}} \tag{9}$$

4.4. Mills Ratio

The mills ratio for the SMPIE is defined as

$$M.R = \frac{G_{SMPIE}(x;\alpha,\lambda)}{R_{SMPIE}(x;\alpha,\lambda)} = \frac{e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})} - \alpha}{1 - e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})}} \tag{10}$$

4.5. Quantile function

Theorem 1. If $X \sim SMPIE(\alpha, \lambda)$ distribution, then the quantile function of X is given as

$$x = \frac{-\lambda}{\log\left[1 - \frac{\log\{u(1-\alpha)+\alpha\}}{\log\alpha}\right]} \tag{11}$$

where u is a uniform random variable, $0 < u < 1$
Proof. Let $G_{SMPIE}(x;\alpha,\lambda) = u$. The quantile function of SMPIE distribution can be obtained as follows.

$$\Rightarrow \frac{e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})} - \alpha}{1-\alpha} = u$$

$$\Rightarrow e^{(\log\alpha)(1-e^{-\frac{\lambda}{x}})} = u(1-\alpha) + \alpha$$

Taking logarithm on both sides and simplifying further, we obtain the required quantile function as

$$x = \frac{-\lambda}{\log\left[1 - \frac{\log\{u(1-\alpha)+\alpha\}}{\log\alpha}\right]} \tag{12}$$

Where u follows a uniform (0,1) distribution. The qth quantile function of SMPIE distribution is given by

$$x_q = \frac{-\lambda}{\log\left[1 - \frac{\log\{u(1-\alpha)+\alpha\}}{\log\alpha}\right]}$$

The median can be obtained as

$$x_{0.5} = \frac{-\lambda}{\log\left[1 - \frac{\log\{0.5(1-\alpha)+\alpha\}}{\log\alpha}\right]}$$

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5. Statistical properties of smpie distribution

Some of the statistical properties of SMPIE will be discussed in this section.

5.1. Moments

The r^{th} moment for SMPIE distribution can be obtained as

$$\mu_r{'} = E(x^r) = \int_0^\infty x^r g_{SMPIE}(x;\alpha,\lambda)\,dx$$

$$E(x^r) = \frac{\sum_{k=0}^{j}\sum_{j=0}^{\infty}(-1)^k\binom{j}{k}(\log\alpha)^{j+1}}{\alpha-1}\int_0^\infty x^r\left(e^{-\frac{\lambda}{x}}\right)^{k+1}\frac{\lambda}{x^2}\,dx$$

$$\Rightarrow \mu_r' = \frac{\sum_{k=0}^{j}\sum_{j=0}^{\infty}(-1)^k\binom{j}{k}(\log\alpha)^{j+1}}{\alpha-1}\lambda^r(k+1)^{r-1}\gamma(1-r) \tag{13}$$

5.2. Harmonic Mean

Harmonic mean is the reciprocal of the arithmetic mean of the reciprocals of the data values. The harmonic mean (H) of SMPIE is given as:

$$\frac{1}{H} = E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{x} g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$\frac{1}{H} = \frac{1}{\alpha - 1} \sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \lambda \int_0^\infty \frac{1}{x^{2+1}} \left(e^{-\frac{\lambda}{x}}\right)^{k+1} dx$$

$$\frac{1}{H} = \frac{1}{\lambda(\alpha - 1)} \sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \frac{1}{(k+1)^2} \tag{14}$$

5.3. Moment Generating Function of SMPIE

Theorem 2. Let X follows the SMPIE distribution, then the moment generating function, $M_X(t)$ of SMPIE distribution is given as

$$M_x(t) = \frac{1}{\alpha - 1} \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{t^r}{r!} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \lambda^r (k+1)^{r-1} \gamma(1-r) \tag{15}$$

Proof. The moment-generating function of SMPIE distribution is defined as

$$M_x(t) = \int_0^\infty e^{tx} g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$M_x(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \dots\right) g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^\infty x^r g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$\Rightarrow M_x(t) = \frac{1}{\alpha - 1} \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{t^r}{r!} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \lambda^r (k+1)^{r-1} \gamma(1-r) \tag{16}$$

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5.4. Characteristic Function of SMPIE distribution

Theorem 3. Let X follows the SMPIE distribution, then the characteristic function, $\phi_X(t)$ of SMPIE distribution is given as

$$\phi_x(t) = \frac{1}{\alpha - 1} \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{j} (-1)^k \frac{(it)^r}{r!} \binom{j}{k} (\log \alpha)^{j+1} \lambda^r (k+1)^{r-1} \gamma(1-r)$$

Proof. The characteristic function of SMPIE distribution is defined as

$$\phi_x(t) = \int_0^\infty e^{itx} g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$\phi_x(t) = \int_0^\infty \left(1 + itx + \frac{(itx)^2}{2!} + \dots\right) g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \int_0^\infty x^r g_{SMPIE}(x; \alpha, \lambda)\, dx$$

$$\Rightarrow \phi_x(t) = \frac{1}{\alpha - 1} \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{(it)^r}{r!} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \lambda^r (k+1)^{r-1} \gamma(1-r) \tag{17}$$

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Lemma 1. Let us suppose a random variable X follows SMPIE (α, λ) with PDF given in equation (6) and let $I_r(t) = \int_0^t x^r g_{SMPIE}(x; \alpha, \lambda) dx$ denotes the r^{th} incomplete moment, then we have

$$I_r(t) = \frac{\sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} (\log \alpha)^{j+1}}{\alpha - 1} \lambda^r \gamma(1 - r, \lambda/t) \tag{18}$$

where $\gamma(a, b) = \int_b^\infty z^{a-1} e^{-z}\, dz$ denotes the upper incomplete gamma function.
Proof. Using the PDF of SMPIE given in equation (6), we have

$$I_r(t) = \frac{1}{\alpha - 1} \sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} (\log \alpha)^{j+1} \int_0^t x^r \left(e^{-\frac{\lambda}{x}}\right)^{k+1} \frac{\lambda}{x^2}\, dx \tag{19}$$

Using substitution ,$\frac{\lambda}{x} = z$ in equation (19), we get

$$I_r(t) = \frac{\sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} (\log \alpha)^{j+1}}{\alpha - 1} \lambda^r \gamma(1 - r, \lambda/t) \tag{20}$$

5.5. Renyi Entropy

The entropy of a random variable is defined as the measure of uncertainty. The Renyi entropy given by [14] is defined as,

$$I_v = \frac{1}{1 - v} \log \int_0^\infty g^v(x)\, dx$$

Using PDF given in Equation (6), we have

$$I_v = \frac{1}{1 - v} \log \left(\frac{\lambda \log \alpha}{\alpha - 1}\right)^v \int_0^\infty \sum_{j=0}^{\infty} \frac{v^j (\log \alpha)^j}{j!} \left(1 - e^{-\frac{\lambda}{x}}\right)^j e^{-\frac{\lambda v}{x}} \frac{1}{x^{2v}}\, dx$$

$$I_v = \frac{1}{1 - v} \log \left(\frac{\lambda \log \alpha}{\alpha - 1}\right)^v \sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} \frac{v^j (\log \alpha)^j}{j!} \int_0^\infty \left(e^{-\frac{\lambda}{x}}\right)^{k+v} \frac{1}{x^{2v}}\, dx$$

$$I_v = \frac{1}{1 - v} \log \left(\frac{\lambda \log \alpha}{\alpha - 1}\right)^v \sum_{k=0}^{j} \sum_{j=0}^{\infty} (-1)^k \binom{j}{k} \frac{v^j (\log \alpha)^j}{j!} (\lambda(k+1))^{1-2v} \gamma(2v - 1)$$

which is required expression of Renyi entropy for SMPIE distribution.

6. Order Statistics

Let $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$ be the random sample of size n and let $X_{i:n}$ denote the i^{th} order statistics, then the PDF of $X_{i:n}$ is given by

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} F(x)^{i-1} (1 - F(x))^{n-i} f(x) \tag{21}$$

Using equation (5) we have,

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} \left(\frac{e^{(\log \alpha)(1 - e^{-\frac{\lambda}{x}})} - \alpha}{1 - \alpha}\right)^{i-1} \left(1 - \frac{e^{(\log \alpha)(1 - e^{-\frac{\lambda}{x}})} - \alpha}{1 - \alpha}\right)^{n-i} g_{SMPIE}(x; \alpha, \lambda)$$

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!(1-\alpha)^{n-1}} \left(e^{(\log \alpha)(1 - e^{-\frac{\lambda}{x}})} - \alpha\right)^{i-1} \tag{22}$$
$$\times \left(1 - e^{(\log \alpha)(1 - e^{-\frac{\lambda}{x}})}\right)^{n-i} \left(\frac{\log(\alpha)}{\alpha - 1} \frac{\lambda}{x^2} e^{(\log \alpha)(1 - e^{-\frac{\lambda}{x}})} e^{-\frac{\lambda}{x}}\right)$$

The expression for PDF of the smallest (minimum) order statistics and the largest(maximum) order statistics of SMPIE distribution are respectively obtained by setting i=1 and i=n in above equation.

7. Estimation of Parameters

In this section, we consider the method of maximum likelihood estimation to estimate the unknown parameters of the SMPIE distribution.
Maximum likelihood estimation (MLE) is a technique used for estimating the parameters of a given distribution using some observed data.
Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample of size n having PDF in equation (6) . Then the likelihood function is given by

$$L = \prod_{i=1}^{n} \frac{\log(\alpha)}{\alpha - 1} \frac{\lambda}{x_i^2} e^{(\log\alpha)(1-e^{-\frac{\lambda}{x_i}})} e^{-\frac{\lambda}{x_i}}$$

The log likelihood function is given by;

$$\log L(x:\alpha,\lambda) = -n\log(\alpha-1) + n\log(\log\alpha) + n\log\lambda - \sum_{i=1}^{n}\frac{\lambda}{x_i} + (\log\alpha)\sum_{i=1}^{n}\left(1-e^{-\frac{\lambda}{x_i}}\right) - 2\sum_{i=1}^{n}\log x_i$$

The MLEs of α, λ are obtained by partially differentiating above equation with respect to model parameters and equating to zero, we have

$$\frac{\partial \ell}{\partial \alpha} = -\frac{n}{\alpha-1} + \frac{n}{\alpha\log\alpha} + \sum_{i=1}^{n}\frac{(1-e^{-\frac{\lambda}{x_i}})}{\alpha} = 0 \tag{23}$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{n}{\lambda} - \frac{1}{\sum_{i=1}^{n} x_i} + \log\alpha\sum_{i=1}^{n}\frac{e^{-\frac{\lambda}{x_i}}}{x_i} = 0 \tag{24}$$

Since the above equations is not in a closed form, therefore we will employ Newton-Raphson method and hence using R software to solve these equations and estimate the parameters.

8. Simulation Study

This section deals with the simulation study using R software to demonstrate the MLE's behavior. The inverse CDF method is used to generate a random sample of size n=25, 75, 150, 300, 500. The process is repeated 1000 times and two different combinations of parameters are chosen as (0.2,0.60) and (0.4, 0.25) with relation to the standard order (α, λ). The average MLE values, bias, and mean square error (MSE) of the ML estimates were calculated for each scenario. Table (1) exhibits the ML estimates, bias, and MSE. The MLEs presented are consistent estimators, which means as the sample size increases, these estimates converge in probability to the true parameter values. The estimates are stable and near the actual parameter values. As the sample size increases the MSE drops for all estimates.

Table 1: *MLE, Bias, and MSE for the parameters α and λ*

Sample size	Parameters		MLE		Bias		MSE	
n	α	λ	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\lambda}$
25	0.20	0.60	1.15185	0.82595	1.02735	0.33423	5.78128	0.21386
75			0.42503	0.66675	0.32059	0.21102	0.40229	0.07628
150			0.29569	0.62366	0.18561	0.15443	0.07317	0.03772
300			0.24847	0.61015	0.12152	0.11157	0.02855	0.02076
500			0.20553	0.60487	0.09484	0.08943	0.01588	0.01424
25	0.40	0.25	1.53164	0.30428	1.30163	0.10692	10.94223	0.01896
75			0.67186	0.26746	0.43151	0.06582	0.48012	0.00724
150			0.53525	0.25733	0.27963	0.04846	0.18152	0.00392
300			0.46257	0.25416	0.17622	0.03342	0.05949	0.00201
500			0.40278	0.25087	0.13152	0.02502	0.03022	0.00106

9. Application

This section tests the flexibility, adaptability, and suitability of the SMPIE model against a few other existing distributions using two real life data sets. To compare the SMPIE model with other fitted distributions, we compare the fits of the SMPIE distribution with the generalized inverted generalized exponential distribution (GIGE) was proposed by [9], Exponentiated inverted exponential distribution (EIE) was introduced by [4], and transmuted inverted exponential distribution (TIE) was obtained by [10]. Using several goodness of fit criteria such as -2ll, Akaike Information criterion (AIC), Bayesian information criterion (BIC), Akaike Information criterion Corrected (AICC), Kolmogorov Smirnov (KS) and P value. In general, the model is to be considered the best for which these goodness of-fit statistics have the least value.

9.1. Data set 1

The first data set [6] represents the failure times, in minutes, of 15 electronic components in an accelerated life test and they are as follows:
1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23.0, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2

Table 2: *Estimates (standard errors), -2ll, AIC, BIC, AICC, K-S statistic and P-value for Data-set 1.*

Model	$\hat{\alpha}$	$\hat{\lambda}$	$-2ll$	AIC	BIC	AICC	K-S	P-value
SMPIE	0.0218	3.7371	134.5441	138.5441	139.9602	139.5441	0.18358	0.6829
	(0.0610)	(2.7082)						
GIGE	0.9107	8.9462	138.03922	142.0392	143.4553	143.03926	0.24898	0.2633
	(0.1654)	(1.0369)						
EIE	3.2399	2.9504	138.1101	142.1101	143.5262	143.1101	0.26314	0.2093
	(913.6348)	(832.0125)						
TIE	-0.7039	2.0491	135.973	139.973	141.3891	140.6789	0.20540	0.4886
	(0.322)	(2.1894)						

9.2. Data set 2

The second data set consists of vinyl chloride data (in mg/L) obtained from clean-up-gradient monitoring wells. The data has been previously used by [12].
5.1, 1.2, 1.3, 0.6, 0.5, 2.4, 0.5, 1.1, 8.0, 0.8, 0.4, 0.6, 0.9, 0.4, 2.0, 0.5, 5.3, 3.2, 2.7, 2.9, 2.5, 2.3, 1.0, 0.2, 0.1, 0.1, 1.8, 0.9, 2.0, 4.0, 6.8, 1.2, 0.4, 0.2.

Table 3: *Estimates (standard errors), -2ll, AIC, BIC, AICC, K-S statistic and P-value for Data-set 2.*

Model	$\hat{\alpha}$	$\hat{\lambda}$	$-2ll$	AIC	BIC	AICC	K-S	P-value
SMPIE	0.0468	0.2523	114.48888	118.488881	118.8759	121.54161	0.0951	0.9179
	(0.1145)	(0.1782)						
GIGE	0.9214	0.5412	118.2569	122.2569	122.64403	125.30966	0.1365	0.5499
	(0.2110)	(0.1285)						
EIE	0.7060	0.8108	118.3860	122.3860	122.7731	125.4388	0.1470	0.4544
	(49.5443)	(56.8950)						
TIE	-0.6300	0.4138	115.8404	119.8404	120.2275	122.8931	0.10350	0.8592
	(0.0782)	(0.1089)						

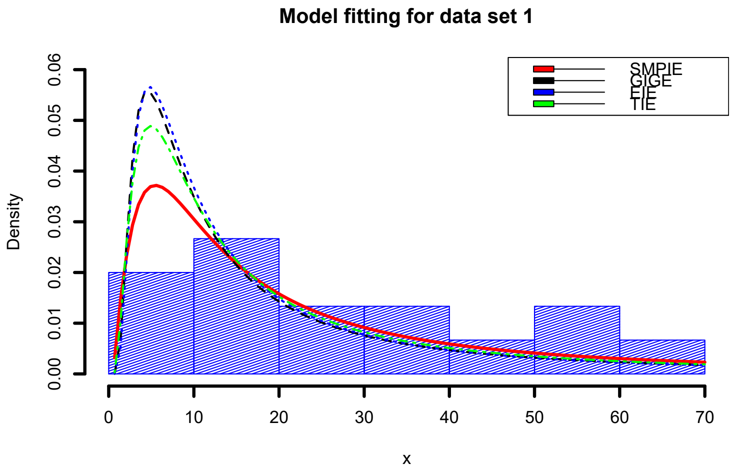

Figure 4: *Fitted density plots for dataset 1*

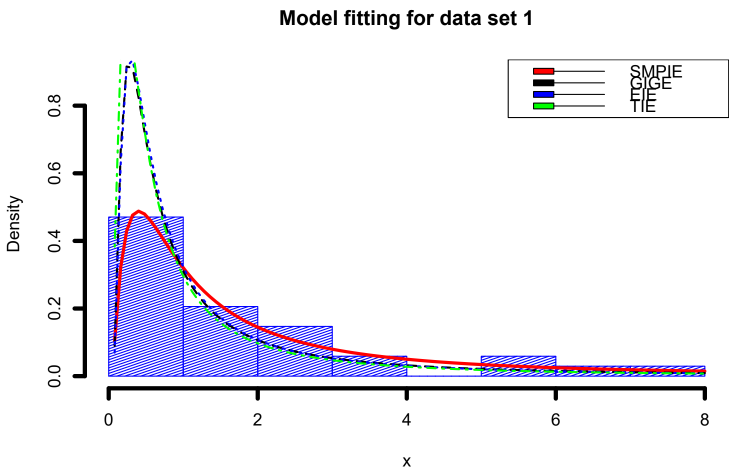

Figure 5: *Fitted density plots for dataset 2*

10. Conclusion

In this manuscript, we have proposed a new distribution called SMP Inverted Exponential (SMPIE) distribution. The model is positively skewed, its shape could be decreasing (depending on the values of the parameters). Various statistical properties of the proposed distribution such as survival function, hazard function, rth moments, quantile function, moment generating function, Renyi entropy, and order statistics were studied. A simulation study was carried out to test the performance of maximum likelihood estimation. The result shows that the mean square error decreases as the sample size increases, i.e. they are consistent estimators. The proposed distribution was applied to two real life data sets and comparing it with well-known standard distributions, and the outcomes are shown in Tables(2) and (3). An application to the real-life data sets shows that the fit of SMPIE distribution is superior to the fits using GIGE, EIE and TIE distributions.

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